Understanding the Condition under which Decapitation Strategy Can Impact on Terrorists' Strength and Sustainability: A Mathematical Perspective

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Abstract: - Since the advent of modern wave of terrorism in the aftermath of 9/11 tragedy, decapitation of terrorists' leadership have been a major counter-terrorism strategy of most countries. Notwithstanding the huge successes recorded in the elimination of prominent terrorists' leaders the world over, terrorist organizations remains more resilient with unperturbed activities. This study presents a novel Mathematical deterministic model, aimed at assessing how and under what condition can "leadership decapitation" impact on the overall strength and sustainability of a given terrorist group. The construct, a set of coupled differential equations whose variables are parameters of the internal and external dynamics of the organization is analytically and numerically simulated. The result of the analysis shows that, decapitation may have greater utility in an effort to fight a given terrorist group but only in combination with a viable "positive incentives". Individually, the strategy is insufficient to guarantee optimal decline of the organizations' strength. The analysis also shows that the "positive incentives alone have a comparatively greater utility than decapitation. Given the veracity of our model, it should be possible to evaluate the efficacy of the various policies of government and hence measure the degree of success or failure of a given counterterrorism measure.

Key Words: Counter-terrorism, Mathematical deterministic model, leadership decapitation, Terrorists' strength & sustainability.

I. INTRODUCTION

Most democratic nations today are facing an increasingly more fluid and complex terrorist landscape, populated by a diverse array of actors employing new technologies and tactics to advance their agendas. Terrorist threats the world over is growing more dynamically diffused, with an increasing number of groups, networks, and individuals exploiting global trends, including the emergence of more secure modes of communications, the expansion of social and mass media, and persistent instability across several regions. Since the first priority of any sovereign state is always the safety and securities of her citizens, then, relentless focus on countering terrorists that jeopardizes its citizens and interests

is a sine-qua-non. To sufficiently mitigate the evolving threats that today's geographically more dispersed and tactically more diversified terrorists pose, the state's approach to counterterrorism must also change. This should involves adjusting the existing strategies to meet the evolving threats and new facts, and discarding those strategies that have not yield sustainable result and applying new approaches informed by experience and judgment.

In the conventional security and defence philosophy, high premium is placed on military offensive approach, targeted directly at terrorist leaders, operational planners, recruiters and individuals deploying their expertise in areas such as IED (explosives), recruitment, cyber operations and propagandas; hence, the dominant leadership decapitation strategy of most world governments. The policy justification for targeting terrorists' leadership is rooted in the beliefs that targeting the leadership of an organization should disrupt operational and strategic functioning of the group. With the destabilization of these core elements of the group, its capacity to conduct operations should diminish, and its cohesiveness should decline. With enough disruption it may even be possible to induce distrust, infighting and atomization of the group, which in turn may lead to the collapse of the organization.

For example, leadership decapitation (kingpin strategy as it is commonly called), or the killing or capturing of the leaders of terrorist organizations, has been a core feature of US counterterrorism policy, since Al-Qaida attack on the World Trade Centre, USA in September 11, 2001. Significantly, in the aftermath of September 11, 2001, the kingpin strategy subsequently yielded an overwhelming over the demise of Osama Bin Laden- the Al-Qaida leader, on May 2, 2011. Since then the United States has targeted and killed or captured many Al-Qaida leaders as part of a general campaign to decapitate the organization. It has employed a variety of military operations to achieve this objective, including raids by Special Operations forces; whose success also saw the death of Abu Musab al-Zarqawi – the leader of Al-Qaida in Iraq. In June 2012, Abu Yahya Al-Libi, then al-Qaida's

deputy leader, was also targeted and killed in Pakistan in a drone strike coordinated by the US Central Intelligence Agency. While on the 5th of October, 2012the US forces also captured and killed Abu Anas Al-Libi, an Al-Qaida leader in a raid in Libya. The US has relied heavily on drone strikes to target Al-Qaida leaders and other militants in Afghanistan, Pakistan, and Yemen. In the aftermath of Al-Libi's death, Peter Bergen argued that Ayman Al-Zawahiri was likely the only remaining influential leader in Al-Qaida[5],[33],[36],[37].

Scholars and policymakers saw the death of these leaders as a significant blow to an already weakened al-Qaida terrorist group[16]. Nine months earlier, a Hellfire missile fired from a US drone killed Anwar Al-Awlaki, a Yemeni-American cleric linked to a number of terrorist plots in the West. On August 22, 2011, Atiyah Abd Al-Rahman, believed to be the organization's second-highest leader, was reportedly targeted and killed in a drone strike in Pakistan[48],[49]. The US drone also targeted and killed Ilyas Kashmiri, reputed to be a senior member of Al-Qaida and the operational commander for Harakat-ul-Jihad Al-Islami, in South Waziristan on June 3. 2011[48],[49]. These examples illustrate the frequency with which the US targeted Al-Qaida leaders and operatives over the last decades, specifically through the use of drone strike. Not only have drone strikes been used to kill terrorist leaders, but they have also been employed against lower-level operatives and anti-government insurgent leaders in countries such as Pakistan and Yemen, often with the agreement of those countries' governments. Despite these and other instances of successful targeting and elimination of Al-Qaida leaders, the organization remains a resilient terrorist organization.

Notwithstanding the prevalence of leadership decapitation as a strategy for states to fight the scourge of terrorism (and other illegal organizations), there is little conclusive evidence that this strategy is successful in disrupting a terrorist campaign, or even of mitigating its destructive effects. There is also practically few quantitative works on identifying the conditions that may impact negatively or positively on the effectiveness of the kingpin strategy, and whether its effectiveness can be generalized to all types of organizational structures. However, consequence upon the resiliencies of terrorist organizations in the face of frequent and severe decapitation of its leadership cadres; evidenced in the insurmountable severity and frequency of attacks, terrorism research scholars have hypothesized the application of psychological motivation theory to counterterrorism measures[3],[24],[67],[68].

Under this theory, the use of combine positive incentives or reward (Carrots) and negative incentive or punishment (sticks) to elicit and coerce behaviour compliance from adaptable adversary is recommended as a complementary strategy to the sole conventional military offensive (attrition) against recalcitrant terrorists' operatives. Positive incentives (carrots) could, for example consist of providing goods and services, or

valuable opportunities to persons or groups of persons that refrain from terrorism - making non-terrorist activity more attractive[21],[22]. Examples of positive incentives can include: direct monetary transfers; economic assistance (e.g. lucrative job placement, educational scholarship opportunities, etc.); provision of goods, services, and opportunities; lifting of sanctions; period of seize-fire, amnesty program, suspension of prison sentences, rehabilitation and reintegration of repented terrorists, and removal of taxes or customs duties[3],[67],[68].

Negative incentives (sticks), the dominant CT approach are intended to increase the cost of terrorism to potential terrorists group or terrorist sponsored states, by such means as imposing trade restrictions, freezing the terrorist's assets, arrest and assassination of operatives; restricting how the terrorist operates, and retaliating militarily. Positive and negative incentives may be interpreted as income effects. Enders and Sandler [18] refer to "freezing terrorist's assets", which reduces their war chest and their overall ability to conduct a campaign of terror - for example, by freezing their bank accounts. Lakdawalla and Zanjani[43] show that "protection reduces the payoff to terrorism"; they argued that "deterrence due to income reduction takes place insofar as private selfprotection raises the level of non-violent activities and lowers the total amount of violent terror investments". Finally, Hausken[27] proposes a model in which defensive investment not only helps defend the government's asset, but also reduces the terrorist's resources, so that the terrorist's attack effort becomes smaller.

However, in this study, we do not explicitly concern about the issue of negotiating with terrorists, since conventional wisdom abhors negotiating with terrorists or other similar adversaries (such as rogue states);though, some states had of-course negotiated with insurgents. In particular, Spector[64],[65] discusses negotiations between Israel and the Palestine Liberation Organization; between the US and Haiti; between the US and North Korea, and between Great Britain and Sinn Fein. Spector[65] argues that "despite the risks inherent in negotiating with terrorists, the risks of following a nonegotiation policy are likely to be more deadly. States need to assess terrorist interests and intentions to find if there are reasonable entry points for negotiation and take advantage of these to transform the conflict." Similarly, Pruitt[58] considers both the peace process in Northern Ireland and negotiations with Islamic terrorist groups. He suggests that the success of negotiations depends on flexible attitudes on the part of both parties, and that "there are many arguments against negotiating with terrorists, but most of them do not apply to secret, backchannel talks, which are usually the method of choice in first approaching these groups." He also observes that "negotiation with non-ideological ethnonationalist terrorists is more common and more successful than with other kinds of terrorists".

In this paper, we are concern about the conditions under which the state would be willing to offer terrorist positive incentives to enhance internal personnel defection and as well as refrain from attacking, and when prospective terrorist group or operatives would be willing to accept such incentives. For instance, in an effort to marshal resources against Al-Qaeda terrorist organization in the aftermath of the September 11, 2001 attacks, the United States had implemented "carrot and stick" approach to help build a coalition and gather credible intelligence for smart targeting of Al-Qaeda's location. Carrots or rewards were provided for voluntary compliance with implied preferences or explicit direction, while sticks or punishment were imposed to coerce compliance.

