Extension of Comparative Analysis of Estimation Methods for Frechet Distribution Parameters

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Abstract: **- Parameter estimation is very significant for any probability distribution and therefore, various estimation methods are frequently studied in the statistical literature. This research examined six methods to estimate the parameters of Frechet distribution (Generalized Maximum Likelihood Estimator, Maximum Product Spacing Estimator, L-Moment Estimator and Method of Moment Estimator). These methods were compared using Bias, Mean Square Error, Mean Absolute Error and Variance criteria as applied to Nigeria Maximum Annual Rainfall (2010-2015). Simulation study was carried out with simulated data set at different sample sizes and different levels of the shape and scale parameters. The simulation study and analysis revealed that the Generalized Maximum Likelihood (GML) Estimation was the best estimation method in terms of the Mean square Error, Mean Absolute Error and Variance; while Maximum Product Spacing Estimation method was the best estimation method with real life data.**

Keywords: **Parameter Estimation, Generalized Maximum Likelihood Estimator, Maximum Product Spacing Estimator, L-Moment Estimator and Method of Moment Estimator**

I. INTRODUCTION

The distribution of extremes such as flood peaks, maximum daily rainfall, maximum daily wind speeds, or minimum daily returns over annual or other time intervals is of common interest to many disciplines including the natural and social sciences. A comprehensive review of the analysis of hydrological extremes is given in Katz et al. [2002] and provides a general introduction to the analysis of extreme values. Murphy (2012) considered the modeling of extremes in insurance, finance, and quantitative risk management. However, for many analyses, there are often limited data available on extremes and so fitting an extreme value distribution can lead to difficulties, particularly for the estimation of extreme quantiles. In such cases, there is a need to better understand how well the asymptotic properties of the parameter estimators of an extreme value distribution, and its estimated quantiles, approximate those in small to medium-size samples. Parameter estimation is very significant for any probability distribution and therefore, various estimation methods are frequently studied in the statistical literature. In the last decade, several authors have compared different estimation methods for different distributions, among whom are: Ramos and Louzada (2016), Ramos et al. (2017); Ramos et al. (2018) for the generalized weighted Lindley distribution, Teimouri et al. (2013) for the Weibull distribution. Traditional estimation methods such as the Maximum Likelihood Estimator (MLE), Method of Moments Estimator (MME) and Least Squares Estimator (LSE) are often opted for parameter estimation, though each has its own merits and demerits. This research considers four methods to estimate the parameters of Frechet distribution. They are: Generalized Maximum Likelihood Estimator (GMLE), Maximum Product Spacing Estimator (MPS), L-Moment Estimator (LME) and Method of Moment Estimator (MOM). The research will attempt to compare the four aforementioned estimators for the two-parameter Frechet distribution, with a view to providing better estimation procedure for the Frechet distribution.

1.2 Frechet Distribution

The Frechet Distribution is a continuous distribution that is used to model maximum values in a data set. It is a special case of the generalized extreme value (GEV) distribution, named after a French mathematician, Maurice Rene Frechet (1878-1973). The Frechet distribution was introduced in 1927 as a possible limit distribution for the largest order statistic (i.e. as a potential asymptotic distribution for the maximum value of a sample distributed according to some other distribution), also known as the extreme value distribution of type II. The Frechet Distribution is an important distribution used for modelling and analyzing several extreme events and real-world phenomena ranging from accelerated life testing to earthquakes, wind speeds, sea currents, rainfall and floods, horse racing, queues in supermarkets, track race records and so on. The Frechet Distribution can be represented in two ways in terms of the parameters of interest to be estimated.

(i) Frechet Distribution [α, β] represents the Frechet Distribution with the shape parameter α and scale parameter β, and

(ii) Frechet Distribution [α, β, μ] represents the Frechet Distribution with the shape parameter α, scale parameter β and location parameter μ

Thus, the Probability Density Function (PDF) and the Cumulative Distribution Function (CDF) of the Frechet Distribution are expressed as follows:

Let the random variable X follow Frechet distribution. That is, X ~ Frechet (α, β). Then, the Probability Density Function (PDF) and the Cumulative Distribution Function (CDF) of Frechet distribution are respectively given by

$$f(x\backslash\alpha,\beta)=\frac{\alpha}{\beta}\left({}^{\beta}/_{x}\right)^{\alpha+1}exp^{-(\beta/x)^{\alpha}},x>0 \quad (1)$$

$$F(x\backslash\alpha,\beta)=exp^{-\left({}^{\beta}/_{x}\right)^{\alpha}},x>0, \quad (2)$$

where α > 0 is the shape parameter and β > 0 is the scale parameter respectively.

