Hamming net based Low Complexity Successive Cancellation Polar Decoder

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Abstract: **This paper aims to implement hybrid based Polar encoder using the knowledge of mutual information and channel capacity. Further a Hamming weight successive cancellation decoder is simulated with QPSK modulation technique in presence of additive white gaussian noise. The experimentation performed with the effect of channel polarization has shown that for 256- bit data stream, 30% channels has zero bit and 49% channels are with a one bit capacity. The decoding complexity is reduced to almost half as compared to conventional successive cancellation decoding algorithm. However, the required SNR of 7 dB is achieved at the targeted BER of 10 ˉ4. The penalty paid is in terms of training time required at the decoding end.**

Keywords: **AWGN, SC, SNR**

I. INTRODUCTION

The theory of probability is an essential mathematical tool in the design of digital communication system. The two important contents of information theory are the compression and the reliable transmission of data. Shannon in his work formalized both these problems for determining their fundamental limits. Since then the basic aim of source and channel coding theory has been to find practical schemes that approach these limits [1].The fundamental concept of error control coding is to add redundancy in information bits at the channel encoder side of transmitter and exploitation of this redundancy is done by channel decoder at the receiver to detect and correct errors. There are two types of codes available which can detect and correct of codes available which can detect and correct errors. The first such code is forward error correction code and the second one is automatic repeat request. Forward error correction is the most frequently used mechanism for error protection against noise. It improves the reliability of data transmission. Hence, error control codes are applied to the domain of space and satellite communication, data storage and transmission, transmission of digital audio and video, file transfer and mobile communication [2,3].A polar code, recently invented by Arikan is one of the forwarded error codes used for correcting and detecting errors in data transmission. The construction of code based upon a phenomenon of channel polarization.

Channel polarization refers to the fact that it is possible to make a synthesis of useful and useless channels. A survey of the literature shows that the development of code can replace the practical turbo and LDPC codes with their remarkable performance for mobile communication [4].

In the literature of Polar encoding and decoding, K. Chen et. al. has discussed how list successive cancellation algorithm simultaneously produces at most L-locally best candidates during the decoding process to reduce the chance of missing the correct codeword. Xia Shao et. al. shows the short frame transmission method of turbo code construction in achieving energy efficient transmission. Rabii Dariti et. al. Briefly describe new families of perfect linear block codes and its significances. Amin Alamdar et. al. presented a simplified successive cancellation decoding algorithm of polar code. Eridal Arikan provides the method of Polar code construction and analysis of their performance in binary symmetric channel. A trade-off between complexity vs. performance for the given block length of polar codes found to be a research problem. A method proposed in reducing the decoding complexity against code efficiency and code rate.

Channel Polarization arranges data bits into a tree-based structure. It encodes the signal bit by bit, starting from most significant bit till least significant bit according to its mutual information. This helps the algorithm for getting the desired bits per sample or to reach required reconstruction quality

with desired reliable ratio. This paper presents the modified hybrid of constructing generator matrix and successive cancellation decoding algorithm. It makes usage of mutual information and weight of the code to get rid of redundancy. It has observed that the code words generated from modified hybrid method contain a stream of more ones. This property facilitates us to make use of successive cancellation decoding algorithm. Use of modified encoding and decoding shows improvement in reliability ratio while preserving the same quality of the reconstructed signal [5].

The paper organized as: Section II talks about polarization effect based on mutual information, the weight of the code and the bound on Bhattacharya parameter for the formation of generator matrix. In section III discussion on designing Hamming net based Successive Cancelation polar decoder (SC) is mentioned. Section IV describes the decoding algorithm with illustrative example. Section V describes the simulation test engine and validation Section VI focuses on experimentation results and conclusions. In the final section, concludes the paper with references.

