Stability Analysis of the Numerical Approximation for HIV-Infection of Cd4+ T-Cells Mathematical Model

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Abstract:- **This study investigated a stability analysis of the numerical approximation of a differential equation model of HIV-infection of CD4+ T-cells which is susceptible to infection, that is target cells T, which through interactions with virus V, become infected. Analytical solution of the model was considered, the steady state solution of the model was investigated and the linear stability analysis of the model was obtained. The numerical solution shows that upon the variation of the rate of production of CD4$^+$Tcells, a mixture of stable and unstable steady state-solutions was found. And by varying the death rate of virus particlesγ, we found a combination of stable and unstable steady state-solutions which became a predominant unstable when the per capita rate of disappearance of infected cells was varied.**

Keywords: **HIV-infection, Steady State, Linear Stability, CD4, T-cells**

I. INTRODUCTION

Over three decades since Medical Scientists discovered the Human Immunodeficiency Virus (HIV) as the causative agent of AIDS. From World Health Organization (WHO) data, tens of millions of people are affected by the HIV pandemic, the world over, even with the use of very active anti-retroviral therapy, the infection cannot be completely treated (Arts, 2012). Owing to the fact that, the HIV provirus can stay unnoticed in the T-cells and thus, remain undetected by the body defense system, and also because of the variable nature of the HIV, it may lead to drug resistance in immune recognition (Arts, 2012).

However, the development and availability of modern methods of investigation led to some understanding of the importance of specific factors of the human body in order to support it, or help limit the replication of the HIV. Some of the interesting and salient businesses of mathematical immunology are to develop models, methods of analysis, control etc of the immune system processes (Kim, 2015). Beginning with the classic research of G.I. Marchuk and R.V. Khokhlova, on immune system modeling and HIV-infection; this research in same vein also intends to examine further certain other properties of the already solved dynamic model, representing the HIV-infection. Knowledge of both the stability of the model and the steady state solutions should further explain the obtained solution of the model which is an infinite power series for appropriate boundary conditions.

The CD4 protein is encoded by the CD4 gene in the human body. CD4+ T helper cells are white blood cells that are an essential part of the human immune system. They are most times called CD4 cells, T4 cells or T helper cells. They are called helper cells because, one of their major jobs is to send signals to other types of immune cells like: CD8 killer cells, which then destroy the infectious particles (Bernard, 1984).

A mathematical model of first order ordinary differential equation of HIV/TB co-infection has been developed and analyzed by Bolarin and Omalola (2016). The researchers that utilized the Bellman and Cooke's theorem in their stability analysis of the model for Endemic Equilibrium State and Disease Free Equilibrium. It is in the opinion of the authors that a patient with HIV may likely be infected by TB when proper care is not taken. Also in the previous work of Bolarin (2012), a sex-structured model was formulated to capture the effect of complacency toward HIV/AIDS. These studies did not include a mix of TB. Early detection and early introduction of antiretroviral drugs was advocated.

Srivastava, Awasthi and Kumar (2014) have studied a dynamical model of HIV infection of CD4$^+$ T-cell using an approximated analytical method of differential transform which is derived from Taylor Series expansion. The solution obtained by this method was compared with other solution obtained by Euler's and RK4 methods. The authors recommended the such mathematical for solving nonlinear problems. Earlier, Perelson and Nelson (1999) constructed a mathematics model of HIV 1 dynamics in vivo, their work shows that a valuable understanding of the dynamic HIV follows some industrial and practical exposures.

Merdan, Gokdogan and Yildirim (2011) examined the dynamics of a model of HIV infection of CD4$^+$ T-cell. They adopted a numerical approach in finding the solution to the three components of the model equations. The authors extended the application of the analytic iteration method. In their study Laplace Transform and Pade approximation was used to obtain an analytic solution in order to improve the accuracy of Vanational Iteration Method (VIM). Upon the application of vanatioral iterative method, Baker, G.A. (1975) established the connection between Admomian decomposition method and pade approximants in the studies of solutions of nonlinear first order differential equations.

A model of HIV consisting of four components of free virus, uninfected healthy $CD4^+$ T-cell, latently infected $CD4^+$ T-cell, and actively infected $CD4^+$ T-cell which are proven valuable in the understanding of the dynamics of HIV infection has been studied by Ding and Ye (2009). The researchers introduced fractional order into the HIV-infection model to establish a non negative solution to the dynamic. They adopted an Adams-Types Predictor-Corrector Method for the purpose of effectively generating a numerical solution for the first order differential equation. They observed that a fractional order system has a positive equilibrium. A model with a mixed compartment of bloodstream of HIV-1, has been analyzed using a fractional calculus for linear and non-linear ordinary differential equation with robust approach to mathematical modeling in immunology by Asquith and Bangham (2003).

