Effect of High Altitude in Facilitated Oxygen Diffusion in Humans

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Abstract: **- The paper analyses the effect of high altitude in facilitated oxygen diffusion in skeletal muscle fibre in steady and extinction states. The human body continuously consumes the oxygen, diffuses in muscle fibre as well as binds with the myoglobin to produce oxy-myoglobin. In absence of external oxygen supply at high altitude, oxy-myoglobin releases oxygen to meet its deficiency. In continuation of the process concentration of oxygen at the center of the muscle becomes zero giving rise to oxygen debt, highly undesirable condition. The condition, which determines the movement of point of anoxia with time depends upon the rate of absorption of oxygen in the medium and its altitude. Finite difference numerical method has been used to find the oxygen concentration at any time at any point in skeletal muscle fibre as a function of rate of oxygen consumption and the facilitated diffusion parameter at different altitude. The obtained results are compared with the results available in literature, which are in close agreement.**

Key words: **High Altitude, Facilitated oxygen Diffusion, Humans.**

Nomenclature

S	Oxygen concentration $[O_2]$
S_0	Oxygen concentration at the surface at sea level
S_a	Oxygen concentration at the surface at height h
C	Oxy-myoglobin concentration [Mb]
e	Myoglobin concentration $[MbO_2]$
e_0	Total myoglobin
f	Rate of uptake of oxygen
D_s	Oxygen diffusion constant
D_c	Oxy-myoglobin diffusion constant
D_e	Myoglobin diffusion constant
t	time
r	radius
σ	SK^+/K^-
σ_1	S_aK^+/K^-
U	C/e_0
O_2	oxygen molecule
V	e/e_0
J	flux of oxygen
J'	Ja/ε_1
ε_1	$D_s/K^+e_0\, a^2$
ε_2	$D_c/K^-\, a^2$
ε_3	$D_e/K^- a^2$
K	K^+ /K^-
T	K^+e_0t
R	r/a
ρ	$\varepsilon_2/\varepsilon_1$
g	consumption parameter
f	oxygen uptake
γ	g/e_0K^-
γ_1	$\gamma/4.0$
h'	height of experiment performed
h_0	Maximum altitude, oxygen available in atmosphere.
h	h'/ h_0

I. INTRODUCTION

High altitude has considerable effect on facilitated oxygen diffusion in humans and hence its study is very important from astronautical point of view. When human body reaches at a height of around 2,100 m (7,000 feet) above sea level, the saturation of oxy-myoglobin begins to decrease [1]. However, the human body has both short-term and long term adaptation to altitude that allow it to partially compensate with the deficiency of oxygen to some extent. Generally, athletes use these adaptation techniques to help their performance at high altitude [2-7], but within the altitude of "death zone". The death zone is referred as the altitude above 26000 feet from sea level from literature, where the amount of oxygen is insufficient to sustain human life. Atmospheric pressure decreases exponentially with altitude, while the amount of oxygen (O_2) fraction remains unchanged to about 100 km, so O_2 decreases exponentially with altitude as well. Travelling to high altitude regions can lead to several medical problems such as cerebral edema, risk of permanent brain damage etc.[8-11]. The human body can perform best at sea level[12], where the atmospheric concentration of oxygen (O_2) in air is maximum (20.9%) and the partial pressure of O_2 (pO_2) is 21.136 kPa. This saturates hemoglobin in healthy individuals as the oxygen-binding red pigment in red blood cell.

The facilitated oxygen diffusion problem is of immediate interest in medical research concerning the uptake of oxygen by tissues of human being living at high altitude. The problem was carried out by Crank and Gupta [13]. They considered the rate of absorption as constant and solved the problem of oxygen diffusion. Martinez and Marquina[14] and Martinez et. al.[15] extended the work by considering that the rate of absorption as function of distance from outer surface, applicable to inhomogeneous media. Roger [16] analysed the moving boundary problem for non homogeneous rate of absorption in the medium. The eminent Physiologists, such as Huxley[17], Hodgkin[18] etc. had taken a keen interest towards the Mathematical approach to solve physiological problems. There are various reactions, which are catalised by enzymes (generally proteins), but they themselves are not changed. Enzymes are efficient in speeding up the biological reaction. It is often the case that the reactants in enzymatic reactions are free to diffuse in the medium. Therefore one has to keep track of both i.e. reaction and diffusion, such type of systems are called reaction diffusion systems. The subject is very useful to understand the facilitated diffusion process[22-35].

Muscle fibre consumes oxygen even at rest state of the body, because of the biological process taking place in muscles. This consumption of energy requires constant metabolisim of sugar, which consumes oxygen. In human beings, the oxygen consumption in live tissues at rest is about 5x10-8 mol/cm3s in human being and the consumption of myoglobin is about 2.8x10-7 mol/cm3. Thus when myoglobin is fully saturated it contains only 5% supply of oxygen. Further, the oxygen at the exterior (surface) of the muscle cell must penetrate to the center of the cell to prevent the oxygen deficiency, to avoid the case of oxygen debt, which is really a common problem at high altitude.When a body is in dynamic state such as exercise, physical work, running etc., consumption rate of oxygen is further increased. Thus, the chances of occurrence of oxygen debt [19-21] is increased more at high altitude, giving rise to various illness[8-11].

The present work is concerned with the effect of high altitude in facilitated oxygen diffusion in skeletal muscle fibre. The amount of oxygen reduces with the increase of height and becomes almost zero at death zone. The presence of myoglobin protects the muscle at the farthest distance from the exterior surface, from oxygen debt by releasing the stored oxygen to a certain time. Model suggested by Crank and Gupta[13], Martinez et. al. [14] and others [26-29] can be obtained as particular cases. Free and bounded oxygen concentrations are obtained at any time at various altitudes as a function of rate of absorption of oxygen and the facilitated diffusion parameter by numerical methods. Numerical algorithms suggested by Ansari [22-23] to obtain the position of the moving boundary and the concentration of oxygen at the given time are well-defined and consistent. Results obtained are in consistent with available results in literature.