II. POLARIZATION EFFECT

A binary-valued random value represents the input and output of a Binary Erasure Channel. The input symbols are equally likely and the output symbols depend on the input according to the erasure probability of error ρ. The conditional probabilities are given by equation (1)

$$P(Y=0|X=0)=1-\rho$$

$$P(Y=1|X=0)=\rho$$

$$P(Y=1|X=1)=1-\rho$$

$$P(Y=0|X=1)=\rho \quad (1)$$

The mutual information about the occurrence of the events zero and one is given by equation (2)

$$I(x_1;y_1)=I(0;0)=log_2\frac{2(1-\rho)}{(1-\rho+\rho)}$$

$$I(x_2;y_1)=I(1;0)=log_2\frac{2*\rho}{(1-\rho+\rho)} \quad (2)$$

If the value of erasure probability is zero and one, the mutual information carried is 1 bit. The output specifies the input with certainty, and the channel is called noiseless. On the other hand, the mutual information between input and output is zero. channel is called useless. [6,7]. The channel capacity is given by equation (3)

$$C=\rho*log_2 2\rho+(1-\rho)*log_2 2(1-\rho) \quad (3)$$

III. HAMMING NET BASED POLAR DECODER

Polar codes are block code [8]. It consists of a set of fixed-length vectors called code words. The length of a code word is the number of elements in the vector denoted by any $N = 2^n$. A block of k=2^{n-1} information bits mapped into a codeword of length N. The block code as an (N, k) code size. The encoding and decoding functions involve the arithmetical operations of addition and multiplication performed on code words. All arithmetical operations perform according to the conventions of the algebraic field [9].

A Bhattacharyya parameter talks about channel reliability. It is given by using recursive formula as given in equation (4)

$$Z_{2k,j}=2*Z_{k,j}-Z_{k,j}^{\;2} \quad for\ 1\le j\le k$$

$$=Z_{k,j-k}^{\;2} \quad for\ k+1\le j\le 2k \quad (4)$$

The selection of rows of generator matrix is equivalent to compute vector Z_8 =[($Z_{8,1}$ $Z_{8,2}$ $Z_{8,3}$ $Z_{8,4}$ $Z_{8,5}$$Z_{8,6}$ $Z_{8,7}$ $Z_{8,8}$]. This is explained for a block length of size N=8.

Case 1: For k=1 and $j \subseteq \{1, 2\}$

$Z_{2,1} = 2Z_{1,1} - (Z_{1,1})^2 = 2*0.5-0.25 = 0.75$

$Z_{2,2} = (Z_{1,1})^2 = 0.25$

Case 2: For k=2 and $j \subseteq \{1,2,3,4\}$

$Z_{4,1} = 2Z_{2,1} - (Z_{2,1})^2 = 2*0.75-0.75*0.75 = 0.9375$

$Z_{4,2} = 2Z_{2,2} - (Z_{2,2})^2 = 2*0.25 -0.25*0.25 = 0.4375$

$Z_{4,3} = (Z_{2,1})^2 = 0.75*0.75 = 0.5625$

$Z_{4,4} = (Z_{2,2})^2 = 0.25*0.25 = 0.0625$

Case 3: For k=4 and $j \subseteq \{1,2,3,4,5,6,7,8\}$

$Z_{8,1} = 2Z_{4,1} - (Z_{4,1})^2 = 2*0.9375-0.9375*0.9375 = 0.996$

$Z_{8,2} = 2Z_{4,2} - (Z_{4,2})^2 = 2*0.4375 -0.4375*0.4375 = 0.683$

$Z_{8,3} = 2Z_{4,3} - (Z_{4,3})^2 = 2*0.5625 - 0.5625*0.5625 = 0.808$

$Z_{8,4} = 2Z_{4,4} - (Z_{4,4})^2 = 2*0.06255 -0.0625*0.0625 = 0.121$

$Z_{8,5} = (Z_{4,1})^2 = 0.9375*0.9375 = 0.879$

$Z_{8,6} = (Z_{4,2})^2 = 0.4375*0.4375 = 0.191$

$Z_{8,7} = (Z_{4,3})^2 = 0.5625*0.5625 = 0.316$

$Z_{8,8} = (Z_{4,4})^2 = 0.0625*0.0625 = 0.004$

This results into Z_8 = [0.996 0.684 0.809 0.121 0.879 0.191 0.316 0.004]. Polar code are linear code based on Galois field of the order 2 that has base generator matrix given as

$$\mathbf{G_2}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$

In this paper, polar encoder is constructed with hybrid method. A hybrid method selects the row that has a minimum value of channel reliabilities and maximum weight. The number of nonzero elements in a codeword is its weight. The set of all weights in a code constitutes the weight distribution of the code.

