Magnetohydrodynamic Convective Oscillatory Flow with Thermal Radiation and Soret Effect

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***Abstract:-* This paper investigates governing equations for unsteady Magnetohydrodynamic convective oscillatory flow with thermal radiation and soret effect through a semi-infinite vertical permeable moving plate with; one stationary and the other non-stationary embedded in a porous medium via heat absorption. The velocity was maintained at a constant value and the flow was subject to a transverse magnetic field. The computed values obtained from the analytical solution for the velocity temperature, concentration field, skin friction coefficient, Peclet number, Schmidt and Reynolds number with their amplitude and phase are presented graphically. After a suitable transformation of the governing partial differential equations was transformed to ordinary differential equation. These equations were solved analytically by using two-term harmonic and non-harmonic functions. The velocity decreases with increase in Peclet number, the magnetic field parameter whereas reverse trend is seen with increasing the heat generation parameter, radiation parameter, porous parameters, soret number and Grashof numbers. The temperature decreases as the values of Peclet number increases and the reverse is seen by increasing the values of thermal radiation parameter, heat source parameter, the concentration increases as the values of the Peclet number increases, despite increase in radiation and Grashof number, heat transfer remains the same.**

I. INTRODUCTION

In recent years, a great deal of interest has been established in the area of heat and mass transfer. The study of unsteady magneto hydrodynamic convection flow in a porous medium has received much attention in recent time owing to diverse new technological developments in modern metallurgical and metal-working processes. Engineers are continuously taking the task to improve the efficiency of the MHD energy systems. Convective heat and mass transfer in porous media has also been a subject of great interest for the last few decades due to its application in various disciplines, such as geophysical, solar power collectors, cooling of electronic systems, chemical catalytic reactors, thermal insulating engineering, high-performance building insulating modeling of packed sphere beds etc. Singh et al. (1989) investigated the effect of permeability variation on free convective flow in a porous medium bounded by a vertical porous wall. Shreekanth et al. (2006) have investigated the effects of time-dependent permeability variation in the free convection flow past a vertical porous wall placed in a porous medium. Raptis et al. (2004) analyzed the steady MHD asymmetric flow of an electrically conducting fluid past a semi-infinite stationary plate in the presence of radiation. Bakier (2001) investigated the effect of radiation on mixed convection from a vertical plate in a saturated porous medium. Makinde and Mhone (2005) have investigated the effect of magnetic field and thermal radiation on oscillatory flow in a channel filled with porous medium.

Cookey et al. (2003) examined the effect of viscous dissipation and thermal radiation on unsteady MHD free convection flow past an infinite heated vertical surface in a porous medium with time-dependent suction.

Nowadays, the studies of fluid flows through porous medium become interesting and inevitable in case of extraction of crude oil from the pores of rocks. The hydromagnetic convection with heat and mass transfer in porous medium has been studied due to its importance in the design of MHD generators and accelerators in geophysics, in design of underground water energy storage system, soil-sciences, astrophysics, nuclear power reactors and so on. Magnetohydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases.

The present work, we make an attempt to study an unsteady oscillatory hydromagnetic mixed convection flow through a porous medium with periodic temperature variation. The work deals with realistic values for the leading parameters. There are four governing equations which includes: continuity, momentum, energy, and concentration equation. We analyzed an unsteady flow of viscous incompressible and electrically conducting fluid through a porous medium with thermal radiation and heat source. The porous medium was bounded by two infinite vertical plates at a distance d apart. One of the plate is stationary and the other non-stationary. The Cartesian system is chosen such that the other plate is neglected to a uniform velocity u and the same constant suction velocity v'. A homogenous magnetic field of strength B_o was applied normal to the plane of the plates. By the infinite nature of the plates, the fluid properties except pressure became function of y and t only.

II. MATHEMATICAL MODELLING

We analyze an unsteady flow of a viscous, incompressible and electrically conducting fluid through a porous medium with thermal radiation and heat source. The Statistical packaged used in this work is Mathematica package (Latex). The porous medium is bounded by two infinite vertical plates at distance d apart. One of the plate is stationary and the other non-stationary. The Cartesian system is chosen such that the other plate is neglected to a uniform velocity U and the same constant suction velocity V'. A homogeneous magnetic field of strength B_o is applied normal to the plane of the plates. By the infinite nature of the plates, the fluid properties except pressure becomes function of y and t'. The original equation was generated first by Israel-Cookey C., Amos Emeka and Nwaigwe C., (2017).

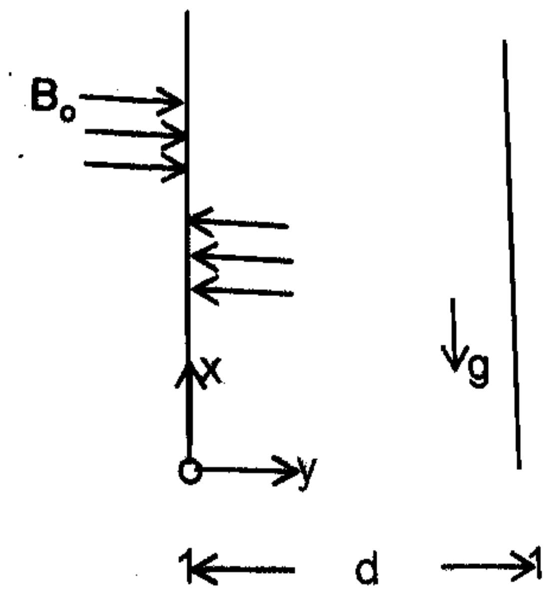

Figure 1: A schematic of the flow region

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial u'}{\partial x'} + \mathrm{v}\frac{\partial^2 u'}{\partial y'^2} - \frac{\mathrm{v}}{k'}u' - \sigma\frac{B_0^2}{\rho}u' + g\beta(T' - T_O') + g\beta(C' - C_O') \quad (2)$$

$$\frac{\partial \mathrm{T}'}{\partial t'} + \mathrm{v}\frac{\partial \mathrm{T}'}{\partial y^1} = \frac{K}{Pc_p}\frac{\partial^2 \mathrm{T}'}{\partial y'^2} - \frac{1}{Pc_p}\frac{\partial \mathrm{q}'}{\partial y'} + \frac{Q'}{Pc_p}(T' - T_o') \quad (3)$$

$$\frac{\partial \mathrm{c}'}{\partial t'} + V'\frac{\partial \mathrm{c}'}{\partial y'} = \mathrm{D}\frac{\partial^2 \mathrm{C}'}{\partial y'^2} + \mathrm{D}_1\frac{\partial^2 \mathrm{T}'}{\partial y'^2} \quad (4)$$

Therefore, the governing equation in dimensionless form for tabulating such flow takes the form,

Subject to:

$u' = 0$, $T' = T_0$, $C' = C_0'$ at $y' = 0$

$u' = U$, $T' = T_w'$, $C' = C_w' = y' = d$

Where p' is the pressure, ρ is the density of the fluid t^1 is the time, u^1 is the velocity, k is the permeability of the porous medium, g is the acceleration due to gravity, υ is the kinematic viscosity of the fluid, β_T and β_c are the coefficient of thermal and concentration expansion respectively. C_p is the specific heat at constant pressure, q^1 is the radiation flux, D is the mass diffusivity, D_1 is the soret coefficient, Q' is the heat source.

Israel-Cookey C., Amos Emeka and Nwaigwe C., (2017) worked on this governing equations without soret coefficient term.

Equation (1), (2), (3) and (4) are continuity equation, momentum equation, energy equation and the concentration equation.

We non-dimensionalized equation (2 – 4) using the dimensionless variables below;

$x = \frac{x'}{d}$, $y = \frac{y'}{d}$, $u = \frac{u'}{U}$, $t = t'\frac{V}{d}$, $\frac{\partial q'}{\partial y'} = 4\alpha^2(T' - T_0')$

$T = \frac{T' - T_0'}{T_w' - T_w'}$, $C = \frac{C' - C_0'}{C_w' - C_0'}$, $M = Bod\left(\frac{\delta}{\mu}\right)^{1/2}$, $P = \frac{P'}{P \cup V}$, $v' = v$

$R_e = \frac{Vd}{\mathrm{v}}$, $G_r = \frac{g\beta o(T_w' - T_0')}{UV^2}$, $G_m = \frac{g\beta o(C_w' - C_0')}{UV^2}$, $S_c = \frac{\mathrm{v}}{D}$,

$S_o = D_1\frac{(T_w' - T_0')}{\nabla C_w^1 - C_0^1}$, (4a)

$S = \frac{Q'd}{\mathcal{P}c_pV}$ $N = 2 \propto \frac{d}{k^{1/2}}$, $K = \frac{K'V}{\mathrm{v}d}$, $\mathcal{P} = \frac{\mu}{\mathrm{v}}$, $P_e = \frac{\mathcal{P}c_pVd}{k}$

Substituting equation (4a) into equation (2). Then, the dimensionless form of equation (2) is;

$$\frac{\partial \mathrm{u}'}{\partial t'} + v'\frac{\partial v'}{\partial y'} = -\frac{1}{p}\frac{\partial \mathrm{p}'}{\partial x'} + \sigma\frac{\partial^2 \mathrm{u}'}{\partial y^{12}} - \frac{\upsilon}{K'}u' - \sigma\frac{B_0^2 u'}{\rho} + g\beta(T' - T_o') + g\beta(C' - C_0)$$

$$\frac{V \cup}{d}\frac{\partial u}{\partial t} + \frac{V \cup}{d}\frac{\partial u}{\partial y} = \frac{1(\rho \cup V)}{\rho}\frac{\partial p}{d\,\partial x} + \frac{\upsilon \cup}{d^2}\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon \cup Vu}{\upsilon kd} - \frac{\sigma B_0^2 \cup u}{\rho} + g\beta(T_w - T_o)T + g\beta_c(C_w - C_o)$$

