Remarks on Commutativity Results for Alternative Rings with $[(x^2y^2 + y^2x^2), x] = 0$

Moharram A. Khan[1], Abubakar Salisu[2], Shu'aibu Salisu[3]

[1]*Department of Mathematics and Statistics, Faculty of Natural and applied Sciences, Umaru Musa Yar'adua University, Katsina. Katsina State, Nigeria*

[2]*Science and Technical Education Board Dutse. Jigawa State, Nigeria*

[3] *Katsina State Science and Technical Education Board, Nigeria*

***Abstract*: In this article, it is shown that the commutativity of alternative ring satisfying the following properties:**

(p_1) $[(x^2y^2 + y^2x^2), x] = 0$.

(p_2) $x(x^2y^2) = (x^2y^2)x$.

***Keywords:* Alternative ring, assosymetric ring, commutator, prime rings.**

I. INTRODUCTION

Throughout R represents an alternative ring, $C(R)$ the commutator, $A(R)$ the assosymetric ring. $N(R)$ the set of nilpotent element. An alternative ring R is a ring in which $(xx)y = x(xy)$, $y(xx) = (yx)x$ for all x, y in R, these equations are known as left and right alternative laws respectively. An assosymetric ring A(R) is one in which $(x, y, z) = (p(x), p(y), p(z))$, where p is any permutation of $x, y, z \in R$. An associator (x, y, z) we mean by $(x, y, z) = (xy)z - x(yz)$ for all $x, y, z \in$ R. A ring R is called a prime if whenever A and B are ideals of R such that $AB = \{0\}$ then either $A = \{0\}$ or $B = \{0\}$. If in a ring R, the identity $(x, y, x) = 0$ i.e. $(xy)x = x(yx)$ for all x, y in R holds then R is called flexible. A ring R is said to be m-torsion tree if $mx = 0$ implies $x = 0$, m is any positive number for all $x \in R$.A non-associative rings R is an additive abelian group in which multiplication is defined, which is distributive over addition on left as well as on right $[(x + y)z = xz + yz$, $z(x + y) = zx + zy, \forall\, x, y, z \in R]$.

Abujabal and Khan [1] proved the commutativity of associative ring satisfies the identity $(xy)^2 = xy^2x$. Gupta [2] established that a division ring R is commutative if and only if $[xy, yx] = 0$. In addition, Madana and Reddy [3] have established the commutativity of non-associative ring satisfying the identities $(xy)^2 = x^2y^2$ and $(xy)^2 \in Z(R) \forall x, y \in R$.Further,

Madana Mohana Reddy and Shobha latha.[4] established the commutativity of non-associative primitive rings satisfying the identities:

$(x(x^2 + y^2)$+$(x^2 + y^2)x \in Z(R)$ and $x(xy)^2 - (xy)^2x \in Z(R)$.

Motivated by these observation it is natural to look commutativity of alternative rings satisfies:(p_1) & (p_2),.

In the present paper we consider the following theorems.

II. THE MAIN THEOREMS

The following are main results.

Theorem 2.1 Let R be a 2-torsion free alternative ring with unity satisfy (p_1), Then R is commutative. .

Now, we begin with the proof of our theorems.

Proof of Theorem 2.1

From the hypothesis (p_1) we have

(1) $x(x^2y^2 + y^2x^2) = (x^2y^2 + y^2x^2)x$

for all $x, y \in$ R.

Replace x by $(x + 1)$in (1), and Apply 2-torsion free, we get

(2) $xy^2 = y^2x$ for all $x, y \in$ R.

Replace y by $(y + 1)$ we find that

(3). $2(xy - yx) = 0$ Apply 2-torsion,

This implies $xy = yx$ and hence R is commutative.

Since R is a commutative ring and satisfies the identities either $(xx)y = x(xy)$ or

$y(xx) = (yx)x$, so that R is an alternative ring. Hence an alternative ring R with identity together with commutativity yields $(x, x, y) = 0 = (y, x, x)$, which completes the proof.

Theorem 2.2 If R is a 2-torsion free alternative ring with unity satisfy (p_2) then R is commutative.

Proof of Theorem 2.2

From the hypothesis (p_2)

Replace x by $(x + 1)$ in (p_2) we have

(4) $(x + 1)[(x + 1)^2y^2] = [(x + 1)^2y^2](x + 1)$

for all $x, y \in$ R.

Using (p_2) in (4) also Apply 2-torsion, we get

(5) $xy^2 = y^2x$ for all x, y ∈ R.

Replace y by $(y+1)$ we find that

(6). $2(xy - yx) = 0$ Apply 2-torsion, for all x, y ∈ R.

This implies $xy = yx$ and hence R is commutative.

Now using the same argument as in last paragraph of the proof of the theorem 2.1.

REFERENCES

[1] H.A.S Abu Jabal and M.A Khan. (1993) "Some Elementary commutativity theorem for Associative Rings", Kyungpook Math J .1: 49-51.

[2] Gupta.R.N. (1970). Nilpotent matrices with invertable transpose,proc.Amer.Math.Soc.,24, 572-575.

[3] Y. Madana Mohana Reddy, G. Shobhatha and D.V Ramin Reddy (2017) "Some Commutativity Theorem for non-associative rings" Math Archive, 5:379-382.

[4] Madana Mohana Reddy and shobha latha.(2020). On Commutativity for certain of Non-Associative Primitive Rings with: $[x(xy)^2 - (xy^2)x \in Z(R)]$.7: 292 − 294.