Life Product Assessment Test Analysis (Case Study of AKT Bulb)

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Abstract:-**This paper work, life product assessment test analysis has its basis in reliability analysis on life data. It is to determine the probability capability of parts, components, equipments, products, system and so on to survive up to certain time. It is based on the application of continuous distributions called lifetime distributions. Though, there are many continuous distributions, but this paper focus on Weilbull distribution. Some concepts associated with reliability theory, such as; reliability function, cumulative density function, hazard function and so on are discussed.**

By linearization, the probability plots of the lifetime distributions are used to estimate the parameters of these distributions with the use of median rank.

Keywords: **reliability function, cumulative density function, hazard function, cumulative hazard function and mean time between failure**

I. INTRODUCTION

In several occasion, often made mention of live data in reliability statistics. Live data can be lifetimes of product in the market place, such as the product operated successfully or the time the product operated before it failed.

These lifetimes can be measured in hours, miles, cycles to failure or any other metric with which life or exposure of a product can be measured. For the purpose of this work, we will limit the examples and discussions to lifetimes of inanimate object, such as equipment, component and system as they apply to reliability statistics.

Reliability statistics can be defined as the probability and capability of parts, component, product and system to survive after time in specified environment.

A. Reliability Theory

When an items stops functioning satisfactory. It is said to have failed. The time-to-failure or life-time of an item is linked to its reliability and this is a characteristic that will vary from item to item even if they are supposedly identical.

Their time to failure will differ. There is some randomness to their failure times so their life-time is random variable whose behaviour can be modeled by a probability distribution.

Reliability analysis enables us to answer such questions:

- What is the probability that a unit will fail before a given time?
- What percentage of items will last longer than a certain time?
- What is the expected life-time of a component?

B. Fundamental Concepts Associated With Reliability Statistics

In the analysis of reliability data, it is the LIFETIME OR TIME TO FAILURE of a unit that is of interest. This characteristics lifetime is a random variable. As a result, it will have a probability density function pdf associated with it. In each case, once the pdf is found, the cumulative density function or cdf, can be determined.

C. Cumulative Density Function (CDF)

This gives the probability that a unit will fail before certain time t. And the cumulative density function cdf is given by:

$$F(t) = \int f(u)\, du$$

$$F(t) = \int_{u=o}^{t} \acute{\alpha}\ \beta^{-\acute{\alpha}} (u-n)^{\acute{\alpha}-1} e^{-[{}^{u-n}/_{\beta}]^{\acute{\alpha}}} d\,u$$

Let $W = [\frac{u-n}{\beta}]^{\alpha} \Rightarrow w^{1/\alpha} = \frac{u-n}{\beta}$

When u = 0, $w = [\frac{-n}{\beta}]^{\alpha}$

u = t, $w = [\frac{t-n}{\beta}]^{\alpha}$

$U - n = w^{1/\alpha}\beta$

$u = w^{1/\alpha\beta} \Rightarrow du = \frac{\beta}{\alpha} w^{1/\alpha - 1}\, d\,w$

$$= \int_{(\frac{-n}{\beta})^{\alpha}}^{(\frac{t-n)^{\alpha}}{\beta}} \alpha\beta^{\alpha} [W^{1/\alpha}\beta)^{\alpha-1}\ e^{-w} \frac{\beta}{\alpha} w^{1/\alpha-1} dw$$

$$= \int_{(\frac{-n}{\beta})^{\alpha}}^{(\frac{t-n}{\beta})^{\alpha}} e^{-w}\, dw = [-e^{-w}]_{[\frac{-n}{\beta}]^{\alpha}}^{[\frac{t-n}{\beta}]^{\alpha}}$$

$$= -e^{-[\frac{t-n}{\beta}]^{\alpha}} - [-e^{-[\frac{-n}{\beta}]^{\alpha}}]$$

$$= e^{[{}^{n}/_{\beta}]^{\alpha}} - e^{-[\frac{t-n}{\beta}]^{\alpha}}$$

$$= e^{[{}^{n}/_{\beta}]^{\alpha}} - [\frac{e^{-({}^{t}/_{\beta}]^{\alpha}}}{e^{-(-{}^{n}/_{\beta}]^{\alpha}}}]$$

$$F(t) = 1 - e^{-[\frac{t-n}{\beta}]^{\alpha}}]$$

D. Reliability Function (Or Survival Function)

The reliability functions given the probability that a unit will survive up to time t. This is denoted by R(t) and known as survivor function. These two states are mutually exclusive, since a unit either fails, or survives. The probabilities of these two mutually exclusive state are F(t) and R(t) respectively, one which must occur. Thus

