Random Hybrid Iterative Algorithms of Jungck-Type And Common Random Fixed Point Theorems with Stability Results

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Abstract: **Let E be a separable Banach space and $S,T:\Omega\times Y\to E$ be a nonself random commuting mappings defined on arbitrary Y satisfying generalized random φ- contractive-like operator $\| T(\omega,x)-T(\omega,y)\|\leq\delta\| S(\omega,x)-S(\omega,y)\|+\varphi(\| S(\omega,x)-T(\omega,x)\|)$, with $T(\omega,Y)\subseteq S(\omega,Y)$ and $S(\omega,Y)$ a complete subspace of $E, 0\leq\delta<1,\varphi:\mathbb{R}^+\to\mathbb{R}^+$ with $\varphi(t)>0\forall t\in(0,\infty)$ and $\varphi(0)=0$. It is shown in this paper, that a stochastic version of hybrid iterative algorithm called a modified random Jungck-Mann hybrid iterative algorithm is introduced and is used to approximate the unique common random fixed point of S and T for a generalized random φ-contractive-like operators in a separable Banach space. Strong convergence results for random Picard-Mann, random Picard iterative schemes for single map T are deduced as corollaries. Stability results are proved and an example is provided to demonstrate the applicability of the hybrid scheme.**

***Keywords*-Random Jungck-Mann iterative schemes, generalized random contractive-like operators, random weakly compatible maps, unique common random fixed point.**

I. INTRODUCTION

Many real world problems are full of uncertainties and ambiguities and an important area of mathematics that deals with probabilistic models to solve these problems is Probabilistic functional analysis. Random nonlinear analysis is a vital area of probabilistic functional analysis that deals with various classes of random operator equations and related problems and solutions. The development of various random methods is on the increase. Random fixed theorems are well known stochastic generalizations of classical fixed point theorems and are usually needed in the theory of random equations, random matrices, random differential equations and different classes of random operators emanating in physical systems [21]. The first result and other generalizations of random fixed point theorems exist in the literature, for instance, see ([6], [7], [8], [10], [13], [14], [15], [19], [21], [27] and several related papers therein). In 2015, Okeke and Kim [21] proved strong convergence and summable $T-$stability of the random Picard-Mann hybrid iterative process for a generalized class of random operators in separable Banach spaces.

We begin with the following well known contractive definitions. Throughout this work E shall denote Banach space and X a metric space.

Definition 1.1. Suppose (Ω,Σ) is a measurable space and C be a non-empty closed convex subset of a separable Banach space E. A transformation $T:\Omega\to C$ is termed measurable if $T^{-1}(\mathcal{B}\cap C)\in\Sigma$ for any Borel set $\mathcal{B}$ of E. A transformation $T:\Omega\times C\to C$ is called a random mapping if $T(\cdot,x):\Omega\to C$ is measurable for every $x\in C$. A measurable function $f:\Omega\to C$ is called a random fixed point for the transformation $T:\Omega\times C\to C$ if $T(\omega,f(\omega))=f(\omega)$.

Definition 1.2. Let (Ω,Σ) be a measurable space and C be a nonempty closed convex subset of a separable Banach space E. A measurable function $f:\Omega\to C$ is called a random coincidence for two random mappings $S,T:\Omega\times C\to C$ if $T(\omega,f(\omega))=S(\omega,f(\omega))$ for all $\omega\in\Omega$. The maps S,T are said to be random weakly compatible if they commute at their random coincidence i.e. if $S(\omega,f(\omega))=T(\omega,f(\omega))$ for every $\omega\in\Omega$, then $S(T(\omega,f(\omega)))=T(S(\omega,f(\omega)))$ or $S(\omega,T(\omega,f(\omega)))=T(\omega,S(\omega,f(\omega)))$.

Definition 1.3. Let (Ω,Σ,μ) be a complete probability measure space and Y a non-empty subset of a separable Banach space E. For two random mappings $S,T:\Omega\times Y\to E$ with $T(\omega,Y)\subseteq S(\omega,Y)$ and C a nonempty closed convex subset of a separable Banach space E, there exists a real number $\delta\in[0,1)$ and a monotone increasing function $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ with $\varphi(0)=0$ and for all $x,y\in C$, we have

$$\| T(\omega,x)-T(\omega,y)\|\leq\delta\| S(\omega,x)-S(\omega,y)\| +\varphi(\| S(\omega,x)-T(\omega,x)\|). \tag{1}$$

II. PRELIMINARIES

The commonly used iterative algorithms for approximating the fixed points of several classes of single and pair of quasi-contractive operators are: Picard, Mann and Ishikawa, Jungck, Jungck-Mann and Jungck-Ishikawa iterations. For example see ([1],[3] [5], [5], [11], [12],[16], [20], [22], [23], [24], [25], and [27]).

Suppose E is a Banach space, K, a nonempty convex subset of E and $T:K\to K$ a self map of K.

Definition 2.1 [27]. Suppose $x_0 \in K$. The Picard iteration scheme $\{x_n\}_{n=0}^{\infty}$ is defined by

$$x_{n+1} = Tx_n,\ n \geq 0. \qquad (2)$$

Definition 2.2 [20]. For any given $x_0 \in K$, the Mann iteration scheme $\{x_n\}_{n=0}^{\infty}$ is defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tx_n \qquad (3)$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Definition 2.3 [12]. Let $x_0 \in K$. The Ishikawa iteration scheme $\{x_n\}_{n=0}^{\infty}$ is defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Ty_n$$
$$y_n = (1-\beta_n)x_n + \beta_n Tx_n \qquad (4)$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ are real sequences in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Observe that if $\beta_n = 0$ for each n, then the Ishikawa iteration process (4) reduces to the Mann iteration scheme (3).