II. MATHEMATICAL MODEL

A differential equation model of HIV-infection of CD4+ T-cells which is susceptible to infection, that is target cells T, which through interactions with virus V, become infected is considered

$$\frac{dT}{dt} = \lambda - \alpha T + rT\left(1 - \frac{T+1}{T_{max}}\right) - k^*VT \quad (1)$$

$$\frac{dI}{dt} = k^*VT - \beta I \quad (2)$$

$$\frac{dV}{dt} = N^*\beta I - \gamma V \quad (3)$$

Where T(t) and V(t) concentration of uninfected, infected and virus population of CD4+ T-cells by HIV in the blood, respectively. $rT\left(1 - \frac{T+1}{T_{max}}\right)$ is logistic growth of healthy CD4+ T-cells, T_{max} is the maximum level of CD$+ T-cells in the human body, r is the rate at which T-cells multiply through mitosis when stimulated by antigen or mitogen, λ is the constant rate which the body produces CD4+ T-cells from precursors in the bone marrow and thymus (i.e. λ is the rate of production of CD4+ T-cells), x is the natural turnover rate T-cells and k^*VT is the incidence of HIV infection of healthy CD4+ T-cells, where k>0 is the rate of infection to T-cells by virus. β is the per capita rate of disappearance of infected cells. $N^*\beta I$ is the rate of production of virions by infected cells, where N^* is the average number of virus particles produced by an infected T-cell and γ is the death rate of virus particles.

III. TEST OF STABILITY

According to linear stability analysis, an equilibrium point is stable if all the eigenvalues of the Jacobian, evaluated at that equilibrium point have negative real parts. The equilibrium point is unstable if at least one of the eigenvalue has a positive real part. Positive eigenvalues contribute to the unbounded growth of the solution trajectories as $t \to \infty$. The negative eigenvalues contribute to the decaying behaviour of the solution trajectories as $t \to \infty$.

Steady State Solution

At a steady state all differential coefficients with respect to the independent variables are equal to zero.

Let $k^* = L$, $N^* = M$, and $T_{max} = \delta$. Then (1), (2) and (3) respectively becomes

$$\frac{dT}{dt} = \lambda - \alpha T + rT\left(1 - \frac{T+I}{\delta}\right) - LVT \quad (4)$$

$$\frac{dI}{dt} = LVT - \beta I \quad (5)$$

$$\frac{dV}{dt} = M\beta I - \gamma V \quad (6)$$

At a steady state $\frac{dT}{dt} = \frac{dI}{dt} = \frac{dV}{dt} = 0$,

from (3) to (6)

Thus, the point $\left(\frac{\gamma}{ML},\ 0,\ 0\right)$ is a steady state solution.

By using the quadratic formula

Thus, the points $\left(\frac{r\delta - \alpha\delta \pm \sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{2r},\ 0,\ 0\right)$ is a steady state solutions.

Thus the point $\left(\frac{\gamma}{ML},\ 0,\ \frac{M^2L^2\lambda\delta - ML\alpha\gamma\delta + MLr\gamma\delta - r\gamma^2}{ML^2\gamma\delta}\right)$ is a steady state solution.

Thus, the point $\left(\frac{\gamma}{ML},\ \frac{M^2L^2\lambda\delta - ML\alpha\gamma\delta + MLr\gamma\delta - r\gamma^2}{MLr\gamma},\ 0\right)$ is a steady state solution.

Test of Stability

From (4), (5) and (6) respectively, let

$$F(T,I,V) = \lambda - \alpha T + rT\left(1 - \frac{T+I}{\delta}\right) - LVT \quad (11)$$

$$G(T,I,V) = LVT - \beta I \quad (12)$$

$$H(T,I,V) = M\beta I - \gamma V \quad (13)$$

From (11), (12) and (13), we have the Jacobian matrix;

$$J = \begin{pmatrix} \frac{rML\delta - \alpha ML\delta - 2r\gamma}{ML\delta} & -\frac{r}{\delta} & \frac{-\gamma}{M} \\ 0 & -\beta & \frac{\gamma}{M} \\ 0 & M\beta & -\gamma \end{pmatrix},$$ The eigen values are;

$$C_1 = 0,\ C_2 = -(\beta + \gamma), \text{and}$$

$$C_3 = \frac{rML\delta - \alpha ML\delta - 2r\gamma}{ML\delta}$$

Provided all the parameters of the eigen values are positive, and

$rML\delta - \alpha ML\delta - 2r\gamma > 0$, then the steady state $\left(\frac{\gamma}{ML},\ 0,\ 0,\right.$.