Equations (1) and (2) can be re-parameterized as equation 3 and 4 respectively:

$$f(x\backslash\alpha,\beta)=\alpha\beta x^{-(\beta+1)}exp^{-\alpha x^{-\beta}},x>0 \quad (3)$$

$$F(x\backslash\alpha,\beta)=exp^{-\alpha x^{-\beta}},x>0 \quad (4)$$

Considering the second representation, that is, Frechet Distribution [α, β, μ]

$$f(x\backslash\alpha,\beta,\mu)=\alpha\beta[(x-\mu)^{-(\beta+1)}]exp^{-\alpha[(x-\mu)^{-\beta}]},\ x>0 \quad (5)$$

$$F(x\backslash\alpha,\beta,\mu)=exp^{-\alpha[(x-\mu)^{-\beta}]}\quad ,x>0 \quad (6)$$

where α > 0 is the shape parameter, β > 0 is the scale parameter and μ is the location parameter respectively. The mean and the variance of Frechet distribution are respectively given as

$$E(X)=\mu+\beta\Gamma\left(1-\frac{1}{\alpha}\right)\quad for\ \alpha>1,\qquad \infty\quad otherwise \quad (7)$$

$$V(X)=\beta^{2}(\Gamma\left(1-\frac{2}{\alpha}\right)-\left(\Gamma\left(1-\frac{1}{\alpha}\right)\right)^{2}\text{for } \alpha>2,\qquad \infty\quad otherwise \quad (8)$$

The Frechet distribution has wide applications in different spheres such as: Accelerated life testing, Natural calamities, Horse racing, Rainfall, Queues in the supermarkets, Sea currents, Wind speeds, Track race records

This research centered on studying how the different estimators of the unknown parameters of a Frechet distribution can behave for different sample sizes and for 21 different parameter values. Here, we are mainly comparing the Generalized Maximum Likelihood Estimator (GMLE), Maximum Product Spacing Estimator (MPS), L-Moment Estimator (LME) and Method of Moment Estimator (MOM) with respect to efficiency, bias, mean absolute error and variance using extensive simulation techniques and R. One can use Newton-Raphson method to solve the non-linear equations.

II. METHODOLOGY

2.1. Introduction

The methods of estimation considered for comparison in this research was studied with respect to efficiency, bias, mean absolute error and variance using extensive simulation techniques and R on both real life data and simulated data. The data for this study was a secondary data on annual maximum rainfall in Nigeria over the period 2010-2015 extracted online from the World Bank site. R Statistical software was used to analyze the real life data which was then compared to the simulated data to ascertain the performance of various estimation methods for Frechet distribution.

2.2. Methods of Estimation for Frechet distribution

2.2.1. Generalized Maximum Likelihood Estimator (GLME)

The GML method is based on the same principle as the ML Method but with an additional constraint on the shape parameter, to eliminate potentially invalid values of this parameter.

As indicated by Gupta and Kundu (2001); Harlow (2002); Mubarak(2011) the GML method is a particular case of the Bayesian approach, where the prior distribution is only specified for the shape parameter. An important advantage, of the use of the GML method, is the possibility to integrate any additional information, such as historical and regional information, to define the prior distributions.

Suppose that the shape parameter α of the GEV distribution with probability density function $f(x\backslash\alpha,\beta)$ is a random variable whose range is $[k_L,k_U]$ with prior density$\pi(\alpha)$, the joint density (or the generalized-likelihood function) is computed as $\mathrm{GL}(\beta,\alpha\backslash x)=L(\beta,\alpha\backslash x)\pi(\alpha)$, which shows the relationship between the generalized-likelihood (GL) function and the likelihood function.

. Thus, $\ln\lfloor GL(\beta,\alpha\backslash x)\rfloor=-nln(\alpha)+\sum_{i=1}^{n}\left[\left(\frac{1}{\alpha}-1\right)\ln(xi)-\right.$ (xi)^$1\alpha+\ln[\pi(\alpha)]$. (9)

The generalized maximum likelihood estimator (GMLE) of j, a, and k can be identified by maximizing the generalized log-likelihood function, which corresponds to the mode of the Bayesian posterior distribution of the parameter [Alkasabeh and Ragab, 2009]. The generalized-likelihood estimators are being computed using Newton-Raphson method.

2.2.2. Maximum Product Spacing Estimator (MPS)

The maximum product spacing (MPS) method is a powerful alternative to ML method for estimating the unknown parameters of continuous univariate distributions.

Maximum Product Spacing method (MPS), is a general method of estimating parameters in continuous univariate distributions, especially for cases where Maximum Likelihood method fails. It gives consistent estimators with asymptotic efficiency equal to MLE estimators when these exist.

