Reliable EMQ Model with Price and Seasonal Time Dependent Demand

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Abstract: **In this paper, an Economic Manufacturing Quantity (EMQ) is set up under the effect of reliability with demand based on the selling price and seasonal time. The profit of the concern is absolutely depending on the manufacturing of the number of products equalizing the seasonal demand created for the product in the particular time and on the other hand based on fixing the selling price matching with the requirements of the customers and the demand. Hence, the production process of time demanded products should be reliable for the same. Due to wear and tear, breakdown of the machine and etc.. as a result, the production system may be severely affected and changed to out-of-control state. The imperfect system may increase the production time and costs and also produces less quality items. At the additional fixed cost, the less quality items are reworked or restored. Therefore every production process should be perfect to meet the unexpected seasonal demand. The time based manufacturing process is the critical one to make the system perfect till the seasonal demand is fulfilled within short span of time. To reduce the increasing cost of production and to supply the products at the required quantity with perfect quality in the desirable time , we consider reliability as a decision variable along with the improvement of cost and the production cost as a function of reliability. The profit function is maximized in this model by applying Euler-Lagrange formula. The numerical example and graphical illustrations are given to point up the model.**

Key words: **Inventory – Product Reliability – EMQ – Profit Maximization - Price and Seasonal Time Dependent Demand .**

I. INTRODUCTION

The numerous research efforts during the past several decades have been undertaken to develop the basic economic manufacturing quantity(EMQ) by integrating various assumptions such that the model more suitable to the real production process in all the situations. Especially, the demand of the seasonal products is vary with time and price. The production before the season to meet the unexpected demand may cause to spoilage as it cannot be stored till the particular time arises or it cannot be perfectly estimated the demand. As a result, firm faces the problem of determining the model for the production of seasonal items, estimating the reliability of the factors of production to undertake the orders from the customers, budgeting the costs to be incurred to set up the EMQ model, analyzing the profits during the particular season and fluctuation of variables relating to the production and supply the items. Hence the manufacturer establishes suitable production process.

Markov Process were presented by *Rishel* and *Olsder* and *Suri*, *Porteus* analysed the imperfect production process relating to quality and the lot size and evaluated an optimal investment in the process of quality improvement and the set up cost reduction.. *Gronevelt* discussed an inventory model with an optimal production lot size and a safety stock level. They considered a probalistic distributed repair time and an exponentially distributed failure time. *Sarkar* et al. discussed on inventory model for the variable reliability. They used Kuhn-tucker method to find the optimal solution for the reliability, the production lot size.

The inventory models concentrating on seasonal products can be classified into three main types of models according to their demand characteristic; stock-dependent demand, time-dependent demand and price-dependent demand. In this paper, the last two demand patterns, we refer the readers to *Urban* for a review of inventory models with stock-dependant demand.

In the classical EOQ model, demand is assumed to be constant. However, in reality, demand for a product may vary with the time and price. Since *Silver* and *Meal* proposed a heuristic solution to determine lot size quantities for the general case of a time-dependent demand, numerous inventory control papers have investigated the time-dependent demand pattern. In this direction, an inventory model with planned shortages and price-dependent demand was discussed by *Burwell et at*. An optimal and heuristic replenishment model for deteriorating item with an exponentially time-varying demand was derived by *Hariga* and *Bankherour*. *Datta* and *Pal* discussed an inventory model with both price and stock dependent demand under a finite time horizon. *Papchristos* and *Skouri* developed an inventory model for deteriorating items with some quantity discounts, pricing and time-dependent partial backlogging. You developed some different types of inventory policies with price and time dependent demand.

An economic production quantity model for deteriorating items with price and stock-dependent demand was found by *Teng and Chang*. *Sana et at.* discussed a volume flexible EMQ model with a selling price reduction in an imperfect production process. A deterministic inventory model over an infinite time horizon with some initial lot size, delay in payments, and the price discount offers was presented by *Sana and Chaudhuri. Sana* presented an inventory model by assuming demand rate as a function of selling price and budget for advertising and the sales-team. *Sana* presented an inventory model over an infinite time horizon for perishable items with price-dependent demand and partial backorder. An inventory model for deteriorating items with stock and price-dependent demand was developed by *Chang et al*. *Khanra et al.* developed an inventory model with a quadratic time dependent demand under permissible delay in payments. *Yang et al*. considered a closed loop inventory model for deteriorating items with multi-retailer and price-sensitive demand.