Khan [18], introduced the following Picard-Mann hybrid iterative scheme for a single nonexpansive mapping T.

Definition 2.4 [18]. For any initial point $x_0 \in K$ the sequence $\{x_n\}_{n=0}^{\infty}$ is defined by

$$x_{n+1} = Ty_n$$
$$y_n = (1-\alpha_n)x_n + \alpha_n Tx_n, n \geq 0, \qquad (5)$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1].

He showed that the hybrid scheme (5) converges faster than Picard (2), Mann (3) and Ishikawa (4) iterative algorithms in the sense of Berinde [5] for contractions.

Jungck [16], was the first to introduce an iteration scheme for a pair of maps, which is now called Jungck iteration scheme [16] to approximate the common fixed points of what is now called Jungck contraction maps. Singh et al. [27] in 2005, introduced the Jungck-Mann iteration procedure and discussed it's stability for a pair of contractive maps. Olatinwo and Imoru [23], Olatinwo [23-24] built on the work of [27] to introduce the Jungck-Ishikawa scheme and used their convergence to approximate the coincidence points (not common fixed points) of some pairs of generalized contractive-like operators with the assumption that one of each of the pairs of maps is injective.

Let E be a Banach space, Y be an arbitrary set and $S, T: Y \to E$ such that $T(Y) \subseteq S(Y)$.

Then we have the following definitions.

Definition 2.5 [16]. For any $x_0 \in Y$, the Jungck iteration is defined as the sequence $\{Sx_n\}_{n=1}^{\infty}$ such that

$$Sx_{n+1} = Tx_n, n \geq 0 \qquad (6)$$

This procedure becomes Picard iteration (2) when $Y = X$ and $S = I_d$ where I_d is the identity map on X.

Definition 2.6 [23]. For any given $u_0 \in Y$, the Jungck-Mann iteration scheme $\{Su_n\}_{n=1}^{\infty}$ is defined by

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tx_n. \qquad (7)$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Definition 2.7 [23]. Let $x_0 \in Y$. The Jungck-Ishikawa iteration scheme $\{Sx_n\}_{n=1}^{\infty}$ is defined by

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Ty_n$$
$$Sy_n = (1-\beta_n)Sx_n + \beta_n Tx_n \qquad (8)$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ are real sequences in [0,1] such that $\sum_{n=0}^{\infty} \alpha_n = \infty$.

The following lemmas will be needed in proving our main results.

Lemma 2.8 [5]: If δ is a real number such that $0 \leq \delta < 1$ and $\{e_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} e_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying $u_{n+1} \leq \delta u_n + e_n, n \in \mathbb{N}$. Then we have $\lim_{n\to\infty} u_n = 0$.

Lemma 2.9. Let $(X, \|.\|)$ be a normed linear space and $S, T: Y \to X$ be nonself random commuting operators on an arbitrary set Y with values in X satisfying (1) such that

$$T(\omega, Y) \subseteq S(\omega, Y),$$
$$\| S(\omega, S(\omega, x)) - T(\omega, S(\omega, x)) \| \leq \| S(\omega, x) - T(\omega, x) \|$$

and

$$\| S(\omega, S(\omega, x)) - S(\omega, S(\omega, y)) \| \leq \| S(\omega, x) - S(\omega, y) \|$$

. Let $\varphi: \mathbb{R}^+ \to \mathbb{R}^+$ be a sublinear, monotone increasing function such that $\varphi(0) = 0$ and $\varphi(u) = (1-\delta)u$ for all $0 \leq \delta < 1, u \in \mathbb{R}^+$. Then for every $i \in \mathbb{N}$ and $x, y \in Y$, we have

$$\| T^i(\omega, x) - T^i(\omega, y) \| \leq \delta^i \| S(\omega, x) - S(\omega, y) \| + \sum_{j=1}^{i} \binom{i}{j} \delta^{i-j} \varphi^j (\| S(\omega, x) - T(\omega, x) \|) \qquad (9)$$

Proof:

We start the proof by showing that if φ is subadditive then each of the φ^j of φ is subadditive. Since we assume that φ is subadditive, then $\varphi(x(\omega) + y(\omega)) \leq \varphi(x(\omega)) + \varphi(y(\omega))$, for every $x, y \in \mathbb{R}^+$. Thus, the subadditivity of φ^2 yields the following:

$$\varphi^2(x(\omega) + y(\omega)) = \varphi(\varphi(x(\omega) + y(\omega)))$$
$$\leq \varphi(\varphi(x(\omega))) + \varphi(\varphi(y(\omega))).$$

Similarly, the the subadditivity of φ^3 yields the following:

$$\varphi^3(x(\omega) + y(\omega)) = \varphi\left(\varphi^2(x(\omega) + y(\omega))\right)$$
$$\leq \varphi(\varphi^2(x(\omega))) + \varphi(\varphi^2(y(\omega)))$$
$$= \varphi^3(x(\omega)) + \varphi^3(y(\omega)).$$

Therefore, in general, φ^n $(n = 1,2,3, \dots)$ is subadditive, and it can be written as:

$$\varphi^n\big(x(\omega)+y(\omega)\big) \le \varphi\left(\varphi^{n-1}\big(x(\omega)\big)\right) + \varphi\left(\varphi^{n-1}\big(y(\omega)\big)\right) = \varphi^n(x(\omega)) + \varphi^n(y(\omega)).$$