In an interview with Arafat loyalist (a former terrorist, a senior commander of al-Fatah, and a Palestine brigadier general), Hoffman[29] reports exemplified another two examples of positive incentives. Although they were provided by the leadership of the terrorist organizations to discipline their members, rather than by the potential victims of terrorist attacks, (the state), the example could be modify so that the state can emulate same in eliciting compliance behaviour from potential terrorist operatives. First, after Hashemite King Hussein of Jordan sought to restore his monarchy's rule by quashing the autonomy of Palestinian organizations, killing tens of thousands of people (mostly Palestinians) from September 1970 to July 1971, Yasir Arafat formed the Black September Organization (BSO), the most elite unit of the Palestine Liberation Organization (PLO), consisting of dedicated, ruthless, loyal, and highly skilled warriors. Their first two operations were the November 1971 assassination of Jordan's Prime Minister Wasfi al-Tal, and the September 1972 seizure of Israeli athletes at the Munich Olympic Games (exemplifying terrorism's ability to transform a cause from obscurity to renown). Two years after, Arafat was invited to address the General Assembly of the United Nations (UN), and thereafter the PLO was granted special UN observer status. Having obtained international recognition, Arafat wanted to "turn Black September off" [29]. The PLO leadership used positive incentives to recruit approximately 100 attractive young Palestinian women to Beirut. As Hoffman reports, "the hundred or so Black Septemberists were told that if they married these women, they would be paid \$3,000; given an apartment in Beirut with a gas stove, a refrigerator, and a television; and employed by the PLO in some nonviolent capacity. Any of these couples that had a baby within a year would be rewarded with an additional \$5,000." Thereafter, the PLO provided periodic tests of these individuals' willingness to return to terrorism, but none strayed, and Black September had been effectively dismantled.

Second, the authorities in Northern Ireland pursued a similar strategy before the 2001 cease-fire: "Hard-core IRA and Loyalist terrorists (mostly in their thirties) serving long prison sentences were given brief furloughs during holiday periods" [28]. Combined with a variety of factors in prison conditions, and the possibility of early release, the objective was to allow these to develop family ties, and "wean these men from terrorism." According to Hoffman, "the program

was so successful that the option could be offered to only a limited number of prisoners, lest the terrorist organizations, fearing the loss of too many senior veterans and commanders, forbid their members to participate in the program". The lesson to be learned, the author argues, is that "creative thinking can sometimes achieve unimaginable ends". "Rather than concentrating on eliminating organizations, as we mostly do in our approach to countering terrorism. We should perhaps focus at least some of our attention on weaning individuals from violence".

Another example where it is reasonable to assume that positive incentives could weaken the terrorist's strength and or prevent or reduce the frequency of terrorist attack, is the amnesty program employed by the Nigerian state to disarmed, demobilize and reintegrate the Militant groups in Nigeria Niger Delta region. While the Niger Delta conflict may not have taken analysts by surprise, the new manifestations of the violence particularly the spaces within which they were occurring, was unfathomable in the early 2000s. The struggle over the control of the Niger Delta resources (which form 90% of Nigerian foreign exchange earnings) has not always been violent initially. Niger Deltan elders/elites, from the very beginning were interested in negotiating and dialoguing the contentious politics of oil production in the area through peaceful and non-violent approach through socio-political and ethno-cultural movements.

However, elders/elites involvement took a violent turn when the Niger Deltan youths suddenly lose confidence and trust in their elders' ability to wills government attention to the plight of the region, and thus, began to challenge the Nigerian state, oil companies, and the elites in the region. The youths became increasingly suspicious of their elders/elites who they accused of being weak, fearful, greedy and generally ineffective in obtaining concessions from government and the oil companies on behalf of the people. They therefore resolved to mobilize and engaging the state and the oil companies[34], through incessant threat and violent protests, which gave rise to many militants groups such as the Niger Delta People Volunteer Force (NDPVF), Movement for the Emancipation of the Niger Delta (MEND), Coalition for Militant Action in the Niger Delta (COMA), and the Martyrs Brigade (MB), and the spread of insurgency in the region[32],[55]. In response to the spreading insurgency, the state first employs the negative incentive strategy, by deploying troops of military combatant and war arsenals to protect the oil and gas facilities and as well as coerce compliance of the militants groups in the area. Nonetheless, governments' effort continued fruitless, with the formation of die-hard militant groups comprising of thousands of wellarmed youths trained in covert military operations and determined to engage the forces of the state in prolonged warfare.

Amongst other demands, the militant groups sought "resource control and self-determination" by every means necessary, including kidnapping of oil workers, seizure of oil facilities, looting of crude oil, and physical engagement with security forces. After years of fruitless government efforts; including arrest, assassination and imprisonment of militants and as well threats to force the militants to long-term inactivity, the federal government under the leadership of President Musa Yar' Adua finally decided to adopt the only alternative of positive incentive strategy - the Amnesty program in 25th June, 2009. The development of the amnesty program which came forth in pursuant of section 175 of the 1999 Constitution of the Federal Republic of Nigeria originated from the mutual cognition and recognition of the input from different key stockholders and ministries. The emphasis was on designing disarmament, demobilization and reintegration (DDR) project for any repentant or groups of repentant militants. The amnesty package include among other incentives, payment of monthly allowance, vocational skill acquisition programs, entrepreneurial training programs, foreign educational scholarship and job placement for potential repented militant. Presently, the DDR projects, which saw many ex-militant being trained in vocational skills, trades, given scholarship to study in higher institution abroad, and funded for other entrepreneurial skill programs are increasingly being used to keep post-conflict Niger Deltan from slipping back into crises and to stimulate economic growth and development the Niger Delta[14]. A result of which is bringing relative peace, security and economic development to the embattled region.

Fatigue and attrition of personnel have been empirically found to impact negatively on the dynamical evolution of terrorist organizations. In interviews with captured or retired terrorists, reveals that they often complained about the psychological stress of their past work, its moral contradictions, and the isolation from relatives and friends [30]. This is part of the reason why terrorist organizations cannot remain inactive (as in a cease fire) for very long time without experiencing irreplaceable loss of personnel due to loss of motivation, and many organizations even resort to coercion against desertion. Not with standing the prevalence of leadership decapitation and the offering of positive incentive (carrot) to adaptable adversary, as possible strategies for states to fight the scourge of terrorism (and other illegal organizations), there is little quantitative evidence that these strategies are successful in disrupting a terrorist campaign, or even of mitigating its destructive effects. There is also practically no quantitative work to identifying the conditions that may affect the effectiveness of this "kingpin" strategy, and whether its effectiveness can be generalized to all types of organizational structures. Knowing when and how these strategies affect the strength and sustainability of a given terrorist organization will be useful for policymakers facing a terrorist threat. Therefore, the crux of this study is to investigate the central questions of counterterrorism: "how and under what condition does leadership "decapitation" affect the overall strength and sustainability of a terrorist organization?"The contextual methodology of this study is to address these questions in the context of a terrorist group's organizational dynamics.

Research findings over the past decades indicates that the strength of any terrorist organization depends on its available labour (human resources) force; the success rate of previous attacks to stimulate recruitment and as well as funding and education to support and execute all the activities of the various classes of operatives over time"[8],[23]. Since manpower for carrying out attacks and sustaining operations is a critical resource for terrorist organizations; then hindering recruitment process strikes a blow on the organization's ability to function. A priority of any nation's campaign against global terrorism should be to move beyond responding to attacks and threats to taking proactive steps toward cripple the terrorist group. One prong of this proactive strategy is to diminish the ability of the organization and its affiliates to recruit new members. That is, meticulously identifying and including the terrorist recruiters' class which is a special class of leadership, and their recruitment institutions as apragmatic target for counterterrorism operation.

This study, a preliminary work on my PhD (Applied Mathematics) research dissertation: "Optimal Allocation of Human Resources in Counter-terrorism Operations and the Cataclysmic Dynamic of Terrorism"; takes a crucial look at the dynamic effects of targeting terrorist leadership and recruiters classes simultaneously, through the instrument of positive incentives, negative incentives (military offensive) and a combination of both approaches, on organizations' strength and sustainability. In the study, we develop a deterministic mathematical model of counter-terrorism; where the state's strategies include the use of viable positive incentives or negative incentives or both to elicit high internal personnel drain within the organization. The tripartite aims of these approaches are: to garner the needed moral supports for building a formidable "in-policing" mechanism with the locals; to gather credible intelligence for smart targeting of terrorist location, and to de-legitimize terrorism and its propagandas among the locals. These will culminate in weakening the overall strength and sustainability of the organization, and thereby rendering it grossly incapacitated to organizing a successful terror attack in the longer term. In the study we present a dynamic model of a terrorist's organizational structure consisting of three classes of operatives (leaders, foot-soldiers and recruiters); and use a set of coupled differential equations to describe the structural evolution of the organization in its simplified terms. Our aim is to examine how the numbers of terrorist operatives in the organization changes over time under a given CT strategy, thus, casting our empirical-based knowledge in precise mathematical language.

Our target is to derive an analytical model of CT measure that would help to degrades or render dysfunction a given terrorist organization, while also serving as a quantitative metrics for evaluating the decree of accomplishment or otherwise of a given CT operations. Refinement of our ideas should enable security agencies to able to state emphatically, for example, that they are 85% certain that they have destroyed or captured or weaken the strength of the terrorist group under

investigation; and there is still a 15% chance that terrorists might commit another deadly attack. The study has the potential to inform both scholars and CT policy makers on the optimal strategy for allocating the available CT resources towards effective CT measures. The simplicity of our model should hopefully make it an attractive target for extensions by enterprising students of military operations Research (OR) and dynamical systems analysis. The study also has the synergy to demonstrate that, terrorism though complex and divergent a socio-economics system is an area where mathematical methods can make an impact in a variety of targets and research problems.