II. FORMULATION OF THE PROBLEM

Considering a muscle fibre as a long circular cylinder of radius ‘a’ where diffusion takes place in the radial direction and distribution of chemical species are radially symmetrical. When oxygen $[O_2]$ passes through the muscles, it reacts with myoglobin [Mb] to produce oxy-myoglobin $[MbO_2]$

$$O_2 + Mb \underset{K^-}{\overset{K^+}{\rightleftharpoons}} MbO_2, \qquad (2.1)$$

where K^+ and K^- are reaction constants in forward and backward directions respectively.

The law of mass action for uptake of oxygen f into oxy-myoglobin is

$$f = - K^- C + K^+ Se, \qquad (2.2)$$

where S, e and C are the concentrations of oxygen $[O_2]$, myoglobin [Mb] and oxy-myoglobin $[MbO_2]$ respectively at point r, from the centre of the medium at any time t.

Let the oxygen concentration at the surface (boundary) is

$$S_a = S_0\left(1 - \frac{h'}{h_0}\right), \qquad (2.3)$$

where S_0 is the maximum oxygen concentration at sea level, h’ the current height of experiment performed and h_0 is the maximum possible height, where oxygen is available.

Governing diffusion equations for oxygen, oxy-myoglobin and myoglobin are respectively given by

$$\frac{\partial S}{\partial t} = D_s \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial S}{\partial r}\right) - g - f, \qquad (2.4)$$

$$\frac{\partial C}{\partial t} = D_c \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) + f, \qquad (2.5)$$

$$\frac{\partial e}{\partial t} = D_e \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial e}{\partial r}\right) - f, \qquad (2.6)$$

where g be the constant consumption rate of oxygen per unit volume in the medium and D_s, D_c and D_e are the diffusion constants of oxygen, oxy-myoglobin and myoglobin respectively.

The boundary conditions to solve the differential equations are

$$\text{at } r = a,\ S = S_a, \quad \frac{\partial C}{\partial r} = 0, \frac{\partial e}{\partial r} = 0, \qquad (2.7)$$

$$\text{at } r = 0,\ \frac{\partial S}{\partial r} = 0, \quad \frac{\partial C}{\partial r} = 0, \frac{\partial e}{\partial r} = 0. \qquad (2.8)$$

Total myoglobin e_0 in the medium is conserved by the reaction. Therefore

$$e + C = e_0. \tag{2.9}$$

Let the non dimensionalised variables are

$$U = \frac{C}{e_0}, \quad V = \frac{e}{e_0}, \quad R = \frac{r}{a}. \tag{2.10}$$

Introducing the non dimensional variables (2.10), equations (2.4),(2.5) and (2.6) reduce to

$$\frac{\partial \sigma}{\partial T} = \varepsilon_1 \left(\frac{\partial^2 \sigma}{\partial R^2} + \frac{1}{R} \frac{\partial \sigma}{\partial R} \right) - \gamma + (U - \sigma V), \tag{2.11}$$

where $\varepsilon_1 = \dfrac{D_s}{a^2 K^+ e_0}, \gamma = \dfrac{g}{e_0 K^-}$, $T = t\, e_0 K^+$ and

$\sigma = \dfrac{K^+ S}{K^-}$,

$$\frac{e_0}{K} \frac{\partial U}{\partial T} = \varepsilon_2 \left(\frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} \right) - U + \sigma V, \tag{2.12}$$

where $\varepsilon_2 = \dfrac{D_c}{a^2 K^-}, K = \dfrac{K^-}{K^+}$.

$$\frac{e_0}{K} \frac{\partial V}{\partial T} = \varepsilon_3 \left(\frac{\partial^2 V}{\partial R^2} + \frac{1}{R} \frac{\partial V}{\partial R} \right) + U - \sigma V, \tag{2.13}$$

where $\varepsilon_3 = \dfrac{D_e}{a^2 K^-}$.

Using non dimensional variables defined in equation (2.10), equation (2.9) reduces to

$$U + V = 1. \tag{2.14}$$

The boundary conditions with respect to new variables becomes

$$\text{At } R = 1, \; \sigma = \sigma_1, \quad \frac{\partial U}{\partial R} = 0, \frac{\partial V}{\partial R} = 0. \tag{2.15}$$

and

$$\text{At } R = 0, \; \frac{\partial \sigma}{\partial R} = 0, \quad \frac{\partial U}{\partial R} = 0, \frac{\partial V}{\partial R} = 0 \quad . \tag{2.16}$$

The molecular weight and molecular structure of both oxy-myoglobin and myoglobin are identical, so the constants ε_2 and ε_3 are approximately taken to be the same. Hence equation (2.13) is superfluous. Therefore only differential equations to be solved are (2.11) and (2.12).

2.1 Steady State

Oxygen diffuses into the muscle fibre freely, where some of the oxygen is absorbed by the medium, thereby being removed from the diffusion process. The concentration of oxygen at the surface of the medium is maximum and constant at particular height, but as the height increases concentration of O_2 decreases. The first phase of the problem continues, until steady state reached in which the oxygen does not change any further in the medium with time at a particular height.

