Strength and Dilatancy of Granular Materials

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***Abstract*: In the present study an attempt is made to evaluate the strength and dilatancy parameters of glass beads and etched (rough) glass beads. A series of direct shear tests were performed on sample of glass beads and etched glass beads. The size of glass beads was varied from 1mm-1.7mm,1.7mm-2.36mm and 2.36mm-4.75mm and mixed together to obtain the required sample. The different relative densities at which tests were conducted for smooth glass beads are 20%,50% and 80% respectively, with corresponding unit weights are 14.1kN/m^3, 14.4kN/m^3 and 14.7kN/m^3 respectively and for etched glass beads the different unit weights at which tests were conducted are 14.65 KN/m^3,14.93 KN/m^3 and 15.2 KN/m^3 with corresponding relative densities of 30.86%, 54.5% and 76.48% respectively. Most of the direct shear tests were conducted to shear strain in excess of 30%. The stress strain response was observed and recorded, and the shear strength and dilatancy parameters were obtained for each relative density and normal stresses. The normal stress was varied from 50 kPa to 400 kPa. The tests were conducted on smooth and etched glass beads; the etched surface of glass beads was obtained by keeping the glass beads in a bath of hydrochloric acid. Also in the present work a correlation between peak friction angle, dilatancy angle and critical state friction angle was obtained for glass beads and etched glass beads. The present data was also compared with those of established correlations by Bolton (1986) and Kumar et.al (2007).**

***Keywords*: Strength, Dilatancy, Relative density, Peak friction angle, Critical state, Correlations, Glass beads**

I. INTRODUCTION

An assembly of particles will form a granular material; its mechanical behavior depends on the size and shape of the particles, their arrangement, particle-to-particle friction, associated pore spaces, and the degree of saturation. When deformations take place in granular materials, the external forces may produce internal fabric changes, which may caused by rolling, particles sliding and interlocking, these changes will produce a different response of the material behavior. Considerations of such material response are extremely significant in the design of soil structures such as retaining walls, foundations systems, and dams, because the analyses of these systems are based on the strength and deformation behavior of the material bottom or near to them. According to Reynolds observations (1885) the dense sand exhibit expansion in volume during shear and loose sand contract during shear deformation. The role of volume changes during shear especially dilatancy was recognized by Taylor (1948).Followed by Taylor ,Skempton and Bishop (1950) attempted to separate the strength component (φ_{cv}) purely on an account of friction from that (φ_p -φ_{cv}) due to expansion of material; where φ_p is the angle of internal friction corresponding to peak stress ratio. Roscoe et al .(1958) proved that the critical friction angle (φ_{cv}) depends only on particle shape and material grading. An Ideal method for calculating φ_{cv} value is plot a graph between φ_p and corresponding rate of dilation. The original stress dilatancy model (Rowe 1962) does not capture important behavioral features such as density and stress level dependencies. Bishop (1966) observed that the stress – strain dilatancy behavior of sand varies remarkable with confining pressures. Bishop (1972) conducted experiments on steel shots and concluded that an increase in confining pressure leads to a reduction in the angle of shearing. Bolton (1986) reviewed a large number of triaxial and plane strain test results and proposed a much simpler relationship among φ_p, φ_{cv} and ψ_p , which he found operationally equivalent to Rowe (1962)'s stress – dilatancy relationship; where ψ_p is the angle of dilatancy which indirectly quantifies the rate of dilation. Bolton (1986) provided the following simplified expressions:

$$\varphi_p = \varphi_{cv} + 0.8\psi_p \tag{1}$$

$$\varphi_p = \varphi_{cv} + 5\ I_R \text{ for plain strain condition} \tag{2}$$

$$\varphi_p = \varphi_{cv} + 3\ I_R \text{ for triaxial condition} \tag{3}$$

The quantity I_R is dilatancy index and its magnitude is related to the relative density (D_R) and the effective stress (σ_v) by the relationship

$$I_R = D_R(Q - \ln(\sigma_v)) - R \tag{4}$$

σ_v expressed in kPa , D_R in decimal and Q and R are constants.

According to Bolton(1986) observations R=1 and Q=10 was obtained. Later Salgado et al.(2000) took values as Q=9 and R=0.49 from their experiments.

Kumar et al. (2007) examined further the correlations between φ_p, φ_{cv}, ψ_p and I_R by conducting series of direct shear tests on Bangalore sand. Kumar et al. (2007) provided the following empirical relations

$$\varphi_p = \varphi_{cv} + 0.932\psi_p \tag{5}$$

$$\varphi_p = \varphi_{cv} + 3.5\ I_R \text{ for plain strain condition} \tag{6}$$

$I_R = D_R(10-\ln(\sigma_v))-1$ with σ_v is expressed in kPa and D_R expressed in decimals.