Multiplying by $\frac{d}{V\cup}$ to obtain

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\upsilon}{Vd}\frac{\partial^2 u}{\partial y^2} - \frac{1}{k}u - \frac{\sigma B_o d}{\rho V}u + \frac{g\beta d}{\cup V}(T_w - T_o)T + \frac{g\beta d}{\cup V}(C_w - C_o)$$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - \frac{1}{k}u - \sigma\frac{B_o \upsilon d}{\mu V}u + \frac{g\beta d}{\cup V}(T_w - T_o)T + \frac{g\beta d}{\cup V}(C_w - C_o)$$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_o \upsilon d}{\mu V}.\frac{Vdu}{\upsilon} + \frac{g\beta d}{\cup V}.\frac{Vdu}{\upsilon}(T_w - T_o) + \frac{g\beta d}{\cup V}.\frac{Vdu}{\upsilon}(C_w - C_o)$$

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\frac{\partial^2 u}{\partial y^2}-\frac{1}{k}u-\frac{M^2}{Re}u+\frac{g\beta\upsilon Re}{\cup V^2}(T_w'-T_o)T\frac{g\beta dRe}{\cup V^2}(C_w-C_o)$$

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\frac{\partial^2 u}{\partial y^2}-\frac{1}{k}u-\frac{M^2 u}{Re}+GrReT+G_mReC$$

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\frac{\partial^2 u}{\partial y^2}-\frac{1}{k}u-\frac{M^2}{Re}u+GrReT+G_mReC \quad (5)$$

Equation (5) is the modified momentum equation in its dimensionless form.

Again the dimensionless form of equation (3) is

$$\frac{\partial T'}{\partial t'}+V'\frac{\partial T'}{\partial y'}=\frac{k}{\mathcal{P}c_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\mathcal{P}c_p}\frac{\partial q'}{\partial y'}+\frac{Q'}{\mathcal{P}c_p}(T'-T_0')$$

$$\frac{\partial T(T_W'-T_0')}{\partial\left(\frac{td}{v}\right)}+v\frac{\partial T(T_W'-T_O')}{\partial(yd)}=\frac{k}{\mathcal{P}c_p}\frac{\partial^2 T(T_W'-T_O')}{\partial(yd)^2}-\frac{1}{\mathcal{P}c_p}4\alpha^2(T'-T_0')+\frac{S\mathcal{P}c_p v}{\mathcal{P}c_p d}T(T_W'-T_W')$$

$$\frac{v}{d}\frac{\partial T(T_W'-T_O')}{\partial t}+\frac{v}{d}\frac{\partial T}{\partial y}(T_W'-T_o')=\frac{k}{\mathcal{P}c_p d^2}\frac{\partial^2 T}{\partial y^2}(T_W'-T_O')-\frac{1}{\mathcal{P}c_p}4\alpha^2(T'-T_o')+\frac{Sv}{d}T(T_W'-T_O')$$

Multiply through by $\frac{1}{T_W'-T_O'}$

$$\frac{v}{d}\frac{\partial T}{\partial t}+\frac{v}{d}\frac{\partial T}{\partial y}=\frac{k}{\mathcal{P}c_p d^2}\frac{\partial^2 T}{\partial y^2}-\frac{4\alpha^2(T_W'-T_O')}{\mathcal{P}c_p v(T_W'-T_O')}+\frac{Sv}{d}T$$

Multiply through by $\frac{d}{v}$

$$\frac{\partial T}{\partial t}+\frac{\partial T}{\partial y}=\frac{k}{\mathcal{P}c_p vd}\frac{\partial^2 T}{\partial y^2}-\frac{4d\alpha^2(T'-T_O')}{\mathcal{P}c_p(T_W'-T_O')}+ST$$

$$\frac{\partial T}{\partial t}+\frac{\partial T}{\partial y}=\frac{1}{Pe}\frac{\partial^2 T}{\partial y^2}-\frac{(2\alpha)^2 d^2}{k}\chi\frac{k}{\mathcal{P}c_P vd}T+ST$$

$$\frac{\partial \mathrm{T}}{\partial t}+\frac{\partial \mathrm{T}}{\partial y}=\frac{1}{P_e}\frac{\partial^2 \mathrm{T}}{\partial y^2}-\frac{N^2}{P_e}\mathrm{T}+\mathrm{ST} \quad (6)$$

Equation (6) is the modified energy equation in its dimensionless form.

And the dimensionless form of equation (4) is;

$$\frac{\partial c'}{\partial t'}+v'\frac{\partial c'}{\partial y'}=\mathrm{D}\frac{\partial^2 \mathrm{C}'}{\partial y'^2}+\mathrm{D}_1\frac{\partial^2 \mathrm{T}'}{\partial y'^2}$$

$$\mathrm{V}\frac{(cw'-C_0{}')}{d}\frac{\partial \mathrm{c}}{\partial t}+\mathrm{V}\frac{(cw'-C_0{}')}{d}\frac{\partial \mathrm{c}}{\partial y}=\mathrm{D}\frac{(cw-C_0)}{d}\frac{\partial^2 \mathrm{c}}{\partial y^2}+\mathrm{D}_1\frac{(Tw-T_0)}{d^2}\frac{\partial^2 \mathrm{T}}{\partial y^2}$$

Multiply by $\frac{d}{V(cw'-C_0{}')}$ to obtain

$$\frac{\partial c}{\partial t}+\frac{\partial c}{\partial y}=\frac{D}{Vd}\frac{\partial^2 \mathrm{c}}{\partial y^2}+\frac{D_1}{Vd}\frac{(Tw'-T_0{}')}{(cw'-C_0)}\frac{\partial^2 \mathrm{T}}{\partial y^2}$$

$$\frac{\partial c}{\partial t}+\frac{\partial c}{\partial y}=\frac{1}{R_eS_e}\frac{\partial^2 \mathrm{c}}{\partial y^2}+\frac{S_o}{R_e}\frac{\partial^2 \mathrm{T}}{\partial y^2} \quad (7)$$

Equation (7) is the modified concentration coefficient in its dimensionless form.

We also non – dimensionalized the boundary conditions as follows;

The mathematical formulation is now complete and the governing equations are

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\frac{\partial^2 u}{\partial y^2}-\frac{1}{k}u-\frac{M^2}{Re}u+GrReT+G_mReC \quad (8)$$

$$\frac{\partial \mathrm{T}}{\partial t}+\frac{\partial \mathrm{T}}{\partial y}=\frac{1}{P_e}\frac{\partial^2 \mathrm{T}}{\partial y^2}-\frac{N^2}{P_e}\mathrm{T}+\mathrm{ST} \quad (9)$$

$$\frac{\partial c}{\partial t}+\frac{\partial c}{\partial y}=\frac{1}{R_eS_e}\frac{\partial^2 \mathrm{c}}{\partial y^2}+\frac{S_o}{R_e}\frac{\partial^2 \mathrm{T}}{\partial y^2} \quad (10)$$

Subject to:

$$u_0=0,\ \partial_0=0,\ \emptyset_0=0 \quad \text{at } y=0 \quad (11)$$

$$u_0=1,\ \partial_0=1,\ \emptyset_0=1 \text{ at } y=1 \quad (12)$$

Method of Solution of equation (5), (6) and (7)

We assume a solution of the form

$$u(y,t)=u_o(y)e^{iwt} \quad (13a)$$

$$C(y,t)=\emptyset_o(y)e^{iwt} \quad (13b)$$

$$\mathrm{T}(y,t)=\theta_0 e^{iwt} \quad (13c)$$

$$-\frac{\partial \mathrm{p}}{\partial u}=Pe^{iwt} \quad (13d)$$

From equation (6)

$$\frac{\partial \mathrm{T}}{\partial t}+\frac{\partial \mathrm{T}}{\partial y}=\frac{1}{P_e}\frac{\partial^2 \mathrm{T}}{\partial y^2}-\frac{N^2}{P_e}\mathrm{T}+\mathrm{ST}$$

$u(y,t)=u_o(y)e^{iwt},\ \ T(y,t)=\emptyset_o(y)e^{iwt},C(y,t)=\emptyset_o(y)e^{iwt},\ -\frac{\partial \mathrm{p}}{\partial u}=Pe^{iwt}$

$$\emptyset_{o_{iw}}e^{iwt}+e^{iwt}\frac{\partial\theta_o}{\partial y}=\frac{1}{P_e}e^{iwt}\frac{\partial\emptyset_o}{\partial y}+Se^{iwt}\emptyset_o-\frac{N^2}{P_e}\emptyset_o e^{iwt}$$

i.e $\quad iw\,\emptyset_o+\frac{\partial\theta_o}{\partial y}=\frac{1}{P_e}\frac{\partial^2\theta_o}{\partial y^2}+\mathrm{S}\emptyset_o-\frac{N^2}{P_e}\emptyset_o$

$$\frac{1}{P_e}\frac{\partial^2\emptyset_o}{\partial y^2}-\frac{\partial\emptyset_o}{\partial y}+\left(s-\frac{N^2}{P_e}-iw\,\emptyset_o=0\right)$$

$$\frac{\partial^2\emptyset_o}{\partial y^2}-Pe\frac{\partial\emptyset_o}{\partial y}+(Pes-N^e-iw\,P_e)\,\emptyset_o=0 \quad (14)$$