The remaining part of the proof of Lemma 2.9, will be done by mathematical induction on i as follows:

Let $i=1$, then contractive condition (9) becomes

$$\| T(\omega,x)-T(\omega,y) \| \le \delta \| S(\omega,x)-S(\omega,y) \| +\varphi(\| S(\omega,x)-T(\omega,x) \|) \quad (10)$$

Next, suppose $i=n$, where $n\in\mathbb{N}$, then (9) becomes

$$\| T^n(\omega,x)-T^n(\omega,y) \| \le \delta^n \| S(\omega,x)-S(\omega,y) \| + \textstyle\sum_{j=1}^{n} \binom{n}{j}\delta^{n-j}\varphi^j(\| S(\omega,x)-T(\omega,x) \|). \quad (11)$$

We now show that the statement is true for $i=n+1$

$$T^{n+1}(\omega,x)-T^{n+1}(\omega,y) \|$$
$$=\| T^n(\omega,T(\omega,x))-T^n(\omega,T(\omega,y)) \|$$
$$\le \delta^n \| S(\omega,T(\omega,x))-S(\omega,T(\omega,y)) \|$$
$$+\sum_{j=1}^{n} \binom{n}{j}\delta^{n-j}\varphi^j(\| T(\omega,S(\omega,x))-T(\omega,T(\omega,x)) \|$$
$$\le \delta^n \| T(\omega,S(\omega,x))-T(\omega,S(\omega,y)) \|$$
$$+\textstyle\sum_{j=1}^{n} \binom{n}{j}\delta^{n-j}\varphi^j(\| T(\omega,S(\omega,x))-T\big(\omega,T(\omega,x)\big) \quad (12)$$

Using (10), we have

$$\| T\big(\omega,S(\omega,x)\big)-T\big(\omega,S(\omega,y)\big) \|$$
$$\le \delta \| S(\omega,S(\omega,x))-S(\omega,S(\omega,y)) \|$$
$$+\varphi(\| S(\omega,S(\omega,x))-T(\omega,S(\omega,x)) \|) \quad (13)$$

$$\| T\big(\omega,S(\omega,x)\big)-T\big(\omega,T(\omega,y)\big) \|$$
$$\le \delta \| S(\omega,S(\omega,x))-S(\omega,T(\omega,y)) \|$$
$$+\varphi(\| S(\omega,S(\omega,x))-T(\omega,S(\omega,x)) \|) \quad (14)$$

Substituting (13) and (14) into (12), we have

$$\| T^{n+1}(\omega,x)-T^{n+1}(\omega,y) \|$$
$$\le \delta^n[\delta \| S(\omega,S(\omega,x))-S(\omega,S(\omega,y)) \|$$
$$+\varphi(\| S(\omega,S(\omega,x))-T(\omega,S(\omega,x)) \|)]$$
$$+\sum_{j=1}^{n} \binom{n}{j}\delta^{n-j}\varphi^j[\delta \| S(\omega,S(\omega,x))-S(\omega,S(\omega,y))$$
$$+\varphi(\| S(\omega,S(\omega,x))-T(\omega,S(\omega,x)) \|)]$$
$$\le \delta^n \| T(\omega,S(\omega,x))-T(\omega,S(\omega,y)) \|$$
$$+\sum_{j=1}^{n} \binom{n}{j}\delta^{n-j}\varphi^j(\| T(\omega,S(\omega,x))-T(\omega,T(\omega,x)) \|)$$
$$=\delta^{n+1} \| S(\omega,S(\omega,x))-S(\omega,S(\omega,y)) \|$$
$$+\delta^n\varphi(\| S(\omega,S(\omega,x))-T(\omega,S(\omega,x)) \|)$$
$$+\sum_{j=1}^{n} \binom{n}{j}\delta^{n+1-j}\varphi^j(\| S(\omega,S(\omega,x))-S(\omega,S(\omega,y)) \|)$$
$$+\sum_{j=1}^{n} \binom{n}{j}\delta^{n-j}\varphi^{j+1}(\| S(\omega,S(\omega,x))-T(\omega,S(\omega,x)) \|)$$
$$\le \delta^{n+1} \| S(\omega,S(\omega,x))-S(\omega,S(\omega,y)) \|$$
$$+\delta^n\varphi(\| S(\omega,S(\omega,x)-T(\omega,S(\omega,x) \|)$$
$$+\sum_{j=1}^{n} \binom{n}{j}\delta^{n+1-j}\varphi^j(\| S(\omega,x)-S(\omega,y) \|)$$
$$+\sum_{j=1}^{n} \binom{n}{j}\delta^{n-j}\varphi^{j+1}(\| S(\omega,x)-T(\omega,x) \|)$$
$$=\delta^{n+1} \| S(\omega,S(\omega,x))-S(\omega,S(\omega,y)) \|$$
$$+\delta^n\varphi(\| S(\omega,S(\omega,x))-T(\omega,S(\omega,x)) \|)$$
$$+\binom{n}{1}\delta^n\varphi(\| S(\omega,x)-T(\omega,x) \|)$$
$$+\binom{n}{2}\delta^{n-1}\varphi^2(\| S(\omega,x)-T(\omega,x) \|)$$
$$+\binom{n}{3}\delta^{n-2}\varphi^3(\| S(\omega,x)-T(\omega,x) \|)+\cdots$$
$$+\binom{n}{n}\delta\varphi^n(\| S(\omega,x)-T(\omega,x) \|)$$
$$+\binom{n}{1}\delta^{n-1}\varphi^2(\| S(\omega,x)-T(\omega,x) \|)$$
$$\binom{n}{2}\delta^{n-2}\varphi^3(\| S(\omega,x)-T(\omega,x) \|)$$
$$+\binom{n}{3}\delta^{n-3}\varphi^4(\| S(\omega,x)-T(\omega,x) \|)+\cdots$$
$$+\binom{n}{n}\varphi^{n+1}(\| S(\omega,x)-T(\omega,x) \|)$$
$$=[\binom{n}{1}+\binom{n}{0}]\delta^n\varphi(\| S(\omega,x)-T(\omega,x) \|)$$
$$+[\binom{n}{2}+\binom{n}{1}]\delta^{n-1}\varphi^2(\| S(\omega,x)-T(\omega,x) \|)$$
$$+[\binom{n}{3}+\binom{n}{2}]\delta^{n-2}\varphi^3(\| S(\omega,x)-T(\omega,x) \|)$$
$$+\cdots+[\binom{n}{n}+\binom{n}{n-1}]\delta\varphi^n(\| S(\omega,x)-T(\omega,x) \|)$$
$$+\varphi^{n+1}(\| S(\omega,x)-T(\omega,x) \|)$$
$$+\delta^{n+1}i(\| S(\omega,S(\omega,x))-S(\omega,S(\omega,y)) \|)$$
$$=[\binom{n+1}{1}\delta^n\varphi+\binom{n+1}{2}\delta^{n-1}\varphi^2+\binom{n+1}{3}\delta^{n-2}\varphi^3+\cdots$$
$$+\varphi^{n+1}] \| S(\omega,x)-T(\omega,x) \|)$$
$$+\delta^{n+1} \| S(\omega,S(\omega,x))-S(\omega,S(\omega,y)) \|$$
$$=\delta^{n+1} \| S\big(\omega,S(\omega,x)\big)-S\big(\omega,S(\omega,y)\big) \|$$
$$+\sum_{j=0}^{n+1} \binom{n+1}{j}\delta^{n+1-j}\varphi^j(\| S(\omega,x)-T(\omega,x)).$$
$$\| T^{n+1}(\omega,x)-T^{n+1}(\omega,y) \|$$