Concentration of oxygen at every point in the medium is constant. The differential equations governing the steady state process are

$$\varepsilon_1 \left(\frac{\partial^2 \sigma}{\partial R^2} + \frac{1}{R} \frac{\partial \sigma}{\partial R} \right) - \gamma + (U - \sigma V) = 0. \tag{2.1.1}$$

$$\varepsilon_2 \left(\frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} \right) - U + \sigma V = 0. \tag{2.1.2}$$

The constant ε_2 in equations (2.1.2) is small enough to warrant the approximation that Quasi steady state holds good in the interior of the muscle fibre. Thus

$$U = \frac{\sigma}{1 + \sigma}. \tag{2.1.3}$$

Steady state solution for concentration distribution in muscle fibre (Keener and Sneyd [24], pp. 43) using condition (2.15) and (2.16) along with (2.1.3) becomes

$$\sigma(R,0) = \frac{1}{2} \left[\gamma_1 (R^2 - 1) + \sigma_1 + \rho U_1 - 1 - \rho + \sqrt{\left(\gamma_1 (R^2 - 1) + \sigma_1 + \rho U_1 - 1 - \rho \right)^2 + 4 \left(\gamma_1 (R^2 - 1) + (\sigma_1 + \rho U_1) \right)} \right] \tag{2.1.5}$$

where σ_1 and U_1 are free and bounded oxygen concentrations at the surface respectively and $\rho = \varepsilon_2 / \varepsilon_1$, $\gamma_1 = \gamma / 4\varepsilon_1$ are facilitated oxygen diffusion parameter and consumption parameter respectively.

Oxygen decreases with the increase of height. Oxygen debt occurs, when σ becomes zero, while marginal oxygen debt occurs when total oxygen concentration falls to zero. The oxygen concentration at the boundary and oxygen content in the myoglobin just enough to prevent oxygen debt at the center is called Critical oxygen concentration σ_0,

$$\sigma_0 + \rho \frac{\sigma_0}{1+\sigma_0} = \frac{\gamma}{4\varepsilon_1} . \tag{2.1.6}$$

2.2 Extinction State

At very high altitude called death zone, free oxygen concentration σ becomes zero. Thus no oxygen passes in the skeletal muscle fibre. The medium continues to absorb the available oxygen already in it due to biological process and as a consequence, oxygen debt occurs at the center of the muscles subsequently. Thus the boundary of zero concentration recedes towards the surface. The oxygen concentration obtained in steady state just at altitude possible to human survival, can be taken as the initial distribution of oxygen for the solution of moving boundary value problem (extinction state.) The governing differential equations of extinction state are

$$\frac{\partial \sigma}{\partial T} = \varepsilon_1 \left(\frac{\partial^2 \sigma}{\partial R^2} + \frac{1}{R} \frac{\partial \sigma}{\partial R} \right) - \gamma + (U - \sigma V) , \tag{2.2.1}$$

$$\frac{e_0}{K} \frac{\partial U}{\partial T} = \varepsilon_2 \left(\frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} \right) - U + \sigma V , \tag{2.2.2}$$

and the boundary conditions are

$$\text{at } R = 1,\ \sigma = 0, \quad \frac{\partial U}{\partial R} = 0, \frac{\partial V}{\partial R} = 0 , \tag{2.2.3}$$

$$\text{at } R = 0,\ \frac{\partial \sigma}{\partial R} = 0, \quad \frac{\partial U}{\partial R} = 0, \frac{\partial V}{\partial R} = 0 , \tag{2.2.4}$$

$$\text{and} \qquad \sigma = \sigma(R,0), \qquad U = U(R,0). \tag{2.2.5}$$

The main aim is to find the point of anoxia and its movement in the muscle fibre. Secondly, the role of myoglobin as oxygen transport to the medium as a function of time, distance, height and consumption parameter. The solution of the problem has been obtained by Finite difference. The rate of change of concentration of oxygen and movement of boundary point(separating oxygen and non oxygen medium in skeletal muscle fibre) in different time and at different height is found to be different. Different Numerical methods are proposed for different time region by earlier Researchers[25-29]. Crank and Gupta[13] suggested that before the disturbance at the surface has an effect on the concentration of oxygen in the neighbourhood of center of the muscle (boundary does not move within specified degree of accuracy), suitable approximation can be obtained using Lagrange Method.

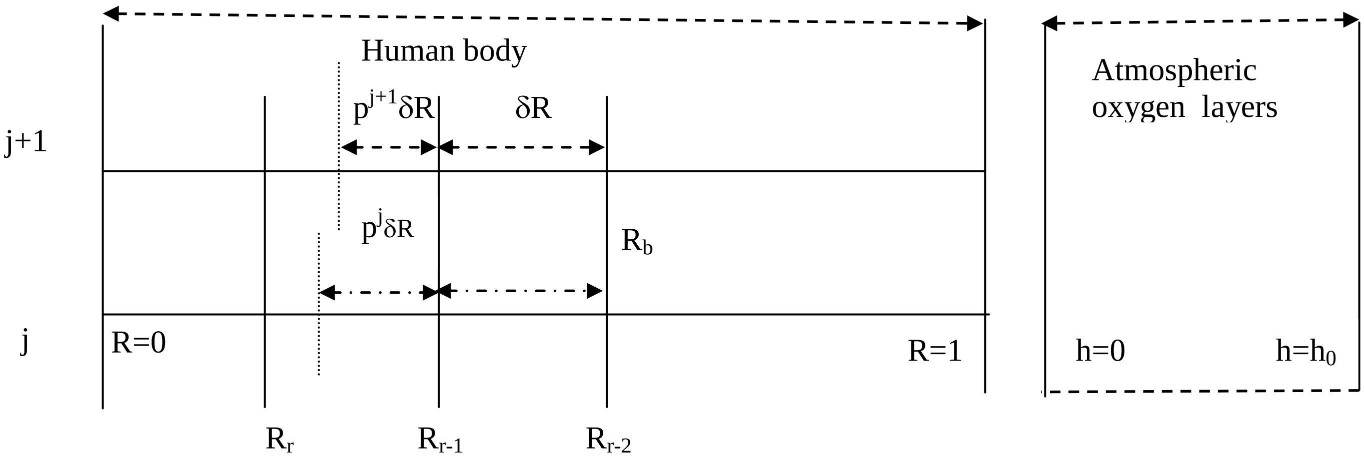
III. NUMERICAL METHODS

The percentage of oxygen concentration of oxy-myoglobin determines the content of oxygen in our blood. At high altitude saturation of oxy-myoglobin decreases[19-21]. The result was found to be more pronounced with the increase consumption rate at high altitude. The continuous absorption of oxygen in skeletal muscle fibre as a spontaneous biological process at death zone results in quick oxygen debt at the centre(farthest distance from surface) of muscle fibre. Subsequently, the point of anoxia, zero concentration of oxygen moves towards exterior surface. Major problem to employ Numerical Methods arises because of abrupt imposition of zero oxygen concentration at the surface at death zone for time T ≥0, which gives discontinuity in the derivative boundary condition. The conditions (2.2.3-2.2.4) show that the concentration at the surface has a negative unit gradient in steady state. The discontinuity in surface-gradients predicts that Numerical Methods based on the finite differences are liable to give inaccurate solution in the surface neighbourhood for short times.

