Squeezing in Stimulated Seven Wave Mixing Process

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***Abstract*- Higher order squeezing in stimulated seven-wave mixing optical process has been studied. It is shown that squeezing increases with the increase in non-classicality associated with higher order squeezing having the same number of photons.**

***Keywords:* Higher order squeezing; Non-classical state, non linear optics**

I. INTRODUCTION

Many theoretical and experimental developments on squeezed states have been taken place in a number of optical processes such as harmonic generation [1-2], multi-wave mixing processes [3]-[5] and optical parametric oscillation [6]-[7]. The non classical effects like squeezed states are a unique set of quantum states for which the Glauber-Sudarshan P-function either goes negative or contains derivatives of delta function [8]. Reduction of quantum fluctuation below the coherent state level corresponds to a non-classical state. The importance of squeezed states in gravitation wave detection [9]-[10], optical communication [11]-[12], quantum computation [13]-[14] and quantum cryptography [15]-[16] is due to its low noise property. Higher order squeezing has drawn the greater attention of the community due to the rapid development of techniques for making higher-order correlation measurements in quantum optics [17]-[19].

In the present work, we have reported that the generation of higher order squeezed state is possible by using stimulated seven-wave mixing process.

II. HIGHER ORDER SQUEEZING

Hong and Mandel [20] and Hillery [21] have introduced the notion of higher order squeezing of quantized electromagnetic field as generalization of normal squeezing. Normal squeezing is defined in terms of the operators

$$X_1=\frac{1}{2}\left(A+A^{\dagger}\right) \quad \text{and} \quad X_2=\frac{1}{2i}\left(A-A^{\dagger}\right)$$

Where X_1 and X_2 are the real and imaginary parts of the field amplitude respectively. A and $A^{\dagger}$ are slowly varying operators defined by

$$A=ae^{i\omega t} \quad \text{and} \quad A^{\dagger}=a^{\dagger}e^{-i\omega t}$$

The operators X_1 and X_2 obey the commutation relation

$$\left[X_1,X_2\right]=\frac{i}{2}$$

Which leads to uncertainty relation ($\hbar$ =1)

$$\Delta X_1 \Delta X_2 \geq \frac{1}{4}$$

A quantum state is squeezed in X_i variable if

$$\Delta X_i < \frac{1}{2} \quad \text{for } i=1 \text{ } or \text{ } 2$$

III. SEVEN –WAVE MIXING PROCESS

In this process, the interaction is looked upon as a process which involves the absorption of two pump photons, each having frequency ω1 and emission of two probe photons of frequency ω2, and three signal photons of frequency ω3 where 2ω1 = 2ω2+3ω3

The Hamiltonian for this process is given as follows (ħ=1)

$$H=\omega_1 a^{\dagger}a+\omega_2 b^{\dagger}b+\omega_3 c^{\dagger}c+g\left(a^2 b^{\dagger 2} c^{\dagger 3}+a^{\dagger 2} b^2 c^3\right) \tag{1}$$

in which g is a coupling constant. A= aexp (iω1t), B= bexp (iω2t) and C = c exp (iω3t) are the slowly varying operators at frequencies ω1, ω2 and ω3, a(a†), b(b†) and c(c†) are the usual annihilation (creation) operators, respectively. The Heisenberg equation of motion for fundamental mode A is given as (ħ=1)

$$\frac{dA}{dt}=\frac{\partial A}{\partial t}+i\left[H,A\right] \tag{2}$$

By using the short-time approximation technique, we expand $A(t)$ by using Taylor's series expansion and retaining the terms up to g2t2 as

$$\begin{aligned} A\left(t\right) = {} & A - 2igtA^{\dagger}B^2C^3 + g^2t^2\Big[2AN_B^2N_C^3 - 9A^{\dagger}A^2N_B^2N_C^2 - 18A^{\dagger}A^2N_B^2N_C \\ & -6A^{\dagger}A^2N_B^2 - 4A^{\dagger}A^2N_BN_C^3 - 36A^{\dagger}A^2N_BN_C^2 - 72A^{\dagger}A^2N_BN_C \\ & -24A^{\dagger}A^2N_B - 2A^{\dagger}A^2N_C^3 - 18A^{\dagger}A^2N_C^2 - 36A^{\dagger}A^2N_C - 12A^{\dagger}A^2\Big] \end{aligned} \tag{3}$$

Where $N(A)=A^{\dagger}A$, $N(B)=B^{\dagger}B$ and $N(C)=C^{\dagger}C$.