$\emptyset''_o P_e - \emptyset'_o + (Pes - N^e - iw P_e)\emptyset_o = 0$

$$\frac{dc}{dt} + \frac{dc}{dy} = \frac{1}{R_e}\left(\frac{1}{Sc}\frac{d^2c}{dy^2} + S_o\frac{d^2T}{dy^2}\right)$$

$$iw + e^{iwt}\frac{d\emptyset_o}{\partial y} = \frac{1}{R_e}\left(\frac{1}{Sc}e^{iwt}\frac{d^2\emptyset_o}{dy^2} + S_o e^{iwt}\frac{d^2\emptyset_o}{dy^2}\right)$$

$$iw\,\emptyset_o + \frac{d\emptyset_o}{\partial y} = \frac{1}{R_e}\left(\frac{1}{Sc}\frac{d^2\emptyset_o}{dy^2} + S_o\frac{d^2\emptyset_o}{dy^2}\right)$$

$$\frac{1}{R_eS_e}\frac{d^2\emptyset_o}{dy^2} - \frac{d\emptyset_o}{\partial y} - iw\,\emptyset_o = -\frac{S_o}{R_e}\frac{d^2\emptyset_o}{dy^2}$$

$$\frac{d^2\emptyset_o}{dy^2} - R_eS_e\frac{d\emptyset_o}{\partial y} - iw\,R_eS_e\,\emptyset_o = -S_oS_c\frac{d^2\theta_o}{dy^2} \tag{15}$$

From equation (5), we have:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{R_e}\frac{\partial^2 u}{\partial y^2} - \frac{1}{k}u - \frac{M^2}{R_e}u + G_r\text{Re}\,T + G_m\text{Re}C$$

$$\frac{\partial}{\partial t}(u_o e^{iwt}) + \frac{\partial u_0}{\partial t}e^{iwt} = \text{P}\,e^{iwt} + \frac{1}{R_e}\frac{\partial^2 u_0}{K} - \frac{M^2}{R_e}e^{iwt}u_o + \text{GrRe}^{iwt}\,\emptyset_o$$

$$iw\,e^{iwt}u_o + e^{iwt}\frac{\partial u_0}{\partial y} = -e^{iwt}\,\text{P} - \frac{e^{iwt}}{R_e}\frac{\partial^2 u}{\partial y^2} - \frac{e^{iwt}u_o}{K} - \frac{M^2}{R_e}e^{iwt}$$

$$iw\,u_o + \frac{\partial u_0}{\partial y} = \text{P} + \frac{1}{R_e}\frac{\partial^2 u_0}{K} - \frac{u-}{k} - \frac{M^2}{R_e}u_o + \text{Gr Re}\,\emptyset_o + G_m R_e\emptyset_o$$

$$iw\,R_e\,u_o + R_e\frac{\partial u_0}{\partial y} = R_e\,\text{P} + \frac{\partial^2 u_-}{K\partial y^2} - \frac{R_e}{k}u_o - M^2u_o + R_e\,\text{P} - \text{Gr Re}^2\,\emptyset_o + G_m R_{e^2}\emptyset_o$$

$$\frac{\partial^2 u_o}{\partial y^2} - R_e\frac{\partial u_0}{\partial y} - iw\,R_e\,u_o - -\frac{R_e}{k}u_o - M^2u_o = -R_e\,\text{P} - \text{Gr Re}^2\,\emptyset_o + G_m R_{e^2}\emptyset_o$$

$$\frac{\partial^2 u_o}{\partial y^2} - R_e\frac{\partial u_0}{\partial y} - \left(\frac{R_e}{k} + M^2 + iw\,R_e\right)u_o = -R_e\,\text{P} - \text{Gr Re}^2\,\emptyset_o - G_m R_{e^2} \tag{16}$$

Subject to:

$u_o = 0,\ \partial_0 = 0,\ \emptyset_o = 0\ \ at\ y = 0$

$u_o = 1,\ \partial_0 = 1,\ \emptyset_o = 1\ \ at\ y = 1$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{R_e}\frac{\partial^2 u}{\partial y^2} - \frac{1}{k}u - \frac{M^2u}{R_e} + G_rR_eT + G_rR_eC \tag{17}$$

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial y} = \frac{1}{P_e}\frac{\partial^2 T}{\partial y^2} - \frac{N^2T}{P_e} + ST \tag{18}$$

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial y} = \frac{1}{R_eS_c}\frac{\partial^2 C}{\partial y^2} + \frac{S_o}{R_e}\frac{\partial^2 T}{\partial y^2} \tag{19}$$

$u = 0\ ,\ T = 0\ ,\ C = O\ ,\ at\ y = 0\quad u = 1\ ,\ T = 1\ ,\ C = 1\ ,$

$at\ y = 1$ (20)

Adopting

u(y,t) $=u_o(y)e^{i\omega t}$ (21)

$T(y,t) = \theta_o(y)e^{i\omega t}$ (22)

C(y,t)$=\emptyset_o(y)e^{i\omega t}$ (23)

Substituting equation (14), (15) and (16) respectively into equation

(10), (11), (12) and (13) gives:

$$u_o'' - R_eu_o' - \left(\frac{R_e}{k} + M^2 + i\omega R_e\right)u_o = -R_eP - G_rR_e^2\theta_o - G_mR_e^2\theta_o \tag{24}$$

$$\theta_o'' - P_e\theta_o' - (N^2 - P_eS + i\omega P_e)\theta_o = 0 \tag{25}$$

$$\emptyset_o'' - R_eS_c\emptyset_o' - i\omega R_eS_c\emptyset_o = -S_oS_c\theta_O'' \tag{26}$$

Subject to ; $u_o = 0\ ,\ \theta_o = 0\ ,\ \emptyset_o = 0$
$at\ \ y = 0\ ,u_o = 1\ ,\ \emptyset_o = 1\ ,\ \theta_o = 1\ at\ y = 1$ (26a)

We first solve equation (18): Let $\theta_o = e^{\alpha y}$ $be\ a\ solution.$

Then the auxillary system is of;

$\alpha^2 - P_e\alpha - A_1 = 0$

Where
$A_1 = N^2 - P_eS + i\omega P_e,$
$and\ \alpha\ having\ two\ roots\ as\ (\alpha_1, \alpha_2) such\ that;$

$$\alpha_1 = \frac{P_e \pm \sqrt{Pe^2 - 4A}}{2}$$

$$\alpha_2 = \frac{P_e - \sqrt{Pe^2 + 4A}}{2}$$

and $\theta_o = C_1e^{\alpha_1 y} + C_2e^{\alpha_2 y}$ (27)

on application of the boundary condition on (3.20) we obtain:

$C_1 = \frac{1}{e^{\alpha_2 y} - e^{\alpha_1 y}}$ $and\ C_2 = -\frac{1}{e^{\alpha_2 y} - e^{\alpha_1 y}}$

$consequently\ the\ solution\ to\ \theta_o\ after$
$simplification\ becomes$:

$$\frac{e^{\alpha_2 y} - e^{\alpha_1 y}}{e^{\alpha_2} - e^{\alpha_1}} \tag{28}$$

We next solve equation (3.19) following similar procedure as in the case of equation

(3.18) we obtain the complimentary function as

$$\emptyset_{oc} = C_3 e^{\alpha_3 y} + C_4 e^{\alpha_4 y} \quad (29)$$

Where C_3 *and* C_4 are constants to be determined from the boundary conditions and

$$\alpha_3 = \frac{ReSc + \sqrt{(ReSc)^2 + 4i\omega ReSc}}{2}$$

$$\alpha_4 = \frac{ReSc - \sqrt{(ReSc)^2 + 4i\omega ReSc}}{2}$$

On application of the boundary conditions on equation (3.23) we obtain, the complete equation (3.19) as nonhomogeneous.

To consider the nonhomogeneous part we write the particular solution (θ_{op}) as

$$\theta_{op} = C_5 \alpha_2 e^{\alpha_2 y} + C_6 \alpha_1 e^{\alpha_1 y}$$

Hence, $\theta'_{op} = C_5 \alpha_2 e^{\alpha_2 y} + C_6 \alpha_1 e^{\alpha_1 y} \quad (30)$

$$\theta''_{op} = C_5 \alpha_2^2 e^{\alpha_2 y} + C_6 e^{\alpha_1 y}$$

On substituting (24) into (19) we have

$$C_5 \alpha_2^2 e^{\alpha_2 y} - C_6 \alpha_1^2 e^{\alpha_1 y} - Re\alpha Sc(C_5 \alpha_2 e^{\alpha_2 y} + C_6 \alpha_1 e^{\alpha_1 y}) - i\omega R_e S_c (C_5 e^{\alpha_2 y} + C_6 e^{\alpha_1 y})$$

$$= -S_o S_C \left(\frac{\alpha^2_{2e}{}^{\alpha_2 y} - \alpha_1^2 e^{\alpha_1 y}}{e^{\alpha_2} - e^{\alpha_1}} \right)$$

Hence, $A_2 = C_5 = \frac{S_o S_c \alpha_1^2}{\alpha_1^2 - Q\alpha_1 - T}$

$$A_3 = C_6 = -\frac{S_o S_c \alpha_2^2}{\alpha_2^2 - Q\alpha_2 - T}$$

Q= ReSc ,T= i$\omega R_e S_C$ with $\emptyset_{o=\emptyset_{oc}+\emptyset_{op}}$ $_{,we\ hav}$

$$\emptyset_o = C_3 e^{\alpha_3 y} + C_4 e^{\alpha_4 y} - \frac{S_o S_c \alpha_2^2}{\alpha_2^2 - Q\alpha_2 - T} + \frac{S_o S_c \alpha_1^2}{\alpha_1^2 - Q\alpha_1 - T} \quad (31)$$

On application of the boundary conditions to (25) we obtain

$$A_4 = c_4 = \frac{1 - A_3(e^{\alpha_3} - e^{\alpha}{}_2) - A_2(e^{\alpha_1} - e^{\alpha_3})}{e^{\alpha_4} - e^{\alpha_3}}$$

$$c_4 = -\left[\frac{1 - A_3(e^{\alpha_3} - e^{\alpha_2}) - A_2(e^{\alpha_1} - e^{\alpha_3})}{e^{\alpha_4} - e^{\alpha_3}} - A_3 + A_2 \right] e^{\alpha_3 y}$$