1.1 Relevance Academic Literature

Like any organization, terrorist groups vary widely in terms of goals, strategies and their organizational structure. A terrorist group's ability to withstand attacks is dependent on the bureaucratization structure of the organization and as well as the communal supports it's enjoying from the locals. Analyzing the effects of certain counterterrorism approaches on terrorists' strength and sustainability; this study differs from existing work by providing a novel mathematical lens through which to evaluate the effectiveness of a given counterterrorism policy. A great deal of research seeks to the understand quantitatively consequences of counterinsurgency on terrorists' behavior its organizational structure[8],[23],[43],[58],[66], but much of these literature looks at one specific strategy, such as leadership removal (decapitation)[23],[36],[40] or multiple targeting[8],[43],[47],[66] through the dominant military offensive strategy. Thoughsome case studies suggest that conciliatory and coercive strategy have distinct effects on insurgent strength and sustainability and thus, its violence rate[8],[16], but such research is only qualitative and rare.

However, the literature on the effectiveness of leadership targeting focuses on five areas: the role of leadership, intelligence, and organizational structure, as well as quantitative findings and counterproductive consequence. Also, much of the optimism surrounding the effect of leadership decapitation is grounded in theories that analyze the role of these leaders within their organizations rather than on the psychological implication of the CT strategy. Theories of charismatic leadership, for example, posit that the susceptibility of terrorist organizations to leadership targeting is a function of qualities inherent to the leader[51]. These qualities not only sustain leaders' legitimacy, but also foster the belief among followers that such leaders are irreplaceable [52]. Theories of charismatic leadership, however, over predicted the success of leadership decapitation while overlooking both organizational variables and social context. In a decentralized organization the leadership has less control over the strategic and operational details. It is assumed that adopting a decapitation strategy here would prove relatively ineffective. Conversely, in a centralized organization, removing the leader or leaders could have catastrophic consequences for the group. However there are many possible and subtle nuances to this sort of theorizing. For example, hierarchies are also more resilient, having procedures for promotion and replacement. In diffused organizations senior commanders may play more critical roles in determining group activity.

Other studies examine the contribution of leaders to organizational strength and cohesion. According to Bryan Price, terrorist groups are clandestine and value-based, leadership succession difficult[6]. organizations are more cohesive and are often led by charismatic leaders, making succession especially difficult. Clandestine organizations are more dependent on their leaders than non-clandestine organizations that are unlikely to institutionalize their operations for both strategic and personal reasons, further complicating succession. Finally, because terrorist organizations are values-based, leaders are harder to replace, and their removal can cause instability. Bryan[6] offers one of the few theoretical explanations for leadership decapitation, but his model does not account for variation in the effectiveness of leadership targeting. By treating all terrorist organizations as values-based, his theory overpredicts the occurrence of success.

The role of the leader can be another predictor of the ability of a decapitation strike to weaken the organization's strength. Michael Freeman argues that the likelihood of success in targeting leaders can be determined by whether they play an operational or inspirational role [52]. Organizations in which the leader plays both operational and inspirational roles are the most likely to collapse after decapitation. Freeman claims that although organizations with charismatic leaders tend to be susceptible to leadership attacks, over time they can become more institutionalized and more resilient in the face of such attacks. Secondly, targeting operations can yield critical intelligence about terrorist group activity and lead to organizational weakening. For example, authorities found documents during the arrest of the leader of Peru's Shining Path, Abimael Guzmán that led to the capture of other members of the group[11].Leaders under arrest can provide information about their organizations' location, capabilities, personnel, and operations[2].

Thirdly, some studies have found a link between organizational structure and group stability[12],[35],[40]. Kathleen Carley[41], among others, argues that decentralized organizations are harder to destabilize than hierarchical organizations. Marc Sageman[47] claims that, given the structure of such organizations, leaders may not be the right targets. In a study of the global Salafi jihad, Sageman argues that social networks provide an opportunity for socially and geographically isolated and alienated individuals to join a community[47]. The social bonds created within these communities create and strengthen the ideological commitment of potential militants, encouraging them to join jihadist movements. The decentralized and local nature of these groups makes them difficult to target.

Fourth, quantitative studies on leadership targeting yield different results regarding whether and when decapitation is effective. Bryan[6] finds that decapitation increases the mortality rate of terrorist organizations. Johnston concludes that decapitation decreases the intensity and frequency of terrorist attacks, increases the chance of war termination, and raises the probability of government victory[39]. Johnston claims that although decapitation can "help break the morale of insurgencies that have been engaged in long, often difficult campaigns"[9], it is more effective as part of a larger campaign. Although Johnston's statistical findings are robust, his article lacks a theoretical discussion of decapitation: it does not provide an explanation for why decapitation is effective, how it can influence a group's operational capacity, and when states should or should not target militant organizations. Furthermore, Johnst on focuses exclusively on insurgencies. Although some terrorist groups are also insurgents, many insurgent organizations do not employ terrorist tactics. Finally, capturing a leader may have quite different effects than killing them; in the first case they may order a reduction in activity to facilitate bargaining for their release, while killing personnel may induce more attacks as a symptom of competition amongst potential successors. Some that leadership targeting may studies argue counterproductive and susceptible to blowback syndrome. Unintended consequences include the creation of a martyrdom effect, Herostratos syndrome, a surge in recruitment, the occurrence of retaliatory attacks, an increase in group resolve and strength, and a rise in the frequency and intensity of attacks.

1.2 The Differential Equation Models of Terrorism

The research methodology of modeling global trends and patterns in terrorism with coupled differential equations, owes much to the seminal work of Lewis Fry Richardson in 1941. Richardson - a physicist and meteorologist, perhaps best known to social scientists for collecting data on conflicts (or deadly quarrels) and modeling arms races using differential equations, also made early contributions to modeling the frequency severity distributions of wars[60],[61]. Richardson showed that the frequency and severity of wars robustly follows a power-law relationship [38,39], where severity is inversely proportional to frequency. A very significant earliest application of differential equations model to warfare dynamics is credited to Lanchester [44], who developed the first conventional warfare dynamic model. Lanchester's warfare dynamic equations was a mile stone in dynamical modeling - the mathematical technique of describing a complex socio-economic system like terrorism with a set of coupled differential equations has enjoyed a wider application. From this mile stone, the last decades saw various dynamic models of terrorism also hitting the waves of differential equations[1],[8],[10],[23],[43],[66]. The primary advantages of these models are that the authors provided their estimated parameter values and differential equations. Although much of their data were "notional assumptions", their models were well developed in a progressive fashion.

Weaver[69]opined that the accuracy of future models would "depend heavily upon model parameterization and the formulation of effect functions".

complement the power-law relationships [38],[39],[60],[61], a differential equation model to studied and identifies the robust patterns in the frequency and severity of violent attacks by terrorist organizations in relationship to their organizational size (numerical strength) and experience was also developed[1]. Applying the group-level static and dynamic analyses of terrorist attacks worldwide from 1968 to 2008 and a simulation of its organizational growth, the authors concluded that "the production rate of violent events by any terrorist organization depends on its available labor (numerical strength); the time between subsequent attacks is inversely proportional to the size of the organization; terrorist organizational growth is driven partly by recruitment processes associated with the production of new events. That is increase violent events lead to more recruitment which in turn leads to organizational growth. As an organization carries out more attacks, the time between subsequent attacks decreases, andthe severity of new attacks depends on available labor pool and thus increases with organizational size and with the number of attacks, respectively[1].

In a nonlinear differential equation model: "A Dynamical model of terrorism"[66], a new paradigm for studying terrorism by examining the "long-term dynamical evolution" of a terrorist organization when direct military and policing interventions are employed was proposed. The authors' methodology was to develop a dynamic model of terrorism in terms of the dynamics of the population of the individuals directly involve with terrorist activities - the terrorists, the susceptible, and the pacifists. The model, solved for fixed points with relevant sensitivity analysis yielded the conclusion "a combination of military/police action and nonviolent/persuasive intervention would yield a fixed point with a lower terrorist population than with any one of these interventions in isolation"[66]. Though the study makes the most comprehensive use of dynamical system analysis applied to terrorism research, however, it was criticized to be monadic. Like most mathematical surveys in terrorism, the study was observed by some CT scholars to be limited in scope and breadth, and neglecting the basics of political science conflict-peace theory.

In a similar system dynamics techniques, Chamberlain[10] sought a better understanding of how Al-Qaida is able to 'produce' terrorists that are willing to risk, and at times, sacrifice their lives and then deploy these individuals to carry out terrorist attacks. By graphically representing the cause and effect interactions of the Al-Qaida terrorist organization through stock-flow diagrams and causal loop diagrams, the author was able to develop a "historical behavior pattern for this organization". The author analysis was extended to the US CT strategies; to include funds allocations for the Department of Defense, Department of Justice, Department of

Transportation, and the Department of Homeland Security. In conclusion, the author observed that "doing something, following either the current US strategy or the US preventive strategy is better than doing nothing at all (the Baseline Strategy)". By implication, the model asserted that "the post 9/11 US counter-terrorism strategy of the US is an effective approach to the global war on terrorism". Chamberlain[10] further predicted that, "regardless of the US strategy followed, Al-Qaida will still be able to conduct at least three additional terrorist attacks, resulting in at least 2,000 additional US citizen deaths before 2010". This conclusion put the entire US security agencies at red alert. However, because this expected attacks was not forthcoming since author's postulation, terrorism scholars commented on the limitations of system dynamics techniques for predictive purposes.