At the steady state $\left(\frac{r\delta - \alpha\delta \pm \sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{2r},\ 0,\ 0\right)$,

Thus, the Jacobian matrix is;

$$J = \begin{pmatrix} \frac{\pm\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{\delta} & -\frac{r}{\delta} & \frac{L\alpha\delta - Lr\delta \pm L\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{2r} \\ 0 & -\beta & \frac{Lr\delta - L\alpha\delta \pm L\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{2r} \\ 0 & M\beta & -\gamma \end{pmatrix}$$

The characteristics equation is;

$$\left(C \pm \frac{\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{\delta}\right)\Big[(C+\beta)(C+\gamma) +$$
$$M\beta L\alpha\delta - Lr\delta \pm L\alpha\delta - r\delta 2 + 4r\lambda\delta 2r = 0$$

The eigen values are;

$$C_1 = \frac{\pm\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{\delta}$$

$$C_2 = \frac{-(\beta+\gamma) + \sqrt{(\beta+\gamma)^2 - \frac{4r\beta\gamma + 2M\beta L\alpha\delta - 2M\beta Lr\delta \pm 2M\beta\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{r}}}{2}$$

and

$$C_3 = \frac{-(\beta+\gamma) - \sqrt{(\beta+\gamma)^2 - \frac{4r\beta\gamma + 2M\beta L\alpha\delta - 2M\beta Lr\delta \pm 2M\beta\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{r}}}{2}$$

For the eigen value C_1 to be real, then $\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}$ must be greater than zero $\left(\text{i.e. } \sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta} > 0\right)$. Also, $(\beta+\gamma)^2$ must be greater than $\frac{4r\beta\gamma + 2M\beta L\alpha\delta \pm 2M\beta\sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{r}$ for the eigen values C_2 and C_3 to be real. Therefore, C_2 and C_3 must be negative.

Provided all the parameters of the eigen values are positive, by considering the positive value of C_1, the steady state $\left(\frac{r\delta - \alpha\delta \pm \sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{2r},\ 0,\ 0\right)$ is unstable because two of the eigen values C_2 and C_3 are negative and the other C_1 is positive. By considering the negative value of C_1 the steady state $\left(\frac{r\delta - \alpha\delta \pm \sqrt{(\alpha\delta - r\delta)^2 + 4r\lambda\delta}}{2r},\ 0,\ 0\right)$ is stable because the three eigen values C_1, C_2 and C_3 are negative.

At the steady state $\left(\frac{\gamma}{ML},\ 0,\ \frac{M^2L^2\lambda\delta - ML\alpha\gamma\delta + MLr\gamma\delta - r\gamma^2}{ML^2\gamma\delta}\right)$, the Jacobian matrix is;

$$J = \begin{pmatrix} \frac{-M^2L^2\lambda\delta - r\gamma^2}{ML^2\gamma\delta} & -\frac{r}{\delta} & \frac{-\gamma}{M} \\ \frac{M^2L^2\lambda\delta - ML\alpha\gamma\delta + MLr\gamma\delta - r\gamma^2}{ML\gamma\delta} & -\beta & \frac{\gamma}{M} \\ 0 & M\beta & -\gamma \end{pmatrix}$$

The characteristic equation is;

$$ML^2\gamma\delta^2C^3 + [ML^2\gamma\delta^2(\beta+\gamma) + M^2L^2\lambda\delta^2 +$$
$$r\delta\gamma 2C2 + \beta + \gamma M2L2\lambda\delta 2 + r\delta\gamma 2 + LrM2L2\lambda\delta 2 - ML\alpha\gamma\delta + MLr\gamma\delta - r\gamma 2C + L\gamma r + \beta\delta M2L2\lambda\delta 2 - ML\alpha\gamma\delta + MLr\gamma\delta - r\gamma 2 = 0$$

IV. NUMERICAL SIMULATION

The following initial conditions and parameter values were considered in perfoming the numerical simulation for this study. result and discussions on the effect of variation of some model parameters on the dynamical system.