$$\leq \delta^{n+1} \parallel S(\omega, S(\omega, x)) - S(\omega, S(\omega, y)) \parallel$$
$$+ \sum_{j=0}^{n+1} \binom{n+1}{j} \delta^{n+1-j} \varphi^j (\parallel S(\omega, x) - T(\omega, x) \parallel) \quad (15)$$

In view of (10) and (15), we have

$$\parallel T^{n+1}(\omega, x) - T^{n+1}(\omega, y) \parallel$$
$$\leq \delta^{n+1} \parallel S(\omega, S(\omega, x)) - S(\omega, S(\omega, y)) \parallel$$
$$+\varphi^{n+1}(\parallel S(\omega, x) - T(\omega, x) \parallel). \quad (16)$$

III. MAIN RESULTS

3.1 Convergence Results in Separable Banach Spaces

We shall define a stochastic version of common fixed point iteration algorithm call random Jungck-Mann hybrid iterative algorithm and prove convergence and stability results with the scheme.

Definition 3.1. Let (Ω, Σ, μ) be a complete probability measure space and E be a nonempty subset of a separable Banach space X. Let $S, T: \Omega \times E \to E$ be two self random operators. Let $F(S,T) = \{p(\omega) \in E: S(\omega, p(\omega)) = T(\omega, p(\omega)) = p(\omega), \omega \in \Omega\}$be the set of random common fixed points of S, T. The random Jungck-Mann hybrid iterative algorithm $\{S(\omega, x_n(\omega))\}_{n=1}^{\infty}$ is defined by

$$S(\omega, x_{n+1}(\omega)) = T(\omega, y_n(\omega))$$
$$S\big(\omega, y_n(\omega)\big) = (1-\alpha_n) S\big(\omega, x_n(\omega)\big)$$
$$+\alpha_n T(\omega, x_n(\omega)) \quad (17)$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a measurable sequence in [0,1].

Remark 3.2.: If S is an identity map in (17), we obtain a modified random Picard-Mann iterative algorithm $\{x_n(\omega)\}_{n=1}^{\infty}$ as follows:

$$x_{n+1}(\omega) = T(\omega, y_n(\omega))$$
$$y_n(\omega)) = (1-\alpha_n) x_n(\omega)$$
$$+\alpha_n T(\omega, x_n(\omega)) \quad (18)$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a measurable sequence in [0,1].

Theorem 3.3. Let E be a separable Banach space and $S, T: \Omega \times Y \to E$ be a nonself random commuting mappings defined on arbitrary Y such that (10) holds with $T(\omega, Y) \subseteq S(\omega, Y)$ and $S(\omega, Y)$ is a complete subspace of E. Let $p(\omega)$ be the random coincidence of S, T, that is, $S(\omega, f(\omega)) = T(\omega, f(\omega)) = p(\omega)$, then the the random-Jungck-Mann hybrid iterative algorithm (17) converges strongly to $p(\omega)$.

In addition, if $Y = E$ and S, T commute at $f(\omega)$ (that is, S, T are randomly weakly compatible), then $p(\omega)$ is the unique common random fixed point of S, T.