R(t) = 1 – F(t)

$R(t) = 1 - [1 - e^{-[\frac{t-n}{\beta}]^{\alpha}}] \qquad \Rightarrow R(t) = e^{-[\frac{t-n}{\beta}]^{\alpha}}$

E. Hazard/Instantaneous Failure Rate Function

This is the rate of failure of units; i.e the rate at which the population survivors at any given instant are "falling over the cliff". It is denoted by h(t) and,

$$h(t) = \frac{f(t)}{R(t)}$$

$$= \frac{\alpha\beta^{-\alpha}(t-n)^{\alpha-1} e^{-(\frac{t-n}{\beta})^{\alpha}}}{e^{-[\frac{t-n}{\beta}]^{\alpha}}}$$

$$h(t) = \alpha\beta^{-\alpha}(t-n)^{\alpha-1}$$

It indicates the "proneness to failure of risk" of a unit after time t has elapsed. However, it is not a probability (it can take values greater than 1)

F. Cumulative Hazard Function

The cumulative hazard function H(t) is used to change hazard function h(t) if someone need to work in probability with help of survivor function R(t).

$$H(t) = -In\, R(t)$$

$$H(t) = -In\, [e^{-(\frac{t-n}{\beta})^{\alpha}}]$$

$$= [-[\frac{t-n}{\beta}]^{\alpha}]\ (-1)\log_{10} e = [\frac{t-n}{\beta}]^{\alpha}$$

$$H(t) = [\frac{t-n}{\beta}]^{\alpha}$$

G. Mean Time Between Failure (MTBF)

The mean time between failures (MTBF) is a concept which is frequently use in reliability work. It is defined to be the "average" or "expected" lifetime of an item, which is denoted as μ or E(T) and is given by MTBF = $\int_0^{\infty} t$ f(t)dt

$$MTBF = \int_0^{\infty} t\, f(t)\, dt$$

$$f(t) = \left\{\alpha\beta^{-1}(t-\eta)^{\alpha-1}\ \ell^{-(\frac{t-\eta}{\beta})^{\alpha}}\right.$$

if t = (t - η), then f(t) becomes

$$f(t) = \alpha\beta^{-1}t^{\alpha-1}\ \ell^{-\left(t/\beta\right)^{\alpha}} \quad \text{.......... (2.1)}$$

Therefore,

$$E(t) = \int_0^{\infty} t\ f(t)\ dt$$

$$E(t) = \int_0^{\infty} \alpha\,\beta^{-1}\, t^{\alpha+1-1}\ \ell^{-\left(t/\beta\right)^{\alpha}}$$

Let $U = (t/\beta)^{\alpha} \Rightarrow t = \beta U^{1/\alpha}$

$$dt = \left(\frac{\beta}{\alpha}\right)U^{(1/\alpha)^{-1}}\, du$$

when t=0, U=0

t= ∞, U= ∞

$$= \alpha\beta^{-\alpha}\int_0^{\infty}\left(BU^{1/\alpha}\right)^{\alpha}\ \ell^{-u}\left(\frac{\beta}{\alpha}\right)U^{1/\alpha-1}du$$

$$E(T) = \alpha\beta^{-\alpha}\ \beta^{\alpha}\frac{\beta}{\alpha}\int_0^{\infty}U^{1/\alpha}\ell^{-u}\, du$$

$$= \beta\int_0^{\infty}U^{1/\alpha}\ \ell^{-u}\, du$$

$$but\ \Gamma(\alpha+1) = \int_0^{\infty} y^{\alpha}\ell^{-y}\, dy$$

$$E(T) = \beta\,\Gamma(1+1/\alpha)$$

$$\therefore E(t-n) = \beta\,\Gamma\left(\frac{\alpha+1}{\alpha}\right)$$

$$E(t) = \eta + \beta\,\Gamma\left(\frac{\alpha+1}{\alpha}\right)$$

Also from equation (2.1)

