The Study of Resistance in Unsteady Flow in Surge

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***Abstract*— A hydroelectric power plant having a long conduit necessitates the installation of a surge tank to reduce the amplitude of water hammer pressure, which may cause due to a change in the load demand. The unsteady continuity equation and momentum in surge when combined become non-linear and hence cannot be solved analytically. Therefore, in the present study, few numerical methods of solution are used viz. Jacobsen's method, Pressel's method and Finite difference method. Author solved the equation by these three numerical solution with variable friction factor 'f ' given by Barr approximate direct solution of Colebrook-White equation (1975) at every time step. Solutions are compared with the available laboratory data and Jacobsen's solution agrees quite well. Again comparison of author's solution with Chattarjee's solution by Pressel's method comes quite closer. Therefore, solution of the equation using variable 'f ', at different time step as suggested by Barr is very essential for a hydraulic engineer to design of different hydraulic structures in a meaningful way.**

***Keywords*— friction factor f, Steady state velocity in the pipe V_0, Unsteady velocity in the surge tank V_s.**

I. INTRODUCTION

In high head hydro projects, a surge is always necessary as shown in Figure 1. A steady state condition prevails with friction loss h_f and velocity V_0. As soon as the valve is closed as shown in Figure 2 a big rise of pressure is being developed causing unsteady flow in the surge tank and in the pipe or tunnel that carrying water from the reservoir. Unsteady continuity equation and momentum in surge when combined, becomes non linear and hence cannot be solved analytically.

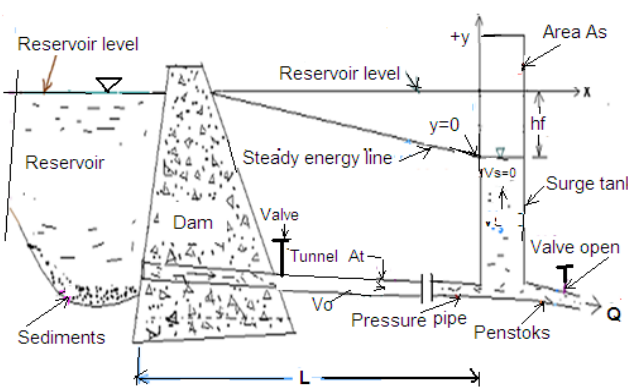

Figure 1 Schematic diagram of steady state flow in hydro project

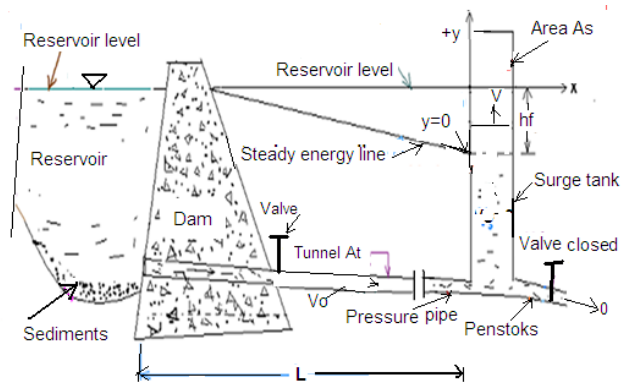

Figure 2 Schematic diagram of steady state flow in hydro project

L=Length of the pipe, D=Diameter of the pipe, A_p=Area of pipe, A_s=Area of surge tank

II. BASIC EQUATIONS

Continuity equation when the valve is completely closed:

$$A_t V = A_s \frac{dy}{dt} \qquad (1)$$

Dynamic equation is:

$$\frac{L}{g}\frac{dV}{dt} + y + \frac{fLV^2}{2gD} = 0 \qquad (2)$$

When equation (1) and (2) are combined, it comes:

$$\frac{d^2y}{dt^2} + \frac{fA_s}{2DA_t}\left(\frac{dy}{dt}\right)^2 + \left(\frac{gA_t}{LA_s}\right) = 0 \qquad (3)$$

All the terms in the above equations are defined in Figure 1 and 2. Thus the final equation (3) is nonlinear and cannot be solved analytically.

III. SOLUTIONS OF THE EQUATION

Various solutions that exist are:

(1) Classical (neglecting friction term)

(2) Thoma Solution

(3) Approximate solution by Jaeger

(4) Graphical solution

(5) Numerical Solution

Classical solution has no practical use without friction. It gives only the surge height y and the trend of sine and cosine curves of unsteady velocity V of the pipe and V_s the surge tank. Similarly solutions by Thoma, Jaeger and graphical give are very much approximate. Numerical solutions are preferred with the advent of computer.

