Asymptotic Approximation Analysis of Optical Bragg Fiber

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Abstract **— In this we are using an asymptotic analysis, we obtain an eigenvalue equation for the general mode dispersion in Bragg fibers. The asymptotic approach is applied to calculate the dispersion properties and the field distribution of TE modes in a Bragg fiber. We made a comparison between the asymptotic results with the exact solutions and find excellent agreement between them.**

I. INTRODUCTION

Bragg fibers, in which light confinement is due to cylindrical Bragg reflection instead of total internal reflection, were proposed by Yeh et al. [1] An especially attracting feature of these fibers is the possibility of guiding light in an air core, which has attracted considerable recent interest.2 –5 Potential advantages of air-core fibers are lower absorption loss and higher threshold power for nonlinear effects. In the original proposal and analysis of Bragg fibers[1]. It was shown that confined modes exist in a Bragg fiber that consists of a low-index core, including air, surrounded by a suitably designed alternating cladding of high- and low-refractive-index media (see Fig. 1). The exact theoretical analysis of Bragg fibers in is considerably more complicated than that of planar Bragg waveguides[1].One main reason is that the Bloch theorem is no longer applies to cylindrically symmetric geometries such as Bragg fibers. Therefore it is difficult to find an equation that determines the confined modes. In this Letter we develop a formalism to analyze guided modes in Bragg fibers in the asymptotic limit. We find that the asymptotic approach greatly simplifies the problem and provides an excellent approximation to the exact solution, even when the radius of the low-index core is relatively small. Therefore we expect that the results in this Letter will greatly facilitate the design and analysis of Bragg fibers.

We consider a Bragg fiber composed of a low index core (refractive index n_c) surrounded by pairs of high- and low-refractive-index layers with n_1 and n_2, respectively. Other parameters of the Bragg fiber are defined in Fig. 1. For guided modes we can factor out the temporal and the z dependence as exp $[i(\beta z-\omega t)]$ and the azimuthal dependence as $\cos(l\theta)$ o $\sin(l\theta)$ [1]. If we use the asymptotic expressions [3] for the Bessel functions at $kr\rightarrow\infty$ to describe the cladding fields, the radial dependence of Ez and Hz becomes

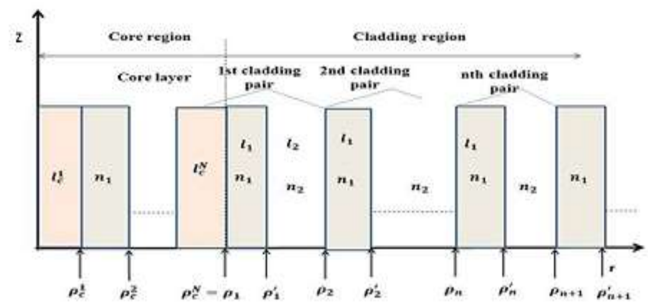

Fig.1. Schematic of r-z cross section of a Bragg fiber. The dielectric layers of a Bragg fiber are separating into two regions, first is the core region and second is the cladding region. The core region composed of N concentric layers each with refractive index n_c^i and thickness l_c^i , i = 1,2,. . .N. The cladding region of fiber consists of pairs of alternating layers of different dielectric materials. Layer I having refractive index n_1 and thickness l_1 and Layer II having refractive index n_2 and thickness l_2 .

In the core region

$$E_z = a_c\, J_l(k_c r) \qquad 0 < r < \rho_1$$

And

$$H_z = c_c\, J_l(k_c r) \qquad 0 < r < \rho_1$$

At the cladding layer I the field distribution E_z and H_z are given as,

$$E_z = \frac{1}{\sqrt{k_1 r}}\left[a_n\, e^{ik_1(r-\rho_n)} + b_n\, e^{-ik_1(r-\rho_n)}\right]$$

$$\rho_n < r < \rho_n + l_1.$$

$$H_z = \frac{1}{\sqrt{k_1 r}}\left[c_n\, e^{ik_1(r-\rho_n)} + d_n\, e^{-ik_1(r-\rho_n)}\right]$$

$$\rho_n < r < \rho_n + l_1, \qquad (1a)$$

Similarly at the cladding layer II the field distribution E_z and H_z are given as,

$$E_z = \frac{1}{\sqrt{k_2 r}}\left[a'_n e^{i2(r-\rho'_n)} + b'_n e^{-ik_2(r-\rho'_n)}\right]$$

$$\rho'_n < r < \rho'_n + l_2,$$

$$H_z = \frac{1}{\sqrt{k_2 r}}\left[c'_n e^{ik_2(r-\rho'_n)} + d'_n e^{-ik_2(r-\rho'_n)}\right]$$

$$\rho'_n < r < \rho'_n + l_2, \qquad (1b)$$

Where

$$k_c = \sqrt{(n_c\,\omega/c)^2 - \beta^2}, \quad k_1 = \sqrt{(n_1\,\omega/c)^2 - \beta^2}, \text{ and}$$
$$k_2 = \sqrt{(n_2\,\omega/c)^2 - \beta^2} \qquad (2)$$