Step 1: Set code rate = 0.5

Step 2: Select number of rows based on maximum weight code.

Step 3: Calculate the Bhattacharya parameter of rows using recursive method.

Step 4: Select number of rows with least value of Bhattacharya parameter and maximum weight.

IV. ILLUSTRATIVE EXAMPLE

The algorithm is divided into two parts, one based on hamming distance between stored pattern with an input pattern and other is SC un-decoded bits based on mutual information. The function of a stochastic dynamic system of a Binary symmetric channel defined over the interval {0,∈, 1} is given by equation (5)

$$f_0(\rho)=\rho*(1-\rho)+\rho*\rho+(1-\rho)*\rho = 1-(1-\rho)^2$$

$$f_1(\rho)=\rho^2 \quad (5)$$

A set of binary stored vectors with dimensionality used for finding hamming net is given in equation (6)

$y_p = \begin{pmatrix} y_{p,1} \\ y_{p,2} \\ y_{p,n} \end{pmatrix} y_{p,i} \in \{-1,+1\}$ for each i =1 to n & $p = 1\ to\ P$

$weight\ matrix\ W = \frac{1}{2} * \begin{pmatrix} y_1^T \\ y_2^T \\ y_p^T \end{pmatrix}$ and $\theta = \begin{pmatrix} -\frac{n}{2} \\ -\frac{n}{2} \\ -\frac{n}{2} \end{pmatrix}$

A new vector presented to polar decoding network

$$O = OUTPUT = W_i + \theta = \frac{1}{2}\begin{pmatrix} i_{1,i} - n \\ i_{2,i} - n \\ i_{p,i} - n \end{pmatrix} \qquad (6)$$

decoding method is used to retrieve the user vital information from the Channel. The bits are decoded successively and only one bit at a given time. This is called Successive Cancellation algorithm [10, 11,12]. The correction is based on decision rule and likelihood ratio as in (7)

$$L_N^i\left(y_i^N, \widehat{m_i}^{i-1}\right) = \log \frac{W_N^i\ (y_i^N, \widehat{m_i}^{i-1} |\ 0)}{W_N^i\ (y_i^N, \widehat{m_i}^{i-1} |\ 1)}$$

$$\widehat{m_i} = \begin{cases} 0 & if L_N^i\left(y_1^N, \widehat{m_1}^{i-1}\right) \geq 0 \\ 1 & otherwise \end{cases} \qquad (7)$$

For i =1

$$L_8^1\left(y_1^8, \widehat{m_1}^0\right) = log_2 \frac{W_8^1\ (y_1^8, \widehat{m_1}^0 |\ 0)}{W_8^1\ (y_1^8, \widehat{m_1}^0 |\ 1)}$$

$$= log_2 \frac{0.9832}{0.0168} = 58.32$$

It detects the bit as $\widehat{m_1} = 0$ since $L_8^1\left(y_1^8, \widehat{m_1}^0\right) \geq 0$

For i =2

$$L_8^2\left(y_2^8, \widehat{m_2}^1\right) = log_2 \frac{W_8^2\ (y_2^8, \widehat{m_2}^1 |\ 0)}{W_8^2\ (y_2^8, \widehat{m_2}^1 |\ 1)}$$

$$= log_2 \frac{0.7575}{0.2425} = 0.49$$

gives $\widehat{m_2} = 0$ since $L_8^2\left(y_1^8, \widehat{m_2}^1\right) \geq 0$