4.1 Results

Table 1: Impact of variation of $r$on the stability of the dynamical system

S/N	r	T	I	V	λ_1	λ_2	λ_3	TOS
0	3.0000	5.7283	516.5859	727.6903	0.0740	-0.4111	-2.4037	Unstable
1	0.1500	22.6970	0.0004	0.0006	0.1255	-0.2158	-0.2158	Unstable
2	0.3000	291.0057	0.0102	0.0107	0.1635	0.5101	-3.2100	Unstable
3	0.4500	1028.0000	127.6487	92.3870	1.8622	-0.7051	-4.3317	Unstable
4	0.6000	0.5034	538.4405	764.6449	-0.2946	-1.7095	-2.3962	Stable
5	0.7500	0.1522	263.5467	376.3818	-0.4271	-0.2908	-2.4003	Stable
6	0.9000	0.3174	162.9490	232.6565	0.1559	-0.3013	-2.4009	Unstable
7	1.0500	0.7806	154.4985	164.8096	0.4774	-0.2991	-2.4025	Unstable
8	1.2000	2.1719	89.5243	127.5400	0.7626	-0.2946	-2.4074	Unstable
9	1.3500	6.7969	74.3540	105.3460	0.9703	-0.2802	-2.4237	Unstable
10	1.5000	23.2839	66.6943	92.7647	1.1262	-0.2295	-2.4805	Unstable
11	1.6500	82.0674	69.1069	90.5232	1.1649	-0.0727	-2.6632	Unstable
12	1.8000	256.0440	98.4618	111.1695	1.0086	0.1247	-3.0861	Unstable

13	1.9500	515.4094	220.1759	204.8458	1.2595	-0.7224	-3.4864	Unstable
14	2.1000	475.5304	535.7887	480.9593	1.2764	-2.6383-0.3891i	-2.6383+ 0.3891i	Unstable
15	2.2500	189.3599	834.9934	869.5671	0.6570	-2.6477-0.9408i	-2.6477+ 0.9408i	Unstable
16	2.4000	51.7418	858.9393	1050.3035	0.0974	-2.3965- 0.6388i	-2.3965+ 0.6388i	Unstable
17	2.5500	17.0189	767.2685	1021.9312	-0.1226	-2.0844- 0.1360i	-2.0844+ 0.1360i	Stable
18	2.7000	8.3092	668.6671	923.3675	-0.1777	-1.1977	-2.3713	Stable
19	2.8500	5.8939	584.8077	819.2760	-0.1423	-0.6773	-2.3960	Stable
20	2.9400	5.5853	542.2873	762.6796	-0.0418	-0.4810	-2.4012	Stable

Table 2: Impact of variation of r on the stability of the dynamical system

S/N	r	T	I	V	λ_1	λ_2	λ_3	TOS
0	3.0000	5.7283	516.5859	727.6903	0.0740	-0.4111	-2.4037	Unstable
1	3.1500	6.9698	462.4045	652.0010	0.4245	-0.3454	-2.4098	Unstable
2	3.3000	10.2044	419.8346	590.5096	0.7608	-0.3245	-2.4192	Unstable
3	3.4500	17.1298	388.0239	542.0584	1.0430	-0.3106	-2.4373	Unstable
4	3.6000	31.1203	367.7047	506.5935	1.2465	-0.2936	-2.4726	Unstable
5	3.7500	58.7056	361.8096	485.5852	1.3339	-0.2746	-2.5384	Unstable
6	3.9000	107.8064	377.3838	483.8739	1.2681	-0.2959	-2.6405	Unstable
7	4.0500	176.6946	423.7130	509.7506	1.1151	-0.5180	-2.7416	Unstable
8	4.2000	239.1146	506.7077	570.0745	1.0281	-1.1175	-2.7277	Unstable
9	4.3500	257.6795	618.5877	668.2231	0.9685	-2.2156- 0.2634i	-2.2156+ 0.2634i	Unstable
10	4.5000	225.5845	731.6492	788.7354	0.8500	-2.3740- 0.7217i	-2.3740+ 0.7217i	Unstable
11	4.6500	172.4127	816.6091	904.0584	0.6840	-2.3977- 0.8110i	-2.3977+ 0.8110i	Unstable
12	4.8000	122.6082	861.8640	993.3005	0.5126	-2.3286- 0.7639i	-2.3286+ 0.7639i	Unstable
13	4.9500	86.7615	875.0667	1049.7231	0.3725	-2.2185- 0.6504i	-2.2185+ 0.6504i	Unstable
14	5.1000	64.0646	864.8180	1073.3133	0.2807	-2.0873- 0.4793i	-2.0873+ 0.4793i	Unstable
15	5.2500	50.2803	842.8131	1073.8930	0.2347	-1.9530- 0.1660i	-1.9530+ 0.1660i	Unstable
16	5.4000	42.5968	815.7708	1059.2580	0.2334	-1.4142	-2.2427	Unstable
17	5.5500	38.4716	789.4798	1038.4849	0.2684	-1.1293	-2.3188	Unstable
18	5.7000	38.2180	763.6636	1011.4625	0.3492	-0.9367	-2.3558	Unstable
19	5.8500	39.6458	741.4413	985.5625	0.4525	-0.8050	-2.3793	Unstable
20	5.9400	41.9873	731.1890	971.6580	0.5187	-0.7612	-2.3890	Unstable