The objective of the present study is to examine further these correlations on glass beads. For this purpose series of direct shear tests were conducted on smooth and etched glass beads and also to find the effect of surface roughness on strength and dilatancy of etched glass beads. Tests were conducted by varying normal stress from 50 kPa to 400 kPa with three different relative densities of sample. All the tests were conducted for both smooth and etched glass beads sample. Each tests was continued till the achievement of the critical state of the sample. The values of φ_p and ψ_p were determined for each test for different combinations of relative density and normal stress values. All the test results were compared with recommendations of Bolton (1986) and Kumar et al.(2007) so as to suggest correlations which provides better estimation with regard to the present experimental data on glass beads.

II. MATERIALS

Materials used in this experiment were smooth glass beads and etched (Rough) glass beads. Three different sized glass beads varying from 4.75 mm -2.36 mm, 2.36 mm-1.7 mm and 1.7 mm-1 mm were mixed together to form the desired sample of glass beads. The particle size distribution of the materials is shown in Figure 1. The specific gravity of smooth glass beads obtained was 2.6 and that for rough glass beads it was 2.58. The maximum and minimum unit weights of smooth glass beads obtained was 14.9 KN/m^3 and 13.9 KN/m^3 respectively and for rough glass beads maximum and minimum unit weights obtained was 15.5 KN/m^3 and 14.3 KN/m^3 respectively. The different parameters obtained from grain size distribution curve of the glass beads are D_{10}= 1.4 mm, D_{30}=2 mm, D_{50}=2.6 mm, D_{60}=2.9 mm, C_u=2.07 and C_c=0.99; D_{10}, D_{30}, D_{50} and D_{60} are respective percentage of finer, and C_u and C_c are the uniformity coefficient and the coefficient of curvature of the glass beads respectively. As per IS 1498-1970 sample is said to be poorly graded. Etched glass beads also giving same particle size distribution curve, as that of smooth glass beads; that means etching process does not affect the uniformity coefficient and the coefficient of curvature values.

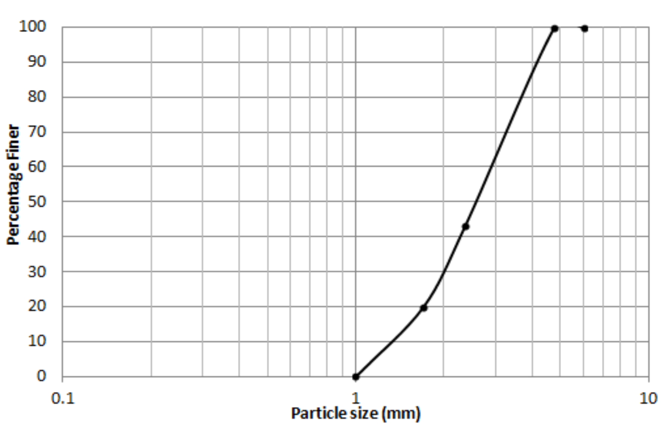


Fig. 1Grain size Distribution Curve of smooth glass beads

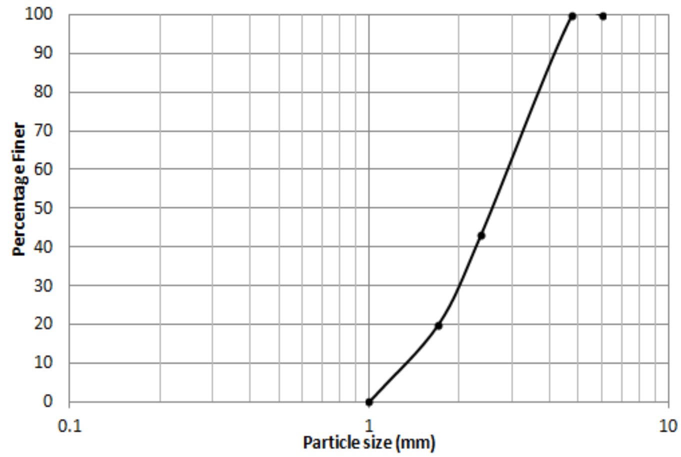
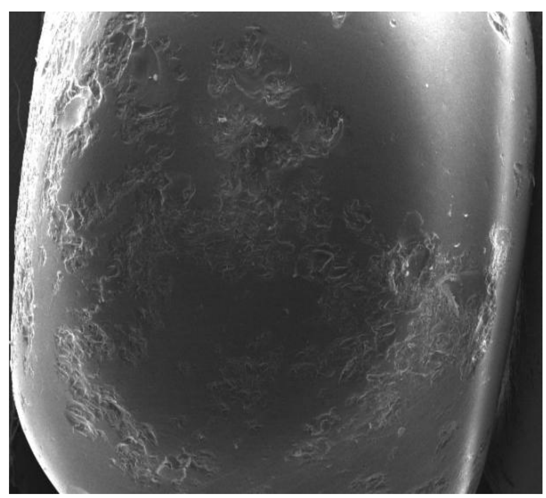