Panda, Senapati, and Basu also developed an inventory model for deteriorating seasonal products with a ramp-type time –dependent function over the season. *Shinn and Hwang* investigated the problem of determining the order quantity in which demand is a convex function of price and the delay in payments is order-size dependent. *Banerjee and Sharma* presented a deterministic inventory model for the product with a price and time dependent demand rate. In the above mentioned inventory model with pricing strategy, pricing decision is determined under the assumption that the times of changing price are pre-specified.

The remainder of this paper is organized as follows. In Section 2, we present the modeling and analysis. In section 3, we provide the numerical example and graphical representation . Finally, conclusions are given in Section 4.

II. MATHEMATICAL MODELING AND ANALYSIS

We use following notations to develop the model suitable for seasonal time and price dependent demandable EMQ model.

Notations:

T - Duration of the selling season.

Q - Constant deterioration rate for item $0 \leq 0 \leq 1$.

K - Constant production rate.

Q(t) - Inventory level at time t, $0 \leq t \leq T$.

C_h - Holding cost per unit time.

$\emptyset$ - Product's reliability (decision variable)

Q - Derivative of Q(t) with respect to time t.

$C_p\ (\emptyset)$ - Development cost production system.

C_m - Material cost

α - Variation constant of food / die costs.

$C_p\ (\emptyset, t)$- Production cost

R - Rework p/ Disposal cost / Deteriorate item

Assumptions:

1. We assume that the demand rate is a function of time and price within the selling period. First, we assume that
$$D\ (t,p) = \ Ae^{b\left(t-(t-\mu\ )\right)1+(t,\mu)} - ap \ldots\ldots\ldots..(1)$$
where A > 0 , a>0 & b > 0 are given constants.
$$\text{H}(\text{t}, \mu) = \begin{cases} 1 \ \ if\ t\ \geq\ \mu \\ 0 \ \ if\ t <\ \mu \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots(2)$$
2. The holding cost increases with time i.e $C_h = C_o + C_1 t$. At t=0, i.e, the initial time the holding cost it fixed at C_o. The reliability $\emptyset$ of the machinery system is defined by
$$\frac{\text{No.of failures}}{Total\ No.of\ working\ \ hours.}$$
3. To make an item more reliable, the material cost depends linearly on reliability $\emptyset$, ie., $C_m = C_2 - C_3\emptyset$
4. Unit production cost is a function of product's reliability $\emptyset$ and production rate P(t) i.e ; $C_p\ (\emptyset, t) = \ C_m + \frac{C_p\ (\emptyset)}{k}\ \alpha\text{P(t)}.$
5. To make the maximum products is perfect. In seasonal time, the production of imperfect items is due to machine failure, labour problems etc.

To reduce the production of imperfect items, we take the production and the development cost as function of the reliability. The reliability of the machinery system is considered at time t as $(t) = e^{-\emptyset t}$. Again $\pi(t) = 1 - F(t) = 1 - \int_0^t f(x)dx$ where F(t) and f(t) denotes the probability distribution and probability density function respectively such that $\int_0^t f(t)dt = 1$ and f(t) $= \frac{d}{dt}\ F(t) = \emptyset e^{-\emptyset t}$ where $\emptyset$ is a design variable and indicates the product's reliability. The lesser value of $\emptyset$ indicates the more reliability of the system i.e the system will be more reliable as the value of $\emptyset$ gets smaller.

The rework cost during new we consider the development cost for production as follows;

$$C_D(\emptyset) = \ C_4 - C_5\emptyset - C_6\emptyset^2, \quad C_4, C_5, C_6 > 0 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$c_p(\emptyset, T) = c_m + \frac{C_D(\emptyset)}{K} + \alpha P(t)$ ………………………..(2)

Therefore, the differential equation of the model is

$$\frac{dQ(t)}{dt} + \theta Q(t) = k - Ae^{bt} + ap, a \le t \ \mu \ldots\ldots\ldots\ldots(i)$$

$\frac{dQ(t)}{dt} + \theta Q(t) = -Ae^{b\mu} + ap$………(ii),

$\mu < t \le T \ with \ I(0) = L \ \& \ I(T) = 0.$

The revenue from selling the items during (0,T) is $\int_0^T \{P - Cp\ \emptyset, P(t) P(t) dt,$

The holding cost during [0,T] is $\int_0^T C_h \ Q \ dt$.