Let $D_i(\alpha,\beta) = F\left(x_{\frac{(i)}{\alpha},\beta}\right) - F\left(\frac{x_{(i-1)}}{\alpha},\beta\right), for\ i = 1,2,\dots,n+1,$ (10)

be the uniform spacings of a random sample from Frechet distribution, where

$F\left(x_{(0)/\alpha,\beta}\right) = 0.\ F\left(x_{(n+1)/\alpha,\beta}\right) = 1$ and $\sum_{i=1}^{n+1} D_{i(\alpha,\beta)=1}$. The maximum product spacings estimators $\hat{\alpha}_{MPS}$ and $\hat{\beta}_{MPS}$ are obtained by maximizing the geometric mean of the spacings

$G(\alpha,\beta) = [\prod_{l=1}^{n+1} D_i(\alpha,\beta)]^\wedge\frac{1}{n+1}$ with respect to α and β, or equivalently, by maximizing the logarithm of the geometric mode of sample spacings

$$H(\alpha,\beta) = \frac{1}{n+1}\sum_{i=1}^{n+1}\log D_{i(\alpha,\beta)} \quad (11)$$

The estimators $\hat{\alpha}MPS$ and $\hat{\beta}MPS\ of\ the\ parameters\ \alpha\ and\ \beta$ can be obtained by solving the following nonlinear equations

$$\frac{\delta(\alpha,\beta)}{\delta\alpha} = \frac{1}{n+1}\sum_{i=1}^{n}\frac{1}{D_{i(\alpha,\beta)}}\left[\eta 1(x_{(i)/\alpha,\beta)-\eta 1(x_{(i-1)/\alpha,\beta}}\right] = 0 \quad (12)$$

$$\frac{\delta H(\alpha,\beta}{\delta\beta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\beta)}\left[\eta 2\left(x_{(i)}/\alpha,\beta\right) - \eta 2(x_{(i-1)/\alpha,\beta)})\right] = 0 \quad (13)$$

MPS has some desirable properties such as asymptotic efficiency and invariance, hence the MPS estimators are asymptotically normally distributed.

2.2.3. L-Moments Estimator (LME)

Hosking [1990] proposed an alternative method of estimation analogous to conventional moments, namely L-moments estimators. L-moments estimators can be obtained by equating the sample with the population L-moments. Hosking [1990] stated that the L-moment estimators are more robust than the usual moment estimators, and also relatively robust to the effects of outliers and reasonably efficient when compared to the MLE for some distributions. For the Frechet distribution, the L-moments estimators can be obtained by equating the first two sample L-moments with the corresponding population L-moments. The first two sample L-moments are

-$l1 = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$ and $l2 = \frac{2}{n(n-1)}\sum_{i=1}^{n}(i-1)x_{(i)} - l1$ respectively

Where $x_{(1)}, x_{(2),\ \dots x_{(n)}}$ denotes the order statistics of a random sample from a distribution function, $F(x/\alpha,\beta)$n

The first two population L-moments are

$$\mu 1(\alpha,\beta) = \int_0^1 Q(p/\alpha,\beta)dp = E(X/\alpha,\beta) = \alpha\frac{1}{\beta}\Gamma(1-\frac{1}{\beta}) \quad (14)$$

and

$$\mu 2(\alpha,\beta) = \int_0^1 Q(p/\alpha,\beta)(2p-1)dp = \alpha\frac{1}{\beta}(2\frac{1}{\beta}-1)\Gamma(1-\frac{1}{\beta}) \quad (15)$$

Where Q (p/α,β) is given as

$$Q(p\backslash\alpha,\beta) = (\frac{1}{\alpha}\log\left(\frac{1}{pi}\right)) \quad (16)$$

The estimator for $\hat{\beta}LME$ can be obtained as

$$\hat{\beta}LME = \frac{\log(2)}{\log(2)+\log(\sum_{i=1}^{n}(i-1)x_{(i)})-\log(n(n-1)x)} \quad (17)$$

2.2.4. Method of Moments Estimator (MME)

The method of moments is one of the oldest methods used for estimating the parameters of the statistical models. Let $X_{1,X_{2,\dots,X_n}}$ be a sample from Frechet (α,β) distribution. The first two moments about origin are given by

$$(\acute{\mu_1}) = E(X) = \mu + \beta\Gamma(1-{}^1/_\alpha) \quad (18)$$

and

$$(\acute{\mu_2}) = E(X^2) = \beta^2(\Gamma\left(1-\frac{2}{\alpha}\right) - (\Gamma\left(1-\frac{1}{\alpha}\right))^\wedge 2) \quad (19)$$

The sample moments are given by

$$(\acute{m_1}) = \frac{1}{n}\sum_{i=1}^{n} X_i \quad (20)$$

and

$$(\acute{m_2}) = \frac{1}{n}\sum_{i=1}^{n} X_{I^2} \quad (21)$$

Thus, by method of moments, we have

$$(\acute{m_1}) = (\acute{\mu_1}) \quad (21)$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} X_{i=\beta\Gamma(1-\frac{1}{\alpha})} \quad (22)$$

and $$(\acute{m_2}) = (\acute{\mu_2})$$

$$\Longrightarrow \frac{1}{n}\sum_{i=1}^{n} X_{i^2} = \beta^2(\Gamma\left(1-\frac{2}{\alpha}\right) - \left(\Gamma\left(1-\frac{1}{\alpha}\right)\right)^2 \quad (23)$$

Solving for α and β, we get the method of moment estimators $(\acute{\alpha})ME$ and $(\hat{\beta})ME$