Once E_z and H_z are calculated, then the other field components E_θ and H_θ can be easily find out from the derivatives of E_z *and* H_z [1]

$$E_\theta = \frac{\omega\mu_0}{k_1\sqrt{k_1 r}}\left[c_n e^{ik_1(r-\rho_n)} - d_n e^{-ik_1(r-\rho_n)}\right]$$

$$\rho_n < r < \rho_n + l_1.$$

$$H_\theta = \frac{-\omega\epsilon_0 n_1^{\,2}}{k_1\sqrt{k_1 r}}\left[a_n e^{ik_1(r-\rho_n)} - b_n e^{-ik_1(r-\rho_n)}\right]$$

$$\rho_n < r < \rho_n + l_1 \qquad (3)$$

Similarly the field components E_θ and H_θ at cladding layer II found as,

$$E_\theta = \frac{\omega\mu_0}{k_2\sqrt{k_2 r}}\left[c'_n e^{ik_2(r-\rho'_n)} - d'_n e^{-ik_2(r-\rho'_n)}\right]$$

$$\rho'_n < r < \rho'_n + l_2,$$

$$H_\theta = \frac{-\omega\epsilon_0 n_2^{\,2}}{k_2\sqrt{k_2 r}}\left[a'_n e^{ik_2(r-\rho'_n)} - b'_n e^{-ik_2(r-\rho'_n)}\right]$$

$$\rho'_n < r < \rho'_n + l_2 \qquad (4)$$

The boundary condition wants E_z, E_θ, H_z and H_θ must be continuous at the interface between two adjacent layers. Keeping the dominant terms of E_θ and H_θ and neglect the rest part in the asymptotic limit, we get the below matrix relations:

$$\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} = \begin{bmatrix} F_{TM} & G_{TM} \\ G_{TM^*} & F_{TM^*} \end{bmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \qquad (5)$$

$$\begin{pmatrix} c_{n+1} \\ d_{n+1} \end{pmatrix} = \begin{bmatrix} F_{TE} & G_{TE} \\ G_{TE^*} & F_{TE^*} \end{bmatrix} \begin{pmatrix} c_n \\ d_n \end{pmatrix} \qquad (6)$$

Where the parameters F_{TE}, G_{TE}, F_{TM} *and* G_{TM} are constants within the complete cladding region and defined as:

$$F_{TE} = e^{ik_1 l_1}\left[i\,\frac{k_1^{\,2} + k_2^{\,2}}{2k_1 k_2}\sin(k_2 l_2) + \cos(k_2 l_2)\right] \qquad (7a)$$

$$G_{TE} = i e^{-ik_1 l_1}\,\frac{k_1^{\,2} - k_2^{\,2}}{2k_1 k_2}\sin(k_2 l_2) \qquad (7b)$$

And

$$F_{TM} = e^{ik_1 l_1}\left[i\,\frac{n_2^{\,4} k_1^{\,2} + n_1^{\,4} k_2^{\,2}}{2n_1^{\,2} n_2^{\,2} k_1 k_2}\sin(k_2 l_2) + \cos(k_2 l_2)\right] \qquad (7c)$$

$$G_{TM} = i e^{-ik_1 l_1}\,\frac{n_2^{\,4} k_1^{\,2} - n_2^{\,4} k_2^{\,2}}{2n_1^{\,2} n_2^{\,2} k_1 k_2}\sin(k_2 l_2) \qquad (7d)$$

We notice that F_{TE}, G_{TE}, F_{TM} *and* G_{TM} all are same for the all cladding layers, by using the application of Bloch theorem to the all cladding fields:

$$\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} = \lambda_{TM} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \qquad (8\text{ a})$$

$$\begin{pmatrix} c_{n+1} \\ d_{n+1} \end{pmatrix} = \lambda_{TE} \begin{pmatrix} c_n \\ d_n \end{pmatrix} \qquad (8\text{ b})$$

From above equation we can easily calculate the value of λ_{TM} and λ_{TE}

$$\lambda_{TM} = Re(F_{TM}) \pm \sqrt{\{[[Re(F_{TM}]^2 - 1]\}} \qquad (8\,c)$$

$$\lambda_{TE} = Re(F_{TE}) \pm \sqrt{\{[[Re(F_{TE}]^2 - 1]\}} \qquad (8\,d)$$

Above results clearly shows that in the asymptotic approximation the properties of cladding of Bragg fiber harmonize with planner Bragg stacks. Ultimately, a number of results of planner Bragg stack can be used directly to Bragg fibers. In case, the condition $Re(F_{TE})$ >1 shows the TE bragg reflection gap, in this λ_{TE} become a real number. And similar to get optimum confinement in Bragg fiber, we must select l_1 and l_2 such that the value of

$$k_1 l_1 = k_2 l_2 = \pi/2 \quad \text{(i.e. Quarter-wave layers)}$$

1. Solution of Propagation constant β and guided modes:

The guided modes in the Bragg fiber are calculated by matching the exact solution in core region[1] with the asymptotic approximation solution in the cladding region (equation 1a - 4).