In parallel to the spontaneous interaction, the stimulated emission is caused due to the coupling of the atom to the other states of the field. So, initially we consider the quantum state of the field amplitude as a product of coherent state for the fundamental mode A and harmonic mode B and vacuum state for mode C i.e.

$$|\psi\rangle=|\alpha\rangle_A|\beta\rangle_B|0\rangle_C \tag{4}$$

Using Equations (3) and (4) a straightforward but strenuous calculation yields

$$\left\langle \psi \left| X_{1A}^2(t) \right| \psi \right\rangle = \frac{1}{4}\Big[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 + 1 - 6g^2t^2(4\alpha^2|\alpha|^2 + 2\alpha^2 + 2\alpha^{*2} + 4\alpha^{*2}|\alpha|^2 + 8|\alpha|^4)(|\beta|^4+4|\beta|^2+2)\Big] \tag{5}$$

and

$$\left\langle \psi \left| X_{1A}(t) \right| \psi \right\rangle^2 = \frac{1}{4}\Big[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 - 6g^2t^2(2\alpha^2|\alpha|^2 + 2\alpha^{*2}|\alpha|^2 + 4|\alpha|^4)(|\beta|^4+4|\beta|^2+2)\Big] \tag{6}$$

Therefore,

$$\left[\Delta X_{1A}(t)\right]^2 - \frac{1}{4} = -3g^2t^2\Big[(3|\alpha|^4+2|\alpha|^2)\cos 2\theta + 3|\alpha|^4)(|\beta|^4+4|\beta|^2+2)\Big] \tag{7}$$

where θ is the phase angle, with $\alpha = |\alpha| e^{i\theta}$ and $\alpha^* = |\alpha| e^{-i\theta}$. The right hand side of Equation (7) is negative, indicating that squeezing will occur in field amplitude in fundamental mode for which $\cos 2\theta > 0$.

Using Equations (3) and (4), the second order amplitude in fundamental mode is expressed as

$$A^2(t) = A^2 - 6g^2t^2(2A^{\dagger}A^3B^{\dagger 2}B^2 + A^2B^{\dagger 2}B^2 + 8A^{\dagger}A^3B^{\dagger}B + 4A^2B^{\dagger}B + 4A^{\dagger}A^3 + 2A^2) \tag{8}$$

For amplitude squared squeezing, the real quadrature component for the fundamental mode is given as

$$Y_{1A}(t) = \frac{1}{2}\left[A^2(t) + A^{\dagger 2}(t)\right] \tag{9}$$

Using Equations (4) and (9), we get the expectation values as

$$\left\langle \psi \left| Y_{1A}^2(t) \right| \psi \right\rangle = \frac{1}{4}\Big[\alpha^4 + \alpha^{*4} + 2|\alpha|^4 + 4|\alpha|^2 + 2 - 24g^2t^2(\alpha^4|\alpha|^2 + 2\alpha^4 + 2|\alpha|^6 + 8|\alpha|^4 + 10|\alpha|^2 + \alpha^{*4}|\alpha|^2 + 2\alpha^{*4} + 2)(|\beta|^4+4|\beta|^2+2)\Big] \tag{10}$$

and

$$\left\langle \psi \left| Y_{1A}(t) \right| \psi \right\rangle^2 = \frac{1}{4}\Big[\alpha^4 + \alpha^{*4} + 2|\alpha|^4 - 24g^2t^2(\alpha^4|\alpha|^2 + 2|\alpha|^6 + \alpha^{*4}|\alpha|^2 + \alpha^{*4} + 2|\alpha|^4 + \alpha^4)(|\beta|^4+4|\beta|^2+2)\Big] \tag{11}$$

Therefore,

$$\left[\Delta Y_{1A}(t)\right]^2 = \frac{1}{4}\Big[4|\alpha|^2 + 2 - 24g^2t^2(\alpha^4 + 6|\alpha|^4 + 10|\alpha|^2 + \alpha^{*4} + 2)(|\beta|^4+4|\beta|^2+2)\Big] \tag{12}$$