Proof:

In view of (17) and (10) coupled with the fact that $T(\omega, f(\omega)) = S(\omega, f(\omega)) = p(\omega)$

$$\parallel S\big(\omega, x_{n+1}(\omega)\big) - p(\omega) \parallel$$
$$= \parallel T(\omega, y_n(\omega)) - T(\omega, f(\omega)) \parallel$$
$$\leq \delta \parallel S(\omega, f(\omega)) - S(\omega, y_n(\omega)) \parallel$$
$$+\varphi(\parallel S(\omega, f(\omega)) - T(\omega, f(\omega)) \parallel)$$
$$= \delta \parallel p(\omega) - S(\omega, y_n(\omega)) \parallel. \quad (19)$$

In view of (19) and (10) coupled with the fact that $T(\omega, f(\omega)) = S(\omega, f(\omega)) = p(\omega)$, we obtain

$$\parallel S\big(\omega, y_n(\omega)\big) - p(\omega) \parallel$$
$$\leq (1-\alpha_n) \parallel S(\omega, x_n(\omega)) - p(\omega) \parallel$$
$$+\alpha_n \parallel T(\omega, x_n(\omega)) - T(\omega, f(\omega)) \parallel$$
$$\leq (1-\alpha_n) \parallel S(\omega, x_n(\omega)) - p(\omega) \parallel$$
$$+\alpha_n[\delta \parallel S(\omega, f(\omega)) - S(\omega, x_n(\omega)) \parallel$$
$$+\varphi(\parallel S(\omega, f(\omega)) - T(\omega, f(\omega)) \parallel)]$$
$$= (1-\alpha_n) \parallel S(\omega, x_n(\omega)) - p(\omega) \parallel$$
$$+\delta\alpha_n \parallel p(\omega) - S(\omega, x_n(\omega)) \parallel. \quad (20)$$

Substituting (20) in (19), we obtain

$$\parallel S\big(\omega, x_{n+1}(\omega)\big) - p(\omega) \parallel$$
$$\leq \delta[(1-\alpha_n) \parallel S(\omega, x_n(\omega)) - p(\omega) \parallel$$
$$+\delta\alpha_n \parallel p(\omega) - S(\omega, x_n(\omega)) \parallel]$$
$$= \delta[1 - \alpha_n(1-\delta)] \parallel S(\omega, x_n(\omega)) - p(\omega) \parallel.$$

Since $0 \leq \delta < 1$ and $[1 - \alpha_n(1-\delta)] < 1$, then $\lim_{n\to\infty} \parallel S(\omega, x_{n+1}(\omega)) - p(\omega) \parallel = 0$.

Therefore, $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to $p(\omega)$.

Next, we show that $p(\omega)$ is unique common random fixed point of S, T.

Suppose there exists another point of coincidence $p^*(\omega) \neq p(\omega)$, such that $T(\omega, f^*(\omega)) = S(\omega, f^*(\omega)) = p^*(\omega)$, then, we show that $p(\omega)$ is unique. Thus, using (10) we have

$$\parallel p(\omega) - p^*(\omega) \parallel$$
$$= \parallel T\big(\omega, f(\omega)\big) - T\big(\omega, f^*(\omega)\big) \parallel$$
$$\leq \delta \parallel S\big(\omega, f(\omega)\big) - S\big(\omega, f^*(\omega)\big) \parallel$$
$$+\varphi\big(\parallel S\big(\omega, f(\omega)\big) - T\big(\omega, f(\omega)\big) \parallel\big)$$
$$= \delta \parallel p(\omega) - p^*(\omega) \parallel.$$

Since $0 \leq \delta < 1$, then $p(\omega) = p^*(\omega)$ and so $p(\omega)$ is unique.

Since, S, T are randomly weakly compatible, then, for $S(\omega, f(\omega)) = T(\omega, f(\omega)) = p(\omega)$, we have $T(\omega, S(\omega, f(\omega))) = S(\omega, T(\omega, f(\omega)))$.

Hence $p(\omega)$ is a coincidence point of S, T and since the point of coincidence is unique, we have

$$T(\omega,p(\omega)) = S(\omega,p(\omega)) = p(\omega).$$

Therefore, $p(\omega)$ is the unique common random fixed point of S,T. This ends the proof.

Theorem 3.3. leads to the following corollary if $E = Y$ and $S = I_d$ (that is, S is an identity map):

Corollary 3.4. Let $(E, \|.\|)$ be a separable Banach space and $T: \Omega \times E \to E$ be a continuous generalized random φ-contractive-like operator with a random fixed point $p(\omega) \in F(T)$ which satisfies $\| T(\omega,x) - T(\omega,y)) \| \leq \delta \| x(\omega) - y(\omega) \| + \varphi(\| x(\omega) - T(\omega,x) \|)$, for each $x,y \in E, 0 \leq \delta(\omega) < 1$ and $\varphi: \mathbb{R}^+ \to \mathbb{R}^+$ a continuous and nondecreasing function with $\varphi(t) > 0 \forall t \in (0,\infty)$ and $\varphi(0) = 0$. Let $\{x_n(\omega)\}_{n=0}^{\infty}$ be the modified random Picard-Mann iterative algorithm defined by (18). Then (18) converges strongly to $p(\omega)$.

3.2 Stability Results in Normed Linear Spaces

Theorem 3.5. Let $(X, \|.\|)$ be a normed linear space and $S,T: \Omega \times X \to X$ be two self random weakly compatible mappings satisfying (10) such that $T(\omega,X) \subseteq S(\omega,X)$, where $\varphi: \mathbb{R}^+ \to \mathbb{R}^+$ is a sublinear monotone increasing function with $\varphi(0) = 0$. Assume that $p(\omega)$ is the unique common random fixed point of S,T. If the iterative algorithm $\{S(\omega,x_n(\omega))\}_{n=0}^{\infty}$ defined by (17) converges to $p(\omega)$, then the modified random Jungck-Mann algorithm (17) is (S,T) $-$stable.