Table 3: Impact of variation of β on the stability of the dynamical system

S/N	β	T	I	V	λ_1	λ_2	λ_3	TOS
0	0.3000	5.7283	516.5859	727.6903	0.0740	-0.4111	-2.4037	Unstable
1	0.0150	1485.2554	4.4256	0.2538	0.2151	-2.9702	-2.6305	Unstable
2	0.0300	1383.3591	105.5604	11.4036	0.3774	-2.8014- 0.1041i	-2.8014+ 0.1041i	Unstable
3	0.0450	601.2645	865.8841	146.3971	0.2859	-1.7057	-2.5773	Unstable
4	0.0600	73.6094	1253.5785	312.9297	0.0486	-0.7489	-2.4262	Unstable
5	0.0750	19.5509	1085.4742	347.2727	0.0180	-0.2901	-2.4097	Unstable
6	0.0900	15.7996	885.9795	342.8636	0.2639	-0.1240	-2.4107	Unstable
7	0.1050	25.6016	721.2321	326.3588	0.5886	-0.1176	-2.4221	Unstable
8	0.1200	58.3716	603.4518	309.5357	0.7663	-0.1229	-2.4595	Unstable
9	0.1350	137.9823	547.2171	305.2630	0.7077	-0.1812	-2.5521	Unstable

10	0.1500	254.7930	574.4471	333.9479	0.6245	-0.5975	-2.6666	Unstable
11	0.1650	308.1811	685.5682	415.3725	0.6491	-1.3342	-2.6252	Unstable
12	0.1800	250.6659	825.1985	543.9924	0.5795	-2.1507- 0.1547i	-2.1507+ 0.1547i	Unstable
13	0.1950	154.0928	919.8314	684.3450	0.4054	-2.1621- 0.4080 i	-2.1621+ 0.4080 i	Unstable
14	0.2100	81.8679	935.7444	791.6829	0.2125	-2.0895- 0.3130i	-2.0895+ 0.3130 i	Unstable
15	0.2250	42.4199	893.9620	850.1927	0.0629	-1.7487	-2.2122	Unstable
16	0.2400	23.1294	822.7193	865.1333	-0.0329	-1.3639	-2.3371	Stable
17	0.2550	13.8290	742.3054	850.9132	-0.0855	-1.0499	-2.3770	Stable
18	0.2700	9.2393	661.8141	818.2391	-0.0997	-0.7673	-2.3928	Stable
19	0.2850	6.8837	585.9557	775.4807	-0.0593	-0.5390	-2.3999	Stable
20	0.2940	6.0918	543.5230	747.1705	0.0088	-0.4491	-2.4024	Unstable