$$E(t^2) = \int_0^{\infty} t^2\ f(t)\, dt$$

$$= \int_0^{\infty} t^2\,\alpha\,\beta^{-1}\,t^{\alpha-1}\ \ell^{-(t/\beta)^{\alpha}}$$

$$= \int_0^{\infty}\alpha\beta^{-1}t^{\alpha+1}\ \ell^{-(t/\beta)^{\alpha}}$$

$$= \alpha\beta^{-\alpha}\int_0^{\infty}\left(BU^{1/\alpha}\right)^{\alpha+1}\ \ell^{-u}\left(\frac{\beta}{\alpha}\right)U^{1/\alpha-1}du$$

$$E(T) = \alpha\beta^{-1}\ \beta^{\alpha+1}\frac{\beta}{\alpha}\int_0^{\infty}U^{2/\alpha}\ \ell^{-u}\, du$$

$$= \beta^2\int_0^{\infty}U^{2/\alpha}\ \ell^{-u}\, du$$

$$E(T) = \beta^2\,\Gamma(1+2/\alpha)$$

$$E\left[(t-\eta)^2\right] = \beta^2\ \Gamma\left(\frac{\alpha+2}{\alpha}\right)$$

$$E(t^2) = \eta^2 + \beta^2\left(\frac{\alpha+2}{\alpha}\right)$$

Therefore, variance will be

$$Var(T) = E(T^2) - E^2(T)$$

$$= \eta^2 + \beta^2\Gamma\left(\frac{\alpha+2}{2}\right) - \left[n + \beta\Gamma\left(\frac{\alpha+1}{2}\right)\right]^2$$

$$= \eta^2 + \beta^2\,\Gamma\left(\frac{\alpha+2}{2}\right) - \eta^2 + \left\{\beta\Gamma\left(\frac{\alpha+1}{2}\right)\right\}^2$$

$$= \eta^2 + \beta^2 \left\{ \Gamma\left(\frac{\alpha+2}{2}\right) - \left[\Gamma\left(\frac{\alpha+1}{\alpha}\right)\right]^2 \right\}$$

Lifetime Distribution

There are a handful of lifetime distribution models that have enjoyed great practical success, served as population modes for failure time, arising from a wide range of products and failure mechanisms. In this work, Weilbull distribution will be used or will be discussed.

H. Lifetime Following A Weibull Distribution

Definition: A random variable t is said to possess a Weilbull distribution if it has the density

$$f(t) = \begin{cases} \alpha\beta^{-\alpha} t^{\alpha-1} \ell^{-\left(t/\beta\right)^{\alpha}}, & if\ t > 0 \\ 0, & if\ t \le 0 \end{cases}$$

Where $\alpha = shape\ parameter$, $\beta = scale\ parameter$, η = location parameter.

The scale parameter, β, reflect the size of the units in which the random variable t is measured. The shape parameter α, causes the shape of the distribution to vary. By changing the value of β, it can generate a widely varying set of curves to model real lifetime failure distributions.

The location parameter, η, locates the distributions along the abscissa. Changing the value of η has the effect of sliding the distribution.

Weibull Model: By taking the two parameter Weibull distrubuted, the CDF can be shown as:

$F(t) = 1 - e^{-(t/\beta)\alpha}$

This function can be linearized (i.e put in the common form of

y = a + bx) as follows;

$f(t) = 1 - e^{-(t/\beta)\alpha}$

$In\ [1 - F(t)] = \ln (e^{-(t/\beta)\alpha})$

$In\ [1 - F(t)] = - (t/\beta) \propto$

Take the log of both side again

$In\ [- In\ (1 - F(t)] = \alpha\ In\ [\frac{t}{\beta}]$

$In\ [In\ \left[\frac{1}{1-F(t)}\right] = \alpha\ In(t) - \alpha\ in\ (\beta)$ ……………. (i)

Putting y in $[In\ \left[\frac{1}{1-F(t)}\right]$………….. (ii)

$and\ x = ln\ (t)$

by substituting equation (ii) into (i), we have

$y = \alpha x - \alpha\ In(\beta)$

Which is now a linear equation with slope α and intercept of and

α In (β)

I. Estimation of Reliability Analysis

Median Ranks

Median ranks are used to obtained and estimate of the unreliability, $F(t_j)$, for each failure. It is the value that the probability of failure, $F(t_j)$ should have at the jth failure out of a sample N units, at a 50% confidence level.