Proof:

Let $\{S(\omega,x_n(\omega))\}_{n=0}^{\infty}$ be the theoretical sequence and $\{S(\omega,z_n(\omega))\}_{n=0}^{\infty}$ be the approximate sequence in X.

Let $\epsilon_n = \| S(\omega,z_{n+1}(\omega)) - T(\omega,u_n(\omega)) \|, n = 0,1,2,\ldots,$

where

$S(\omega,u_n(\omega)) = (1-\alpha_n)S(\omega,z_n(\omega)) + \alpha_n T(\omega,z_n(\omega))$and let $lim_{n\to\infty}\epsilon_n = 0$.

Then, we shall prove that $lim_{n\to\infty}S(\omega,z_n(\omega)) = p(\omega)$ using the generalized $\varphi-$ contractive-like operator satisfying condition (10).

That is,

$$\| S(\omega,z_{n+1}(\omega)) - p(\omega) \| \leq \| S(\omega,z_{n+1}(\omega)) - T(\omega,u_n(\omega)) \| + \| T(\omega,u_n(\omega)) - p(\omega) \|$$

$$\leq \epsilon_n + \| T\big(\omega,u_n(\omega)\big) - p(\omega) \|. \qquad (22)$$

Applying contractive condition (10) on (22), we have

$$\| S\big(\omega,z_{n+1}(\omega)\big) - p(\omega) \| \leq \epsilon_n$$
$$+\delta \| S(\omega,p(\omega)) - S(\omega,u_n(\omega)) \|$$
$$+\varphi(\| S(\omega,p(\omega)) - T(\omega,p(\omega)) \|). \qquad (23)$$

Since $S(\omega,p(\omega)) = T(\omega,p(\omega)) = p(\omega)$ and $\varphi(0) = 0$, then (24) becomes

$$\| S\big(\omega,z_{n+1}(\omega)\big) - p(\omega) \| \leq \epsilon_n$$
$$+\delta \| p(\omega) - S(\omega,u_n(\omega)) \|. \qquad (24)$$

From (25),

$$\| p(\omega) - S\big(\omega,u_n(\omega)\big) \|$$
$$= \| (1-\alpha_n+\alpha_n)p(\omega) - (1-\alpha_n)S(\omega,z_n(\omega)) - \alpha_n T(\omega,z_n(\omega)) \|$$
$$= \| (1-\alpha_n)(p(\omega) - S(\omega,z_n(\omega))$$
$$+\alpha_n(p(\omega) - T(\omega,z_n(\omega)) \|$$
$$\leq (1-\alpha_n) \| p(\omega) - S\big(\omega,z_n(\omega)\big) \|$$
$$+\alpha_n \| T(\omega,p(\omega)) - T(\omega,z_n(\omega)) \|$$
$$\leq (1-\alpha_n) \| p(\omega) - S(\omega,z_n(\omega)) \|$$
$$+\alpha_n[\delta \| S(\omega,p(\omega)) - S(\omega,z_n(\omega)) \|$$
$$+\varphi(\| S(\omega,p(\omega)) - T(\omega,p(\omega)) \|)]$$
$$= (1-\alpha_n) \| p(\omega) - S\big(\omega,z_n(\omega)\big) \|$$
$$+\alpha_n[\delta \| p(\omega) - S(\omega,z_n(\omega)) \|$$
$$= (1-\alpha_n+\alpha_n\delta) \| p(\omega) - S(\omega,z_n(\omega)) \|. \qquad (25)$$

Substituting (25) in (24), we have

$$\| S\big(\omega,z_{n+1}(\omega)\big) - p(\omega) \|$$
$$\leq \delta[1-(1-\delta)\alpha_n] \| p(\omega) - S(\omega,z_n(\omega)) \| + \epsilon_n \qquad (26)$$

Since $0 \leq \delta < 1$, using Lemma 2.8 in (26), we obtain $lim_{n\to\infty}S(\omega,z_n(\omega)) = p(\omega)$.

Conversely, let $lim_{n\to\infty}S(\omega,z_n(\omega)) = p(\omega)$, we show that $lim_{n\to\infty}\epsilon_n = 0$ as follows:

$$\epsilon_n = \| S(\omega,z_{n+1}(\omega)) - T(\omega,u_n(\omega)) \|$$
$$\leq \| S\big(\omega,z_{n+1}(\omega)\big) - S\big(\omega,p(\omega)\big) \|$$
$$+\| T(\omega,p(\omega)) - T(\omega,u_n(\omega)) \|$$
$$\leq \| S\big(\omega,z_{n+1}(\omega)\big) - S\big(\omega,p(\omega)\big) \|$$
$$+\delta \| S(\omega,p(\omega)) - S(\omega,u_n(\omega)) \|$$
$$+\varphi(\| S(\omega,p(\omega)) - T(\omega,p(\omega)) \|)$$
$$+\varphi(\| S(\omega,p(\omega)) - T(\omega,p(\omega)) \|)$$
$$+\varphi(\|S(\omega,p(\omega)) - T(\omega,p(\omega))\|)$$
$$= \| S\big(\omega,z_{n+1}(\omega)\big) - S\big(\omega,p(\omega)\big) \|$$
$$+\delta \| p(\omega) - S\big(\omega,u_n(\omega)\big) \| \qquad (27)$$