Douglas and Gallie[18] introduced a Numerical method of variable time step, keeping the size of the space mesh fixed. Murray and Landis[19] used a variable space mesh and kept the time step fixed. Ehlrich[20] employed implicit formula at the intermediate points and Taylor's expansions near the moving boundary in both time and space directions. Lotkin[21] used subdivided differences, while Crank[22] suggested a three-point Lagrange interpolation formula near the moving boundary.

In the present analysis, the concentrations at the intermediate points between the two boundaries have been calculated by using simple explicit finite-difference formula. Near the moving boundary a Lagrange formula has been used, as suggested by Crank[22] because of convenience in calculation. The location of the moving point itself is determined by a Taylor's series. The whole region, $0 \le R \le 1$, is subdivided into M intervals each of width δR and taking $R_r = i\delta R$ where $0 \le i \le M$ ($M\delta R = 1$).

Let the concentrations at each of the grid points at the j^{th} time level are known and the position of the moving boundary at that time is somewhere in the i^{th} interval between R_{i-1} and R_i, given by $R_b = (i-1)\,\delta R + p^j \delta R$, $0 \le p^j < 1$ along radial direction. Fig. 1 presents the boundary point R_b.



(a) (b)

Fig. 1: (a) Discretisation of oxygen medium in human body, (b) Oxygen layer of in the atmosphere.

Then the concentrations at the $(j+1)^{th}$ time level can be calculated using the well known explicit formula.

The oxygen concentration at the sealed surface is given by

$$\sigma_{ij+1} = \sigma_{ij} + \delta t\left[\frac{\varepsilon_1(\sigma_{i+1j} - \sigma_{ij})}{dR}\left(\frac{2}{dR}+1\right) - \gamma + U_{ij} - \sigma_{ij}V_{ij}\right], \qquad (3.1)$$

while at the other intermediate points the explicit formula becomes

$$\sigma_{ij+1} = \sigma_{ij} + \delta t\left[\frac{\varepsilon_1}{dR}\left(\frac{(\sigma_{i+1j} - 2\sigma_{ij} + \sigma_{i-1j})}{dR} + \frac{(\sigma_{i+1j} - \sigma_{i-1j})}{2R_i}\right) - \gamma + U_{ij} - \sigma_{ij}V_{ij}\right]. \qquad (3.2)$$

Concentrations in the neighbourhood of the boundary as suggested by Crank and Gupta[8] using the appropriate finite-difference replacement leads to have the following equations.

When boundary does not move within the specified degree of accuracy

$$\sigma_{ij+1} = \sigma_{ij} + \delta t\left[\frac{\varepsilon_1}{dR}\frac{2}{dR}\left(\frac{\sigma_{i-1j}}{p_s+1} - \frac{\sigma_{ij}}{p_s}\right) - \gamma + U_{ij} - \sigma_{ij}V_{ij}\right], \qquad (3.3)$$

where $p_s = \dfrac{\sqrt{2\sigma_{ij}}}{dR}$,

-when boundary starts moving

$$\sigma_{ij+1} = \sigma_{ij} + \delta t\left[\frac{\varepsilon_1}{dR}\left(\frac{2}{dR}\left(\frac{\sigma_{i-2j}}{p_s+1} - \frac{\sigma_{i-1j}}{p_s}\right) + \frac{\sigma_{i-1j} - \sigma_{i-2j}}{R_i}\right) - \gamma + U_{ij} - \sigma_{ij}V_{ij}\right], \qquad (3.4)$$

where $p_s = \dfrac{\sqrt{2\sigma_{i-1j}}}{dR}$.

Similarly, the bounded oxygen concentration at the surface, intermediate points and in the neighbourhood of moving boundary respectively becomes

$$U_{ij+1} = U_{ij} + \delta t\frac{K}{e_0}\left[\frac{\varepsilon_2(U_{i+1j} - U_{ij})}{dR}\left(\frac{2}{dR}+1\right) - U_{ij} + \sigma_{ij}V_{ij}\right]. \qquad (3.5)$$

$$U_{ij+1} = U_{ij} + \delta t\frac{K}{e_0}\left[\frac{\varepsilon_2}{dR}\left(\frac{(U_{i+1j} - 2U_{ij} + U_{i-1j})}{dR} + \frac{(U_{i+1j} - U_{i-1j})}{2R_i}\right) - U_{ij} + \sigma_{ij}V_{ij}\right], \qquad (3.6)$$

-when boundary does not move

$$U_{ij+1} = U_{ij} + \delta t\frac{K}{e_0}\left[\frac{\varepsilon_2}{dR}\frac{2}{dR}\left(\frac{U_{i-1j}}{p_u+1} - \frac{U_{ij}}{p_u}\right) - U_{ij} + \sigma_{ij}V_{ij}\right], \qquad (3.7)$$

where $p_u = \dfrac{\sqrt{2U_{ij}}}{dR}$.

when boundary starts moving

$$U_{ij+1} = U_{ij} + \delta t \frac{K}{e_0}\left[\frac{\varepsilon_2}{dR}\left(\frac{2}{dR}\left(\frac{U_{i-2j}}{p_u+1} - \frac{U_{i-1j}}{p_u}\right) + \frac{U_{i-1j} - U_{i-2j}}{R_i}\right) - U_{ij} + \sigma_{ij}V_{ij}\right] \tag{3.8}$$

where $p_u = \dfrac{\sqrt{2U_{i-1j}}}{dR}$.