$+$

$$\left\{ 1 - A_3(e^{\alpha_3} - e^{\alpha_2 y}) - A_{2(e^{\alpha_1} - e^{\alpha_3})} \right\} e^{\alpha_4 y} - A_3 e^{\alpha_2 y} + A_3 e^{\alpha y} + A_{2e^{\alpha_1 y}}$$

Simplifying further, we have

$$\emptyset_o = (A_3 - A_2 - A_4) e^{\alpha_3 y} + A_4 e^{\alpha_3 y} - A_3 e^{\alpha_2 y} + A_2 e^{\alpha_1 y} \quad (32)$$

We next consider equation (17)

$$u''_o - R_e u'_o - \left(\frac{R_e}{k} + M^2 + iwR_e \right) u_0 = -R_e P - G_r R_e^2 \theta_o - G_m R_e^2 \emptyset_o$$

$$u''_o - R_e u'_o - T_o u_o = -R_e P - G_r R_e^2 \theta_o - G_m R_e^2 \emptyset_o$$

Where $T_{1=} \frac{R_e}{k} + M^2 + i\omega R_e$

The solution to the auxillary equation is

$$u_{oc} = E_1 e^{\alpha_5 y} + E_1 e^{\alpha_6 y} \quad (33)$$

$$\alpha_6 = \frac{R_e - \sqrt{R_e{}^2 + 4T_1}}{2}$$

For the particular integral to (17) we set the RHS as:

$$-R_e P - G_r R_e^2 \left[\frac{e^{\alpha_2 y} - e^{\alpha_2 y}}{e^{\alpha_2} - e^{\alpha_1}} \right] — G_m R_e^2 A_{7e^{\alpha_3 y}} + A_8 [(A_{3-} A_2 - A_4) e^{\alpha_3 y} + A_4 e^{\alpha_4 y} - A_3 e^{\alpha_2 y} + A_2 e^{\alpha_1 y}]$$

$$so\ that\ u_{op} = B + A_5 e^{\alpha_2 y} + A_6 e^{\alpha_1 y} + A_8 e^{\alpha_4 y}$$

$$u'_{op} = A_5 \alpha_2 e^{\alpha_2 y} + A_6 \alpha_1 e^{\alpha_1 y} + A_7 \alpha_3 e^{\alpha_3 y} + A_8 \alpha_4 e^{\alpha_4 y} \quad (34)$$

$$u''_{op} = A_5 \alpha_2^2 e^{\alpha_2 y} + A_6 \alpha_1^2 e^{\alpha_1 y} + A_7 \alpha_3^2 e^{\alpha_3 y} + A_8 \alpha_4^2 e^{\alpha_4 y}$$

Substituting (28) into equation (17) we obtain: $A_5 \alpha_2^2 e^{\alpha_2 y} + A_6 \alpha_1^2 e^{\alpha_1 y} + A_7 \alpha_3^2 e^{\alpha_3 y} + A_8 \alpha_4^2 e^{\alpha_4 y} - R_{e[A_5 \alpha_2 e^{\alpha_2 y} + A_6 \alpha_1 e^{\alpha_1 y} + A_7 \alpha_3 e^{\alpha_3 y} + A_8 \alpha_4 e^{\alpha_4 y}]} - T_4 [B + A_5 e^{\alpha_2 y} + A_6 e^{\alpha_1 y} + A_8 e^{\alpha_4 y}] = - - R_e P - G_r R_e \frac{e^{\alpha_2 y}}{e^{\alpha_2} - e^{\alpha_1}} + \frac{G_m R_e^2 e^{\alpha_1 y}}{e^{\alpha_2} - e^{\alpha_1}} - G_m Re^2 A_3 e^{\alpha_1 y} {}_- G_m Re^2 (A_3 - A_{2-} A_4) e^{\alpha_3 y} - G_m Re^2 A_4 e^{\alpha_4 y}$

Consequently, after simplification, we have:

$$A_5 = \frac{G_r R_e^2 T_2 + G_m R_e^2 A_3}{\alpha_2^2 - R_e \alpha_2 - T_1}, A_6 = \frac{G_r R_e^2 T_2 + G_m R_e^2 A_2}{\alpha_1^2 - R_e \alpha_1 - T_1}, B = \frac{R_e p}{T_1}, A_7 = \frac{G_m R_e^2 (A_3 - A_2 - A_4)}{\alpha_3^2 - R_e \alpha_3 - T_1},$$

$$A_8 = \frac{G_m Re^2 A_4}{\alpha_4^2 - R_e \alpha_4 - T_1}$$

Hence, the general solution is:

$$u_o = u_{oc} + u_{op} = E_1 \alpha_5 y + E_2 e^{\alpha_6 y} + \frac{R_e p}{D} + \left(\frac{G_m Re^2 A_3 - G_r Re^2 T_2}{\alpha_2^2 - Re\alpha_2 - T_1} \right) e^{\alpha_2 y} + \left(\frac{G_r R_e^2 T(\ \ )_2 + G_m R_e^2 A_2}{\alpha_1^2 - R_e \alpha_1 - T_1} \right) e^{\alpha_1 y} - \frac{G_m Re^2 (A_3 - A_2 - A_4) e^{\alpha_3 y}}{\alpha_3^2 - Re\alpha_3 - T_1} - \frac{G_m R_e^2 A_4 e\alpha^{\alpha_4 y}}{\alpha_4^2 - Re\alpha_4 - T_1} \quad (35)$$

Applying the boundary conditions on equation (26a) we obtain:

$$E_1 = -E_2 - B - A_5 - A_6 + A_7 + A_8$$

$E_2 = 1 - \left(A_8 + A_7 - A_6 - A_5 - \frac{R_e p}{T_1}\right) e^{\alpha_5} - B - A_5 e^{\alpha_2} - A_6 e^{\alpha_1} + A_7 e^{\alpha_3} + A_8 e^{\alpha_4}$

So that

$$u_o = (A_9 e^{\alpha_5} + A_{10}) e^{\alpha_5 y} + \left(\frac{1 - A_9 e^{\alpha_5} - A_{10}}{e^{\alpha_6} - e^{\alpha_5}}\right) e^{\alpha_6 y} + B + A_5 e^{\alpha_2 y} + A_6 e^{\alpha_1 y} - A_7 e^{\alpha_3 y} \quad (36)$$

Where $A_9 = B + A_5 e^{\alpha_2} + A_6 e^{\alpha_1} - A_7 e^{\alpha_3} - A_8 e^{\alpha_4}$

$A_{10} = A_8 + A_7 - A_6 - A_5 - B$

Based on equation (14-16) our general solution is

$$U = \left[(A_9 e^{\alpha_5} + A_{10}) e^{\alpha_5 y} + \left(\frac{1 - A_9 e^{\alpha_5} - A_{10}}{e^{\alpha_6} - e^{\alpha_5}}\right) e^{\alpha_6 y} + B + A_5 e^{\alpha_2 y} + A_6 e^{\alpha_1 y} - A_7 e^{\alpha_3 y}\right] e^{i\omega t} \quad (37)$$

$$\theta = \left[\frac{e^{\alpha_2 y} - e^{\alpha_1 y}}{e^{\alpha_2} - e^{\alpha_1}}\right] e^{i\omega t} \quad (38)$$

$$\emptyset = [(A_3 - A_2 - A_4) e^{\alpha_3 y} + A_4 e^{\alpha_4 y} - A_3 e^{\alpha_2 y} + A_2 e^{\alpha_1 y}] e^{i\omega t} \quad (39)$$

$cf = \left(\frac{\partial u_o}{\partial y}\right) |_{y=o} = \alpha_5 (A_9{}^{\alpha_5} + A_{10}) + \alpha_6 (1 - A_9 e^{\alpha_5} + A_{10}) + \alpha_2 A_5 + \alpha_1 A_6 - \alpha_3 A_7$

$Nu = \left(\frac{\partial \theta_o}{\partial y}\right) |_{y=o} = \left[\frac{\alpha_2 e^{\alpha_2 y} - \alpha_1 e^{\alpha_1 y}}{e^{\alpha_2} - e^{\alpha_1}}\right]$

$Sh = \left(\frac{\partial \emptyset_o}{\partial y}\right) |_{y=o} = \frac{\alpha_2 - \alpha_1}{e^{\alpha_2} - e^{\alpha_1}}$

III. DISCUSSION OF RESULTS

The significance of the problem under consideration is discussed with realistic values for the leading parameter.

Figure 1 shows the influence of soret on the temperature profile. It is observed that increase in Soret increases the temperature. Physically, increase in Soret shows in diffusive motion which leads to increase in temperature.

The effect of pedlet number on the temperature is shown in figure 2. The profile indicates that increase in the pedlet number decreases the temperature. Physically this indicates increased dominance of advection over diffusion. In between the parameter values, the effect is more pronounced at the center of the flow region.

The influence of radiation on the temperature is shown in figure 3. It is shown that increase in the radiation parameter decreases the temperature. Physically this is an indication that the momentum boundary layer thickness is decreased as a result of the increase in radiation.

Figure 4 illustrates the effect of the Reynolds number on the concentration. It is observed that increase in the Reynolds number increases the concentration. This is because of increase in the ratio of inertial forces to viscous forces.

The effect of the pedlet number on the fluid concentration is shown in figure 5. We noticed that increase in Pedlet number increases the concentration of the fluid. This effect is more pronounced at higher values of the pedlet number, which is an indication of advectively dominated flow.

Figure 6 shows the effect of the Schmidt number in the concentration. The profile indicates that increase in the Schmidt number leads to increase in the concentration as a result of decreased momentum diffusivity as compared to man diffusivity.

Figure 7 shows that the fluid concentration decreases with increase in radiation.