2.3. Simulation Study

Simulation is an important method of analysis which is easily verified, communicated and understood. Across industries and disciplines, simulation modeling provides valuable solutions by giving clear insights into complex systems. This simulation study was conducted to compare the accuracy of the Generalized Maximum Likelihood Method of Estimation, Maximum Product Spacing Method of Estimation, L Moment Method of Estimation and Method of Moment estimation approaches with respect to estimating the parameters of generalized extreme value distribution type II (Frechet). To this end, we simulated samples from the generalized extreme value distribution type II (Frechet) for variety of combinations of sample sizes and parameter values. The dimension used throughout the simulation study is two (2), that is, the parameters of interest are two (shape =α and scale =β). For each combination, the simulation process was replicated 1000 times. For each combination, we also estimated the parameters in the six methods mentioned above. We then selected the best method on the basis of Bias, Mean Absolute Error (MAE), Mean Square Error (MSE) and Variance.

The simulations are structured as follows:

(i) We generated N=1000 samples of sizes n = 15, 20, 30, 50, 100, 150, 200, 250 and 300 from Frechet. The parameters were selected in three perspectives; α = β, α < β, α > β. For α = β, the values are (3.5, 5, 6). For α > β, the combinations α, β = (2, 0.5), (3, 1.5), (3.5, 2), (4.5, 4). For α < β, the combinations α, β = (3.5, 3. 8), (5.5, 4.8)

(ii) Using the observed and the predicted values of α and β compute

a. $BIAS = \frac{\sum_{i=1}^{n}(y_i - x_i)}{N}$

b. $MSE = \frac{1}{N}\sum_{i=1}^{n}(\mu_i - \hat{y}_i)^2$

c. $MAE = \frac{\sum_{i=1}^{n}|y_i - x_i|}{N}$

d. Variance = MSE + BIAS^2

III. DATA ANALYSIS AND RESULTS

The results of the simulation study carried out are given as follows:

Note: Values in bold fonts are the least and hence the best for each of the categories.

Performance of parameter estimation methods in terms of **Bias** as the sample size and parameter dimension varied.

Table 1: Performance of estimation methods via Bias for α = 2, β = 0.5

Parameters	GMLE	MPS	LMOM	MOM
	0.1413	0.147	0.1258	1.3414
	0.1204	0.1208	0.107	1.2364
α = 2	0.0906	0.089	0.0785	1.1824
	0.0657	0.0611	0.0544	1.2008
	0.0405	0.0308	0.0269	1.3165
	0.0348	0.0217	0.0188	1.3743
β = 0.5	0.0339	0.0169	0.0153	1.3974
	0.0288	0.0148	0.0129	1.4669
	0.0269	0.0144	0.0132	1.4885

Table 2: Performance of estimation methods via Bias for α = 3, β = 1.5

Parameters	GMLE	MPS	LMOM	MOM
	0.0906	0.214	0.1209	1.6018
	0.0713	0.1702	0.0879	1.495
ρεϖισεα = 3	**0.0564**	0.1375	0.075	1.4299
	0.0491	0.1003	0.0536	1.4181
	0.0332	0.0565	0.0268	1.4785
	0.0209	0.0428	0.0187	1.5065
β = 1.5	0.0139	0.0351	0.0182	1.5152
	0.0135	0.0311	0.0127	1.5539
	0.0124	0.0285	0.0112	1.5615

Table 3: Performance of estimation methods via Bias for $\alpha = 3.5, \beta = 2$

Parameters	GMLE	MPS	LMOM	MOM
	0.0902	0.2507	0.1031	1.6292
	0.1083	0.1976	0.0698	1.5193
$\alpha = 3.5$	0.0947	0.1628	**0.0625**	1.4497
	0.0886	0.1204	**0.0448**	1.4293
	0.0576	0.0693	**0.0196**	1.4734
	0.0405	0.0531	**0.0136**	1.4954
$\beta = 2$	0.0308	0.0438	0.0143	1.5017
	0.0251	0.0388	0.0097	1.5347
	0.0224	0.0351	0.0089	1.5408

Table 4: Performance of estimation methods via Bias for $\alpha = 4.5, \beta = 4$

Parameters	GMLE	MPS	LMOM	MOM
	0.195	0.4719	0.1025	2.5155
	0.2386	0.3619	0.0568	2.4546
$\alpha = 4.5$	0.1847	0.3105	**0.0639**	2.4494
	0.1557	0.2332	**0.0496**	2.4821
	0.1092	0.1377	**0.0153**	2.5818
	0.0755	0.1074	**0.0111**	2.6343
$\beta = 4$	0.0557	0.0894	**0.0148**	2.6465
	0.0432	0.0791	**0.0102**	2.7041
	0.0376	0.0702	**0.0108**	2.718

Table 5: Performance of estimation methods via Bias for $\alpha = 3.5, \beta = 3.8$