The total profit function

F = $\int_0^T c^{-pt} \int \left(P - C_p(\emptyset, P(t))\right) P(t) - C_h \ Q \ R \ \beta e^{\emptyset t} P(t)\} \ dt$

$= \int_0^T e^{-pt} \{(P - C_m - R\beta e^{\emptyset t})(\dot{Q} + D) - \alpha (\dot{Q} + D)^2 - ChQ - CD(\emptyset) \ dt$

$= \int_0^T X(\dot{Q}, Q, t) dt$ ………………………………………(3)

where X $(\dot{Q}, Q, t) = e^{-pt} \{P - C_m - R\beta e^{\emptyset t})(\dot{Q} + D) - \alpha(Q+D)2 - CnQ - CD(\emptyset)$

In this model, we consider a price and seasonal time demand

D(t,p) = $Ae^{b[(t-(t-\mu))\ H\ (t,\mu)]} - ap$ with I(0) = I (T) = 0 .

The solution of the 2nd differential equation of (ii) is

$$\frac{d}{dt}\left(e^{\theta t} Q(t)\right) = -Ae^{b\mu + \theta t} + Qpe^{\theta t}$$

Integrating from f to T and using I (T) = 0,

Q(t) = $\frac{Ae^{b\mu}}{\theta}\left[e^{\theta(T-t)} - 1\right], \mu \le t \le T$

$similarly, (i) \Rightarrow \frac{d}{dt}\left(e^{\theta(T-t)}\right) = Ke^{\theta t} - Ae^{(b+\theta)t} + ape^{\theta t}$

Integrating it from 0 to t and using I (0) = 0,

K(t) = $\frac{K+ap}{\theta}\left[1 - e^{\theta t}\right] - \frac{A}{\theta+b}\left[e^{bt} - e^{\theta t}\right], 0 \le t < \mu.$

Also, from the boundary condition at time μ, we obtain

$$\frac{Ae^{b\mu} - ap}{\theta}\left[e^{\theta(T-\mu)} - 1\right] = \frac{K + ap}{\theta}\left[1 - e^{-\theta\mu}\right] - \frac{A}{\theta + b}\left(e^{b\mu} - e^{-\theta\mu}\right).$$

Therefore the solution is,

$\left\{Ae^{b\mu}\left(e^{\theta T} - e^{\theta\mu}\right) - K\left(e^{-\theta\mu} - 1\right) + \frac{A\theta}{\theta+b}\left(e^{(\theta+b)\mu} - 11a(e\theta T-1)\right.\right.$

……………………………………..…………(4)

Now substituting D(p,t) is $X(\dot{Q}, Q, T)$, we have $X(\dot{Q}, Q, T) = e^{-pt}\left[(p - C_m - R\beta e^{\theta t})\ (\dot{Q} + Aebt-t-\mu H(t,\mu) - ap - \alpha \quad Q + Aebt - (t-\mu \quad H \quad (t,\mu) - ap2 - Q - CD(\emptyset)\right.$………………………………………(5)

Now our objective is to obtain the optimal path of Q(t)$l\ K$ substitute F is maximized.

The profit function F has a maximum value in the interval (O,T). We consider a curve E_o for which the profit function will be at maximum and consider another curve E_w is considered as Q = $Q_w(t) = Q^0(t) + w\eta(t), 0 \le t \le T$ & E_0 is considered as Q = $Q^0(t); 0 \le t \le T, \eta(t)$ is similarly arbitrary differential functional of t & w is a small quantity.

F(w) = $\int_0^T X_w \ dt$, $where \ X_w = \lambda(Q^0(t) + w\eta(t), Q^0(t) + w\eta t, t$

$$\frac{dF(w)}{dw} = \int_0^T \left\{\eta(t)\frac{\partial X_w}{\partial Q} + \dot{\eta}(t)\frac{\partial X_w}{\partial \dot{Q}}\right\} at$$
$$= \int_0^T \eta(t)\frac{\partial X_w}{\partial Q} dt + \left[\eta(t)\frac{\partial X_w}{\partial \dot{Q}}\right]_0^T - \int_0^T \eta(t)\frac{d}{dt}\left(\frac{\partial X_w}{\partial \dot{Q}}\right) dt$$