Table 4: Impact of variation of β on the stability of the dynamical system

S/N	β	T	I	V	λ_1	λ_2	λ_3	TOS
0	0.3000	5.7283	516.5859	727.6903	0.0740	-0.4111	-2.4037	Unstable
1	0.3150	5.1759	454.2914	678.3060	0.2779	-0.3672	-2.4064	Unstable
2	0.3300	5.0913	398.5337	628.6410	0.5019	-0.3577	-2.4089	Unstable
3	0.3450	5.3606	349.0214	579.8787	0.7217	-0.3598	-2.4120	Unstable
4	0.3600	5.9667	305.2723	532.8668	0.9288	-0.3660	-2.4160	Unstable
5	0.3750	6.9943	266.9162	488.2363	1.1197	-0.3731	-2.4217	Unstable
6	0.3900	8.5283	233.3727	446.2765	1.2933	-0.3794	-2.4297	Unstable
7	0.4050	10.7313	204.2576	407.2878	1.4485	-0.3833	-2.4413	Unstable
8	0.4200	13.8652	179.0014	371.1105	1.5857	-0.3832	-2.4575	Unstable
9	0.4350	18.2810	157.3327	337.9712	1.7035	-0.3769	-2.4819	Unstable
10	0.4500	24.5869	139.0068	308.1851	1.7995	-0.3614	-2.5162	Unstable
11	0.4650	33.2497	123.6442	281.3226	1.8739	-0.3342	-2.5646	Unstable
12	0.4800	45.2326	111.0286	257.5923	1.9237	-0.2916	-2.6306	Unstable
13	0.4950	61.7019	101.0851	237.1955	1.9452	-0.2303	-2.7193	Unstable
14	0.5100	83.9665	93.7149	220.1663	1.9356	-0.1488	-2.8345	Unstable
15	0.5250	113.4395	88.8886	206.6291	1.8933	-0.0491	-2.9787	Unstable
16	0.5400	151.4342	86.6213	196.8059	1.8214	0.0604	-3.1522	Unstable
17	0.5550	199.1361	87.0820	191.0665	1.7321	0.1593	-3.3530	Unstable
18	0.5700	256.5060	90.6639	189.7908	1.6581	0.2068	-3.5747	Unstable
19	0.5850	322.7817	97.6717	193.7810	1.6481	0.1457	-3.8085	Unstable
20	0.5940	365.4815	103.6311	198.8156	1.6860	0.0427	-3.9488	Unstable

Table 5: Impact of variation of γ on the stability of the dynamical system

S/N	γ	T	I	V	λ_1	λ_2	λ_3	TOS
0	2.4	5.7283	516.5859	727.6903	0.0740	-0.4111	-2.4037	Unstable
1	0.1200	0.0130	16.2530	3882.8583	-7.5393	-0.2985	-0.1186	Unstable
2	0.2400	0.1086	17.0418	1220.7574	-0.3781-0.0963i	-0.3781+ 0.0963i	-0.1343	Unstable
3	0.3600	3.9797	20.9560	513.5912	1.5498	-0.2417	-0.4326	Unstable
4	0.4800	125.4352	78.2493	360.0893	1.6612	0.0954	-1.1870	Unstable
5	0.6000	316.9159	451.9842	786.6562	1.6544	-1.9350- 1.2046i	-1.9350+ 1.2046i	Unstable

6	0.7200	69.0734	782.6828	1667.6520	0.6042	-2.8523	-2.1862	Unstable
7	0.8400	8.8666	758.2880	2074.2738	-4.1437	-0.0751	-1.0939	Stable
8	0.9600	1.9342	696.8268	2078.8290	-4.0296	-0.2429	-1.0218	Stable
9	1.0800	0.8011	651.5127	1934.1988	-3.5476	-0.2759	-1.1052	Stable
10	1.2000	0.5381	620.4359	1754.6904	-2.9999	-0.2835	-1.2173	Stable
11	1.3200	0.4963	600.0731	1584.8138	-2.4984	-0.2848	-1.3380	Stable
12	1.4400	0.5438	586.5568	1435.1692	-0.2836	-2.0589	-1.4677	Stable
13	1.5600	0.6582	578.5259	1308.9520	-0.2806	-1.6466- 0.0887i	-1.6466+ 0.0887i	Stable
14	1.6800	0.8568	573.7556	1201.6861	-0.2753	-1.5601- 0.0186i	-1.5601+ 0.0186i	Stable
15	1.8000	1.1267	569.9226	1108.3145	-0.2673	-1.2134	-1.7762	Stable
16	1.9200	1.5260	564.8346	1023.2699	-0.2535	-0.9790	-1.9061	Stable
17	2.0400	2.0724	556.1972	942.0985	-0.2280	-0.7650	-2.0314	Stable
18	2.1600	2.8516	543.9315	864.1775	-0.1741	-0.5830	-2.1554	Stable
19	2.2800	4.0024	530.0345	791.9914	-0.0694	-0.4658	-2.2792	Stable
20	2.3520	4.9457	521.7902	752.3907	0.0144	-0.4274	-2.3538	Unstable