IV. RESULTS AND DISCUSSION

Numerical solution is carried out using finite difference method at intermediate points and suitable numerical approximation at the boundary points are also taken to avoid the inconsistency with practical results.

Fig 2 represents the oxygen distribustion throughout the skeletal muscle fiber from centre (R = 0) to the outer surface (R = 1) for consumption parameter γ_1, = 3.587 and facilitated parameter ρ= 3.00 at different height h = 0.0 to 0.9. Oxygen concentration is found to be highest at height h =0.0 i.e. at the sea level and hence the best performance of human is found to be at see level only. As the human body climbs at higher altitude, oxygen concentration decreases and becomes zero at h=1.0, called the Death zone**,** where survival is not possible. The oxygen concentration is very low at height h=0.9. At high altitude, deficiency of oxygen causes several illness [8-11]. Fig.2 shows how the oxygen concentration decreases with the increase of height and hence one can take precautionary measure accordingly. At high altitude, it is advised to increase the breathing rate so that more oxygen enters into the body. Thus with increase of facilitated oxygen diffusion parameter ρ =5.00, oxygen distribution in human is found to be increased (see Fig. 3), keeping other parameter fixed. The effect of physical exercise and daily physical work at high altitude has been presented in Fig. 4. Figure shows that as the consumption rate γ_1 of oxygen increases in body and hence oxygen distribution is further decreased with the increase of height. All these observations gives very important information of crisis of oxygen to take care, before one collapse due to chest pain and effect on heart. When one is at high altitude and such situation arises, he should reduce the physical work at once and breath fast to meet the immediate requirement of oxygen. Technically, decrease of rate of consumption of oxygen γ_1 and increase of facilitated oxygen diffusion parameter ρ increases the oxygen distribution in human body.

There are some situation, when free oxygen reduces to very low level either due to increase to height or other physical problem, oxygen debt arises. In such case oxy-myoglobin releases the bounded oxygen to meet the requirement of oxygen in the body in both states (see Fig 5).

Extinction state: Extinction state is an analysis of oxygen diffusion in Death zone. Some times it happens that human body reaches as death zone knowingly or unknowingly, where surface oxygen concentration in air is zero. The human body continuously consumes the total oxygen available inside the body, due to biological process even in static state. After some time, oxygen debt arises at the centre of the human body. Subsequently, the zero oxygen concentration point recedes towards the outer surface of the body. This is the actual situation of death with time, due to lack of oxygen. A time dependent analysis of oxygen distribution in human body at the centre, mid and exterior points is presented in Fig. 6 at death zone for facilitated oxygen parameter ρ=3.00 and consumption parameter γ_1= 3.587. Figure shows that oxygen concentration decreases with the time T. The effect was found to be more pronounced (see Fig 7), if one is involved in some physical work (i.e. increase of consumption parameter γ_1). Fig. 8 presents the oxygen distribution with time for facilitated oxygen parameter ρ =5.00 and the oxygen distribution is found to greater in this case. Hence, oxy-myoglobin plays very important role at hyper anoxia situations.

Fig. 9 presents the oxygen distribution against height h at time T=0 (steady state) and time T= 10 (extinction state) at R=1(exterior surface), R =0.5 (mid point) and R= 0.0(centre). Oxygen concentration reduces with time and effect is found to be more at the exterior surface at sea level. The effect is less with time at high altitude. When oxygen concentration σ reduces to very low level at high altitude, oxy-myoglobin releases the bounded oxygen to meet the oxygen requirement in the body rapidly with time (see Fig. 10).

Figure 11 compares oxygen concentration with the result obtained by Crank and Gupta[8], when rate of consumption per unit volume is constant in one-dimension cartesian coordinate system, in the absence of oxygen uptake function *f*. The obtained results are in good agreement with those of Crank and Gupta[13]. The model presented by Martinez *et al*.[14-15] can be obtained as a special case of the present model, by taking rate of consumption per unit volume as a function of distance (i.e. √distance) in the one-dimension. A comparative study with different models presented by Martinez *et al*.[10], Crank and Gupta[13] and Hansen and Hougaard [30] is shown in Table 1. Figure 12 compares the critical oxygen concentration at the surface against oxygen consumption rate parameter γ_1 for facilitated parameter ρ = 0, 5 and 10 with the result of Keener and Sneyd [24] at sea level. Figure 12 shows that as the consumption rate increases, the critical oxygen concentration at the surface has to be increased to prevent oxygen debt.

V. CONCLUSION

The study of facilitated oxygen diffusion in skeletal muscle fibre at mountain has wider applications from medical point of view. Oxygen concentration decreases with the increase of height at atmosphere. The information presented here is of immediate interest for humans living at high altitude of mountain. The present problem analyses the combined effect of rate of consumption of oxygen and along with the increase of height. The role of myoglobin is to prevent the deficiency of oxygen either due to high altitude or increase of oxygen consumption parameter in the muscle fibre. The results obtained are in good agreement with the modelling results available in literature and will be useful for practical purposes.