The influence of frequency of oscillation is shown in figure 8. Increase in the frequency of oscillation leads to decrease in concentration. This indicates that intra molecular forces in the fluid medium are weakened by increased frequency of oscillation. The effect is more pronounced in higher oscillations.

The effect of radiation on the heat transfer is shown in figure 9. It is observed that despite increase in the radiation parameter, and the Grashof number, the heat transfer remains the same. This observation is the same as shown in figure 10.

We have studied MHD convective oscillatory flow with thermal radiation and soret effects. To gain insight into the problem we have used realistic values on the physical parameters such as magnetic fluid M, Schmidt number S_c, frequency of oscillation w, Grashof number G_r, Soret parameter S_{o1}, modified Grashof number G_m. To present the results in graphical form and subsequently their interpretation. Figure 11 shows the effect of the magnetic field on the velocity profile. It is observed that increase in the magnetic field decreases the velocity. Physically this is as a result of the action of the Lorentz force which retards the flow.

In figure 12. The effect of the frequency of oscillation is presented. The profile reveals that increase in the frequency of oscillation reduces the flow velocity.

The effect of the Schmidt number on the velocity is presented in figure 13. Increasing Schmidt number leads to increase in the velocity. The profile reveals that very little increase leads to significant increase in the velocity.

Physically this is true, increase in S_c means decrease of molecular diffusion.

Figure 14 shows the effect of surds in the velocity profile. The profile shows that increase in the soret parameter increases the velocity. This is true physically since increase in soret increases the driving force for man diffusion.

It is observed in figure 15 that increase in porosity leads to increase in the velocity. The increase is not very significant in this study.

The effect of the graph of linear velocity is shown in figure 16. It is noted in the profile that increase in the graph of number shows the velocity. This illustrates the fact that their thermal buoyancy force enhances velocity. Similar effects are noted in the modified graph of number (fig 17) where the increase in velocity is more significant.

Figure 4.18 shows the effect of the radian parameter on the velocity. It reveals that increase in radian is not very significant as increase in velocity. Though the parameter values used here are close to each other.

Fig 19 shows the effect of the Reynolds molecules in the velocity. It is observed that the velocity increases in a result of increase in the Reynolds number.

Temperature Profile

Figure 20 shows the effect of the graph of number on the skin friction. The profile shows that increasing the air leads to increase in the skin friction. The increase in the soret causes a linking relationship to be established.

The effect of the magnetic field on the skin friction is shown in fig 21. It is observed that increase in the magnetic field leads to a reduction in the skin friction.

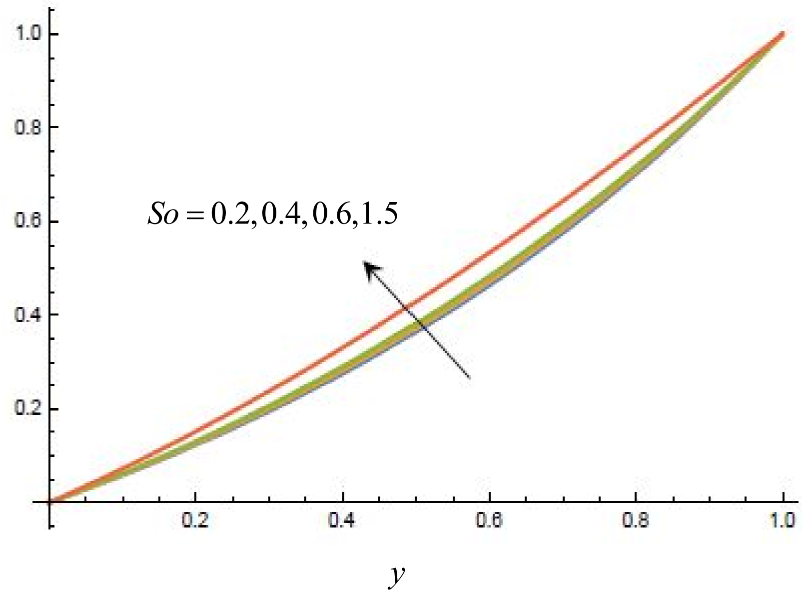

Figure 1: Effect of Soret on temperature for $\omega = 0.50, t = 0.1, Pe = 1.0, N^2 = 0.5, S = 1.0$

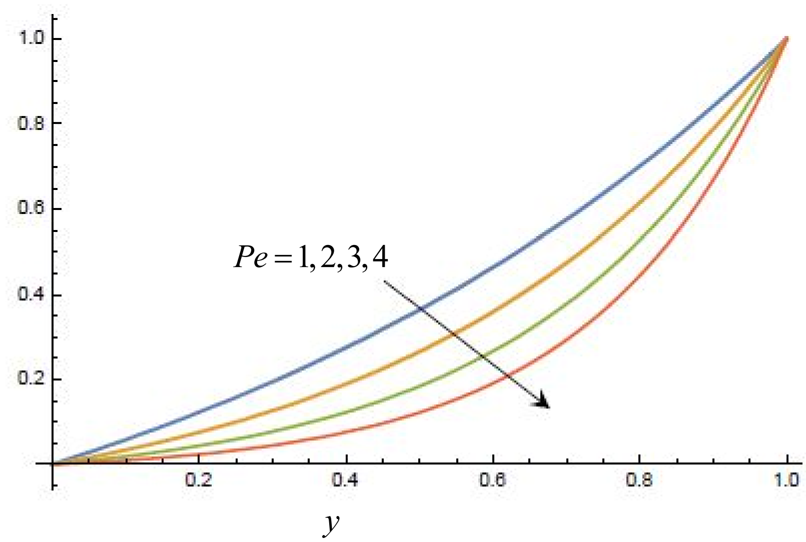

Figure 2: Effect of Peclet number on temperature for $\omega = 0.50, t = 0.1, N^2 = 0.5, S = 1.0$

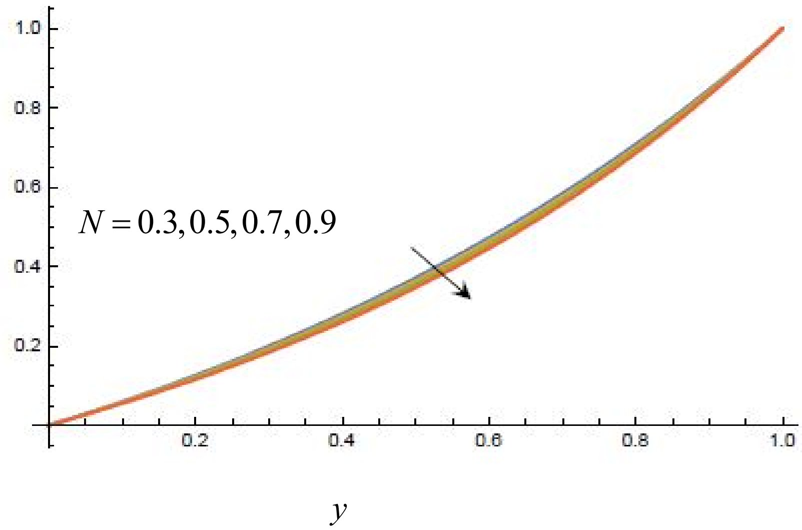

Figure 3: Effect of radiation on temperature for $\omega = 0.50, t = 0.1, Pe = 1.0, S = 1.0$

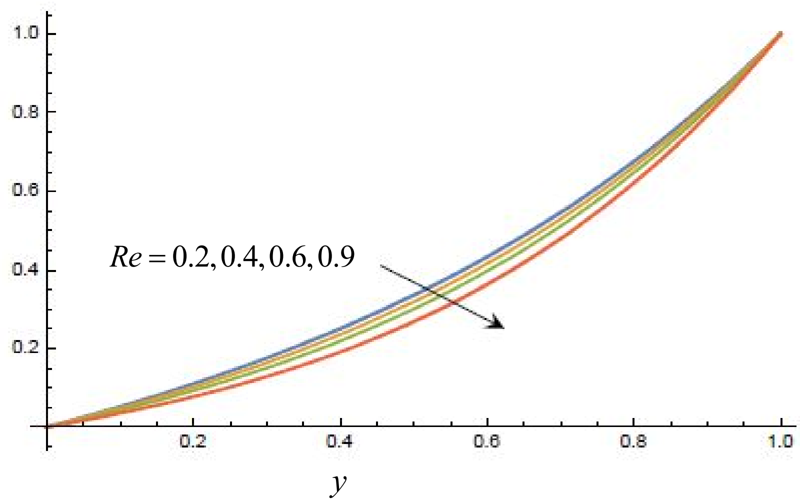

Figure 4: Effect of Renolds number on concentration for $\omega = 0.50, t = 0.1, Pe = 1.0$, $N^2 = 0.5, S = 1.0, So = 0.5, Sc = 0.5$

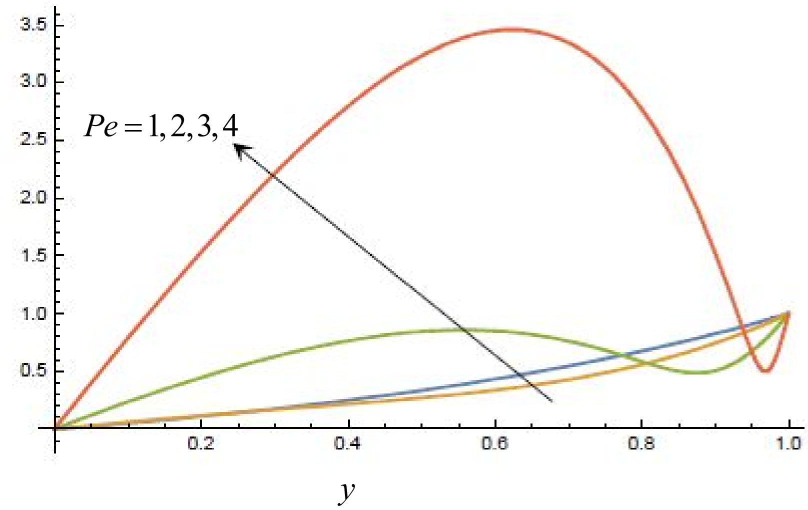

Figure 5: Effect of Peclet number on concentration for $\omega = 0.50, t = 0.1, Re = 1.0$, $N^2 = 0.5, S = 1.0, So = 0.5, Sc = 0.5$