Another effort in the application of differential equation to describe and model the behavior of terrorist organization was Gutfraind's [23] dynamic model, whose methodology was creating a very simple dynamic model of a terrorist organization structure from a human resources point of view; focusing on examining how the numbers of terrorist operatives in the organization change with time. In this work, the author evolved and solves analytically a system of two linear ODEs, whose result concluded that, "in a general system, his modeled terrorist organization, "would collapse if its strength and its pool of foot-soldiers decline simultaneously". And in contrast, "a simultaneous decline in its strength and its pool of leaders is often insufficient and short-termed as well to guarantee the eminent collapse of the organization" [23].

Perhaps, the presumed gap created by the above Gutfraind's [23]conclusion motivated Butler[8] to attempt an analytical study of the viability and the indispensable hidden catalytic roles of the terrorist recruiters in the organization's dynamics. Butler's dynamic model: "Hezbollah: The Dynamics of Recruitment"[8] centered on deriving a differential equation model of Hezbollah's recruitment process, by blending the deterministic mathematical modeling techniques with qualitative techniques of case study analysis from the political science discipline. His monograph highlighted the logistic growth as a driver of adaptation for Hezbollah's recruitment process; hence the utility of using the logistic growth model to estimate the potential recruitment pool of susceptible youth's population in the absence of validated data was illustrated. However, while drawing analogy on the role of terrorist recruiters to that of catalytic enzymes in biochemical kinetics, Butler[8]evolved and solved analytically, a system of four non-linear ODEs popularly called "Michaelis-Menten equations of enzymes kinetics"[15]. These equations represent the dynamical evolution of four classes of terrorist operatives (the restless, the recruiters, the recruits and the radicals). The author's analytical results concluded that "the growth rate and strength of the Hezbollah terrorist organization under study (and indeed most terrorist organizations) is dependence on the number of recruiters; the number of recruits (footsoldiers); the success rate of previous terrorist attacks to stimulate recruitment; and as well as funding and education to support and execute all of these activities by the various populations over time"[8]. To proffer solution to his identified terrorist's strength drivers, the author suggested that a reduction in "non-suicide violence through education, and the numbers of recruiters (in order of preference) may contribute to the degradation of recruiter's ability to conduct the recruitment process", and hence the collapse of the organization.

Furthermore, guided by the efficacy of credible intelligence gathering in ideal CT operation, the war of attrition model by Kress & Szechtman[42] was also developed. The model, also a differential equation model, dynamically addresses the relationship between intelligence gatherings; collateral casualties in the population during terror war of attrition; recruitment to the insurgency and reinforcement to the government force during CT operations. The author's aims was to examine the effect of the collateral damage that often occurs when large-scale US military operations take a highforce buck-shot approach toward eradicating terrorists' cells within a locality. The consideration was that such collateral damage caused to the general population resulting from poor human targeting, often result in mass killing of innocent civilians' population. This may generate adverse response against the government and security operatives and create popular support for the insurgents; which may result in higher recruitment to the insurgency. Conventionally, collection of sufficient and credible intelligence to build up detail knowledge of the location of terrorists or threats to a country's security should be at the heart of every government's security and intelligence experts. Intelligence operations protect society against the threat of terrorism. "Without sufficient and credible intelligence, governments cannot protect their citizens. Without accurate knowledge of who the terrorists are, governments cannot know where they are. Without knowing where the suicide bombers are, the security agency cannot prevent planned suicide bombing attacks" [25], [26]. Therefore, sufficient and credible intelligence is the heart and soul of counter-terrorism operations.

Applying conventional warfare dynamic concept[44], Kress and Szechtman[42] evolved and solved analytically, a system of two ODEs describing the interaction between two explicit players in CT operation (the insurgents and the government forces), that battled each other, at the detriments of one implicit player (the general population) that are usually victims of government's military offensives and provides new recruits to the insurgency. From their findings, the authors concluded that "government would always lose if there is no reinforcement to its force because the intelligence capabilities of the government degrade with the attrition of its force, causing many innocent casualties and indirectly strengthening the insurgency which eventually takes over". Proffering solution to their identified error, the authors suggested that "an influence campaign operation could reverse this reinforcing loop"[42].

However, without specifying the type or details of their suggested influence operation, the authors' expectation is that, with a well-articulated and credible influence campaign operations, the numbers of terrorist defectors would increase. An increased numbers of defectors would provide more accurate intelligence for smart-targeting of insurgent locations, and that civilian deaths could be reduced if the selective. killing were more Though, Kress Szechtman[42] recognize that terror war of attrition is problematic and cost oriented, however, they maintained that increasing killing accuracy is the best-case scenario for ideal CT measures. However, the authors concluded that "government cannot totally eradicate the insurgency by force, the best it can do is to contain it at a certain fixed level". Kress and Szechtman [42] also recognized that ideology might play a role in waging an anti-insurgent "war of attrition" but excluded ideology from their model because it was too difficult to measure.

Furthermore, given the high correlation between ideological strength and insurgent community size, and given insurgents' employment of ideology as a currency to recruit supporters; the size of the insurgent community may serve as a proximal metric of its ideological strength. Therefore, as an extension of the idea of "smart-target" eradication of ideology, Weaver[69]developed a dynamic model whose methodology was an extension of Kress and Szechtman[42] model; by adding the concept of insurgent ideologies as a CT target. Moving the discussion beyond the "reduction of killing" approach, the author though transparent in her approach, uses system parameters and functions which were highly notional. Since the author foresaw that a model could test alternative influence campaigns, she extended the terror war of attrition model [42] to include the impact of the influence campaign operations, by defining "defection messaging" as an antiinsurgent influence campaign that could target ideology[70]. She however, was not aware of any empirically supported models of such an intervention.

II. THE MATHEMATICAL MODEL

To develop our model, we consider a potential numerical terrorists' strength S(t) at any time (t) to consist of the population of three potential classes of operatives; Leaders Y(t), Foot-soldiers X(t) and recruiters Z(t); with their differential derivatives, $Y^{'}(t)$, $X^{'}(t)$ and $Z^{'}(t)$, representing their respective rate of growth. Since the leaders Y(t), with their valuable skills, wealth of experience and financial sovereignty contribute more to the overall strength S(t) of any organization than an equivalent number of other operative; then S(t) can be taken as a weighted sum of leaders, Y(t) recruiters, Z(t) and foot-soldiers X(t); with leaders, Y(t) having more weight, say t 1 than other variables [23]. Mathematically,

$$S(t) = mY(t) + X(t) + Z(t) \tag{1}$$

2.1 The Birth Process of the Organization

- (a) The leaders: Theoretically, from the history of Al-Qaeda, Hezbollah, ISIS and other terrorist organizations, the pool of terrorist leaders and experts Y(t) grows primarily when foot-soldiers acquire battlefield experience or receive training (internally, or in terrorist-supporting states[8],[66]. Consequently, the pool of terrorist leaders Y(t) is provisioned with new leaders at a rate proportional to the number of foot soldiers (i.e. Y' = pX(t)), (where the proportionality constant (p) denote the "promotion" process).
- (b) The Foot-soldiers: Unlike the dynamic of leaders, research also shows that the growth in the number of terrorist footsoldiers (rank-and-file) X(t) through recruitment process (r)is driven primarily by two factors, (i) the intensity and success rate of terrorist propaganda, and (ii) effect of blowback action occasioned by the collateral damages and mass killing of innocent civilian population during military offensive against terrorists' cells within a locality. This often incites the spirit of disaffection and animosity among the local population and, hence causing a "blowback action". The blowback action provoke "Herostratos syndrome" in the disaffected youth population (the source of new insurgents). This help to pumpup new recruits and even increasing the number of new insurgents groups[4],[8]. Thus, the growth in the number of foot-soldiers X(t) varies with the strength S(t) of the organization and with the number of commissioned recruiters Z(t) as well as the recruitment and training facilities (i. e. X' = rS(t) + cZ(t)),(where the proportionality constant (c) denote the "commission" process). This assumption capture the often seen cycle where every military attacks leads to greater recruitment (increase (r)) which leads to greater strength (increase S(t)) and more attacks from the terrorist[1],[9],[19],[20],[70].
- (c) The Recruiters: Research also shows that the growth in the number of potential terrorist recruiters Z(t) increases primarily when "non-suicide" ex-jihadists acquire training by radical religious clerics, or socio-cultural or clandestine organization; usually funded by external states or financially sovereign individuals[8]. Hence, the increase in number of terrorist recruiter varies with the strength, S(t) of the organization and with the number leaders, Y(t) in the organization (i.e. Z' = cSt + pY(t). This process is termed "commission", C > 0.

2.2. The Death Process of the Organization

The growth rates of terrorist operatives are often opposed by two processes: (i) internal personnel loss due to demotivation, fatigue, desertion as well as in-fighting and splintering[30]. This phenomenon is modeled as a loss of a fraction, (d > 0) of the number of operatives per unit time. (ii) Counterterrorism measures targeted specifically at arresting, assassinations, kidnapping and as well as efforts to disrupt communications, financial asset and threat to force the operatives into long-term inactivity. We modeled this as a

deduction of a fixed fraction, say, $(\alpha > 0)$ from leaders, $(\omega > 0)$ from the foot-soldiers and $(\sigma > 0)$ from the recruiters' classes, per unit time. Mathematically:

$$Y' = pX(t) - dY(t) - \alpha Y(t) X' = rS(t) + cZ(t) - (d + \omega)X(t) Z' = cS(t) + pY(t) - (d + \sigma)Z(t)$$
(2)

However, in this paper we are considering the impact of boosting the internal personnel drain factor $(i.e.d \gg 0)$ through the "Carrot" approach (positive incentive measures) and using the conventional CT measures the "Stick" approach (negative incentive measures), on the terrorist strength and sustainability.