Table 1

Comparative study of free oxygen concentration in extinction state as a function of time T at the surface

Time T	Matinez et. al. Model		Crank and Gupta Model		Hansen and Hougaard Model	
	Ref. [15]	Present	Ref. [13]	Present	Ref. [24]	Present
0.04	0.2759	0.2758	0.2759	0.2743	0.2743	0.2741
0.10	0.1449	0.1445	0.1449	0.1432	0.1432	0.1430
0.18	0.0235	0.0238	0.0235	0.0213	0.0218	0.0217
0.19	0.0109	0.0111	0.0109	0.0082	0.0090	0.0091

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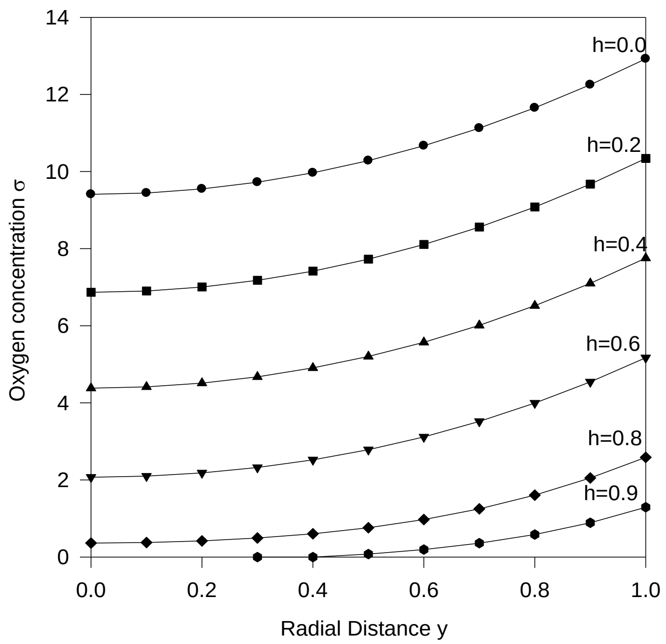

Fig 2. Free oxygen concentration σ vs. radial distance y for facilitated diffusion parameter ρ =3.00 and consumption parameterγ_1= 3.587.

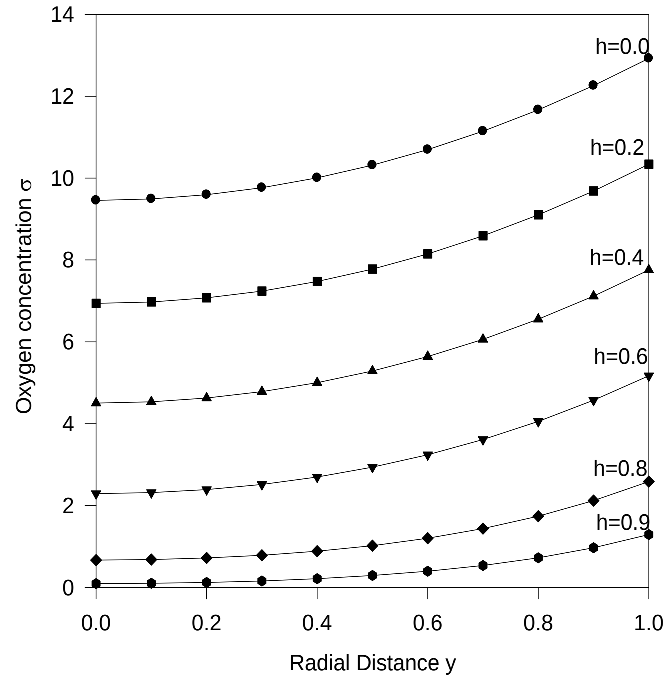

Fig 3. Free oxygen concentration σ vs. radial distance y for facilitated diffusion parameter ρ =5.00 and consumption parameter γ_1= 3.587

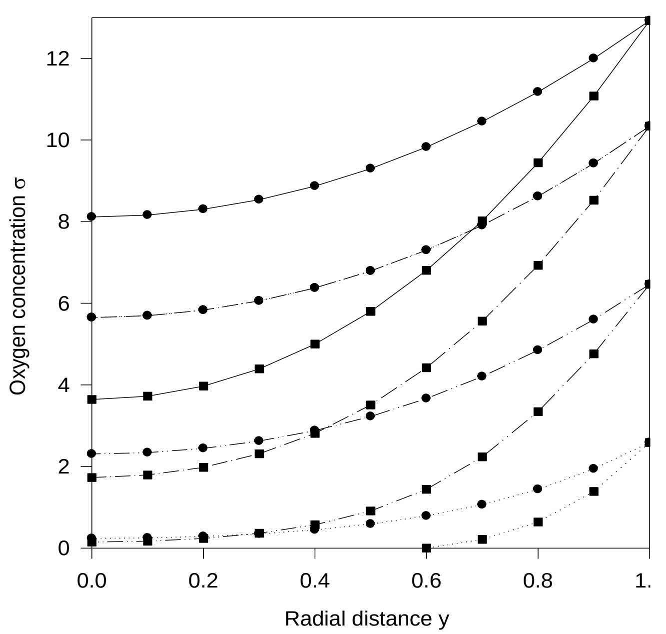

Fig 4. Free oxygen concentration σ vs. radial distance y for facilitated diffusion parameter ρ =5.00 and consumption parameterγ_1= 5.00: ● and γ_1= 10.00: ■

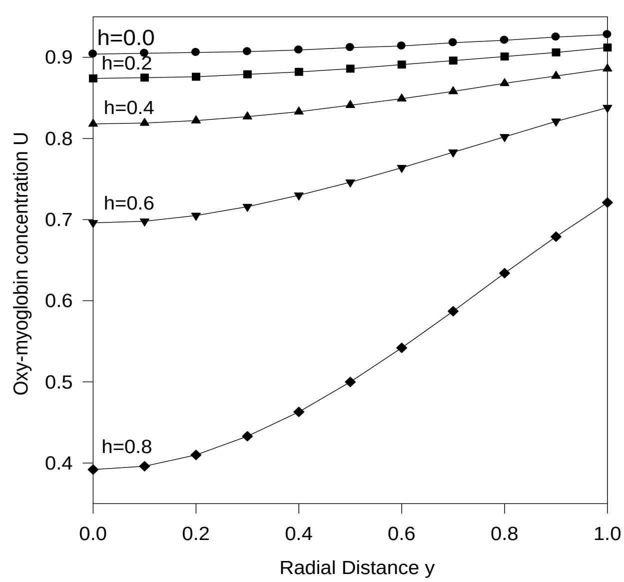

Fig 5.Oxymyoglobin concentration U vs. radial distance y at different height h for facilitated diffusion parameter ρ =5.00 and consumption parameter γ_1= 3.587.