This estimate is based on a solution of the binomial distribution.

$$P = \sum_{k=j}^{N} \binom{N}{K} Z^k (1 - Z)^{N-k}$$

When N is the sample size and the order number. The median rank is obtained by solving the following equation for Z at P = 0.5

$$0.50 = \sum_{k=j}^{N} \binom{N}{K} Z^k (1 - Z)^{N-k}$$

A quick and less account approximation of the median ranks also given by

$$MR = \frac{j - 0.3}{N + 0.4} \text{ x } 100$$

This approximation of the median ranks is also known as Bernard's approximation.

Data Set Problem

Ten units of AKT 60 watts bulbs were tested to failure and the failure times were collected for record purpose as follow:

52, 71, 89, 108, 130, 149, 167, 186, 205, 223

Solution

First, rank the times-to-failure in ascending order as shown next.

Table 1: Rank Table

Time to failure (00 hrs)	**Failure order Number of a Sample size 10**
52	1
71	2
89	3
108	4
130	5
149	6
167	7
186	8
205	9
223	10

Median rank positions are used instead of other ranking methods because median ranks are at a specific confidence level (50%).

The times-to-failure, with their corresponding median ranks, are shown next.

Table 2: Percentage Media Rank

Time to failure (00hrs)	Median Rank (%)
52	6.73
71	16.35
89	25.96
108	35.56
130	45.19
149	54.81
167	64.42
186	74.04
205	83.65
223	93.27

TABLE 3: Estimation of the Data

	ti	Xi-In(ti)	F(ti)	Yi	(In ti)^2	Y:^{2}	(In ti)yi
1.	52	3.9512	0.0673	-2.6636	15.6120	7.0948	-10.5244
2.	71	4.2627	0.1635	-1.7232	18.1706	2.9694	-7.3455
3.	89	4.4886	0.2596	-1.2020	20.1475	1.4448	-5.3953
4.	108	4.6821	0.3556	-0.8223	21.9221	0.6762	-3.8501
5.	130	4.8675	0.4556	0.4974	23.6926	0.2474	-2.4211
6.	149	5.0039	0.5481	-0.2303	25.0390	0.0530	-1.1524
7.	169	5.1180	0.6442	0.0329	26.1939	0.0011	0.1684
8.	186	5.2257	0.7404	0.2991	27.3079	0.0895	1.5630
9.	205	5.3230	0.8365	0.5938	28.3343	0.3526	3.1608
10.	223	5.4072	0.9327	0.9927	29.2378	0.9855	5.3677
		48.3299		-5.1538	235.6577	13.9143	-20.4289

$$\hat{b} = \frac{\sum_{1=1}^{10} (Inti)yi - \left(\sum_{1=1}^{10} Inti\right)\frac{\left(\sum_{1=1}^{10} Inti\right)}{10}}{\sum_{1=1}^{10} (Inti)^2 - \frac{\left(\sum_{1=1}^{10} Inti\right)^2}{10}}$$

$$\hat{b} = \frac{-20.4289 - 48.3299\left(\frac{-5.1538}{10}\right)}{235.6577 - \left(\frac{48.3299}{10}\right)^2}$$

$$\hat{b} = \frac{-20.4289 + 24.9083}{235.6577 - 233.5779}$$

$$= \frac{4.4794}{2.0798} = 2.1538$$

$$\hat{a} = \bar{y} - \hat{b}\bar{t} = \frac{\sum yi}{N} - \hat{b}\frac{\sum Inti}{N}$$

$$= \frac{5.1538}{10} - 2.1538\left(\frac{48.3299}{10}\right)$$

$$= -0.51538 - 10.4093$$

$$\hat{a} = -10.9247$$

$$\therefore \hat{\alpha} = \hat{b} = 2.1538$$

$$\hat{\beta} = e^{-\frac{\hat{a}}{b}} = e^{-\frac{\hat{a}}{\hat{b}}} = 5e^{-\left(\frac{-10.9247}{2.1538}\right)}$$