$= \left[\eta(t)\frac{\partial X_w}{\partial \dot{Q}}\right]_0^T + \int_0^T \eta(t)\left\{\frac{\partial X_w}{\partial Q} - \frac{d}{dt}\left(\frac{\partial X_w}{\partial \dot{Q}}\right)\right\} dt$

$= \int \eta(t)\left[\frac{\partial X_w}{\partial Q} - \frac{d}{dt}\left(\frac{\partial X_w}{\partial \dot{Q}}\right)\right] dt$, − − − − − − − − − − − −(6)

$\eta(0) = \eta(T) = 0 \ at \ w = 0.$

$Therefore, (6) \Rightarrow \frac{\partial X}{\partial Q} - \frac{d}{dt}\left(\frac{\partial X}{\partial \dot{Q}}\right) = 0$ which is necessary condition for the optimum value of the profit function F.

$$\frac{d^2F(w)}{dw^2} = \int_0^T \left(\eta\frac{\partial^2 X_w}{\partial Q^2} + 2\eta\dot{\eta}\frac{\partial^2 X_w}{\partial Q \partial \dot{Q}} + \dot{\eta}^2\frac{\partial^2 X_w}{\partial \dot{Q}^2}\right) dt$$

As w is very small quantity, we obtain,

$\frac{d^2F(w)}{dw^2} = \int_0^T \left(\eta^2\frac{\partial^2 X}{\partial Q^2} + 2\eta\dot{\eta}^2\frac{\partial^2 X}{\partial Q \partial \dot{Q}} + \eta 2\ \partial 2X \partial Q2\ dt\right.$………………………(7)

For finding the optimal path, differentiating X partially with respect to Q and $\dot{Q}$, we get

$\frac{\partial X}{\partial Q} = -e^{-pt}C_h$, $\frac{\partial^2 X}{\partial Q^2}$ =0

$\frac{\partial X}{\partial \dot{Q}} =$
$e^{-R^t}\left[\left(p - C_m - R\beta e^{\emptyset t}\right) - 2\alpha\left(\dot{Q} + Ae^{b[(t-(t-\mu))H(t,\mu)]} - ap\right)\right.$

$$\frac{\partial^2 X}{\partial \dot{Q}^2} = -2\alpha e^{-\rho t}, \qquad \frac{\partial^2 X}{\partial Q \partial \dot{Q}} = 0$$

Using these in (7) , we have;

$$\frac{d^2F}{dw^2} = \int_0^T \{\eta^2 . 0 + 2\eta\dot{\eta}^2 . 0 + \dot{\eta}^2 (-2\alpha e^{-\rho t}) dt$$

$$= -2\alpha \int_0^T e^{-\rho t} dt < 0$$

Hence the sufficient condition $\frac{\partial^2 F}{dw^2} < 0$ *indicates* that the profit function F has a maximum value in [0,T]. Now from the necessary conditions,

$$\frac{d}{dt}\left(\frac{\partial X}{\partial \dot{Q}}\right) - \frac{\partial X}{\partial Q} = 0 \text{ ……………………………………..(8)}$$

$\Rightarrow \ddot{Q} - \rho\dot{Q} = -\frac{1}{2\alpha}\left[(C_2\rho - C_3\phi\rho) - \rho\left(Ae^{b[t-(t-\mu)]} - \right.\right.$
ap+Rβ eϕt−ϕeϕt+ddt (Aebt−t−μ−ap)

The complete solution for Q(t) is

Q(t) = M + $Ne^{\rho t} - \frac{1}{2\alpha}\left[\frac{R\beta e^{\phi t}(1-\phi)}{\phi(\phi-\rho)} - \frac{1}{\rho}\left[\frac{K+ap}{\theta}\left(1 - e^{-\theta t}\right) - \right.\right.$
Aθ+b ebt−e−θt− 1ρ V2t2+ U+ Vρ t ………..(9)

where M & N are arbitrary constants, using the boundary conditions Q(0) = L,Q(T) = 0 .