Parameters	GMLE	MPS	LMOM	MOM
	0.1659	0.4176	0.1233	2.6196
	0.1895	0.321	0.0776	2.5651
$\alpha = 3.5$	0.1507	0.2736	**0.0775**	2.5708
	0.1281	0.2045	**0.0582**	2.6195
	0.0848	0.1201	**0.0222**	2.7482
	0.0573	0.0935	**0.0158**	2.8116
$\beta = 3.8$	0.0417	0.0778	**0.0182**	2.8279
	0.0317	0.0689	**0.0127**	2.8958
	0.0278	0.0614	**0.0126**	2.912

Table 6: Performance of estimation methods via Bias for $\alpha = 3.5, \beta = 3.5$

Parameters	GMLE	MPS	LMOM	MOM
	0.1531	0.3901	0.1199	2.4545
	0.1774	0.3004	0.0763	2.3908
$\alpha = 3.5$	0.1413	0.2551	**0.075**	2.3839
	0.1215	0.1906	**0.056**	2.4212
	0.0802	0.111	**0.0217**	2.5357
	0.0545	0.0866	**0.0154**	2.5922
$\beta = 3.5$	0.0398	0.0721	**0.0176**	2.6068
	0.0307	0.0639	**0.0122**	2.669
	0.0269	0.057	**0.012**	2.6835

For the above level of parameters, the L-Moment has the least value of Bias for almost all the sample sizes implying that in terms of bias, L-Moment is the best estimator.

Performance of parameter estimation method in terms of **Mean Square Error** [MSE] as the sample size and parameter dimension varied.

Table 7: Performance of estimation methods via MSE for $\alpha = 2, \beta = 0.5$

	Total					
Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.0715	0.1726	0.1227	0.1787	0.1222	9.5058
	0.0452	0.1187	0.0907	0.1231	0.0901	7.4084
$\alpha = 2$	**0.0311**	0.0623	0.0646	0.0674	0.0639	7.4296
	0.0179	0.0304	0.0397	0.0328	0.039	9.2576
	0.0225	**0.0136**	0.0231	0.0143	0.0225	9.6235
	0.0399	**0.0087**	0.0171	0.009	0.0165	7.8168
$\beta = 0.5$	0.0427	**0.007**	0.0138	0.0071	0.0134	6.4742
	0.0359	**0.0054**	0.0115	0.0056	0.0113	5.9692
	0.0356	**0.0044**	0.0099	0.0045	0.0096	5.41

Table 8: Performance of estimation methods via MSE for $\alpha = 3, \beta = 1.5$

	Total					
Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.2303	0.3085	0.2972	0.3692	0.2958	8.0257
	0.1514	0.211	0.2097	0.2418	0.2084	6.3802
$\alpha = 3$	**0.0998**	0.123	0.1413	0.1454	0.1403	5.4957
	0.0611	0.0663	0.0826	0.0775	0.0817	5.2218
	0.0352	**0.0314**	0.0424	0.0351	0.0419	4.8852
	0.0241	**0.0198**	0.0286	0.0218	0.0282	4.304
$\beta = 1.5$	0.0195	**0.0155**	0.0218	0.0169	0.0216	3.8454
	0.0128	**0.0123**	0.0175	0.0134	0.0173	3.7439
	0.0107	**0.0102**	0.0145	0.011	0.0143	3.5386

Table 9: Performance of estimation methods via MSE for $\alpha = 3.5, \beta = 2$

	Total					
Parameters	**GMLE**	**MLE**	**PWM**	**MPS**	**LMOM**	**MOM**
	0.3588	0.4263	0.4306	0.5278	0.429	7.8102
	0.2459	0.294	0.3033	0.344	0.3018	6.1855
$\alpha = 3.5$	**0.156**	0.1745	0.2017	0.2104	0.2008	5.1303
	0.0892	0.0963	0.1169	0.1143	0.1161	4.609
	0.046	0.0462	0.0587	0.0523	0.0583	4.1631
	0.0287	0.0291	0.0389	0.0325	0.0387	3.7102
$\beta = 2$	**0.0218**	0.0226	0.0294	0.0249	0.0292	3.3666
	0.0174	0.0181	0.0235	0.0198	0.0234	3.3029
	0.0145	0.015	0.0194	0.0163	0.0194	3.1451

Table 10: Performance of estimation methods via MSE for $\alpha = 4.5, \beta = 4$

	Total					
Parameters	**GMLE**	**MLE**	**PWM**	**MPS**	**LMOM**	**MOM**
	1.5659	1.5927	1.782	2.0975	1.7762	21.6561
	1.0561	1.1055	1.2502	1.3574	1.2454	18.103
$\alpha = 4.5$	**0.6805**	1.1921	0.8219	0.8546	0.8193	15.6423
	0.3941	**0.3919**	0.4733	0.4777	0.4711	13.7699
	0.1936	0.3361	0.2339	0.2214	0.2331	12.1839
	0.1214	**0.1208**	0.153	0.137	0.1525	10.927
$\beta = 4$	**0.0922**	0.0931	0.1144	0.1042	0.1141	10.0066
	0.0747	0.075	0.0925	0.0833	0.0923	9.8595
	0.0622	0.0623	0.0762	0.0687	0.076	9.4273