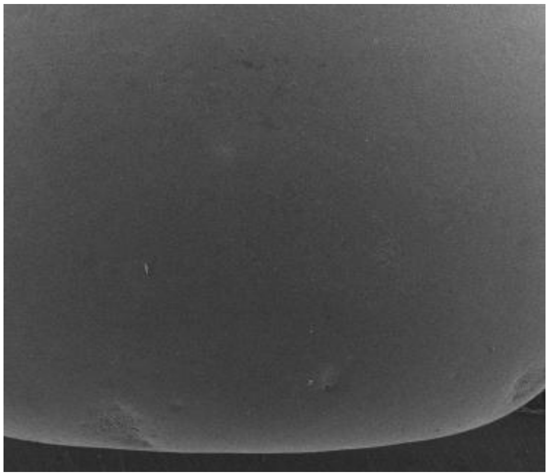
Fig. 2Grain size Distribution Curve of rough glass beads

III. EXPERIMENTAL PROCEDURE

A series of direct shear tests were conducted on smooth glass beads at three different relative densities namely 20%, 50% and 80% the corresponding unit weights are 14.1 KN/m^3, 14.4 KN/m^3 and 14.7 KN/m^3 respectively and on rough glass beads at different unit weights namely 14.65 KN/m^3,14.93 KN/m^3 and 15.2 KN/m^3 with corresponding relative densities 30.86%,54.5% and 76.48% respectively. The size of shear box was 60 mm X 60mm and the sample height was kept at 25 mm for all the tests. All the tested samples were sheared at a constant shear strain rate of 0.625 mm/minute between the two halves of shear box. The normal stresses for all tested samples was varied from 50 kPa to 400 kPa. The samples of a given density were prepared by either raining the materials from a constant height of fall (for loose and medium dense condition) or with the tamping technique using a fixed number of blows (for dense condition). All the tests were continued up to u/H =40%; where u is the horizontal displacement at any time and H is the initial height of the sample.



(a)



(b)

Fig. 3. SEM photographs of (a) rough glass bead and (b) smooth glass bead

Figure 3 shows SEM photographs of rough and smooth glass bead. It is noticed from the Fig.3, the change in surface texture of rough glass beads, then when compared to smooth glass beads.

IV. TEST RESULTS

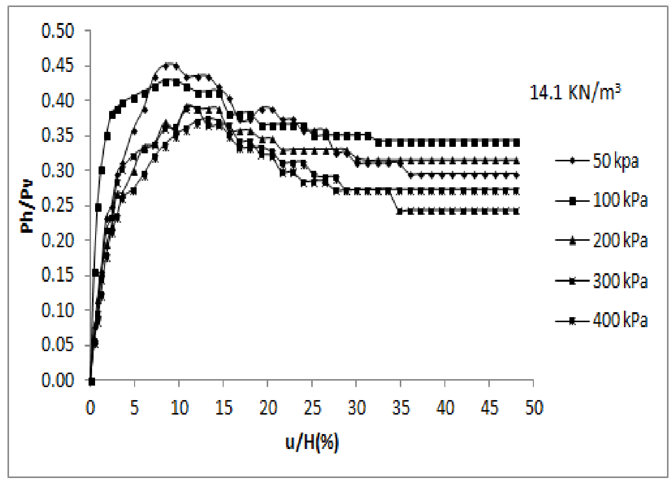
For all the tests, the variation of the horizontal (shear) force (P_h) and the corresponding change (v) in the vertical height of the sample with increase in the horizontal displacement (u) was continuously monitored at regular time interval; volumetric strain simply becomes equal to v/H. The corresponding test results are shown in Figures 3 - 8 in terms of (i) the variation of P_h/P_v with u/H, and (ii) the variation of v/H with u/H; where P_v is the magnitude of the vertical force. From these plots the values of friction angles (φ) and dilatancy angles (ψ) were determined using the following expressions:

$$\Phi = \tan^{-1}\left(\frac{Ph}{Pv}\right) \tag{7}$$

$$\Psi = \tan^{-1}\left(\frac{\delta v}{\delta u}\right) \tag{8}$$

Φ_p and ψ_p are the peak values of Φ and ψ respectively. Variations of Φ_p and ψ_p with variations of normal stress (σ_v) is illustrated in fig .9 for smooth and rough glass beads.

- The peak values of φ and ψ invariably occur almost at the same value of the horizontal displacement in the case of rough glass beads ,but in the case of smooth glass beads peak values are not occurring at the same horizontal displacement. An increase in the relative density of the material causes a marginal decrease in the value of u/H associated with φp.
- For a given relative density of the material, the behaviour of the material at low stress level always remains typically that of a dense sand which indicates a well defined peak corresponding to φ_p and then followed by a decrease in the shear stress which ultimately leads to the critical state of the material at very high values of horizontal displacement; in such cases the material initially shows a decrease in volume followed by an increase in volume it is observed both in the case of smooth glass beads and rough glass beads. At low values of σ_v, the rate of dilation becomes maximum corresponding to φ_p and subsequently the value of dilatancy angle again decreases and finally becomes equal to zero in the critical state. On the contrary at very high values of σ_v, the behaviour of the material remains similar to that of loose sand where the shear stress increases continuously to yield the critical state at very high values of horizontal displacement. In such cases the material experiences a continuous decrease in volume until reaching the critical state.