M = $\frac{Le^{\rho T}}{e^{\rho T}-1} + \frac{1}{2\alpha(e^{\rho T}-1)}\left\{\frac{R\beta(1-\phi)}{\phi(\phi T-\rho T)}\left(e^{\rho T} - e^{\phi T}\right) - \right.$
1ρ Aebμeθ T−eθμ−keθμ−1+ Aθθ+b e(θ+b)μ−11a(eθT−1)−1ρ V2T2+U+VeT

N =
$\frac{L}{e^{\rho T}-1} + \frac{1}{2\alpha(e^{\rho T}-1)}\left\{\frac{R\beta(1-\phi)}{\phi(\phi T-\rho T)}\left(e^{\phi T} - 1\right) - \frac{1}{\rho}\left(\left\{Ae^{b\mu}\left(e^{\theta T} - \right.\right.\right.\right.$
eθμ−keθμ−1+Aθθ+b (e(θ+b)μ−1)1a(eθT−1)+ V2T2+ U+ VpT

where U = $C_2\rho - C_3\phi\rho + C_0$ *and* $V = C_1 t$.

Therefore, the optimal path is

P(t) = $\dot{Q}$ + D= $\dot{Q}$ + A$e^{b[(t-(t-\mu))]\mu(t,\mu)} - ap$

$$= N\rho e^{\rho t} - \frac{R\beta e^{\phi t}(1-\phi)}{2\alpha\phi(\phi-\rho)} + A_2 t^2 + A_3 t + A_4$$

where $A_2 = \frac{V}{4\alpha\rho}$, $A_3 = \frac{(U+\frac{V}{\rho})}{2\alpha\rho}$, $A_4 = \frac{1}{2\alpha\rho}\left\{Ae^{b\mu}\left(e^{\theta T} - e^{\theta\mu}\right) - k\left(e^{\theta\mu} - 1\right) + \frac{A\theta}{\theta+b}\left(e^{(\theta+b)\mu} - 1\right)\right\}\frac{1}{a(e^{\theta T}-1)} + Ae^{b\mu} - ap$

Substituting these values in (3) , we obtain

F = $\int_0^T X(Q,\dot{Q},T)dt = \left[A_1 N\rho T - A_1 A_2\left(\frac{e^{-\rho T}}{\rho}T^2 + \right.\right.$
2e−ρTρ2T+ 2e−ρTρ3− 2ρ3− A1A3 e−ρTρT−e−ρTρ2−1ρ2+ A1A5 e−ρTρ+ A1A5 e−θ+ρ−1θ+ρ− AA1θ+bb−ρ eb−ρT−1+ AA1θ+bθ+ρe−θ+ρT−1− Rβpn2α− Rβϕ eϕT−1− αN2ρeρT−1−R2β24α p1+ αA22 (T4p1+4T3p2+12T2p3+24Tp4+24p5−24ρ3 + αA32T2p1+2Tp2+2p3−2ρ3−αA42e−ρT−1+NRβA2A3A 4p6T3p7−3T2p8+ 6Tp9− 6p10+ 6ϕ4−
$R\beta A_2 A_3 A_4 p_{11}\left(T^3 p_{12} - 3T^2 p_{13} + 6Tp_{14} - 6p_{15} + \frac{6}{(\emptyset-\rho)^4}\right) + \alpha 2 A_2 A_3 A_4 (T^3 p_1 + 3T^2 p_2 + 6Tp_3 + 6p_4 - \frac{6}{\rho^4}) + 2\alpha A_3 A_4\left(Tp_1 + p_2 - \frac{1}{\rho^2}\right) - \frac{M}{\rho}(e^{-\rho T} - 1) + NT - \frac{R\beta}{2\alpha}p_{16} - \frac{A_5}{2\alpha\rho^2}(e^{-\rho T} - 1) + \frac{A_5 p_{17}}{2\alpha\rho} - \frac{A}{2\alpha\rho(\theta+b)}(p_{18} + p_{17}) - A_6\left(T^2 p_1 + 2Tp_2 + 2p_3 - \frac{2}{\rho^3}\right) - A_7(Tp_1 + p_2 - \frac{1}{\rho^2})]$ ………………………………………………………..(10)

where $A_1 = p - C_2 - C_3\emptyset$, $A_5 = \frac{K+ap}{\theta}$, $A_6 = \frac{V}{4\alpha\rho}$, $A_7 = U + \frac{V}{\rho}$, $p_1 = \frac{e^{-\rho T}}{\rho}$,