For i=3

$$L_8^3\left(y_3^8, \widehat{m_3}^2\right) = \log \frac{W_8^3\ (y_3^8, \widehat{m_3}^2 |\ 0)}{W_N^i\ (y_3^8, \widehat{m_3}^2 |\ 1)}$$

$$= log_2 \frac{0.6514}{0.3486} = 0.271$$

since $L_8^3\left(y_3^8, \widehat{m_3}^2\right) \geq 0$ results in $\widehat{m_3} = 0$

For i=4

$$L_8^4\left(y_4^8, \widehat{m_4}^3\right) = \log \frac{W_8^4\ (y_4^8, \widehat{m_4}^3 |\ 0)}{W_8^4\ (y_4^8, \widehat{m_4}^3 |\ 1)}$$

$$= log_2 \frac{0.1655}{0.8345} = -0.70$$

Therefore $\widehat{m_4} = 1$ since $L_8^4\left(y_4^8, \widehat{m_4}^3\right) < 0$

For i=5

$$L_N^5\left(y_5^8, \widehat{m_5}^4\right) = log \frac{W_8^5\ (y_5^8, \widehat{m_5}^4 |\ 0)}{W_8^5\ (y_5^8, \widehat{m_5}^4 |\ 1)}$$

$$= log_2 \frac{0.5021}{0.4975} = 0.604$$

$\widehat{m_5} = 0$ since $L_8^5\left(y_5^8, \widehat{m_5}^4\right) \geq$

For i=6

$$L_8^6\left(y_6^8, \widehat{m_6}^5\right) = \log \frac{W_8^6\ (y_6^8, \widehat{m_6}^6 |\ 0)}{W_8^6\ (y_6^8, \widehat{m_6}^6 |\ 1)}$$

$$= log_2 \frac{0.0866}{0.9134} = -1.02$$

$\widehat{m_6} = 1$ since $L_8^6\left(y_6^8, \widehat{m_6}^5\right) < 0$

For i=7

$$L_8^7\left(y_7^8, \widehat{m_6}^6\right) = \log \frac{W_8^7\ (y_7^8, \widehat{m_7}^6 |\ 0)}{W_8^7\ (y_7^8, \widehat{m_7}^6 |\ 1)}$$

$$= log_2 \frac{0.0505}{0.9495} = -1.27$$

$\widehat{m_7} = 1$ since $L_8^7\left(y_7^8, \widehat{m_7}^6\right) < 0$

For i=8

$$L_8^8\left(y_8^8, \widehat{m_8}^7\right) = \log \frac{W_8^8\ (y_8^8, \widehat{m_8}^7 |\ 0)}{W_8^8\ (y_8^8, \widehat{m_8}^7 |\ 1)}$$

$$= log_2 \frac{0.0006}{0.9994} = -3.22$$

$\widehat{m_8} = 1$ since $L_8^8\left(y_8^8, \widehat{m_8}^6\right) < 0$

The detected bits are [0 0 0 1 0 1 1 1].

V. SIMULATION TEST ENGINE AND VALIDATION OF RESULTS

This section presents simulation test engine for simulation of the proposed algorithm. The experiment is carried with QPSK modulation techniques. The performance evaluation parameters used for comparing the polarization rate is the frozen bits [13]. The quality of the reconstructed signal depends on the channel capacity as in equation (8)

C = max I(X;Y)

$$= \max \sum_{j=0}^{3} \sum_{i=0}^{2} P\left(x_j\right) * P(y_i \left| x_j\right) * \log_2 \frac{P(y_i|x_j)}{P(y_i)} \qquad (8)$$

Figure1 shows the block diagram of Simulation test engine setup.