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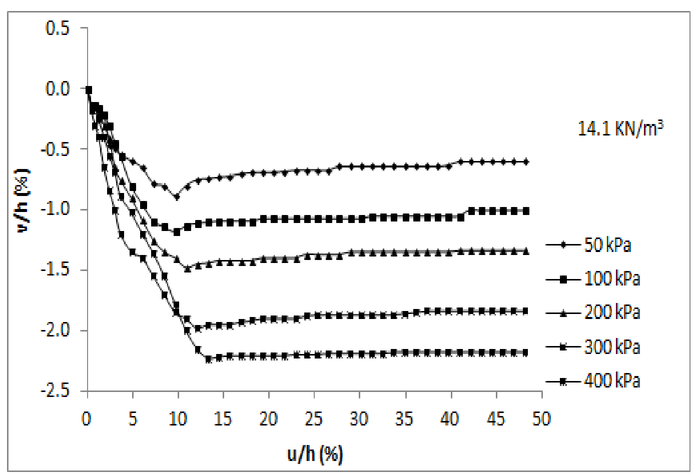
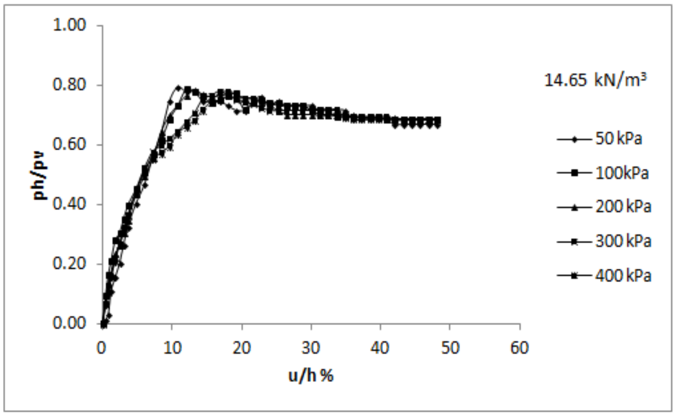
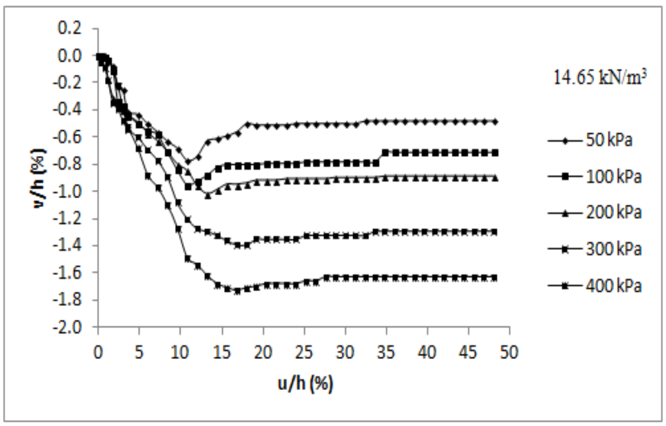


(b)

Fig. 3 For γ = 14.1 kN/m3 smooth glass beads, the observed variation of (a) Ph/Pv with u/H, and (b) v/H with u/H



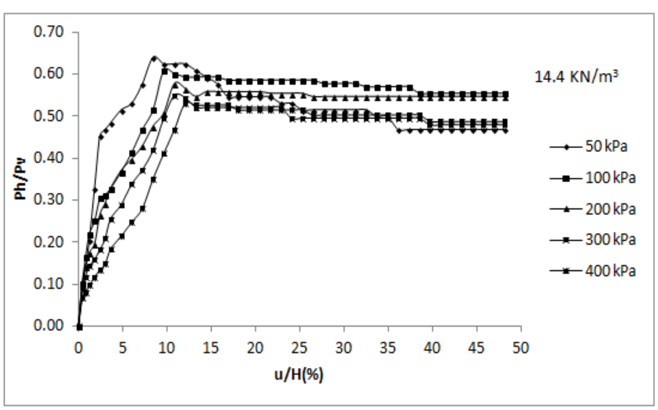
(a)



(b)

Fig.4 For γ = 14.65 kN/m3 rough glass beads, the observed variation of (a) P_h/P_v with u/H, and (b) v/H with u/H

- The values of φ_p as well as ψ_p decrease with increase in the value of σ_v. As compared to loose sand, the effect of σ_v on the changes in the values of φ_p and ψ_p was seen to be more significant in the case of dense sand.
- For a given value of σ_v, an increase in the relative density of the material causes an increase in the values of both φ_p and ψ_p.