Table 6: Impact of variation of γ on the stability of the dynamical system

S/N	γ	T	I	V	λ_1	λ_2	λ_3	TOS
0	2.4	5.7283	516.5859	727.6903	0.0740	-0.4111	-2.4037	Unstable
1	2.5200	8.1528	506.4181	673.9670	0.2124	-0.3883	-2.5292	Unstable
2	2.6400	11.5501	499.8052	629.6774	0.3293	-0.3789	-2.6564	Unstable
3	2.7600	16.0323	497.1151	593.9670	0.4192	-0.3760	-2.7853	Unstable
4	2.8800	21.4415	498.6324	566.0809	0.4810	-0.3770	-2.9255	Unstable
5	3.0000	27.7194	504.3662	545.2812	0.5169	-0.3820	-3.0467	Unstable
6	3.1200	34.6893	512.7181	528.8385	0.5362	-0.3897	-3.1785	Unstable
7	3.2400	42.0538	523.6136	516.3084	0.5414	-0.4005	-3.3103	Unstable
8	3.3600	49.5221	536.4685	506.6797	0.5366	-0.4142	-3.4415	Unstable
9	3.4800	56.9127	550.7033	499.3489	0.5251	-0.4305	-3.5719	Unstable
10	3.6000	64.1415	565.8760	493.3978	0.5098	-0.4489	-3.7014	Unstable
11	3.7200	71.0727	581.6413	488.6630	0.4922	-0.4692	-3.8093	Unstable
12	3.8400	77.6545	597.7546	484.4997	0.4737	-0.4905	-3.9574	Unstable
13	3.9600	83.9910	613.9055	480.9711	0.4550	-0.5131	-4.0842	Unstable
14	4.0800	90.0676	629.9751	477.6156	0.4368	-0.5362	-4.2104	Unstable
15	4.2000	96.0306	645.7576	474.3324	0.4196	-0.5598	-4.3361	Unstable
16	4.3200	101.9001	661.1804	470.8779	0.4036	-0.5833	-4.4616	Unstable
17	4.4400	107.7665	676.1102	467.6141	0.3886	-0.6077	-4.5863	Unstable
18	4.5600	113.7005	690.5028	464.1067	0.3750	-0.6320	-4.7119	Unstable
19	4.6800	119.7400	704.2640	460.3645	0.3629	-0.6563	-4.8371	Unstable
20	4.7520	123.4493	712.2050	458.0473	0.3562	-0.6709	-4.9122	Unstable