IV. NUMERICAL SOLUTIONS

There are few numerical methods of solution. Authors solved the equations by the following three numerical solutions with variable friction factor f given by Barr approximate direct solution of Colebrook- White equation (1975) at every time step.

(1) Jacobsen's method (1922)

(2) Pressel's method (1909)

(3) Finite Difference method

The variable direct solution of friction factor f equation of Barr (1980) obtained from the implicit Colebrook and White is used in the solution at every time step to take care of the flow from turbulent to laminar. The equation of Barr used is:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{1}{3.71\frac{D}{k}} + \frac{5.1286}{R_e^{089}}\right] \qquad (4)$$

V. PREVIOUS WORKS ON THE NUMERICAL SOLUTION

AIT, Bangkok (1969) presented a numerical solution similar to Jakobsen. Solution advanced only for 23 sections with an integration step of 1 second. It gives only first maximum upsurge and the first minimum down surge as practically these two values are required to design the surge tank height.Chattarjee (1965) developed direct step by step finite difference integration of the equation. He solved up-to 135 seconds with integration time step of 5 seconds. Colebrook and White (1937) first observed liquid in the flow system attains flow states from turbulent, transition to laminar. The equation they developed for friction factor is implicit that needs trial and error every time the friction is calculated. Barr (1975) modified Colebrook-White equation to determine the friction directly at any state of flow given by equation (4). Borthakur (1997), Das (1997), Das Mimi (1999), Das Mimi, Sarma and Das M M (2005) studied on the numerical solution. Das and Borthakur (1996) performed laboratory data in a physical model in Assam Engineering College Hydraulics laboratory to produce data up to 300 seconds to compare their solutions.

VI. THE AUTHOR'S WORKS

The authors solved the equations numerically by finite difference, modified Jacobsen finite difference methods. Solutions are compared with the available laboratory data of Borthakur and Das (1966). The results and comparison are shown graphically.