From (27),

$$\| p(\omega) - S\big(\omega,u_n(\omega)\big) \|$$
$$\leq (1-\alpha_n) \| p(\omega) - S\big(\omega,z_n(\omega)\big) \|$$

$$+\alpha_n \parallel T(\omega,p(\omega)) - T(\omega,z_n(\omega)) \parallel$$
$$\leq (1-\alpha_n) \parallel p(\omega) - S(\omega,z_n(\omega)) \parallel$$
$$+\alpha_n[\delta \parallel S(\omega,p(\omega)) - S(\omega,z_n(\omega)) \parallel$$
$$+\varphi(\parallel S(\omega,p(\omega)) - T(\omega,p(\omega)) \parallel$$
$$= (1-\alpha_n) \parallel p(\omega) - S(\omega,z_n(\omega)) \parallel$$
$$+\alpha_n \delta \parallel S(\omega,p(\omega)) - S(\omega,z_n(\omega)) \parallel$$
$$= (1-\alpha_n+\alpha_n\delta) \parallel p(\omega) - S(\omega,z_n(\omega)) \parallel$$
$$= [1-(1-\delta)\alpha_n] \parallel p(\omega) - S(\omega,z_n(\omega)) \parallel \tag{28}$$

Substituting (28) in (27), we have

$$\epsilon_n \leq \parallel S(\omega,z_{n+1}(\omega)) - p(\omega) \parallel$$
$$+\delta[1-(1-\delta)\alpha_n] \parallel S(\omega,z_n(\omega)) - p(\omega) \parallel. \tag{29}$$

Since $lim_{n\to\infty} S(\omega,z_n(\omega)) = p(\omega)$ by our assumption, then we have $lim_{n\to\infty}\epsilon_n = 0$.

Therefore, the modified random Jungck-Mann hybrid iterative scheme (17) is (S,T)-stable. This ends the proof.

Theorem 3.6 yields the following corollary:

Corollary 3.7. Let $(X, \|.\|)$ be a normed linear space and $T: \Omega \times X \to X$ be a self random mapping satisfying the contractive-like condition

$$\parallel T(\omega,x) - T(\omega,y)) \parallel$$
$$\leq \delta \parallel x(\omega) - y(\omega) \parallel$$
$$+\varphi(\parallel x(\omega) - T(\omega,x) \parallel)$$

where $\delta \in [0,1)$ and $\varphi: \mathbb{R}^+ \to \mathbb{R}^+$ a sublinear monotone increasing function with $\varphi(0)=0$. Assume that $p(\omega)$ is the unique random fixed point of T. If the iterative algorithm $\{x_n(\omega)\}_{n=0}^{\infty}$ defined by (18) converges to $p(\omega)$, then the modified random Picard-Mann algorithm (18) is $T -$ stable.

Corollary 3.8. Let $(X, \|.\|)$ be a normed linear space and $T: \Omega \times X \to X$ be a self random mapping satisfying the contractive-like condition

$$\parallel T(\omega,x) - T(\omega,y)) \parallel \leq \delta \parallel x(\omega) - y(\omega) \parallel$$
$$+\varphi(\parallel x(\omega) - T(\omega,x) \parallel)$$

where $\delta \in [0,1)$ and $\varphi: \mathbb{R}^+ \to \mathbb{R}^+$ a sublinear monotone increasing function with $\varphi(0)=0$. Assume that $p(\omega)$ is the unique random fixed point of T. If the random Picard iterative scheme $\{x_n(\omega)\}_{n=0}^{\infty}$ defined by (2) converges to $p(\omega)$, then the random Picard scheme (2) is $T -$ stable.

Example 3.9. Let $(X,d) = ([0,20], |.|)$.

Define S and T by

$$S(\omega,x) = \begin{cases} 3 & if \quad x \in (0,2] \\ 0 & if \quad x \in \{0\} \cup (2,20] \end{cases}$$

and

$$T(\omega,x) = \begin{cases} 0 & if \quad x = 0 \\ x+8 & if \quad x \in (0,2] \\ x-2 & if \quad x \in (2,20] \end{cases}$$

Then

(i). $S(\omega,x) = T(\omega,x)$ iff $x=0$,

$S(\omega,T(\omega,0)) = T(\omega,0) = 0, \quad T(\omega,S(\omega,0)) = S(\omega,0) = 0.$ Therefore S and T are weakly compatible.

Example 3.10. Let (Ω,Σ) be a measurable space and C be a nonempty closed convex subset of a separable Banach space E and $f: \Omega \to C$ a random coincidence for two random mappings $S,T: \Omega \times C \to C$. Consider the equation $g(\omega,x) = 0$, where g is the real random function defined on interval $[0,\frac{\pi}{2}]$ by $g(\omega,x) = x^2 - \left(\frac{\pi}{2}\right)^2 \cos(x)$. g can be decomposed as $g = \frac{\pi}{2}(S-T)$, where the maps S and T are the self random mappings in $[0,\frac{\pi}{2}]$ defined by $S(\omega,x) := \frac{2}{\pi}x^2$ and $T(\omega,x) := \frac{\pi}{2}\cos(x)$. They satisfy inequality (10). They coincide at $f(\omega) \approx 1.0792$ and we have $p(\omega) = S(\omega,f(\omega)) = T(\omega,f(\omega)) \approx 0.7415$. Thus, $f(\omega)$ is solution to the equation $f(\omega,x) = 0$. From **Theorem 3.3**, the random Jungck-Mann iterative algorithm $\{S(\omega,x_n(\omega))\}$ defined in (18) converges to $p(\omega) = S(\omega,f(\omega))$. Using MATHEMATICA 10.2, we have the following table:

n	$x_n(\omega)$	$S(\omega,x_n(\omega))$
0	0.1000	0.1000
1	1.0484	0.6996
2	1.0769	0.7401
3	1.0793	0.7414
.	.	.
.	.	.
.	.	.
5	1.0792	0.7415
.	.	.
.	.	.
.	.	.
7	1.0792	0.7415

Thus, $\{S(\omega,x_n(\omega))\}$ converges to $S(\omega,f(\omega))$ implies that the sequence $\{x_n(\omega)\}$ converges to $f(\omega)$, the zero of g.