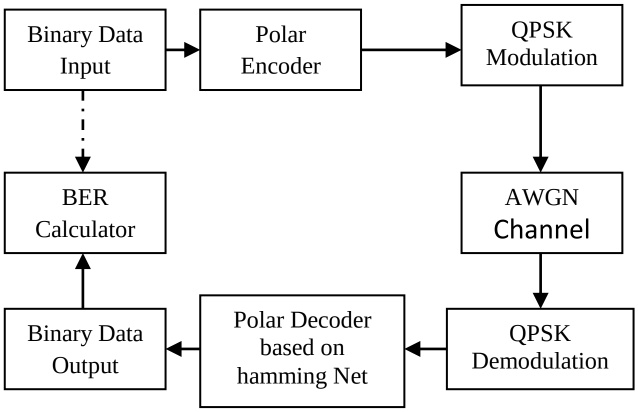

Figure1 Simulation Test Engine with QPSK Modulation

In the direction of testing the algorithm, the data is generated random function. These bits are fed to a half rate polar encoder. The coded words are modulated with QPSK modulation techniques. Table 1-3 shows the effect of polarization with different values of erasure probabilities. A hamming net based on weight of the received signals are used to detect the valid code-word.

Table 1. Channel Polarization effect with erasure probability of 0.6

Sr. No.	Data Bit Stream (N)	Frozen Bits	Non-Frozen Bits (K)	Energy Efficiency (%)
1	8	4	4	50
2	16	10	6	37.5
3	32	19	13	40.62
4	64	38	26	40.62
5	128	77	51	39.84
6	256	154	102	39.84

Table 2. Channel Polarization effect with erasure probability of 0.5

Sr. No.	Data Bit Stream (N)	Frozen Bits	Non-Frozen Bits (K)	Energy Efficiency (%)
1	8	4	4	50
2	16	8	8	50%
3	32	16	16	50%
4	64	36	36	50%
5	128	64	64	50%
6	256	128	128	50%

Table 3. Channel Polarization effect with erasure probability of 0.4

Sr. No.	Data Bit Stream (N)	Frozen Bits	Non-Frozen Bits (K)	Energy Efficiency (%)
1	8	4	4	50
2	16	6	10	62.5
3	32	13	19	59.37
4	64	26	38	59.37
5	128	51	77	60.15
6	256	102	154	60.15

An energy efficiency achieved is 39.83% with erasure probability of 0.6. There is a significant improvement in results with the proposed hybrid encoding and hamming net based decoding algorithms.

The plot of the recursion level against erasure probability is as in Figure 2. The graph helps the designer to select the rows of generator matrix with desired values of mutual information and capacity. Further this optimum level of a recursive point on the graph helps to trade-off code rate, the value of erasure probability and the bit stream length. It has been observed that the polarization of frozen bit gets improved. with an increase in the length of bit stream. Polarization generates recursive tree structure arrangement for frozen and unfrozen bits. In this tree, few bits are with low channel capacity and remaining are with high capacity. If the output specifies the input with certainty, the channels are called noiseless. On the other hand, if the mutual information between input and output is zero, such channels are called useless.