2.2.1`Assumptions of the Model

The model is predicated on the following assumptions:

- (i.) On the average, the numerical strength of a terrorist organization at any interval of time is nourished by the combined processes of promotion (p>0), recruitment, (r>0) and commission (c>0) and drained through desertion (d>0)and CT measures $(\alpha, \omega, \sigma>0)$ targeted on the respective classes of operatives.
- (ii.) Once a foot-soldier X(t) is promoted to leadership Y(t) cadre, a new foot-soldier is recruited as a replacement. If in some organizations such recruitment isn't automatic, then the current model is still valid for these organizations as long as (r > p)
- (iii.) Similarly, once a leader Y(t) is assigned a recruiters' Z(t) responsibility, a new foot-soldier X(t) is promoted to a leadership position as a replacement. Also if in some organizations such promotion isn't automatic, then the current model is still valid for these organizations as long as (p > c). In any case the drain due to promotion is marginal because foot-soldiers are far more numerous than leaders even in relatively "top heavy" organizations.
- (iv.) The ingenuity of recruiters Z(t), like a catalytic enzyme, to speed-up the recruitment and transformation process of the recruits into radical foot-soldiers without being directly targeted by a viable CT measures, allows terrorist organizations to grow through a sustained support-base that is constantly supplying new personnel for the possible conversion into radical foot-soldiers[8].
- (v.) As implicit assumptions, the model also assumes:
 - (a) A state of stable gradual change, such that the effect of one terrorist or interdiction process is smoothed. This should be acceptable in all cases where the terrorist organization is not very small and thus changes are not very stochastic.
 - (b) That an organization's growth is constrained only by the available manpower and factors such as money or weapons do not impose an independent constraint.

(c) That the growth in foot-soldiers is not constrained by the availability of potential recruits since in most ideologically driven terrorism, willing recruits are always plentiful.

Combining equation (1), equation (2) and the above array of assumptions, we arrived at our mathematical model represented by the system of three ODEs:

$$S(t) = mY(t) + X(t) + Z(t) Y' = pX(t) - (d + \alpha)Y(t) X' = rmY(t) + (r - d - \omega)X(t) + (r + c)Z(t) Z' = (mc + p)Y(t) + cX(t) + (c - d - \sigma)Z(t) (m > 1; 0 < p, r, c, d, \alpha < 1)$$
(3a)

The system of ODEs (3) will be subjected to the initial conditions

$$Y(t_0) = Y_0; X(t_0) = X_0; and Z(t_0)$$

= Z_0 (3b)

Table 1: Model Variables, Parameters and their Description

Variable/	•	
Parameters	Description	Values
Y(0)	Number of terrorist leaders at time (t) .	5.0
X(0)	Number of terrorist foot-soldiers at time (t) .	120
Z(0)	Number of terrorist recruiter at time (t) .	2.0
m > 1	Weight of the leadership (over other operatives),	10.0
0	Proportion of foot-soldier promoted to a leader, 3%	0.03
0 < r < 1	Proportion of foot-soldiers recruited, 12%	0.12
0 < <i>c</i> < 1	Proportion of commissioned recruiters 1%	0.01
$0 < d_0 < 1$	Proportion of internal personnel drain, 3%	0.03
$0 < d_1 < 1$	Proportion of operative interdicted by "carrots", 20%	0.2
$0 < \alpha < 1$	Proportion of leaders interdicted by "sticks" 20%	0.2
0 < ω<1	Proportion of foot-soldiers interdicted by "sticks" 20%	0.2
0 < <i>σ</i> <1	Proportion of Recruiters interdicted by "sticks" 20%	0.2

III. ANALYSIS OF THE MODEL

Before studying the solution paths of equation (3a), in this section, we first analyze equation (3a) to find the terrorist's equilibrium points and their local stability. To determine the equilibrium points, let the dynamic of each population in equation (3a) equals zero, (i.e.Y' = X' = Z' = 0):

$$0 = pX(t) - (d + \alpha)Y(t)$$

$$0 = rmY(t) + (r - d - \omega)X(t) + (r + c)Z(t)$$

$$0 = (mc + p)Y(t) + cX(t) + (c - d - \sigma)Z(t)$$
(4a)

At this state, we assume that there is no terrorist activity, and thus, no interdiction process except the internal personnel drain. This proves that $E_0 = (Y(t), X(t), Z(t)) = (0, 0, 0)$ is the terrorist-free equilibrium state (TFES) of the model. To determine the stability of the equilibrium state by the linearization stability method, we have that

$$f_{1} = pX(t) - (d + \alpha)Y(t)$$

$$f_{2} = rmY(t) + (r - d - \omega)X(t) + (r + c)Z(t)$$

$$f_{3} = (mc + p)Y(t) + cX(t) + (c - d - \sigma)Z(t)$$
(4b)

$$\begin{aligned} &\frac{\partial f_1}{\partial Y} = -(d+\alpha); \ \frac{\partial f_1}{\partial X} = p; \ \frac{\partial f_1}{\partial Z} = 0; \\ &\frac{\partial f_2}{\partial Y} = rm; \ \frac{\partial f_2}{\partial X} = (r-d-\omega); \ \frac{\partial f_2}{\partial Z} = (r+c); \\ &\frac{\partial f_3}{\partial Y} = (mc+p); \ \frac{\partial f_2}{\partial X} = c; \ \frac{\partial f_2}{\partial Z} = (c-d-\sigma); \end{aligned}$$

Therefore the Jacobian matrix is

$$J = \begin{bmatrix} -(d+\alpha) & p & 0\\ rm & (r-d-\omega) & (r+c)\\ (mc+p) & c & (c-d-\sigma) \end{bmatrix}$$
(4b)

Therefore,

$$|J - \lambda I|$$

$$= \begin{vmatrix} -(d+\alpha) - \lambda & p & 0 \\ rm & (r-d-\omega) - \lambda & (r+c) \\ (mc+p) & c & (c-d-\sigma) - \lambda \end{vmatrix}$$

$$= 0$$
(4c)

Solving the expression (4c) for λ , we also have

$$\lambda_{1} = \frac{1}{6} \begin{pmatrix} 48\alpha r\sigma + 48\alpha \omega c - 48\alpha \omega \sigma + \\ 108mpc^{2} + 84\alpha c^{2} + \\ 108rp^{2} + 108cp^{2} + \cdots \end{pmatrix}$$

$$\lambda_{2} = \frac{-1}{12} \begin{pmatrix} 48\alpha r\sigma + 48\alpha \omega c - 48\alpha \omega \sigma + \\ 108mpc^{2} + 84\alpha c^{2} + \\ 108rp^{2} + 108cp^{2} + \cdots \end{pmatrix}$$

$$\lambda_{3} = \frac{-1}{12} \begin{pmatrix} 48\alpha r\sigma + 48\alpha \omega c - 48\alpha \omega \sigma + \\ 108mpc^{2} + 84\alpha c^{2} + \\ 108rp^{2} + 108cp^{2} + \cdots \end{pmatrix}$$

By Routh-Hurwitz criteria, $\lambda_1 > 0$, $\lambda_2 < 0$, and $\lambda_3 < 0$, the terrorist free equilibrium state $E_0 = (0, 0, 0)$ is marginally stable; a common characteristics of most terrorist organizations.

3.1. Numerical Simulation

Given the experimental data set on table-1, we analyze our model (3), to study the evolutional dynamics of the terrorist strength when the state offer positive incentive to enhance internal personnel drain in the organization, and as well as employ leadership decapitation strategy simultaneously. To achieve this aim, we first analyze the impact of positive incentives on the terrorist strength without decapitation, and then with decapitation. Mathematically, the solution path of each sub-model corresponds to the question of whether at some future time the value of the variables Y(t), X(t) and Z(t) would reach zero, i.e.

$$\lim_{t_0 \to t} Y(t) = \lim_{t_0 \to t} X(t) = \lim_{t_0 \to t} Z(t) = 0$$

$$\Rightarrow \lim_{t_0 \to t} S(t) = 0$$
(5)

For experimental analysis, we consider a notional terrorist group with initial strength of 5 leaders, 120 foot-soldiers and 2 recruiters. Research findings and journalistic accounts of how

Al-Qaida and its affiliates developed and grows over the last decades indicate that; constraint by the heterogeneity of the terrain, the asymmetric nature of the warfare, the insufficient credible intelligence and as well as limited human/material resources at the disposal of CT operatives and decision makers; a notional terrorist organization under the influence of an orchestrated CT measure can at-most be nourished at an average rate of 3% per leaders, 12% per foot-soldiers and 1% per recruiters annually. This dynamics is hampered by internal personnel drained factor of 3% and CT measure of 20% per class of operatives[8],[10],[23],[42],[66]. Thus, in this analysis, we are hypothesizing a positive incentive measures (Carrot) that would boost the internal personnel drain factor (d) to at-least 20%.

3.1.1. Terrorist Evolution under Positive Incentive Strategy

The first condition analyzes the strength at inherent internal drain $(d_0 = 0.03, \alpha = \omega = \sigma = 0)$, represented by the blue curve. This denotes when there is no positive incentive from government. While the second condition $(d_1 = 0.2, \alpha = \omega = \sigma = 0)$ represented by the red curve, denote when there is viable positive incentives from government to incite high internal personnel drain. Under these parameters, the solution of equation 3 is given by figure 1 (a,b,c,d), below.