$p_2 = \frac{e^{-\rho T}}{\rho^2}$ $p_3 = \frac{e^{-\rho T}}{\rho^3}$, $p_4 = \frac{e^{-\rho T}}{\rho^4}$, $p_5 = \frac{e^{-\rho T}}{\rho^5}$, $p_6 = \frac{(1-\emptyset)}{(\emptyset-\rho)}$, $p_7 = \frac{e^{\emptyset T}}{\emptyset}$, $p_8 = \frac{e^{\emptyset T}}{\emptyset^2}$, $p_9 = \frac{e^{\emptyset T}}{\emptyset^3}$, $p_{10} = \frac{e^{\emptyset T}}{\emptyset^4}$, $p_{11} = \frac{(1-\emptyset)}{\rho(\emptyset-\rho)}$, $p_{12} = \frac{e^{(\emptyset-\rho)T}}{(\emptyset-\rho)}$, $p_{13} = \frac{e^{(\emptyset-\rho)T}}{(\emptyset-\rho)^2}$, $p_{14} = \frac{e^{(\emptyset-\rho)T}}{(\emptyset-\rho)^3}$

$p_{15} = \frac{e^{(\emptyset-\rho)T}}{(\emptyset-\rho)^4}$, $p_{16} = \frac{(1-\emptyset)(e^{(\emptyset-\rho)T}-1)}{\emptyset(\emptyset-\rho)^2}$, $p_{17} = \frac{(e^{-(\emptyset+\rho)T}-1}{(\emptyset+\rho)}$, $p_{18} = \frac{(e^{(b-\rho)T}-1}{(b-\rho)}$

III. NUMERICAL EXAMPLES AND GRAPHICAL REPRESENTATION

Example 1:

The input parameters are : T=5, ∅ =0.05, K = 100, h = 0.01, c = 30, A = 100, b = 0.05, μ = 2 and a = 1.(In equation (4))

$\hat{p} = \frac{A}{a}$=100.

$p^* = 69.69$.

Fig. 1. Graphical Representation of the Profit Function in Example 1.

Example 2:

α = \$ 0.10/ unit, β = 0.5 year, C_2= \$10, C_3 = \$4, R = \$ 30 / items, C_0= \$ 0.3/unit/ year, C_1= \$ 0.2 / unit / year, p=\$15/ item, ρ =0.05 , T= 340 / 365 years.

Therefore, F = \$ 827369 , $\emptyset$ = 0.327569.

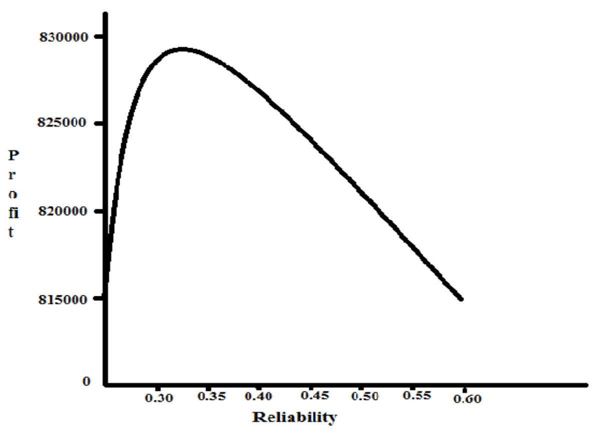

Fig.2. Graphical Representation of the Profit Function in Example 2.

IV. CONCLUSIONS

We consider an Economic Manufacturing Quantity model applicable under the price and seasonal time dependent demand. Generally, every production system produces perfect quality items at the beginning of the production. The cost of maintenance of the system is very low at the starting period of the process. During long-run process, the system begins to produce defective items due to machine failure, labour problems, changing the production method and etc.. The production of defective items increases with the increase of time. Hence the production process should be analysed to prove the reliability of the production. The seasonal demand are short period in nature. The manufacturing model should be perfect to meet the demand. Hence it is essential to analyse on the costs incurred on maintenance of the system including the manpower. If there is appropriate control on the costs and expenditures, the selling price can be fixed at the higher level to yield more profit. The perfect or suitable production system can face the uncontingencies in supply of products.

A numerical example is given and the graphical representation shows the concave nature of the objective function. In our model, the major new contributions are that the system of seasonal production which are influenced by the time and price, can be organized when the reliability exists in it and the profit of the seasonal items are dependable function of the production factors. The estimation of required inventory, costs and selling price and prior analysis of the system are the factors to determine the reliability of the production. Hence our model has a new managerial insight that helps a manufacturing system to gain the profit at optimal level. The numerical example gives a promising result in practice.

The model can be extended by considering deterioration of products and can be developed by analyzing the EOQ for deciding the economic manufacturing quantity of the seasonal products.

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