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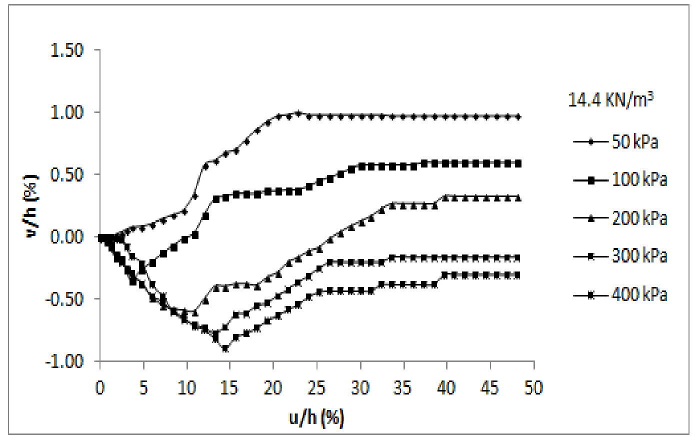


(b)

Fig.5 For γ = 14.4 kN/m3 smooth glass beads, the observed variation of (a) P_h/P_v with u/H, and (b) v/H with u/H

Correlation between Φ_p and Ψ_p

For varying values of σ_v and relative density (D_r) of the material, obtained values of φ_p were plotted against corresponding values of ψ_p. All the data points are indicated in Fig.11 (a) and (b) for smooth and rough glass beads respectively. The following correlations are obtained for the present data :

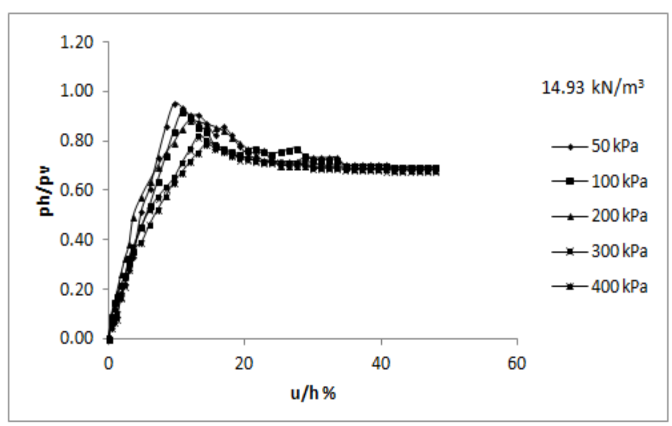
$\varphi_p = \varphi_{cv} + 1.3434\ \psi_p$

for smooth glass beads (9)

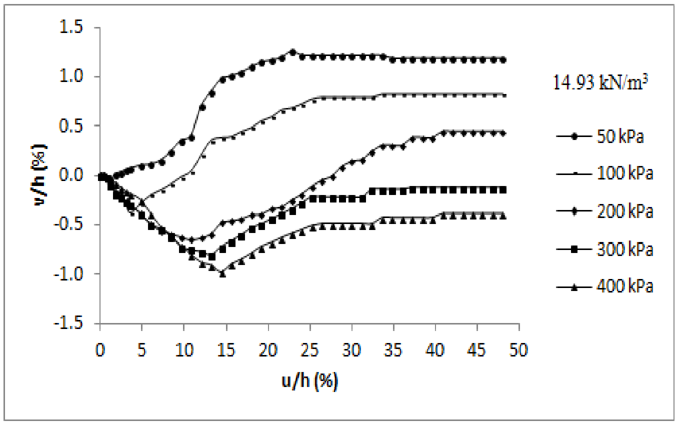
$\varphi_p = \varphi_{cv} + 0.8577\psi_p$

for rough glass beads (10)

It is very well known from the figure 11 that the value of φ_{cv} for the chosen smooth glass bead sample is found to be equal to 21.471^o and for rough glass bead sample found to be 36.72 (that is τ/σ_v = 0.8). It can also be noticed from Figures 3-8 that the value of τ/σ_v at very large value of u/H (30-40%) remains very close to 0.80 indicating the achievement of the same critical state in all the tests for rough glass beads.



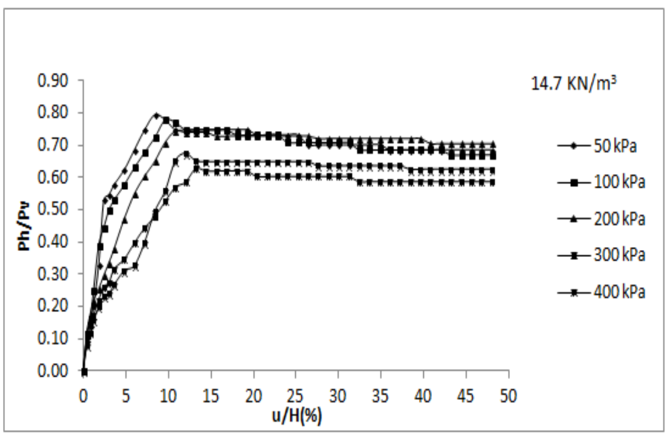
(a)



(b)

Fig.6 For γ = 14.93 kN/m3 rough glass beads, the observed variation of (a) P_h/P_v with u/H, and (b) v/H with u/H

A comparison of Equations (1),(5),(9) and (10) indicates that the variation of Bolton (1986) recommendation was more for smooth glass beads but it compares very well with respect to rough glass beads.