Using Equation (3) number of photons in mode A may be expressed as

$$N_{1A}(t) = A^{\dagger}(t)A(t) = A^{\dagger}A - 6igt\left(A^{\dagger 6}B - A^6B^{\dagger}\right) - 6g^2t^2A^{\dagger 6}A^6 \tag{13}$$

Using Equations (4) and (13), we get

$$\left\langle N_{1A}(t) + \frac{1}{2} \right\rangle = \frac{1}{4}\Big[4|\alpha|^2 + 2 - 24g^2t^2(4|\alpha|^4)(|\beta|^4+4|\beta|^2+2)\Big] \tag{14}$$

Subtracting Equation (14) from Equation (12), we get

$$\left[\Delta Y_{1A}(t)\right]^2 - \left\langle N_{1A}(t) + \frac{1}{2} \right\rangle = -12g^2t^2[|\alpha|^4(\cos 4\theta + 1) + 5|\alpha|^2 + 1)(|\beta|^4 + 4|\beta|^2 + 2)] \tag{15}$$

The right hand side of Equation (15) is negative and thus shows the existence of squeezing in the second order of the field amplitude in stimulated interaction for all values of θ for which $\cos 4\theta > 0$.

IV. RESULT

The presence of squeezing in stimulated seven-wave mixing is shown in Equations (7) and (15) respectively. Figures 1 and 2 show that squeezing increase nonlinearly with $|\alpha|^2$. We also observe that with increase in value of $|\beta|^2$ squeezing also increases and hence increase the non classicality of the field amplitude. Thus we can conclude that the degree of squeezing directly depends upon the photon number of the fundamental mode as well as on the harmonic mode.

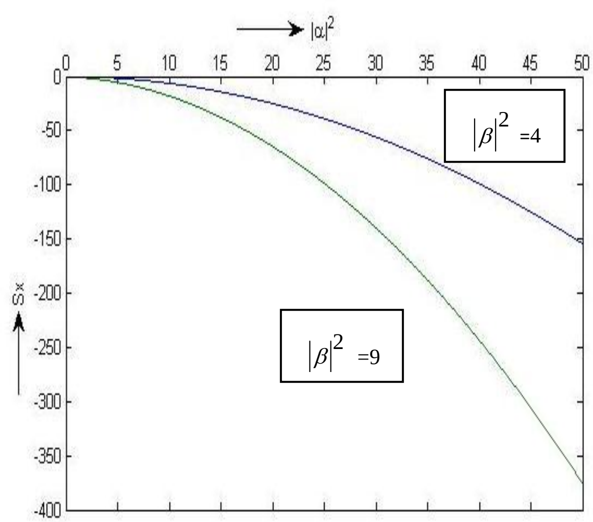

Fig. 1 Dependence of first order squeezing on ($|\alpha|^2$) and ($|\beta|^2$) in stimulated seven wave mixing process

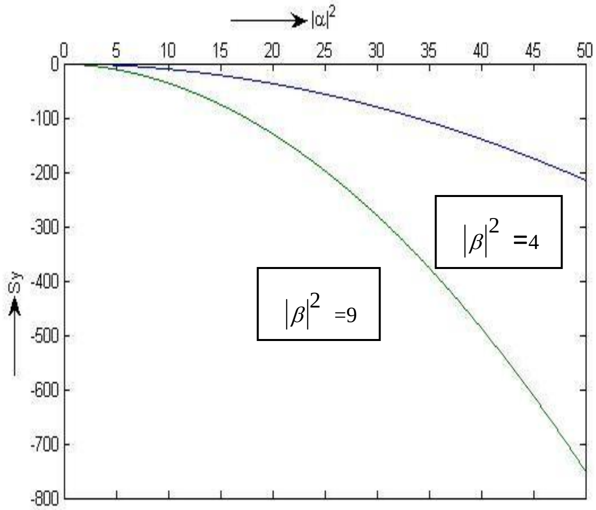

Fig. 2 Dependence of higher order squeezing on ($|\alpha|^2$) and ($|\beta|^2$) in stimulated seven wave mixing process

V. CONCLUSION

The results show the presence of normal and higher order squeezing in stimulated seven-wave mixing process. A comparison between results of spontaneous and stimulated processes shows the occurrence of multiplication factor $(|\beta|^4+4|\beta|^2+2)$. Thus, it implies that squeezing in the fundamental mode in stimulated interaction is greater than corresponding squeezing in spontaneous interaction [5].

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