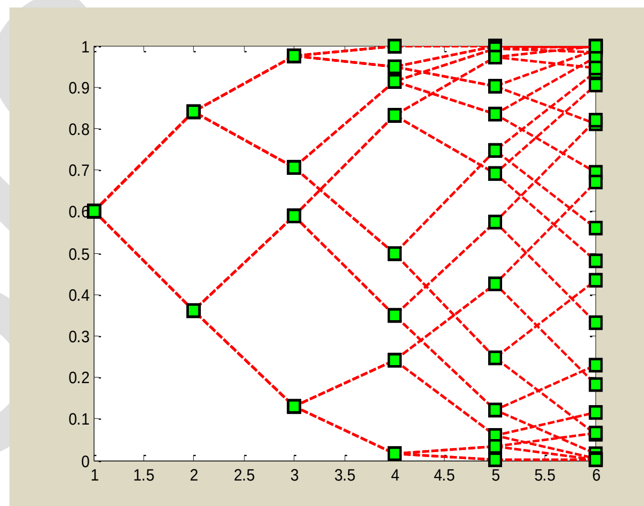

Figure.2 Recursive Tree of Polarization effect

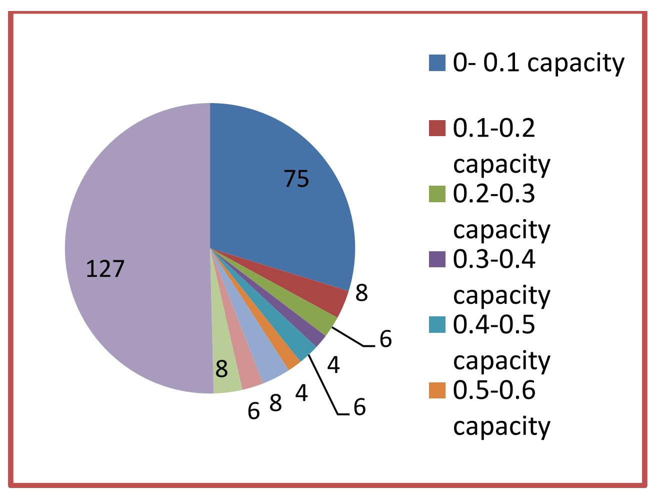

Figure.3: Channel capacity distribution of 256 bits

It is observed that 75 bits are having zero channel capacity and 127 bits having one channel capacity. Figure3 explain the channel capacity distribution of 256 bits of the data stream.

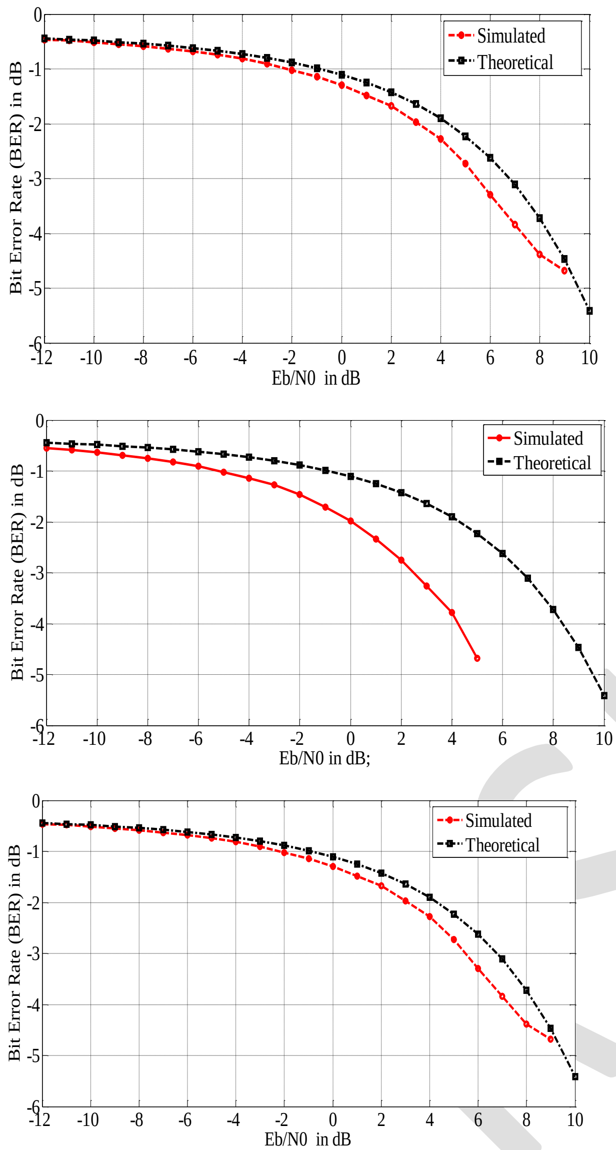

Figure 4. Hamming net based Polar decoder

The simulation results of our proposed decoding algorithm achieves SNR of 7 dB at targeted BER of 10^{-4}. The reason is due to the polarization effect of polar code and the decoding algorithm based on hamming net. The complexity of decoding algorithm is reduced almost to half the computations as compare to conventional SC decoding algorithm. The penalty paid is terms of latency required to train the decoder with the known sets of information. This information is available in look up table to reduce the number of computations. The proposed algorithm is well suited for mobile voice application in present LTE system.