$$\hat{\beta} = e^{5.0723} = 159.54$$

$$\simeq 160$$

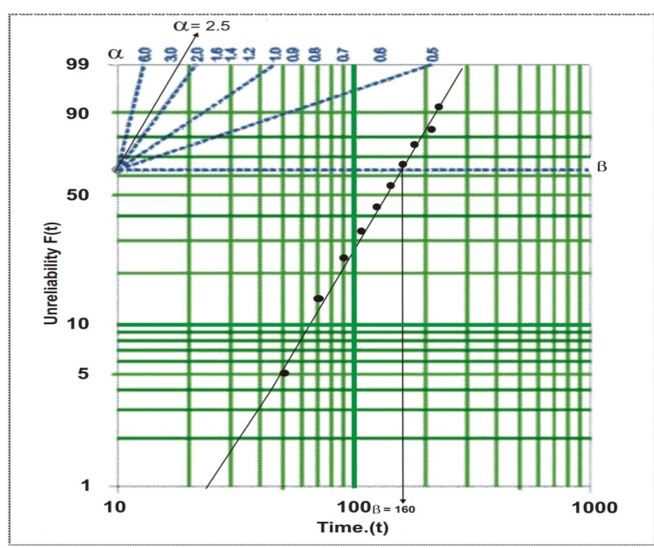

Fig 1: Probability plot of the data

J. Discussion

Base on our result from the graph method above, the slope $\alpha = 2.5$, Q(t) = 63.2% and $\beta = 16000$ hours. The slope (α) of Weibull plot simplify which member or class of the family of Weibull failure distribution best fit or describes the data and which group of failure present.

The slope indicate that the wear out failure exist in the AKT 60 watt bulbs since slope is greater than one.

K. The Correlation Coefficient

The estimator of ρ is the sample correlation coefficient ρ given by

$$\hat{\rho} = \frac{\sum(xi - \bar{x})(yi - \bar{y})}{\sqrt{\sum(xi - \bar{x})^2 \sum(yi - \bar{y})^2}}$$

Table 4: Correlation coefficient of the Data

Xi = In(ti)	Yi	(xi - $\bar{x}$)	(yi - $\bar{y}$)	(xi – $\bar{x})^2$	(yi – $\bar{y})^2$	(xi- $\bar{x})(yi-\bar{y})$
3.9512	-2.6636	-0.8818	-2.1482	0.7776	4.6148	1.8943
4.2627	-1.7232	-0.5703	-1.2078	0.3252	1.4588	0.6889
4.4886	-1.2020	0.3444	-0.6866	0.1186	0.4714	0.2365
4.6821	-0.8223	-0.1509	-0.3069	0.0228	0.0942	0.0463
4.8675	-0.4974	0.0345	0.0180	0.0012	0.0003	0.0006
5.0039	-0.2303	0.1709	0.2851	0.0292	0.0813	0.0487
5.1180	0.0329	0.2850	0.5483	0.0812	0.3006	0.1563
5.2257	0.2991	0.3927	0.8145	0.1542	0.6634	0.3199
5.3230	0.5938	0.4900	1.1092	0.2401	1.2303	0.5435
5.4072	0.9927	0.5742	1.5081	0.3297	2.2744	0.8660
$\bar{x} = 4.8330$	$\bar{y} = -0.5154$			2.0798	11.1895	4.8010

$$\rho = \frac{4.8010}{\sqrt{2.0798x11.1895}} \quad \frac{4.8010}{\sqrt{23.2719}} = \frac{4.8010}{4.8241}$$

$$\rho = 0.9952$$

It shows that the linear regression model fit the data since $\rho \simeq 1$, it indicate a perfect fit with a positive slope.

II. CONCLUSION

With the help of reliability analysis, it enables us to know life time or time-to-failure at specific period such as infant mortality, random failure or wear out failure of a unit products, components, system and so on. It help companies and manufacturers to know the optimum burn-in time or break-in period, as well as optimum preventive replacement time of component in a repairable system. It also help the manufacturer to give specific time for warranty and guarantee of a particular product, after the final production have been made.

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