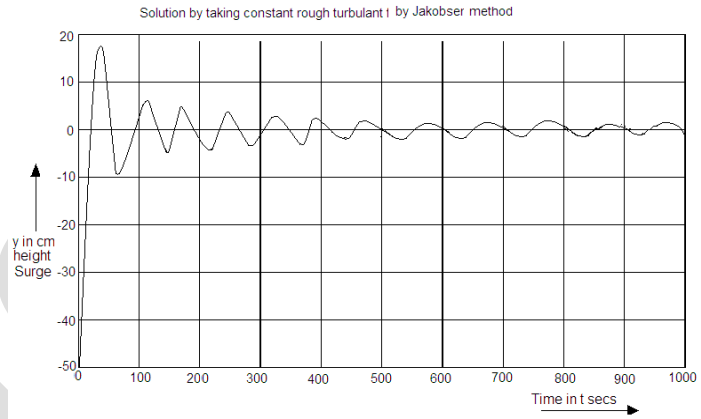

Figure 3 Surge height y against time t with rough turbulant resistace equation

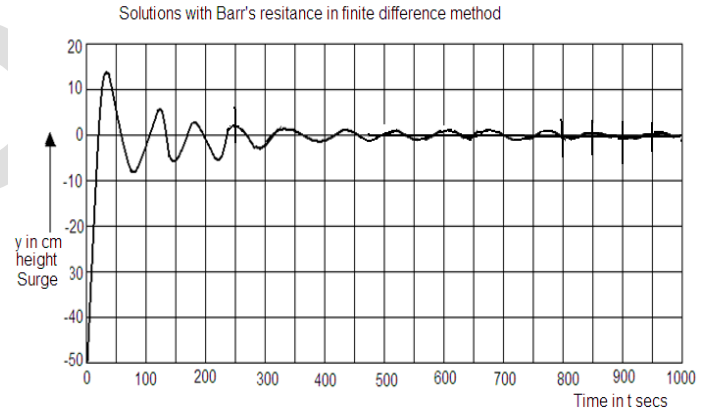

Figure 4 Surge height fluctuatis wit time by Barr's resistance equation

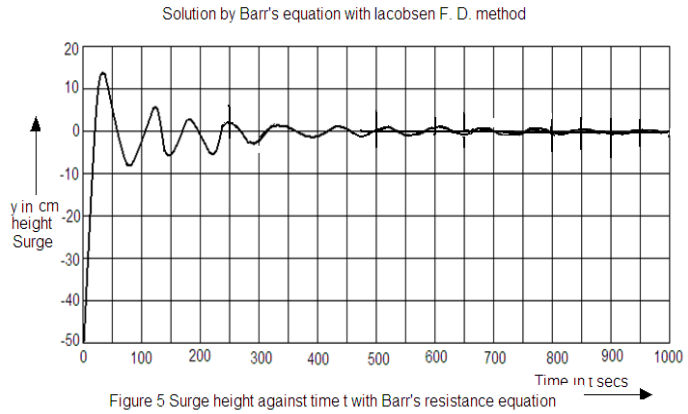

Figure 5 Surge height against time t with Barr's resistance equation

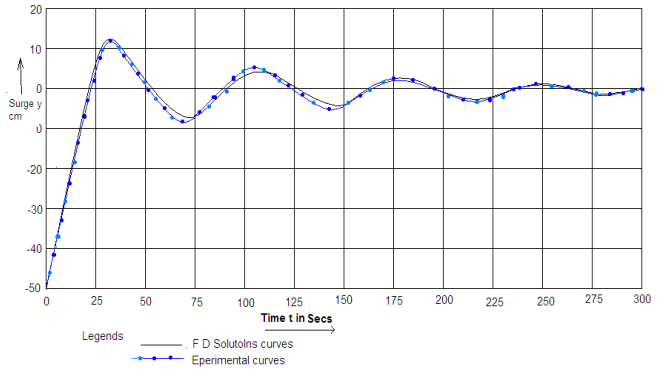

Figure 6 Comparison of eperimental curve with that of Jakopsen method

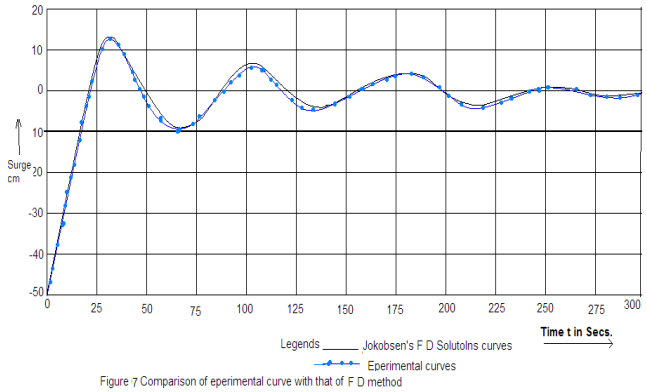

Figure 7 Comparison of eperimental curve with that of F D method

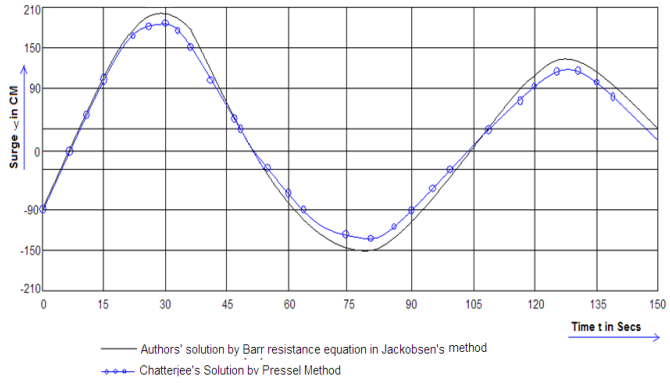

Figure 8 Comparison of Autors's solution with Chatterjee's solution by Pressel method

VII. CONCLUSIONS

The plot of Figure 3 with constant rough turbulent friction factor f shows high value of surge height and slow damping of surges whereas the plots of Figure 4 and 5 present similar plot of initial surge height and more damping effect when Barr's equation is used. Damping of surge height becomes almost a straight line. Comparison of experimental data in Figure 6 and 7 with FD and Jacobsen's solution agrees quite well. Out of all Jacobsen method is found to be much better. Again comparison of author's solution with Chattarjee's solution by Pressel's method in Figure 8 comes quite closer. All these results confirm the numerical solution of the equation using variable f at different time step as suggested by Barr is essential. Use of constant turbulent friction factor at all time steps is not recommended.

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