Table 11: Performance of estimation methods via MSE for $\alpha = 3.5, \beta = 3.8$

	Total					
Parameters	**GMLE**	**MLE**	**PWM**	**MPS**	**LMOM**	**MOM**
	1.173	1.2214	1.3883	1.6014	1.3829	23.9546
	0.798	0.8446	0.9697	1.0327	0.9648	20.1394
$\alpha = 3.5$	**0.5045**	1.1873	0.6411	0.6496	0.6381	17.9031
	0.2909	0.2979	0.3705	0.3621	0.368	16.3937
	0.1423	0.6326	0.1839	0.1675	0.1827	14.9239
	0.09	0.0913	0.1205	0.1034	0.1198	13.3329
$\beta = 3.8$	**0.0698**	0.0705	0.0904	0.0789	0.0899	12.1072
	0.0564	0.057	0.0727	0.0632	0.0723	11.8873
	0.0469	0.0472	0.06	0.052	0.0597	11.3239

Table 12: Performance of estimation methods via MSE for $\alpha = 3.5, \beta = 4.8$

	Total					
Parameters	**GMLE**	**MLE**	**PWM**	**MPS**	**LMOM**	**MOM**
	2.2135	2.2179	2.4208	2.9183	2.4155	19.6822
	1.534	23.1837	1.7087	1.8981	1.7045	16.2422
$\alpha = 3.5$	**0.9799**	1.3265	1.1161	1.1946	1.1143	13.5379
	0.561	**0.5473**	0.6401	0.6684	0.6387	11.3631
	0.2729	**0.2699**	0.3158	0.3109	0.3155	9.6737
	0.0426	**0.0426**	0.0517	0.0504	0.0516	2.4416
$\beta = 4.8$	0.1303	**0.1302**	0.1541	0.146	0.1539	7.9723
	0.1052	**0.1048**	0.1255	0.1165	0.1253	7.8678
	0.0876	**0.0873**	0.1031	0.0961	0.1029	7.5361

Table 13: Performance of estimation methods via MSE for $\alpha = 3.5, \beta = 3.5$

	Total					
Parameters	**GMLE**	**MLE**	**PWM**	**MPS**	**LMOM**	**MOM**
	1.0014	1.0553	1.1874	1.3765	1.1828	20.568
	0.6836	0.7296	0.8299	0.8881	0.8257	17.2123
$\alpha = 3.5$	**0.4308**	0.9637	0.5489	0.5574	0.5464	15.2237
	0.2481	0.2556	0.3173	0.3104	0.3151	13.9216
	0.1214	0.3193	0.1576	0.1426	0.1566	12.6666
	0.0767	0.0783	0.1034	0.0885	0.1028	11.3144
$\beta = 3.5$	**0.0595**	0.0605	0.0776	0.0676	0.0772	10.2736
	0.0482	0.0488	0.0624	0.0541	0.0621	10.0865
	0.0401	0.0405	0.0515	0.0445	0.0512	9.6082

Table 14: Performance of estimation methods via MSE for $\alpha = 5 \ \beta = 5$

	Total					
Parameters	**GMLE**	**MLE**	**PWM**	**MPS**	**LMOM**	**MOM**
	1.8261	1.8423	1.979	2.4053	1.9749	15.181
	1.2686		1.399	1.5671	1.396	12.3501
$\alpha = 5$	**0.8135**	1.0286	0.913	0.9843	0.9118	10.095
	0.4635	**0.4507**	0.5231	0.5498	0.5222	8.3609
	0.2264	**0.2221**	0.2582	0.2557	0.258	7.0454
	0.1406	**0.14**	0.1689	0.1588	0.1688	6.2927
$\beta = 5$	0.1074	**0.1073**	0.1261	0.1201	0.126	5.7964
	0.0866	**0.0863**	0.1028	0.0956	0.1026	5.7194
	0.0722	**0.0717**	0.0844	0.079	0.0843	5.4775

Table 15: Performance of estimation methods via MSE for $\alpha = 6, \beta = 6$

	Total					
Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	2.8552	2.9122	3.0338	3.7761	3.0308	15.0272
	2.0275	36.9186	2.1601	2.4761	2.1584	11.9537
$\alpha = 6$	1.3169	**1.2423**	1.4005	1.5534	1.4006	9.3286
	0.7449	**0.7099**	0.7989	0.8682	0.799	7.2484
	0.367	**0.3516**	0.3951	0.4053	0.3955	5.755
	0.2301	**0.2222**	0.2588	0.2523	0.2591	5.0536
$\beta = 6$	0.1714	**0.1695**	0.1929	0.1897	0.193	4.6419
	0.138	**0.1361**	0.1582	0.1511	0.1581	4.5666
	0.115	**0.1134**	0.1298	0.1248	0.1297	4.3553

For the above level of parameters, the GMLE has the least value of Mean square error followed by the MLE, implying that in terms of mean square error, GMLE is the best estimator.