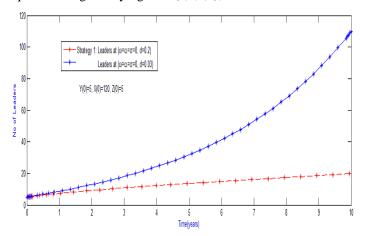


Figure 1(a): Evolution of Leaders underpositive incentives

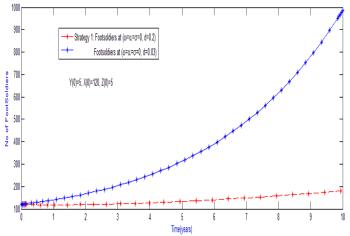


Figure 1(b): Evolution of Foot-soldiers underpositive incentives

The figure 1(a), (b) above and 1(c) below shows that, without positive incentives, the population of the leaders, foot-soldiers and recruiters, grows exponentially from its initial 5 leaders, 120 foot-soldiers and 2 recruiters to a corresponding 110 leaders, 1000 foot-soldiers and 87 recruiters within a period of ten years. This gives an increase of 10.5 leaders, 88 foot-soldiers and 8.5 recruiters annually. While with positive incentives, the population of the operatives declined linearly to 20 leaders, 180 foot-soldiers, and 16 recruiters, giving an increase of only 1.5 leaders, 6.0 foot-soldiers and 1.4 recruiters annually. Thus, the positive incentives alone have the propensity of interdicting an average of 9.0 leaders, 82 foot-soldiers and 7.1 recruiters annually.

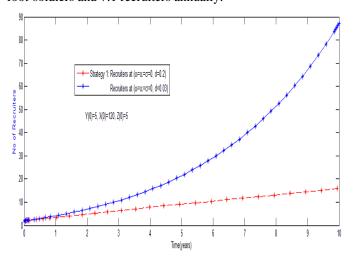


Figure 1(c): Evolution of Recruiters under Positive incentives

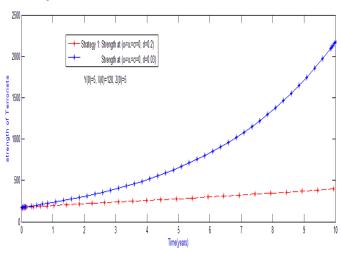


Figure 1(d):Evolution of Strength under positive incentives

Similarly, figure 1(d) above summarized strength evolution of the organization: without positive incentive measures, the overall terrorist strength grow exponentially from initial 172 operatives to 2171 operatives (an increase of 199.9 operatives annually). While with positive incentives, the overall strength of the terrorist declined to 395 operatives (an average interdiction of 177.6 operatives annually).

3.1.2 Terrorist Evolution under the Combination of Positive Incentives plus Decapitation Approaches

The first condition analyzes the evolution of terrorists' strength at the inherent internal personnel drain factorplus decapitation CT measures $(d_0 = 0.03, \alpha = 0.2, \omega = 0, \sigma = 0.2)$; represented by the blue curve. This denotes when the state employed decapitation strategy only (targeted at leaders and recruiters simultaneously). The second condition analyzes the terrorists' evolution at high internal personnel drain factorplus "decapitation" $(d_1 = 0.2, \alpha = 0.2; \omega = 0, \sigma = 0.2)$, represented by the red curve. This denotes when the state offered positive incentives to incite high internal defection of operative and also employed decapitation for recalcitrant operatives. Under these parameters, the solution of equation 3 is given by figure 2 (a,b,c,d), below.

The figure 2(a), (b) and (c) below shows that, with only decapitation measures, the population of the organization grows from the initial 5 leaders, 120 foot-soldiers and 2 recruiters to a corresponding 50 leaders, 688.5 foot-soldiers and 32.6 recruiters within a period of ten years. This gives an increase of 4.5 leaders, 56.85 foot-soldiers and 3.06 recruiters annually. While a combination of positive incentives with the decapitation, the population of the operatives dropped to 9.1 leaders, 125.8 foot-soldiers and 5.9 recruiters. This gives an average interdiction of 4.09 leaders, 56.27 foot-soldiers and 2.67 recruiters annually.

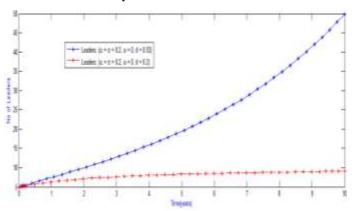


Figure 2(a): Evolution of leaders underdecapitation strategy

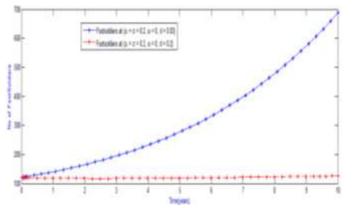


Figure 2(b): Evolution of Foot-soldiers under decapitation strategy

Similarly, figure 2(d) below compares the evolution of the organization's strength under positive incentives and decapitation approaches. With the decapitation measures, the overall terrorists' strength appreciates from initial 172 operatives to 1220 operatives (an increase of 104.8 operatives annually). While with a combination of decapitation approach with positive incentives, the strength declined appreciatively to 222.8 operatives, giving an interdiction of 99.7 operatives annually.

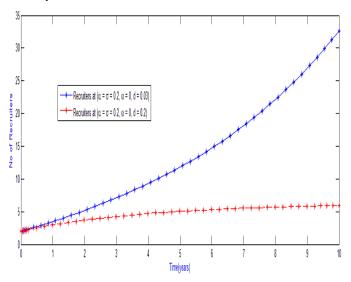


Figure 2(c): Evolution of Recruiter under decapitation strategy

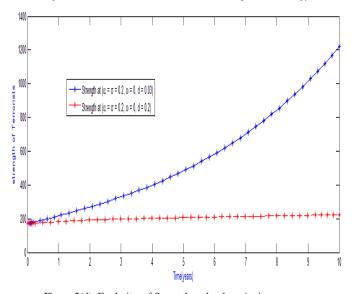


Figure 2(d): Evolution of Strength under decapitation strategy

3.1.3 The Variability of Terrorists' Evolution with CT Strategy

Table 2 below, shows a summary of the variability of the organization's growth and interdiction rates with respect to "decapitation" and "carrots" approaches.

Table 2: Characteristics of Decapitation and Carrot Approaches

Variable	Leaders	Foot- soldiers	Recruiters	STRENGTH
Initial Population	5.0	120	2.0	172
Growth at d_0 (No Carrot)	110	1000	87	2171
Growth at d_1 , (Carrots)	20	180	16	395
Growth at d_0 + Decapitation	50	688.5	32.6	1220
Growth at d_1 + Decapitation ,	9.1	125.8	5.9	222.8
% Growth at No Carrot	210%	73.3%	425%	116.2%
% Growth at Carrot only	30%	5%	70%	13%
% Growth at Decapitation only	90%	47.4%	153%	60.9%
% Growth at Decapitation+ carrots	8.2%	0.48%	19.5%	2.95%
% Interdiction at Carrots only	81.8%	82%	81.6%1	81.8%
%Interdiction at Decapitation only	54.5%	31.2%	62.5%	43.8%
% Interdiction at Carrot + Decapitation	91.7%	87.4%	93.2%	89.7%

The table 2 above shows that the lowest terrorists' strength is obtains under the combine "carrots and decapitation" regime, with 222.8 operatives, representing an annual growth rate of 2.95% operatives only. This is followed by the individual "carrots" and "decapitation" regimes with 395 and 1220 operatives respectively, representing an annual interdiction rate of 13% and 60.9% operatives respectively. Correspondingly, the combine "carrots and decapitation" regime also yield the lowest operatives' population, with 9.1 leaders, 125.8 foot-soldiers and 5.9 recruiters; representing an annual growth rate of 8.2% leaders, 0.48% foot-soldiers and 19.5% recruiters. This is followed by the individual "carrots" regime with the terrorists' population of 20 leaders, 180 footsoldiers and 16 recruiters; representing an annual growth rate of 30% leaders, 5% foot-soldiers and 70% recruiters. While the "decapitations" approach alone yielded operatives' population of 50 leaders, 688.5 foot-soldiers and 32.6 recruiters. This represents an annual interdiction rate of 190% leaders, 47.4% foot-soldiers and 153% recruiters. Thus, the "carrots" approach has the propensity to interdict 60% more leaders and 51% more recruiters annually, than the "decapitation" approach.

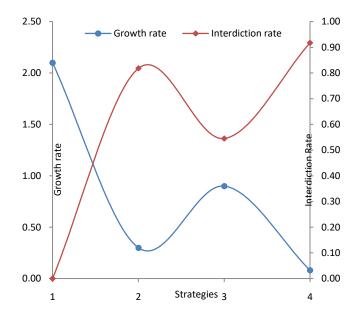


Figure 3(a): Variability of Leaders with Strategies

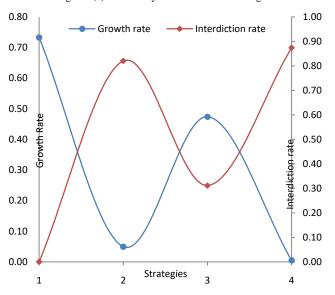


Figure 3(b): Variability of Foot-soldiers with Strategies

The figure 3 above shows the variability of the terrorist annual growth rate under the combine "decapitation" and "carrots" approaches. The figure 3(a) shows that the annual growth rates of the leaders are lowest under the combine "decapitation and carrots" regime, with 8% leaders; representing the highest annual interdiction rate of 92%. This is follows by the "carrots" regime with annual growth rate of 30%leaders; representing an annual interdiction rate of 82%. The figure 3(b) shows that the annual growth rate of the footsoldiers is also lowest under combined "decapitation" and "carrots" regime, with 0.0% foot-soldiers; representing an annual interdiction rate of 87%. The "carrots" regime also yielded the second lowest foot-soldiers' growth rate of 5% annually; representing an annual interdiction rate of 82%.