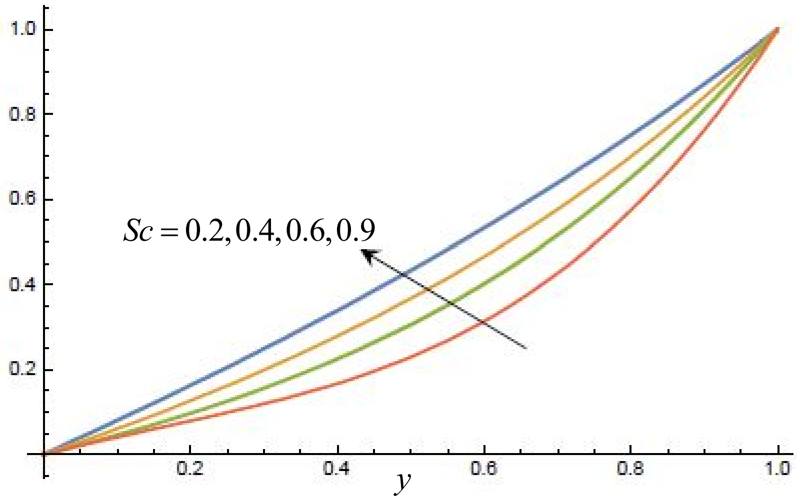

Figure 6: Effect of Schmidt number on concentration for $\omega = 0.50, t = 0.1, Pe = 1.0, N^2 = 0.5, S = 1.0, So = 0.5, Re = 1.0$

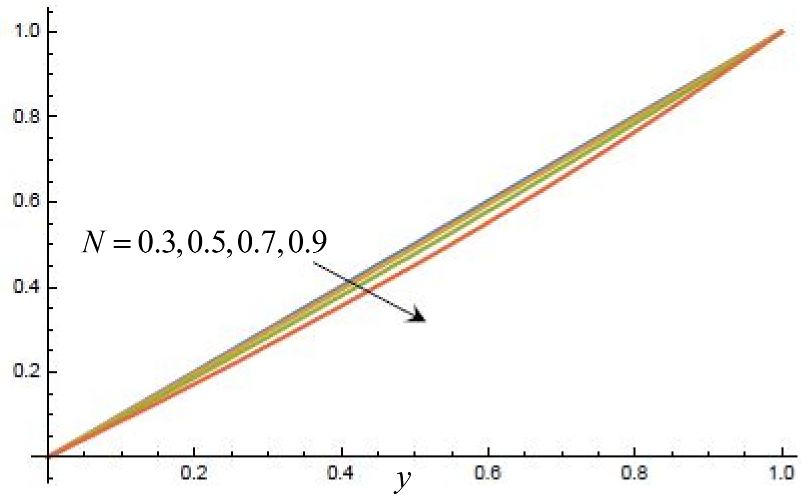

Figure 7: Effect of radiation parameter on concentration for $\omega = 0.50, t = 0.1, Pe = 1.0, Re = 0.5, S = 1.0, So = 0.5, Sc = 0.5$

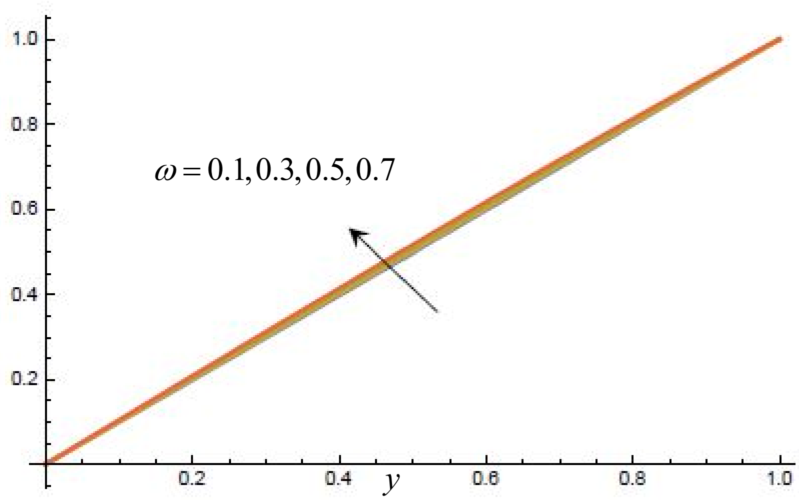

Figure 8: Effect of oscillatory parameter on concentration for $Re = 1.0, t = 0.1, Pe = 1.0, N^2 = 0.5, S = 1.0, So = 0.5, Sc = 0.5$

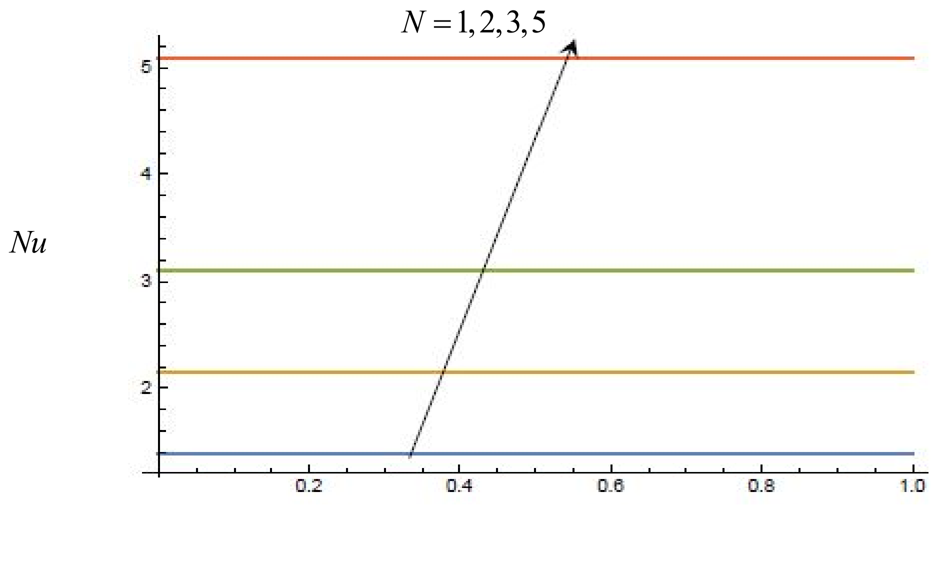

Figure 9: Effect of Radiation parameter on heat transfer for $\omega = 0.50, t = 0.1, Pe = 1.0, N^2 = 0.5, S = 1.0$

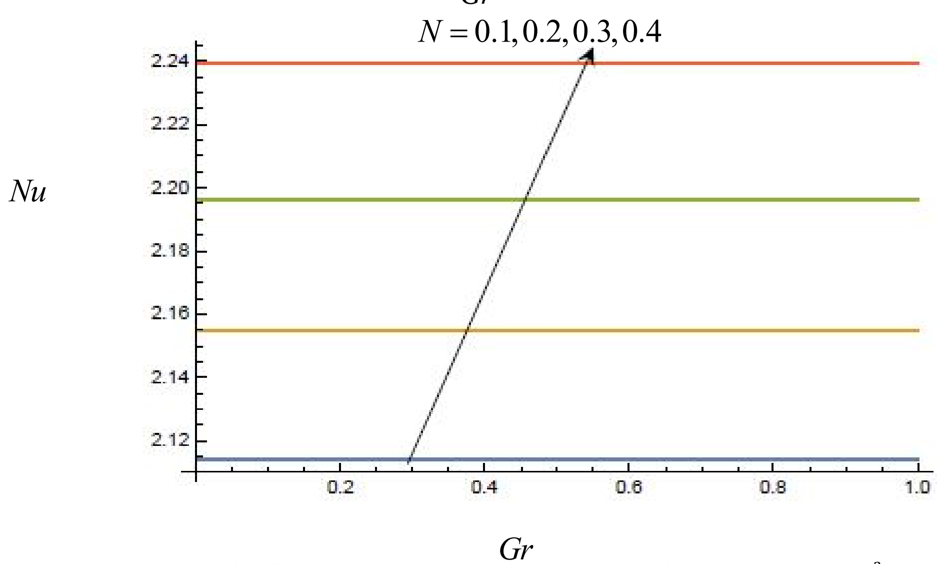

Figure 10: Effect of Radiation parameter on heat transfer for $\omega = 0.50, t = 0.1, Pe = 1.0, N^2 = 0.5, S = 1.0$

Velocity Profile

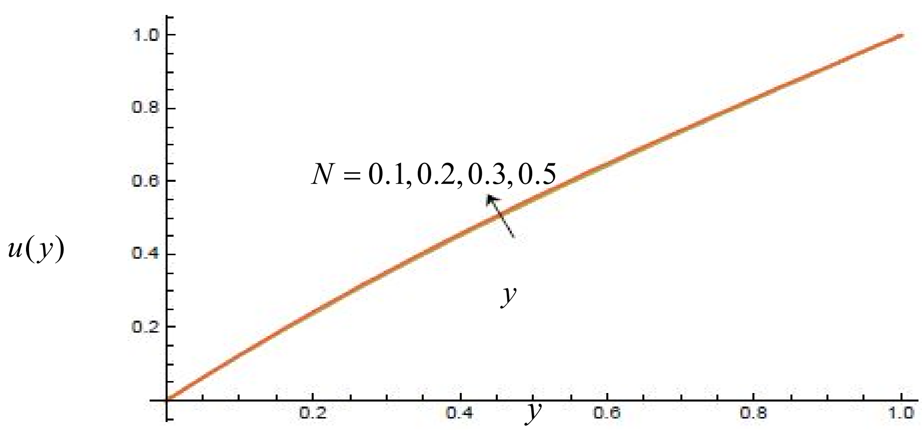

Figure 11: Effect of thermal radiation on velocity for $Gr = 10, Gm = 10, \omega = 3, Sc = 0.5, P = 1, K = 0.5, M = 1, Re = 1, So = 0.5, Pe = 1$