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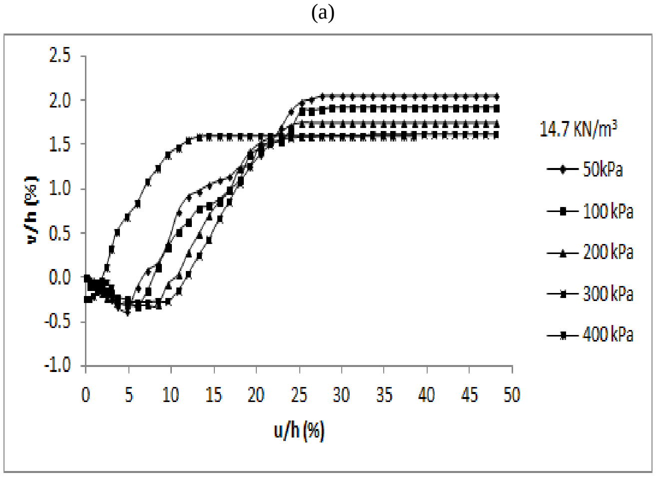
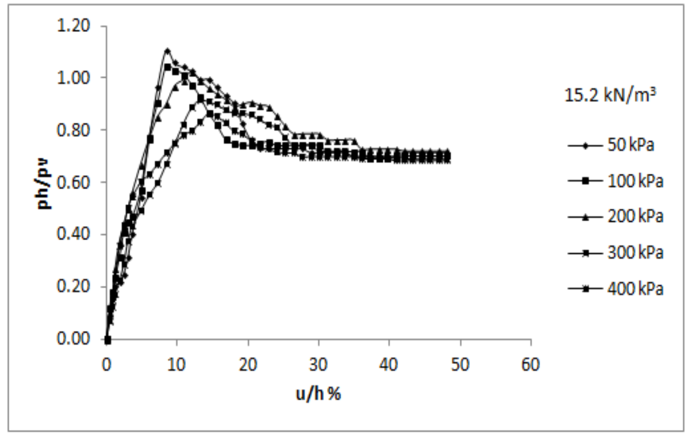
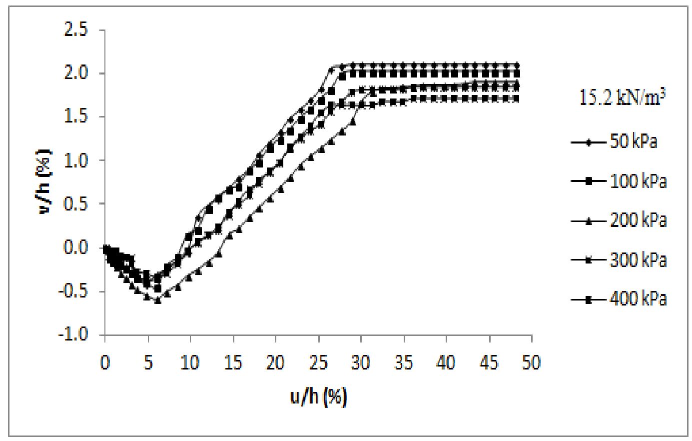


(b)

Fig.7 For γ = 14.7 kN/m3 smooth glass beads, the observed variation of (a) P_h/P_v with u/H, and (b) v/H with u/H



(a)



(b)

Fig.8 For γ = 15.2kN/m3 rough glass beads, the observed variation of (a) P_h/P_v with u/H, and (b) v/H with u/H

Correlation between φ_p and σ_v

As seen earlier from Figure 8 that the value of φ_p reduces with increase in the value of σ_v. According to Bolton's observations obtained equation (2) and also according to Kumar et al equation (6) obtained, where I_R (dilatancy index) is defined by Equation (4) with Q = 10 and R = 1. From the regression analysis following equation is suitable for present data.

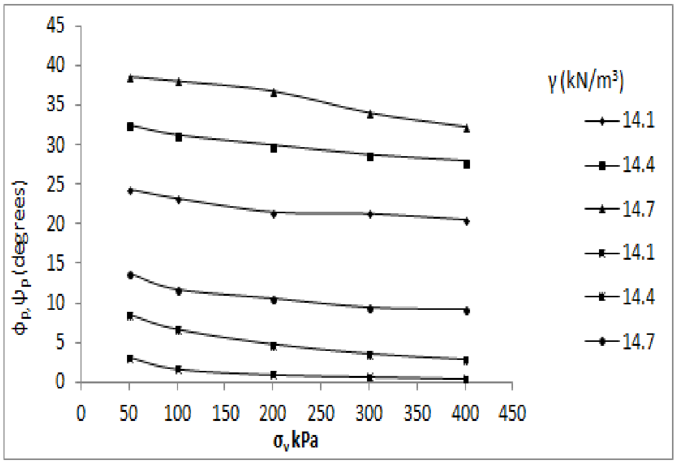
$\varphi_p = \varphi_{cv} + 4.7\ I_R$

for smooth glass beads (11)