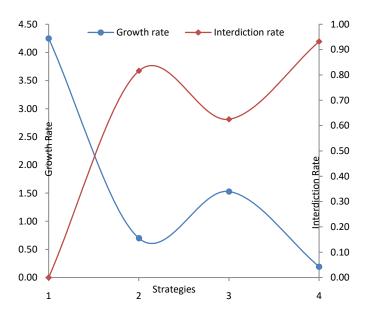


Figure 3(c): Variability of Recruiter with Strategies

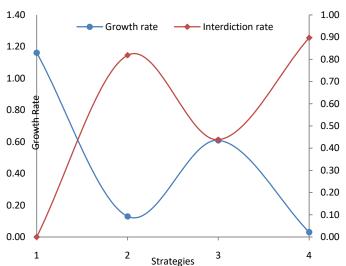


Figure 3(d): Variability of Terrorists' Strength with Strategies

Similarly, figure 3(c) above shows that the growth rate of the recruiters is lowest under the combined "decapitation" and "carrots" regime, with 20% recruiters; representing the highest interdiction rate of 93% recruiters annually. In figure 3(d), the combined "decapitation" and "carrots" approach also yielded the lowest system growth rate of 3% operatives; representing the highest system interdiction rate of 90% operatives annually. This is follows by the "carrots" regime with a system growth rate of 13% operatives; representing an annual system interdiction rate of 82%. Comparatively, the individual "carrots" approach alone has the propensity of interdicting 60% more leaders, and 51% more recruiters annually than the "decapitation" approach.

3.1.4 Organizational Resilience

To predict whether the understudied counterterrorism strategies or approaches would be sufficient to degrade or

drive the organization to a long term in activity, suffice us to analyze the organization's resilience index of each of the strategy. The "organizational resilience" or the capability of a terrorist group to rebound back and still engage in viable terrorist activity notwithstanding the high degree of decapitation of its operatives is a striking characteristic of most terrorist organization. It is the organization's recovery potential after destruction, which emphasizes the ability of recover and develops in a state of uncertainty, discontinuity, and emergency rather than a resistance to unexpected event. Lengnick-Hall et al[45] defined organizational resilience as a firm's ability to effectively absorb, develop situation-specific responses to, and ultimately engage in transformative activities to capitalize on disruptive surprises that potentially threaten organization survival. The terrorist organizational resilience index is scaled using a minimum-maximum rescaling method:

nomalized
$$(X_i)$$

$$= \left| \frac{x_i - X_{min}}{X_{max} - X_{min}} \right|$$
(6)

Where X_i , (i = 1,2,3) denote thein dividual values of strength at each strategy, X_{min} and X_{max} denote the minimum and maximum values of X in each strategy, respectively. The min-max rescaling method adapted from the University of Notre Dame Global Adaptation Index, compare values for resilience and vulnerability, and then place the terrorist in one of four quadrants, illustrated in Figure 5, below. Using the data on table 2 on equation 6 above, the organization's resilience indices are. "*Carrots*" $X_C = 0.111$ (high vulnerability); "Decapitation", $X_D = 0.52$, (high resilience) "Carrots + Decapitation", $X_{C+D} = 0.025$ vulnerability). This implies that, the organization under "carrot" approach has only 11.1% probability to rebound back to active terrorist activities after major drain in its strength, while under the "decapitation" approach the organization has 52% probability of rebounding back to active terrorist activities. And under the combined approach, the organization has only 2.5% probability of rebounding back to active terrorism after major threat on its strength and sustainability.

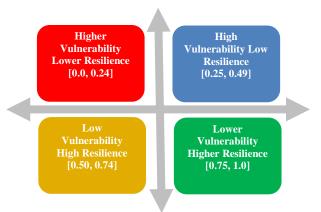


Figure 5: Resilience and Vulnerability Ouadrants

IV. RESULT OF THE ANALYSIS

Given the high bureaucratization structure and the levels of popular supports most ethno-religious ideology driven terrorist organization enjoy from its host community, the study shows that the decapitation strategy may not be strategically viable and sufficient to degradea given terrorist organization. This is evidence in the high growth rates of the organization under this strategy. Thus, given the CT strategy of targeting the leaders and recruiters simultaneously, the analysis shows that only in combination with a viable "carrots" that the "decapitation" approach can yield a high interdiction rate of 89.7% operatives annually. But in the absence of a viable "carrots", the operatives population would grows exponentially at an annual rate of 100% leaders, 57.4% foot-soldier and 163% recruiters; giving an overall system interdiction rate of only 43.8% annually. This may probably account for the resiliency of the organization under the decapitation strategy.

Tactically, the "carrots" approach is more efficient than the "decapitation" approach; with the propensity of interdicting an average of 60% more leaders and 51% more recruiters annually, than the "decapitation". This implies that the inherent hindrances and difficulties of identifying and accessing the clandestine terrorist recruiters' and their recruitment institutions for proper targeting in the dominant military offensive CT strategy, can be address by the introduction of a viable "carrots". Significantly, though the "carrots" approach may be cost intensive and challenging, yet it has the propensity yielding the lowest leadership growth rate of 30%, lowest foot-soldiers' growth rate of 15%, and the overall terrorist strength to 13% operatives annually, than decapitation approach. This gives the "carrot" approach an edge over the "decapitation" approaches; beside its tendency to rejuvenate confidence and moral supports of the local population. The analysis also shows that, the simultaneous targeting of the leaders and recruiters can also result in an appreciable decline in the population of the foot-soldiers. This implies that every proactive step toward identifying and attacking the recruiters' class simultaneously would lead to a considerable decline in the foot-soldiers' growth rate.

Since the evolution of the operatives' population is responsive to "carrots", the study suggest that the high cost of combining "carrots" and decapitation strategy and the inherent tendency to abuse and misconstrued the objectives of the "carrots" by most terrorist leaders, can be address by selective offering of the "carrots" to only the recruiters' and foot-soldiers classes. Finally, though yielded high interdiction rates; considering the characteristic nature of the evolutionary curves (figure 2); and the high resilience factor of the organization under the "decapitation" strategy, such strategy is sufficient to guarantee optimum decline in the overall strength and sustainability of the organization.

In fact in today's ethno-religious ideology driven terrorism, the strategy of targeting terrorist leadership can be morally provocative and contemptuous, hence vulnerable to high degree of blowback action and incitement of Herostratos syndrome in the susceptible youth population. As most of the terrorist leaders are revered political or traditional or religious leaders, highly referenced and supported by locals, hence, any attack on them is perceived as not only a serious threat but an attempt on the collective aspiration and survival of the community or religion. Thus, any attack on these leaders usually creates disaffection, rancor, disharmony and tension among the local population, who sees the cause of these leaders as representing the collective aspiration and interest of the community. This help to fuel and heighten animosity and as well as fanning the amber of enmity between the government and the locals; thus, often times causing uproar and blowback actions often experienced after every major arrest or assassination of a terrorist leader. Blowback actions could provoke "Herostratos syndrome" among the youth population, which help to pump-up more support and recruitment to the terrorist organization[4],[7],[70].

Unlike the hypothetical "carrots and sticks" approach, the "carrots" and decapitation" approach may have greater utility in an effort to fight a given terrorist group only in the short term. In the longer term, such an approach can be counterproductive if the terrorists perceived it to be unfairly applied for an ulterior motive. Therefore, the ultimate aim for combining "carrots" approach should be as part of a more comprehensive effort to give the locals more legitimacy and concession in order to gain their cooperation and supports, which is needed (i) for the initiation of an all-inclusive, proactive and efficient "in-group policing" mechanism between the security agencies and the local population; (ii) to take away the local populations' supports from a given terrorist group (i.e. de-legitimization of terrorism and its propagandas among the local population), and (iii) to encourage and facilitate sufficient and credible intelligence gathering for smart targeting of recalcitrant terrorists' location[30]. Such effort must demonstrate with all sincerity that the state's motive for offering the "carrots" is unprejudiced and must create the conditions that convince the terrorists to believe that such motive is for their own advantage.

V. CONCLUSION

We develop a differential equation model for countering terrorism; where the state's counterterrorism approaches include the use of positive (carrot) incentives to incite high internal personnel drain within the organization and the use of military offensive against terrorist leadership in an effort to weaken the overall terrorists' strength. While the "carrots" approach is best at eliciting behaviour compliance, cooperation and moral supports required for building a formidable "in-policing" mechanism with the locals; gather sufficient and credible intelligence for smart targeting of terrorist location, and de-legitimize terrorism and its

propagandas among the locals. The decapitation strategy is solely aimed at weaken the overall terrorists' strength, there by forcing the organization to a long term inactivity, or possibly rendered it grossly incapacitated to organize a successful terror attack even in the distance future. The model which demonstrates the "strength-strategy" relationships of decapitation and positive incentives strategies for influencing the behavior of intelligence and adaptable adversaries, also bemoaned the insufficiency of leadership decapitation to result in significant drain in the overall terrorists' strength. While it also recognizes the power of positive incentives toward inciting high internal personnel drain within the operatives' population.