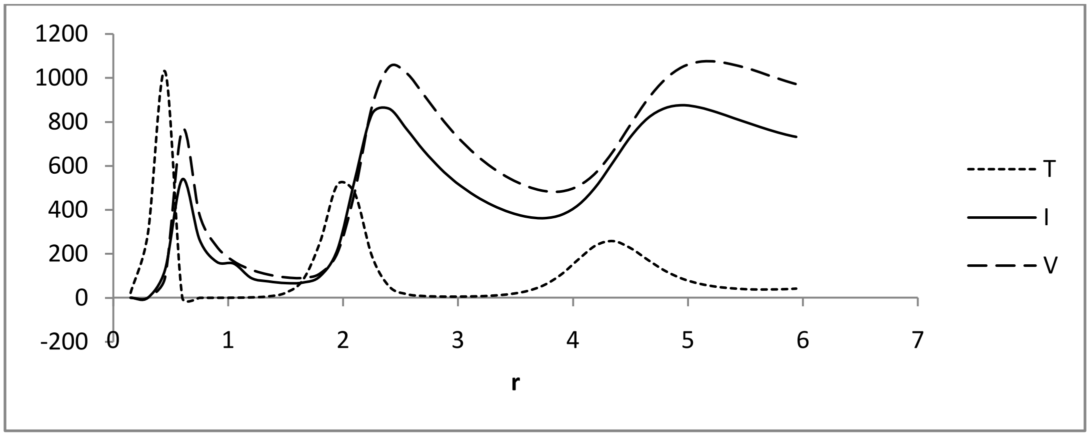

Figure 1: Impact of variation of r on the stability of the dynamical system

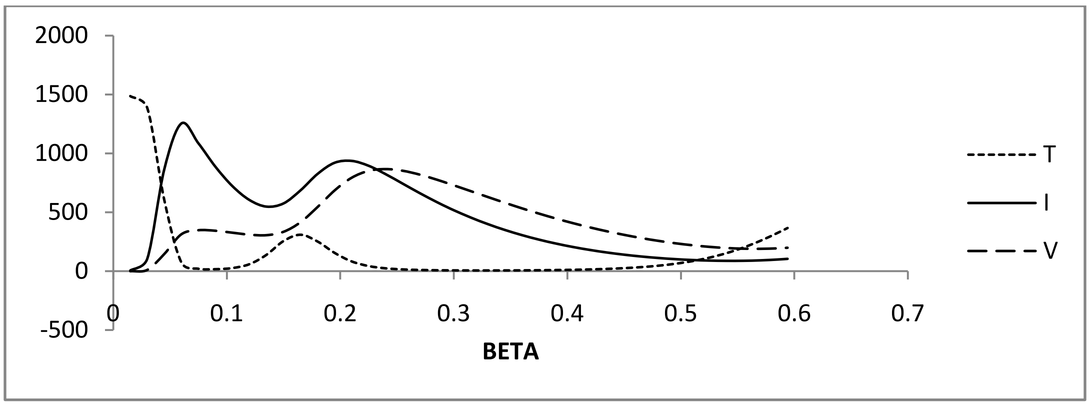

Figure 2: Impact of variation of β on the stability of the dynamical system

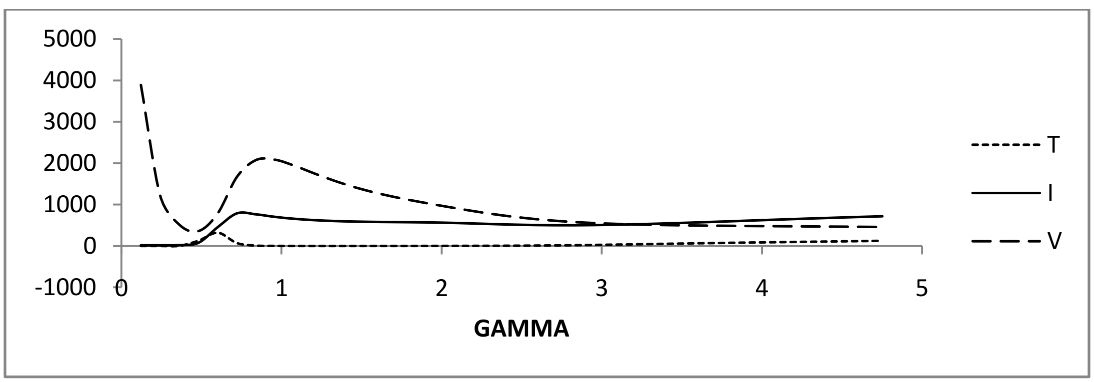

Figure 4.3: Impact of variation of γ on the stability of the dynamical system

V. DISCUSSION

In Tables 1 – 4, an initial value of a given parameter was considered, 5% of this value was then considered and monotonically increased. For instance, $r = 3$, was initially considered in Tables 1 and 2,$\beta = 0.3$ was initially considered in Tables 3 and 4, while in Tables 5 and 6, $\gamma = 2.4$ was initially considered. Tables 1, 3 and 5 show a mixture of stable and unstable steady state-solutions. For instance, in Table 1,when $r = 0.6000$, 0.7500, 2.5500, 2.7000, 2.8500 and 2.9400, the corresponding stable steady-state solutions are (0.5034, 538.4405, 764.6449), (0.1522, 263.5467, 376.3818), (17.0189, 767.2685, 1021.9312), (8.3092, 668.6671, 923.3675), (5.8939, 584.8077, 819.2760) and (5.5853, 542.2873, 762.6796) respectively.

According to linear stability analysis, a steady-state solution is said to be stable if all the eigenvalues of the Jacobian have negative real parts. If at least one of the eigenvalues of the Jacobian has a positive real parts, the steady-state solution is unstable. Similarly, in Table 3 $\beta = 0.2400$, 0.2550, 0.2700 and 0.2850 have stable steady-state solutions. On the contrary, Tables 2, 4 and 6 show unstable steady-state solutions.

Conclusion

This research work carried out a stability analysis of the numerical approximation for HIV-infection of CD4+ T-cells, the steady state solution of the model was investigated and the linear stability analysis of the model was obtained.

By varying the rate of production of $CD4^{+}$Tcells, we have found a mixture of stable and unstable steady state-solutions

By varying the death rate of virus particlesγ, we found we have found a combination of stable and unstable steady state-solutions

Also by varying the per capita rate of disappearance of infected cells, a predominant unstable steady state solution was found.

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