Performance of parameter estimation method in terms of **Mean Absolute Error** as the sample size and parameter dimension varied.

Table 16: Performance of estimation methods via MAE for $\alpha = 3, \beta = 1.5$

	Total					
Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.4796	0.6162	0.5746	0.6554	0.5728	1.9069
	0.3886	0.5046	0.4898	0.5301	0.488	1.7392
$\alpha = 3$	**0.3133**	0.3899	0.4028	0.4111	0.4011	1.6103
	0.2403	0.2844	0.3129	0.3012	0.3113	1.5575
	0.1764	0.1947	0.2262	0.2042	0.2249	1.5868
	0.1452	0.1542	0.1858	0.1603	0.1848	1.6033
$\beta = 1.5$	**0.1309**	0.1364	0.163	0.1416	0.1623	1.6016
	0.1159	0.1215	0.1472	0.1248	0.1466	1.6359
	0.1071	0.1109	0.1342	0.1147	0.1335	1.6396

Table 17: Performance of estimation methods via MAE for $\alpha = 2, \beta = 0.5$

	Total					
Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.264	0.4211	0.376	0.4386	0.3746	1.5752
	0.215	0.3433	0.3264	0.3573	0.3249	1.444
$\alpha = 2$	**0.1776**	0.2611	0.274	0.2723	0.2721	1.3642
	0.1395	0.1872	0.2184	0.1948	0.216	1.3646
	0.1121	0.1265	0.1657	0.1307	0.1634	1.4582
	0.1005	0.1008	0.1399	0.1032	0.1377	1.4912
$\beta = 0.5$	0.0951	**0.0896**	0.125	0.0912	0.1234	1.4945
	0.0863	**0.0791**	0.1152	0.0803	0.1136	1.5471
	0.0815	**0.0723**	0.1056	0.0736	0.1041	1.5568

Table 18: Performance of estimation methods via MAE for $\alpha = 3.5, \beta = 2$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.6148	0.7128	0.6701	0.7628	0.6687	1.9983
	0.5104	0.5844	0.5697	0.6161	0.5682	1.8079
$\alpha = 3.5$	**0.4046**	0.4532	0.4657	0.4793	0.4644	1.65
	0.2987	0.3321	0.3598	0.3536	0.3587	1.5697
	0.2138	0.2284	0.2572	0.2404	0.2564	1.5735
	0.1715	0.1806	0.2097	0.1884	0.209	1.5832
$\beta = 2$	**0.1519**	0.1593	0.1836	0.1662	0.1831	1.5781
	0.1367	0.1423	0.1657	0.1466	0.1653	1.607
	0.1261	0.1298	0.1507	0.1347	0.1502	1.6085

Table 19: Performance of estimation methods via MAE for $\alpha = 4.5, \beta = 4$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	1.1802	1.2497	1.2214	1.3584	1.2195	3.1883
	0.9756	1.0255	1.0341	1.0924	1.032	2.9556
$\alpha = 4.5$	**0.789**	0.8279	0.8397	0.8599	0.838	2.7598
	0.5872	0.5978	0.6469	0.6445	0.6457	2.6568
	0.407	0.4269	0.4567	0.4402	0.4559	2.6822
	0.3215	0.3268	0.37	0.3438	0.3695	2.7172
$\beta = 4$	**0.2814**	0.2873	0.3224	0.3024	0.3219	2.7156
	0.2549	0.2586	0.2916	0.2683	0.2913	2.7692
	0.2341	0.236	0.2652	0.247	0.2647	2.7783

Table 20: Performance of estimation methods via MAE for $\alpha = 3.5, \beta = 3.8$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	1.0202	1.1167	1.0949	1.2115	1.0925	3.1733
	0.8429	0.9163	0.9268	0.9738	0.9243	2.9765
$\alpha = 3.5$	**0.6709**	0.7439	0.755	0.7655	0.7528	2.8313
	0.4992	0.5331	0.5836	0.5724	0.5818	2.7784
	0.3517	0.3988	0.4141	0.3911	0.4129	2.8501
	0.2808	0.2907	0.3367	0.3054	0.3357	2.9
$\beta = 3.8$	**0.2483**	0.2558	0.2937	0.269	0.2929	2.9043
	0.2247	0.2303	0.2652	0.2385	0.2646	2.9681
	0.2066	0.2101	0.2416	0.2195	0.2409	2.9797