In a swift response to the frequent holocaust of terrorist attacks in the aftermath of 9/11, most liberal democratic states resolved to deploy their armed forces toward fighting terrorism. Notably Israel against successive Palestinian groups, Britain in Northern Ireland from 1969-98, Nigerian state against Boko Haram from 2008- date or indeed America and its allies against Al-Qaeda and affiliated groups from 2001-date[31]; with predominant decapitation of terrorist leadership. Although decapitation may continue to produce short-term effects on the operational capacity of some terrorist organization, however, the approach is unlikely to result in significant impact on the organization overall strength and sustainability. The "organizational resilience" or the capability of a terrorist group to still engage in terrorist activity notwithstanding the high degree of decapitation of its leadership cadre is relatively high with such approach.

In view of the bureaucratic structure of today's terrorist organizations, organizational management expert observed that leadership decapitation is unlikely to result in the dissolution of groups that are highly bureaucratized or that have high levels of popular support because leaders matter less in such circumstances. Bureaucratized terrorist groups are diversified, have a clear division of administrative responsibilities and functions, follow rules and procedures, and are thus more likely to withstand the sudden removal of a leader or leaders. Bureaucracies have universalistic rules that are critical when delegating responsibilities within an organization [53]. Michael Crozier argues that "impersonal rules" create a kind of self-enforcing equilibrium by delimiting in great detail the function of every individual within the organization[53]. These results prescribe the behavior to be followed in all possible events. Bureaucracies contain specific features that increase organizational stability and efficiency, making them more resilient to leadership attacks. First, they are characterized by organizational diversification, and they maintain a clear delineation between duties[50],[51]. As organizations become larger, more complex, and more specialized, they are likely to develop diversified functions that increase their stability

Popular support is essential to a terrorist group's ability to maintain organizational strength and capacity following an attack on its leadership. Organizations with high levels of support have an easier time acquiring the resources necessary to carry out effective campaigns. Scholars including Roger Petersen[59] and Scott Atran[62] have argued that effective insurgencies require vast amounts of popular support. Militant organizations have recognized the importance of local community support[57]. Groups with public support are likely to be seen as legitimate by their communities, further increasing their strength and effectiveness. As a result, counterinsurgency strategy that focused on winning the "hearts and minds" of the locals, reduces the desire for rebellion. Underlying this approach is the idea that by identifying and addressing the grievances (causes of insurgency), counterinsurgents will gain local support that could otherwise help insurgents[17],[67]. Because ethnoreligious ideology driven and separatist organizations often represent the views and beliefs of the community from which they emerge, they often have higher levels of communal support than ethno-political ideology driven organizations.

Terrorist groups that provide social services to their local communities may experience increased public support, and thus a boost to their public image. Popular support contributes to terrorist group resolve and stability in many ways. It allows the group to recruit, raise money, provide critical resources, and ensure its ability to operate as a covert organization, encourage more violent behavior, and maintain political and ideological relevance. Thus, supporters can provide useful information and be a source of recruits. Petersen suggests that the provision of resources, information, and recruits by the local community is fundamental to understanding the success of rebellions[59],[62]. The study also reiterates the morale vulnerability of the "decapitation" approach to incite "Herostratos" syndrome and a blowback effect in the susceptible youth population.

Furthermore, in today's ethno-religious ideology driven terrorist organization, the use of decapitation strategy in counterterrorism can be problematic, morally provocative and contemptuous; hence vulnerable to high degree of blowback action and incitement of Herostratos syndrome in the susceptible youth population. As most of the terrorist leaders are revered political or traditional or religious leaders, highly referenced and supported by the locals, hence, any attack on them is seen as not only a serious threat but an attempt on the collective aspiration and survival of the community or religion. Thus, any attack on such leaders usually creates disaffection, rancor, disharmony and tension among the local population, who sees the cause of these leaders as representing the collective aspiration and interest of the community. This help to fuel and heighten animosity and as well as fanning the amber of enmity between the government and the locals; thus, often times causing uproar and blowback actions often experienced after every major arrest or assassination of a terrorist leader. Blowback actions could provoke "Herostratos syndrome" among the youth population, which help to pump-up more support and recruitment to the terrorist organization[4],[7],[70]. One has only to look at the post-2001 "War on Terror" and the criticisms expressed by foreign governments, members of Congress, human rights activists, journalists, and academics regarding the treatment of detainees at Guantanamo Bay, Cuba; the use of unmanned aerial vehicles (UAVs) in air-strikes in Pakistan; and the civilian casualties caused by US and North Atlantic Treaty Organization (NATO) military operations in Afghanistan, and multi-national joint military operations in North-eastern Nigeria[31].

Similarly, in an effort to marshal resources toward building coalition against international terrorism, countries and international bodies heavily rely on strategies that are intended to change the incentives of both the terrorists and their supporters. Key among these Strategies is the "carrots and sticks" approaches. Other nations also capitalized on this strategy to incite or encourage a high internal personnel defection rate in local militia and terrorist group; with the hidden objectives of gather sufficient and credible intelligence for smart targeting of terrorist location, taking the local populations' supports away from the terrorists and as well as building a credible "in-policing" mechanism with the locals. Taking the local populations supports away from the terrorist's organization will creates serious havoc for the terrorists and its cause receives less attention and therefore becomes delegitimized.

The "carrots" which is provided to motivate voluntary compliance in either the terrorist operatives or the local population with implied preferences or explicit direction; is rooted in the belief that reward would have a more positive or long-lasting effect, and would be viewed either as a due compensation for desired performance or as unprejudiced payment for services rendered. Unfortunately, with the inherent psychological inclination of abused and misconstrued of the objectives of the "carrots"; the perception from the recipients may be of a bribe tainting both the donor and recipient or of equally distasteful payment from a master to a servant. If "carrots" is offered arrogantly by a strong state without proper cognizance of the recipients who may see the "carrots" as bribes or arm-twisting, the results may be short termed and counter-productive in the longer term. To harvest the desired utility of the "carrots", the state must be unprejudiced and proportional in its application of rewards and punishments, which in turn should be aligned with an unprejudiced motive. Such motive must be just in the eyes of the locals, the terrorists and other stakeholders, not simply in the eyes of the donor state. In such a situation, the reward for compliance would not be seen as a bribe or a payment for ransom, but simply as an expected consequence of working for a just cause. To make the states unprejudiced motive more compelling and attractive to the terrorists and the locals, their different values and perceptions must be taken into consideration.

The most obvious positive incentives (*carrots*) employed in recent times includes economic assistance; lifting of sanctions; award of educational scholarships; direct monetary payments; and supply of food and services, and other refugee

assistance. All of these privileges can be rewarding, but they may also be more limited and temporary in value than anticipated. For instance, if the terrorists or people felt the sanctions were improperly imposed, then lifting them will be perceived as a correction of a wrong, rather than a gracious act, and thus, may not motivate the desired actions. The use of food or financial assistance to directly influence or elicit compliance or cooperative behaviour from the terrorist or the locals may allow the state to be by-passed to some degree. The altruistic motives of the state may be genuine, but if the hungry recipients perceive that the food aid or financial assistance is given simply to bring a coalition together, or elicit behaviour compliance for selfish intension, then the "carrots" would appear simply as a bribe rather than a just reward. If the terrorist is apocalyptic in objectives or ethnoreligious ideology driven, then the appearance of such a "carrot" may not alter their views towards the state at all. In fact, they may receive the "carrots" with an increased contempt for the state. The ultimate challenge is to make the state's motives unprejudiced in the eyes of the terrorists, the local, and stakeholders whose physical and moral assistance is needed in the war against terror.

However, even if the "carrots" is perceived as a bribe, there may be some utility in a simple bribe or use of force, but the returns could only be temporary and potentially short term as observed previously. If peradventure the state's motive does not fall in line with the motives of the recipients, the intermediate position is to at least make a compelling case that the state motive is just and unprejudiced, and to use the "carrots and sticks" approaches as commensurable rewards or punishments to support that motive. To complement the application of positive and negative incentives or to avoid the challenges of their misconception, some of the root causes of the terrorism and the peoples' disgruntlement must be properly identified and addressed; or the potential "carrots" given as a redress of the root causes of the terrorism and the peoples' disgruntlement. When the terrorist is extremely advantaged (i.e., with a low unit cost of attack, taking into account any increases in unit cost due to negative incentives, or a high asset valuation), the "carrots" cannot incite meaningful internal defection or deter the terrorist from attacking, because the asset is too valuable to the terrorist relative to the cost of attacking. However, a less advantaged terrorist can be successfully deterred by weakening its strength through high internal defection incited by a viable "carrots". If the terrorist violates the trust associated with positive incentives, which may occur when compliance cannot be adequately monitored and enforced, the state is assumed to eliminate the use of positive incentives. But if the terrorist is sufficiently disadvantaged, it is already deterred from attacking, therefore, offering of positive incentives may be of no economic significance.

We also demonstrate a complex relationship between leadership decapitation and positive incentives, which we believe has not been quantitatively appraised and modeled. In particular, we find that positive incentives can play a role when decapitations have an intermediate impact on the terrorist. When decapitations have too little impact, the terrorist is too advantaged for affordable levels of positive incentives to be effective at achieving deterrence. This accords with Pruitt[58], observations that negotiations with terrorists are likely to be successful only when the goals of the terrorist group are relatively modest or pragmatic. By contrast, apocalyptic terrorists may be so "advantaged" by their fanatic devotion to their cause that the benefits of any positive incentives the state might be willing to offer would be pale by comparison. From this perspective, "appeasement" is not necessarily always undesirable, but will be so when the state has underestimated the goals and or devotion of the adversary. Conversely, when negative incentives have a sufficiently large impact, the terrorist is already disadvantaged and deterred by negative incentives alone, and there is no need for the state to offer positive incentives

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