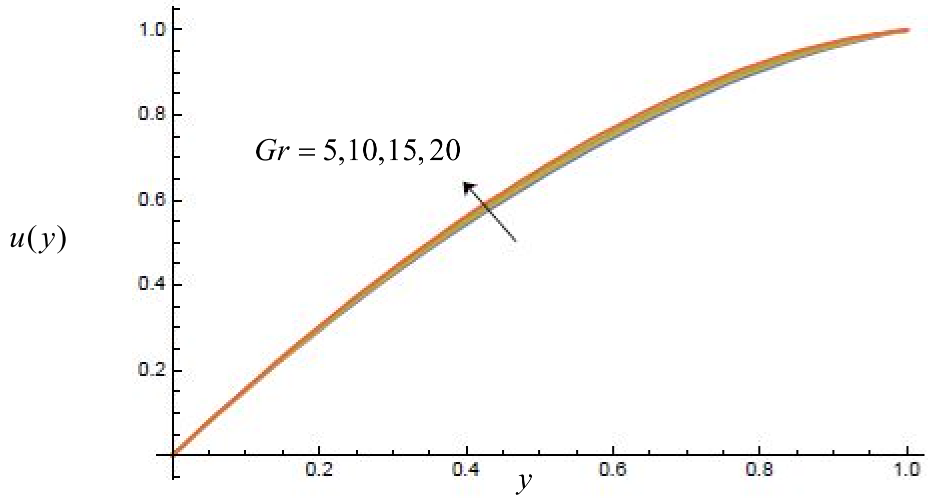

Figure 12: Effect of Grashof on velocity for $N = 0.5, Gm = 10, \omega = 3, Sc = 0.5, P = 1, K = 0.5, M = 1, Re = 1, So = 0.5, Pe = 1$

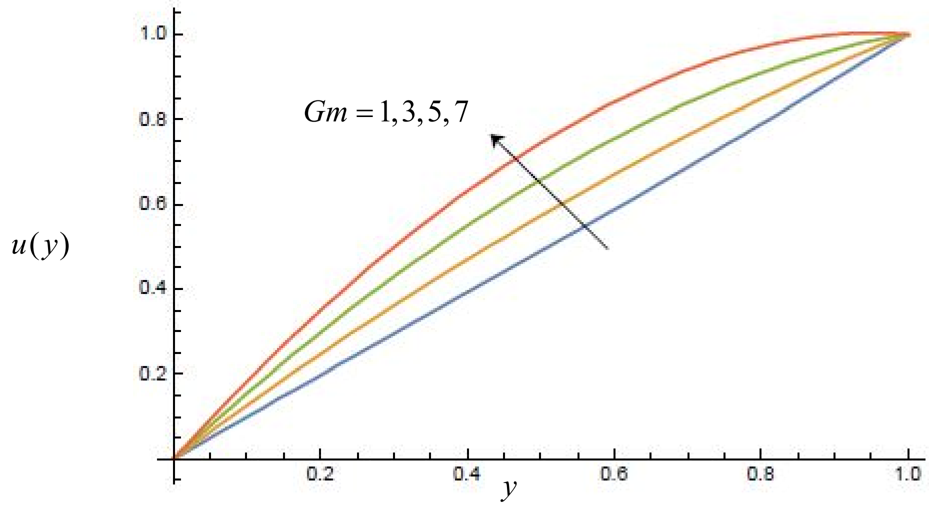

Figure 13: Effect of modified Grashof number on velocity for $Gr = 10, N = 0.5, \omega = 3, Sc = 0.5, P = 1, K = 0.5, M = 1, Re = 1, So = 0.5, Pe = 1$

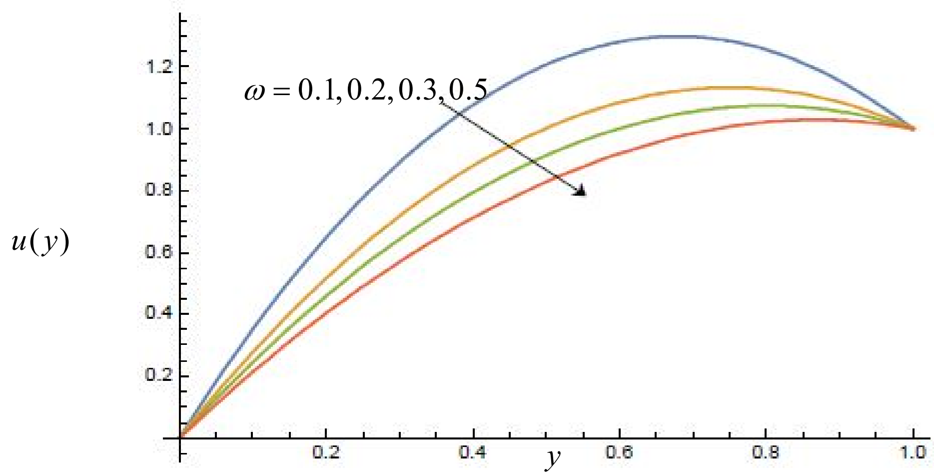

Figure 14: Effect of frequency of oscillations on velocity for $Gr = 10, Gm = 10, N = 0.5, Sc = 0.5, P = 1, K = 0.5, M = 1, Re = 1, So = 0.5, Pe = 1$

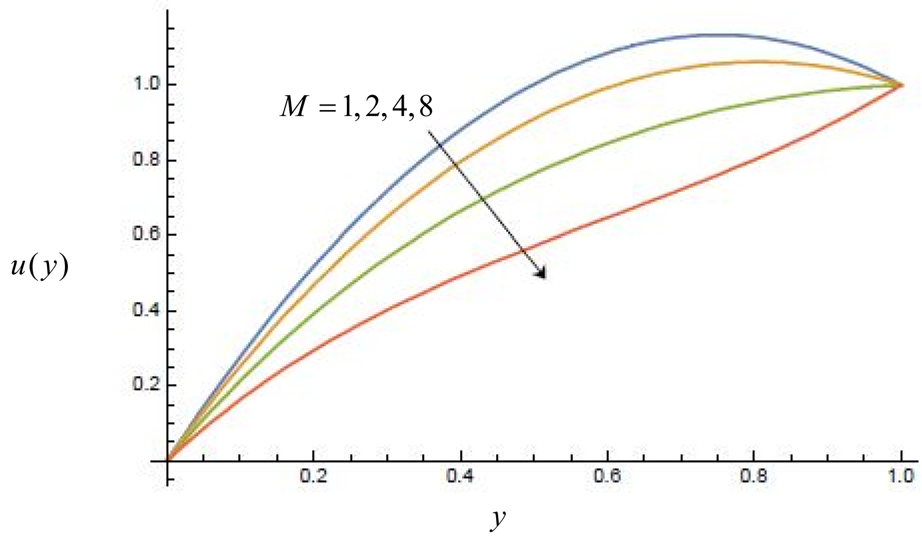

Figure 15: Effect of magnetic field on velocity for $Gr = 10, Gm = 10, \omega = 3, Sc = 0.5, P = 1, K = 0.5, N = 0.5, Re = 1, So = 0.5, Pe = 1$

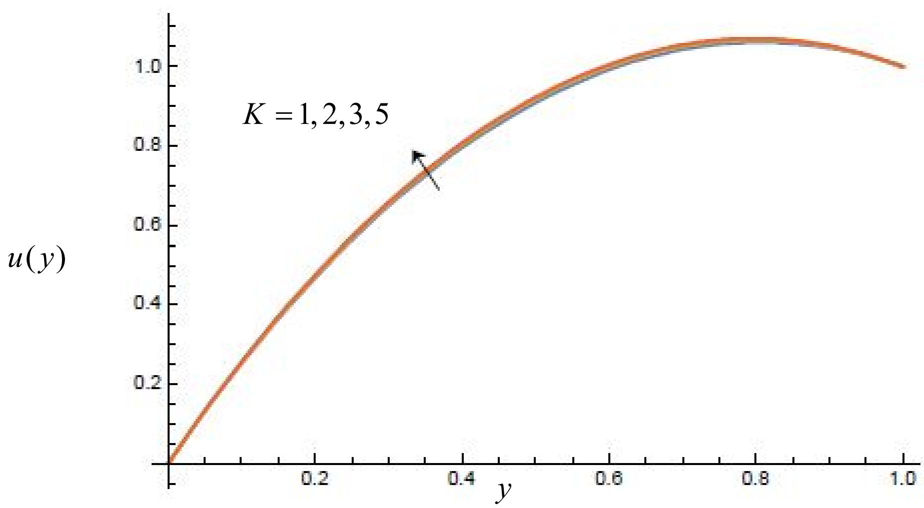

Figure 16: Effect of porosity on velocity for $Gr = 10, Gm = 10, \omega = 3, Sc = 0.5, P = 1, N = 0.5, M = 1, Re = 1, So = 0.5, Pe = 1$