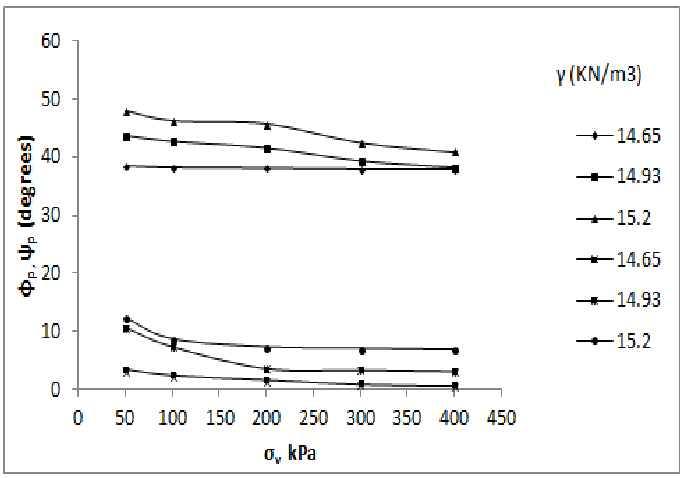
$\varphi_p = \varphi_{cv} + 3.2\ I_R$

for rough glass beads (12)

where $I_R = D_R(10-\ln(\sigma_v))-1$ with σ_v expressed in kPa and D_R in decimal.

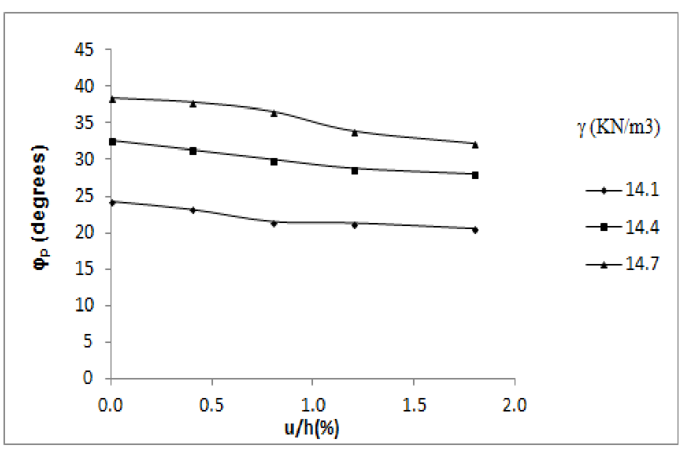


(a)



(b)

Fig. 9 The variation of φ_p and ψ_p with σ_v (a) smooth glass beads and (b) rough glasss beads



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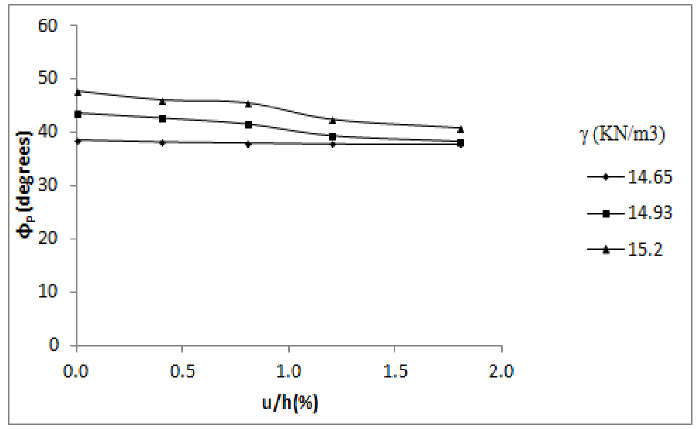
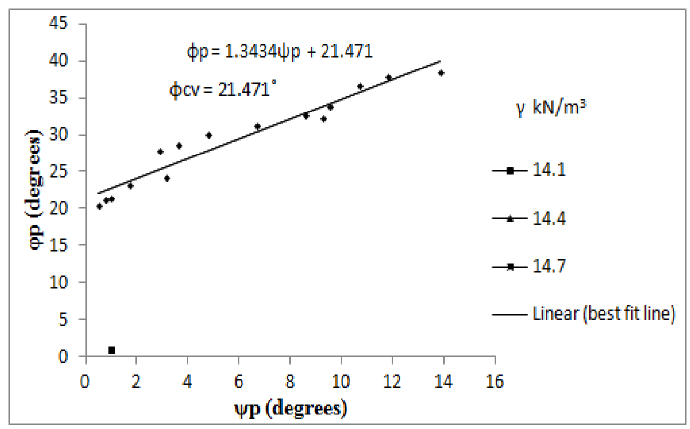