REFERENCES

[1]. H. Akewe, *Approximation of fixed and common fixed points of generalized contractive-like operators*. University of Lagos, Lagos, Nigeria, Ph.D. Thesis, (2010), 112pp.

[2]. H. Akewe, G. A. Okeke, *Convergence and stability theorems for the Picard-Mann hybrid iterative scheme for a general class of contractive-like operators*. Fixed Point Theory and Applications, **66**(2015).

[3]. H. Akewe, *The stability of a modified Jungck-Mann hybrid fixed point iteration procedure*. International Journal of Mathematical Analysis and Optimization: Theory and Applications, **1**(2016), 95–104.

[4]. V. Berinde, *On the stability of some fixed point procedures*, Buletinul Stiintific al Universitatii din Baia Mare. Seria B. Fascicola Mathematica-Informatica, **XVIII(1)**(2002), 7–14.

[5]. V. Berinde, *Iterative approximation of fixed points*, Baia Mare, Editura Efemeride.(2002)

[6]. A. T. Bharucha-Reid, *Fixed point theorems in probabilistic analysis*, Bull. Amer. Math. Soc., **82**(1976), 641–657.

[7]. B. S. Choudhury, *Convergence of a random iteration scheme to a random fixed point*, Journal of Appl. Math. Stochastic Anal., **8**(1995), 139–142.

[8]. B. S. Choudhury, *Random Mann iteration scheme*, Appl. Math. Lett., **16**(2003), 93–96.

[9]. A. Djoudi and Aliouche, *Common fixed point theorems of Gregus type for weakly compatible mappings satisfying contractive conditions of integral type*, Journal of Mathematical Analysis and Applications, **329**(2007), 31–45.

[10]. O. Hans, Random operator equations, *In Proc. Berkely Symposium on Mathematical, Statistics and Probability*, University of California Press, Berkely, **11(1)**(1961), 185â€"-200.

[11]. C. O. Imoru and M. O. Olatinwo, *On the stability of Picard and Mann iteration*, Carpathian Journal of Mathematics, **19**(2003), 155–160.

[12]. S. Ishikawa, *Fixed points by a new iteration method*, Proceedings of the American Mathematical Society, **44**(1974), 147–150.

[13]. Ismat Beg and N. Shahzad, *Random Fixed Point Theorems on Product spaces*, Journal of App Math. and Stochastic Analysis, **6**(1993), 95–106.

[14]. S. Itoh, *Random fixed point theorems with application to random differential equations in Banach spaces*, Journal of. Math. Anal. Appl., **67**(1979), 261–273.

[15]. S. Itoh, *Measurable or condensing multivalued mappings and random fixed point theorems*, Kodai Math. J., **2(3)**(1979), 293–299.

[16]. G. Jungck, *Commuting mappings and fixed points*, American Mathematical Monthly, **83(4)**(1976), 261–263.

[17]. G. Jungck, *Compatible mappings and common fixed points*, International Journal of Mathematics and Mathematical Sciences, **9(4)**(1986), 771–779.

[18]. S. H. Khan, *A Picard-Mann hybrid iterative process*, Fixed Point Theory and Applications, **69**(2013), 10pp.

[19]. T. C. Lin. *Random approximation and random fixed point theorems for nonself mappings*, Proc. Amer. Math. Soc., **103**(1988), 1129–1135.

[20]. W. R. Mann, *Mean value methods in iterations*, Proceedings of the American Mathematical Society, **44**(1953), 506–510.

[21]. G. A. Okeke and J. K. Kim, *Convergence and summable almost T-stability of the random Picard-Mann hybrid iterative process*, Journal of Inequalities and Applications, **290**(2015), 14 pp.

[22]. J. O. Olaleru and H. Akewe, *The convergence of Jungck-type iterative schemes for generalized contractive-like operators*, Fasciculli Mathematici, **45**(2010), 87–98.

[23]. M. O. Olatinwo and C. O. Imoru, *Some convergence results for the Jungck-Mann and Jungck-Ishikawa iteration process in the class of generalized Zamfirescu operators*, Acta Math. Univ. Comenianae, **77(2)**(2008), 299–304.

[24]. M. O. Olatinwo, *Some stability and strong convergence results for the Jungck-Ishikawa iteration process*, Creative Mathematics and Informatics, **17**(2008), 33–42.

[25]. E. Picard. *Sur l'application des methodes d'approximations successives a l'etude de certaines equations differentielles ordinaires*, Journal de Mathematicas, **9**(1893), 217–271.

[26]. R. A. Rashwan and H. A. Hammad, *Stability and strong convergence results for random Jungck-Kirk-Noor iterative scheme*, Fasciculi Mathematici, **58**(2017), 167–182.

[27]. S. L. Singh, C. Bhatnagar and S. N. Mishra, *Stability of Jungck-type iterative procedures*, International Journal of Mathematics and Mathematical Sciences, **19**(2005), 3035–3043.