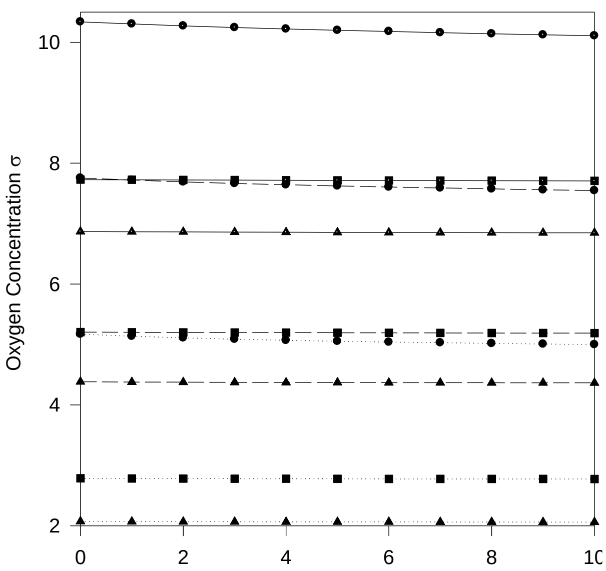

Fig 6.Oxygen concentration σ vs. time T for facilitated diffusion parameter ρ =5.00 and height h =0.2: , h=0.4: and h=0.6: at y=1.0: ●,y=0.5: ■ and y=0.0: ▲ consumption parameter γ_1= 3.587.

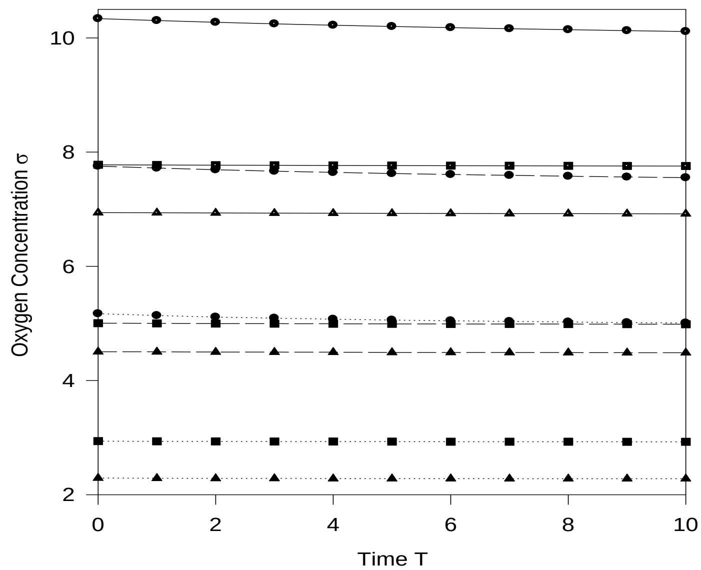

Fig 8.Oxygen concentration σ vs. time T for facilitated diffusion parameter ρ =5.00 and height h =0.2: , h=0.4: and h=0.6: at y=1.0: ●,y=0.5: ■ and y=0.0: ▲ consumption parameter γ_1= 3.587.

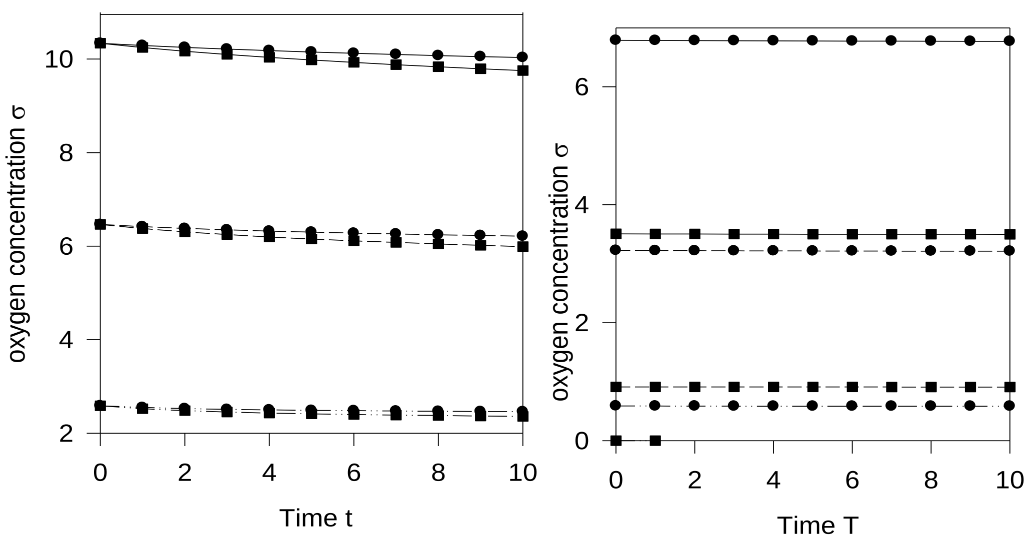

Fig.7A (Surface y=1.0) Fig. 7B (Mid point y=0.5)

Fig 7. Free Oxygen concentration σ vs. time T for facilitated diffusion parameter ρ =5.00 and height h =0.2: , h=0.5: and h=0.8: atconsumption parameter γ_1=5.0: ●,γ_1=10.0: ■