Table 4 . Validation of results

Comparison 0f results	**No of successive iterations for decoding**	**Drawback**	**Decoding Complexity**
	Block size 4 by 8		
[10]	8	No training required. Works on mutual information	More
[12]	8	No training required. Works on mutual information	More
Proposed Method Application is for mobile	4	Works also on mutual information as well as Decoder needs to be trained initially once with specified block length.	Less

V. RESULTS AND CONCLUSION

The work presented in this paper gives high conformity reliability technique of error detection and correction. A hybrid method of polarization to map the rows in the generator matrix has revealed the best combination of reliability ratio and signal quality at the time of reconstruction. Following observations are obtained:

- The recursive algorithm based on hybrid method produces 100 percentage selections of rows for generator matrix that has high channel capacity.
- A simulation result shows that the effect of channel polarization improves with the length of data bit stream. Further the number of channels with zero channel capacity has increased from 30% to 40% whereas with one channel capacity increased from 49% to 59%.
- The required SNR of 7 dB at targeted BER of 10^{-4} is achieved at half the decoding complexity.

The proposed algorithm is well suited for mobile voice application in present LTE system. This will help to replace present turbo code with this upcoming polar code for error detection and correction.

REFERENCES

[1]. John G Proakis, Digital Communication Mcgraw-Hill higher education, 2001.
[2]. Xia Shao, Weidang Zhang, Ping Li,” Decreasing the BER of turbo codes with short frame by optimizing the energy allocation,” Int. J. of Information and Communication Technology, Vol. 7, No.2/3 pp. 316 – 323, 2015.
[3]. D. A. Huffman, “A method for the construction of minimum-redundancy codes,” Proc. IRE, vol. 40, p.p-1098-1101,1152.
[4]. Erdal Arikan, “Channel polarization: A method for constructing capacity- Achieving codes for symmetric Binary – Input memoryless channels,” IEEE Transactions on Information Theory, vol. 55, no. 7, pp. 3051-3073, Jul. 2009.
[5]. Satish Babu Korada, “Polar codes for channel and source coding”, Ph.D. dissertation, EPFL, Suisse, 2009.
[6]. C.E.Shanon,”A mathematical theory of communication,” Bell system Tech. J; vol. 27, pp. 379-423, 623625, July and October 1948.
[7]. Shu Lin, Daniel J Costello, Error Control Coding, Pearson Education Inc., Prentice hall, 2005.

[8]. Rabiî Dariti, El Mamoun Souidi, "New families of perfect linear error-block codes," Int. J. of Information and Coding Theory, Vol. 2, No.2/, pp. 84 – 95, 2013.

[9]. J. Carmelo Interlando, "Algebraic decoding of the ternary (37, 18, 11) quadratic residue code," Int. J. of Information and Coding Theory, Vol. 2, No.1, pp. 59 – 65, 2011.

[10]. K. Chen, K. Niu and J.R.Lin, "List successive cancellation decoding of Polar codes," IEEE Electronics Letter, vol. 48, no. 9, pp. 500-501, Apr. 2012.

[11]. Naveen Goela, Satish Babu korada & Michael Gastpar, "On LP decoding of Polar codes," IEEE Information Theory Workshop, Dublin, pp. 1-5, Aug. – Sept. 2010.

[12]. Amin Alamdar, Yazdi & Frank, R. Kschischang, "A simplified successive cancellation Decoder for Polar codes," IEEE Communication Letters, vol. 15, no. 12, pp. 1378 - 1380, Dec. 2011.

[13]. Erdal Arikan, "Systematic Polar coding," IEEE Communication Letters, vol. 15, no. 8, pp. 860-862, Aug. 2011.

[14]. John G. Proakis, Masaud Salehi, Contemporary Communication Systems using MATLAB, Thomson Learning, 2000.