On reconciliation fields at the interface between two different dielectric layers, we can calculate the value of a_n, b_n, c_n and d_n are given as

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = \lambda_{TM}^{\,(n-1)} \begin{bmatrix} G_{TM} \\ \lambda_{TM} - F_{TM} \end{bmatrix} \qquad (9a)$$

$$\begin{bmatrix} c_n \\ d_n \end{bmatrix} = \lambda_{TE}{}^{(n-1)} \begin{bmatrix} G_{TE} \\ \lambda_{TE} - F_{TE} \end{bmatrix} \qquad (9b)$$

To find the solution of propagation constant β, we have to match the values of E_z, E_θ, H_z, and H_θ using equation (6.7) and (6.10 to 6.15) at boundary interface r = ρ_1, which governs,

$$\frac{\omega^2}{c^2} n_c^2 \left[\frac{J_l'(k_c\rho_1)}{J_l(k_c\rho_1)} + i\frac{k_c n_1^2}{k_1 n_c^2}\left(\frac{\lambda_{TM} - F_{TM} - G_{TM}}{\lambda_{TM} - F_{TM} + G_{TM}}\right)\right]$$
$$\times \left[\frac{J_l'(k_c\rho_1)}{J_l(k_c\rho_1)} + i\frac{k_c}{k_1}\left(\frac{\lambda_{TE} - F_{TE} - G_{TE}}{\lambda_{TE} - F_{TE} + G_{TE}}\right)\right] = \frac{\beta^2 l^2}{k_c^2 \rho_1^2} \qquad (10)$$

It is clearly shown that for any $l \neq 0$, the guided modes are the combination of TE and TM modes. For l =0 the guided modes are categorized as either TE or TM modes.

Equation (10) also gives the regular mode dispersion in any Bragg fiber. To simplify the analysis we bound ourselves for the TE modes. The core having the low refractive index i.e., air ($n_c = 1$) and the cladding having alternate high and low refractive indices $n_1 = 3$ and $n_2 = 1.5$ respectively. We select the hollow fiber core radius to be $\rho_1 = 1$ µm and cladding parameters are such as $l_1 = 0.130$µm and $l_2 = 0.265$µm. These parameters are chosen in such a way so that the cladding of Bragg fiber forms a quarter wave stack for $\lambda = 1.55$µm light. For TE modes, put l = 0 in equation (10) which gives,

$$\left[\frac{J_0'(k_c\rho_1)}{J_0(k_c\rho_1)} + i\frac{k_c}{k_1}\left(\frac{\lambda_{TE} - F_{TE} - G_{TE}}{\lambda_{TE} - F_{TE} + G_{TE}}\right)\right] = 0 \qquad (11)$$

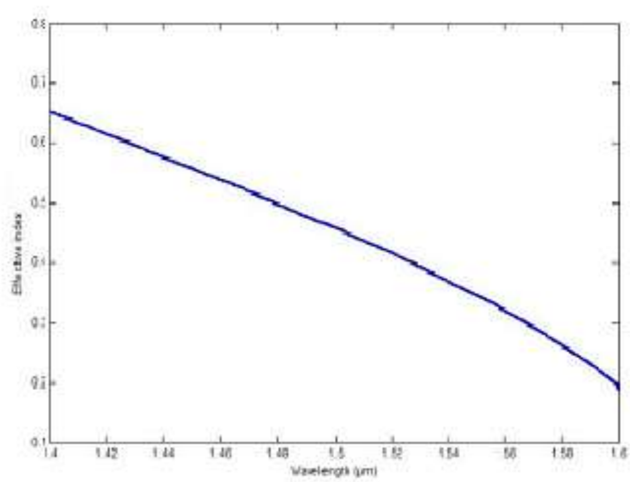

Fig 2. Dispersion of the fundamental TE mode in an air core Bragg fiber

2. Calculation of field distribution

The field amplitude in class II layer of the nth cladding pair can be found very easily on applying the condition of E_z, E_θ, H_z, and H_θ, are continuous at r = ρ_n', with the help of equation (9a) and (9b) which gives the value of a_n', b_n', c_n' and d_n'.