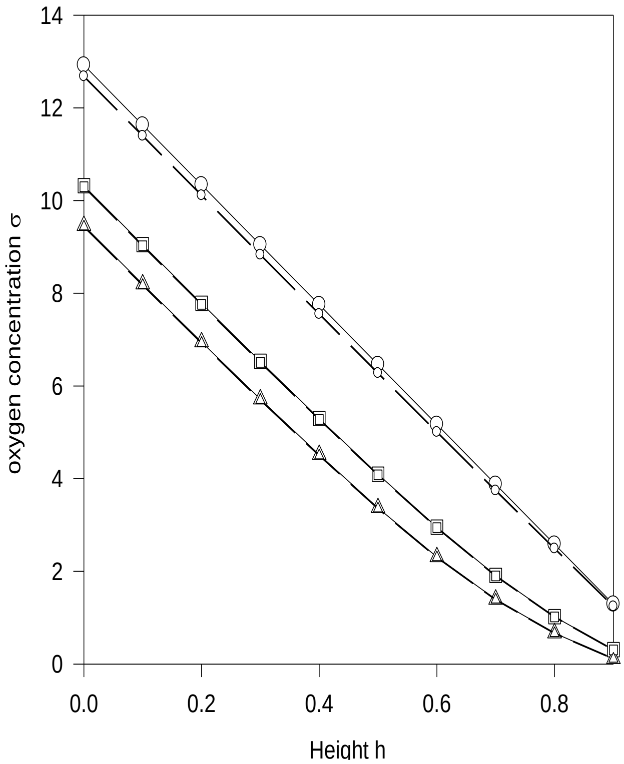

Fig 9. Free Oxygen concentration σ vs. height h for time T =0.0——, and T=10: - - - - for facilitated diffusion parameter ρ =5.00 and y=1.0: ○, y=0.5: □ and y=0.0: △ , atconsumption parameter γ_1=3.587

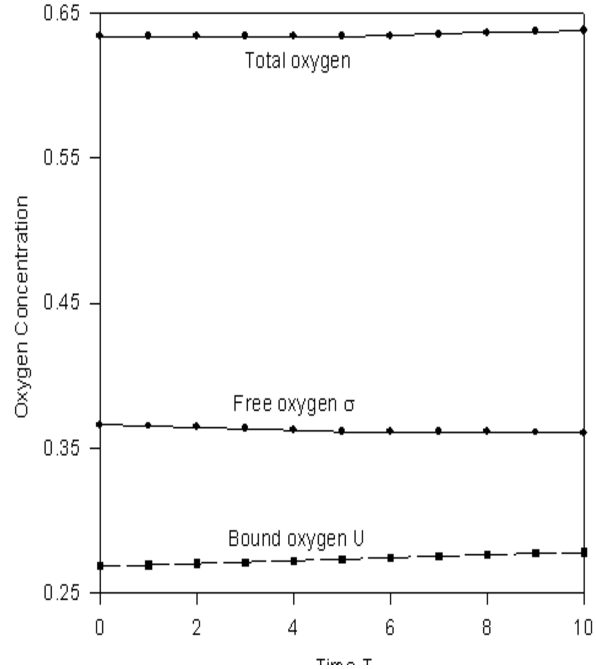

Fig. 10 Release of bounded oxygen by oxy-myoglobin when free oxygen concentration is not sufficient at sea level for facilitated Oxygen diffusion parameter ρ=5.217 and consumption parameter γ_1=16.0

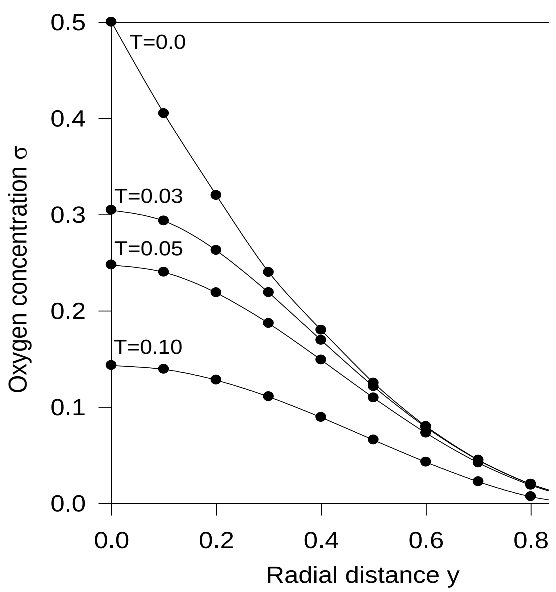

Fig 11.Comparision of oxygen concentraionσ (●) with Crank and Gupta [13]

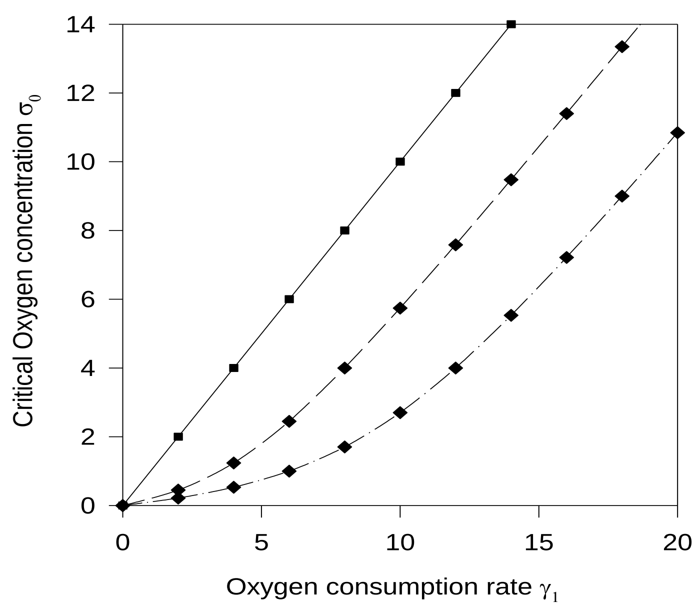

Fig. 12 ■values shows the comparision of critical oxygen concentration with Keener and Sneyd [24] for ρ=0: ——, ρ=5: ——, ρ=10: ------