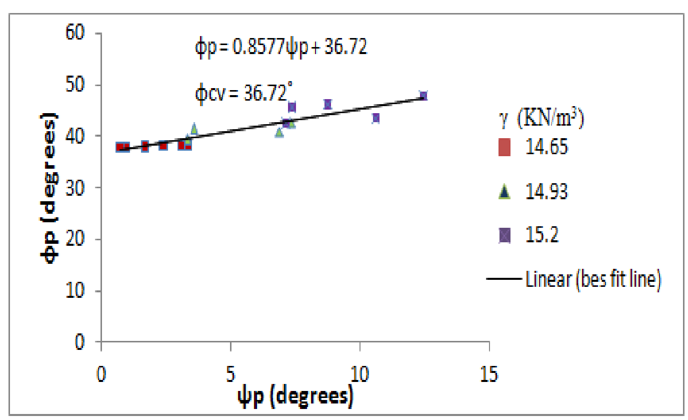
(b)

Fig. 10 The correlation between φ_p and shear strain from all the test results (a)for smooth glass beads (b) for rough glass beads

Experimentally measured values of φ_p - φ_{cv} is plotted against estimated values of the observations of Bolton (1986), Kumar et al (2007) and present correlation was shown in Fig.12, and it was found that the present data on smooth and rough glass beads reasonably compares well with that of Bolton(1986) and Kumar et.al., (2007)

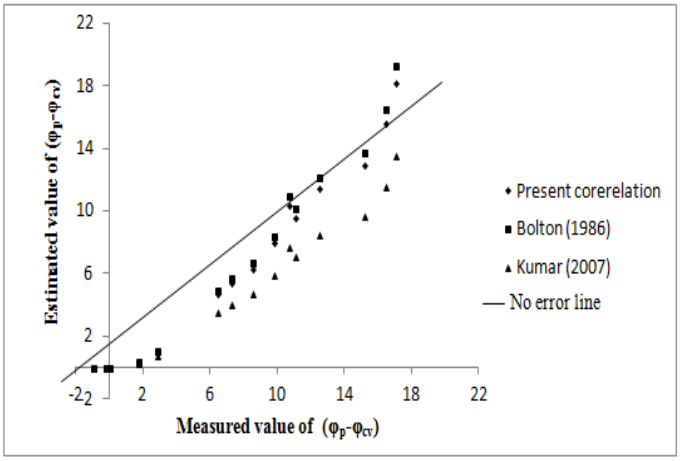


(a)

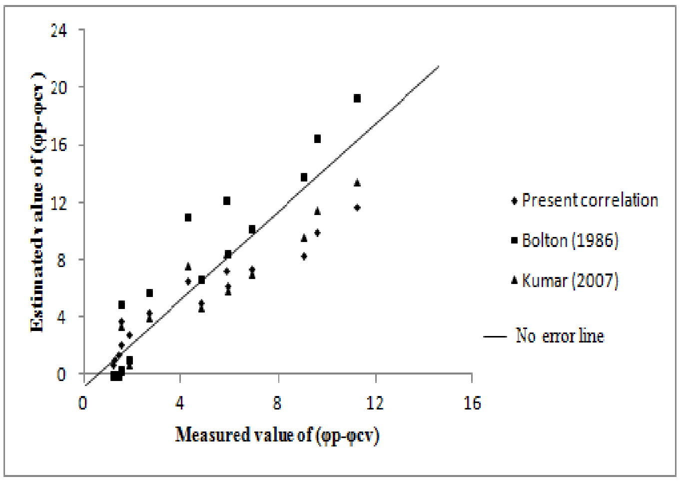


(b)

Fig. 11 The correlation between φ_p and ψ_p from all the test results (a) for smooth glass beads (b) for rough glass beads



(a)



(b)

Fig 12 The prediction of (φ_p-φ_{cv}) using different formulae against measured values of (φ_p-φ_{cv}) for all the tests (a) for smooth glass beads and (b) for rough glass beads

V. CONCLUSIONS

Based on a number of direct shear tests on smooth and rough glass beads, at different density states and stress level, an empirical relationship correlating ϕ_p, ϕ_{cv} and I_R similar to that recommended by Bolton (1986), Kumar et al. (2007), has been suggested. Using this relationship from the knowledge of relative density (D_r) and critical state friction angle (ϕ_{cv}), the value of peak friction angle can be determined for any required effective stress level (σ_v). Further, an expression correlating ψ_p with ϕ_{cv} and ϕ_p has also been provided for smooth and rough glass beads on the basis of which the value of ψ_p can also be predicted. The suggested expressions are found to match well with the test results. Based on the test results, it can be concluded that decrease in σ_v leads to an increase in the values of ϕ_p and ψ_p which necessitates the need of employing secant values of ϕ_p rather than tangent ϕ_p, with some small value of an apparent cohesion. It was also concluded from the test results the values of ϕ_p and ψ_p not only depends on stress level and density but also on the surface texture of glass beads.

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