For the confined mode TE, $E_z = 0$ and λ_{TE}, as given in equation (8 b), must be a real number having an absolute value less then one, which gives a_n', b_n' for the TM component.

$$\begin{bmatrix} a_n' \\ b_n' \end{bmatrix} = \frac{1}{2}\sqrt{\frac{k_2}{k_1}} \begin{bmatrix} \left(1 + \frac{n_1^2 k_2}{n_2^2 k_1}\right) e^{ik_1 l_1} & \left(1 - \frac{n_1^2 k_2}{n_2^2 k_1}\right) e^{-ik_1 l_1} \\ \left(1 - \frac{n_1^2 k_2}{n_2^2 k_1}\right) e^{ik_1 l_1} & \left(1 + \frac{n_1^2 k_2}{n_2^2 k_1}\right) e^{-ik_1 l_1} \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} \qquad (12)$$

And c_n' , d_n' for the TE component,

$$\begin{bmatrix} c_n' \\ d_n' \end{bmatrix} = \frac{1}{2}\sqrt{\frac{k_2}{k_1}} \begin{bmatrix} \left(1 + \frac{k_2}{k_1}\right) e^{ik_1 l_1} & \left(1 - \frac{k_2}{k_1}\right) e^{-ik_1 l_1} \\ \left(1 - \frac{k_2}{k_1}\right) e^{ik_1 l_1} & \left(1 + \frac{k_2}{k_1}\right) e^{-ik_1 l_1} \end{bmatrix} \begin{bmatrix} c_n \\ d_n \end{bmatrix} \qquad (13)$$

Once we find out the values of $a_n, b_n, c_n, d_n, a_n', b_n', c_n'$, and d_n' by combining equation (6.) through (13), which gives the electromagnetic field distribution in entire cladding region.

For a confined TE mode, $E_z = 0$, and H_z is calculated from equations (1a, 1b, 9 and eq. 13) and we get,

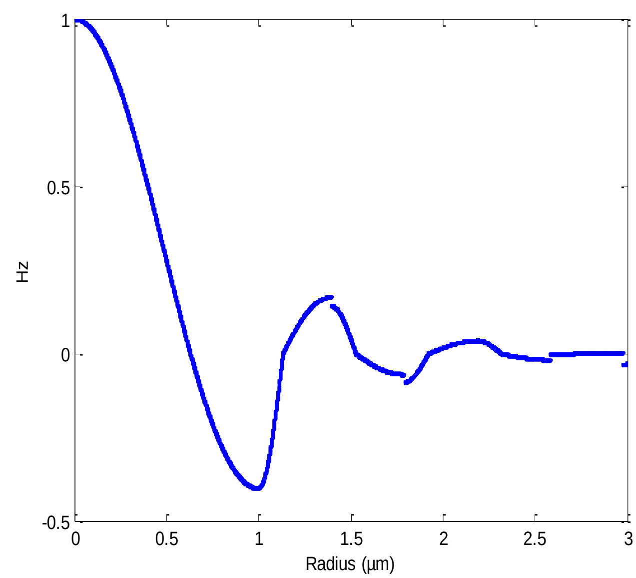

Fig 3. Field distribution of the guided Bragg fiber of TE mode with component H_z at $\lambda = 1.55\mu m$

Similarly E_θ is calculated from the derivative of H_z, we calculate both H_z and E_θ field for the guided TE mode at λ= 1.55µm.

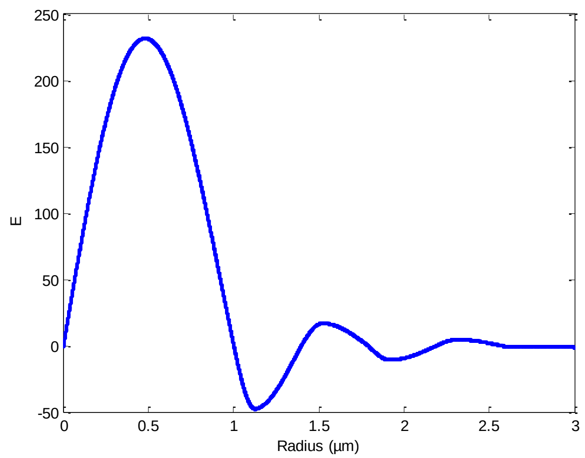

Fig.4. Field distribution of the guided Bragg fiber of TE mode with component E_θ at $\lambda = 1.55\mu m$

II. CONCLUSION

There is only difference between these two approaches, exact and asymptotic approach is that, for the exact and true solution, the field distribution within Bragg fiber is calculated from equation[1]. Hence within the core region, these two approaches gives similar result. In the cladding region, these two approaches differ from each other, and this difference represents, how will this asymptotic approximation works. However, this difference in the cladding region in between these two approaches is very small.

Finally, the conclusion is that this asymptotic approach gives an accurate description of the field distribution of the guided modes, and the agreement between these two solution is actually excellent.

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