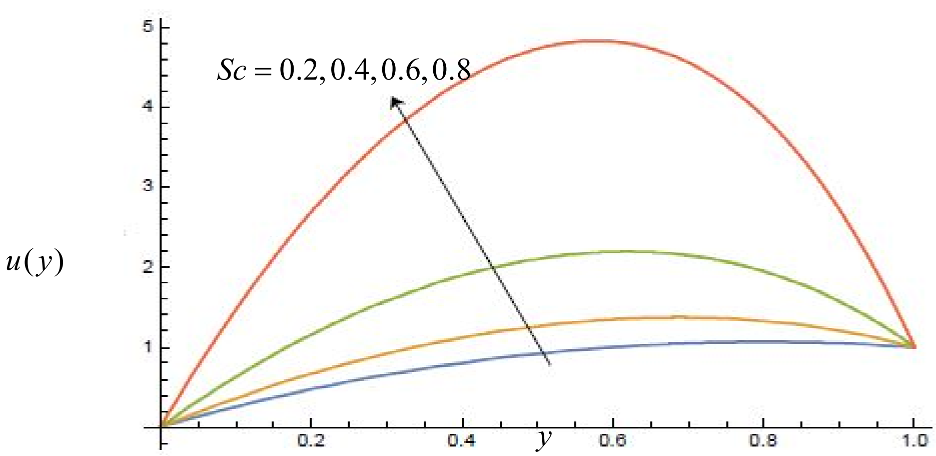

Figure 17: Effect of Scmidt number on velocity for $Gr = 10, Gm = 10, \omega = 3, N = 0.5, P = 1, K = 0.5, M = 1, Re = 1, So = 0.5, Pe = 1$

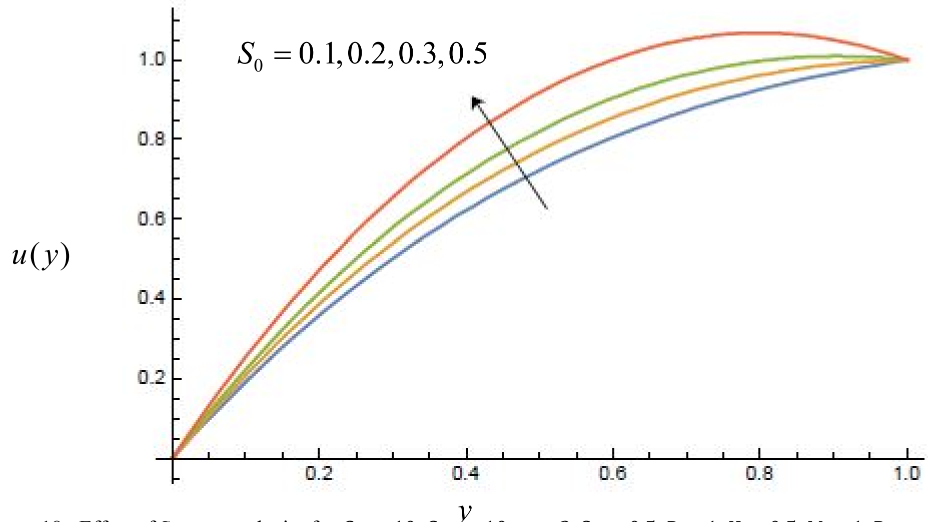

Figure 18.: Effect of Soret on velocity for $Gr = 10, Gm = 10, \omega = 3, Sc = 0.5, P = 1, K = 0.5, M = 1, Re = 1, N = 0.5, Pe = 1$

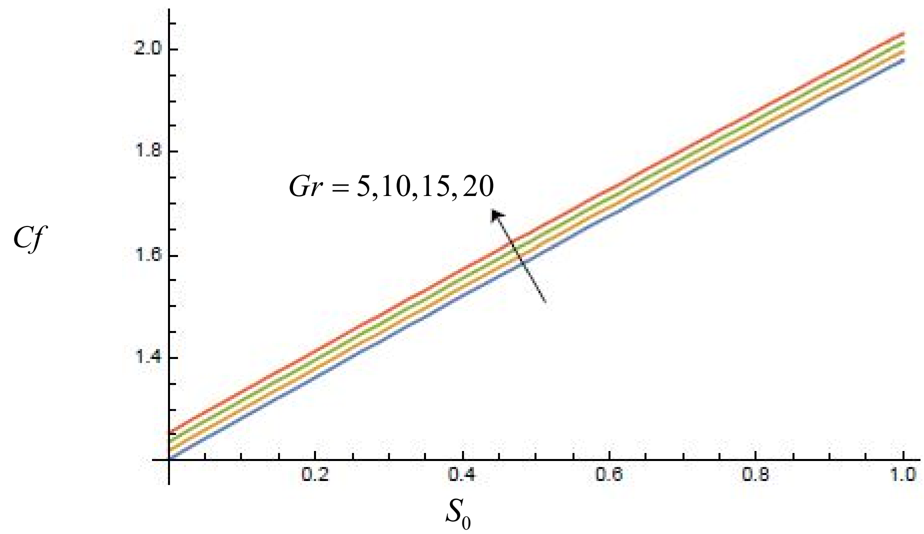

Figure 19: Effect of Grashof number and Soret on the skin friction for $N = 0.5, Gm = 10, \omega = 3, Sc = 0.5, P = 1, K = 0.5, M = 1, Re = 1, So = 0.5, Pe = 1$

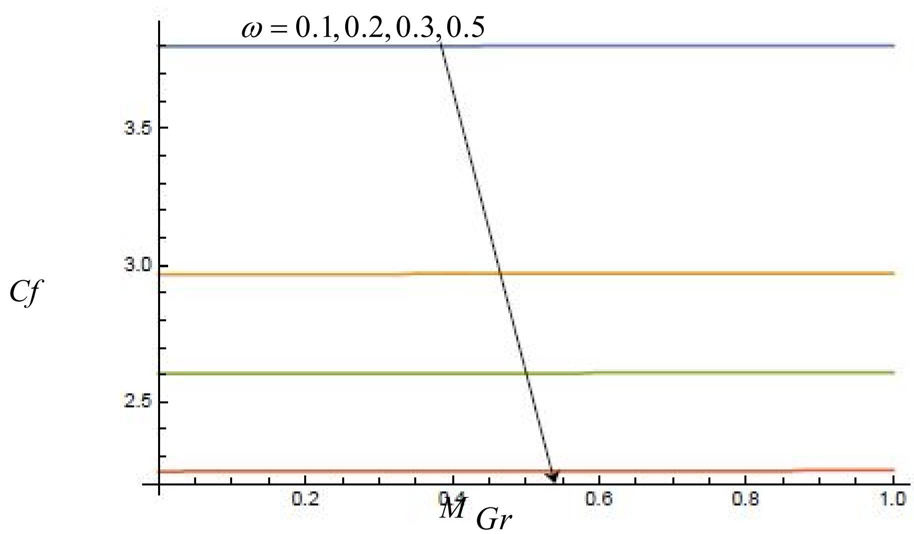

Figure 20: Effect of frequency of oscillation and Grashof number on the skin friction for $Gm = 10, Sc = 0.5, P = 1, K = 0.5, M = 1, Re = 1, So = 0.5, Pe = 1$

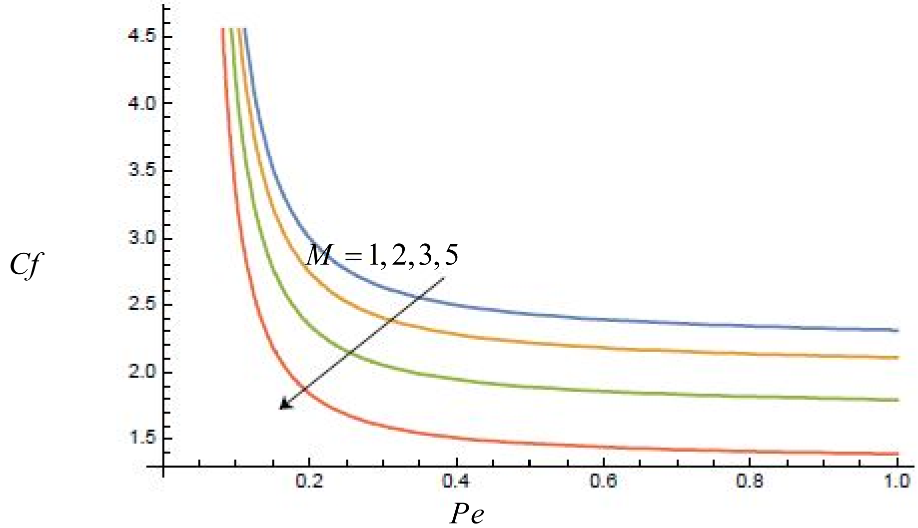

Figure 21: Effect of magnetic field and Peclet number on velocity for $Gr = 10, Gm = 10, \omega = 3, Sc = 0.5,$

$P = 1, K = 0.5, \quad Re = 1, So = 0.5.$

IV. CONCLUSION

The governing equations for unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat absorption was formulated. The plate velocity was maintained at a constant value and the flow was subject to a transverse magnetic field. The computed values obtained from analytical solutions for the velocity, temperature, concentration fields as well as skin-friction coefficient, Nusselt number and the Sherwood number with their amplitude and phase are presented graphically and in tabular form. After a suitable transformation, the governing partial differential equations were transformed to ordinary differential ones. These equations were solved analytically by using two-term harmonic and non-harmonic functions. We conclude the following after analyzing the graphs: The velocity decreases with increasing the Prandtl number, and magnetic field parameter whereas reverse trend is seen with increasing the heat generation parameter, radiation parameter, porous parameter, Soret number, thermal and solutal Grashof numbers. The temperature decreases as the values of Prandtl number increase and reverse trend is seen by increasing the values of the thermal radiation parameter, heat source parameter. The concentration decreases as the values of the chemical reaction parameter and B Schmidt number whereas concentration increases with increase the value of Soret 1 number.

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