Table 21: Performance of estimation methods via MAE for $\alpha = 3.5, \beta = 4.8$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	1.3986	1.4362	1.3992	1.5665	1.398	3.2217
	1.1721	1.3586	1.1855	1.2605	1.1842	2.9386
$\alpha = 3.5$	0.9494	**0.9489**	0.9597	0.9941	0.9588	2.6754
	0.707	**0.6894**	0.7367	0.7462	0.7362	2.5041
	0.4836	**0.4799**	0.5177	0.5094	0.5174	2.4669
	0.0928	**0.0915**	0.1011	0.0993	0.1012	0.6351
$\beta = 4.8$	**0.3305**	0.3315	0.3637	0.3497	0.3636	2.473
	0.2984	0.2986	0.3301	0.3104	0.3299	2.5171
	0.2736	**0.2725**	0.2998	0.2856	0.2997	2.5231

Table 22: Performance of estimation methods via MAE for $\alpha = 3.5, \beta = 3.5$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.9523	1.0501	1.0241	1.1369	1.0219	2.9775
	0.7885	0.8614	0.8673	0.9141	0.865	2.7818
$\alpha = 3.5$	**0.6264**	0.6968	0.7068	0.7179	0.7048	2.6344
	0.4658	0.4995	0.5463	0.5361	0.5446	2.5769
	0.3279	0.3592	0.388	0.3654	0.3868	2.6374
	0.262	0.2723	0.3155	0.2859	0.3146	2.6805
$\beta = 3.5$	**0.2321**	0.2398	0.2754	0.2519	0.2746	2.6832
	0.2101	0.2156	0.2486	0.2232	0.2481	2.7413
	0.1932	0.1967	0.2265	0.2054	0.2258	2.7512

Table 23: Performance of estimation methods via MAE for $\alpha = 5, \beta = 5$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	1.2922	1.3233	1.2805	1.4407	**1.2796**	2.9121
	1.0851	1.2909	1.0855	1.16	**1.0845**	2.6302
$\alpha = 5$	0.8814	**0.8705**	0.8786	0.9139	0.878	2.365
	0.656	**0.6331**	0.6738	0.6848	0.6735	2.1872
	0.448	**0.4404**	0.4735	0.4672	0.4733	2.1332
	0.3497	**0.3473**	0.3823	0.3656	0.3822	2.1385
$\beta = 5$	0.3046	**0.3044**	0.3327	0.3207	0.3326	2.1265
	0.2745	**0.274**	0.3019	0.2844	0.3019	2.162
	0.2517	**0.25**	0.2741	0.2619	0.2741	2.1656

Table 24: Performance of estimation methods via MAE for $\alpha = 6, \beta = 6$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	1.5901	1.6083	1.5495	1.7547	**1.5493**	3.0593
	1.3504	1.5439	1.3152	1.4127	**1.3148**	2.7097
$\alpha = 6$	1.1063	**1.0402**	1.061	1.116	1.0612	2.3614
	0.8299	**0.7711**	0.8101	0.8381	0.8106	2.0885
	0.5705	**0.5382**	0.5686	0.572	0.569	1.9567
	0.4449	**0.425**	0.4572	0.448	0.4575	1.9298
$\beta = 6$	0.3816	**0.3716**	0.3974	0.3917	0.3977	1.9061
	0.3423	**0.3342**	0.3621	0.3478	0.3623	1.9333
	0.3132	**0.3052**	0.3288	0.3201	0.3289	1.9328

For the above level of parameters, the GMLE has the least value of mean absolute error followed by the MLE implying that in terms of mean absolute error, GMLE is the best estimator. The Performance of parameter estimation method in terms of **Variance** as the sample size and parameter dimension varied.

Table 25: Performance of estimation methods via Variance for $\alpha = 2, \beta = 0.5$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.0549	0.1705	0.1007	0.167	0.0997	8.5003
	0.0346	0.1166	0.0758	0.1148	0.0748	6.4322
$\alpha = 2$	**0.0247**	0.0616	0.0549	0.0632	0.0539	6.3455
	0.0146	0.0302	0.0344	0.0308	0.0335	7.9592
	0.021	**0.0135**	0.0208	0.0138	0.0201	7.9291
	0.0391	**0.0086**	0.0158	0.0088	0.0152	5.948
$\beta = 0.5$	0.042	**0.007**	0.0128	0.007	0.0124	4.5551
	0.0354	**0.0054**	0.011	0.0054	0.0106	3.8709
	0.0353	**0.0044**	0.0095	0.0044	0.0092	3.2623

Table 26: Performance of estimation methods via Variance for $\alpha = 3, \beta = 1.5$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.2259	0.2991	0.2879	0.3438	0.2863	6.5702
	0.1467	0.2029	0.2044	0.2266	0.2029	4.933
$\alpha = 3$	**0.0978**	0.1201	0.1378	0.1347	0.1369	3.9255
	0.0599	0.0653	0.0809	0.0715	0.0801	3.465
	0.0347	**0.0311**	0.0419	0.033	0.0414	2.8324
	0.0239	**0.0197**	0.0283	0.0205	0.028	2.103
$\beta = 1.5$	0.0194	**0.0154**	0.0216	0.0159	0.0213	1.6043
	0.0127	**0.0123**	0.0173	0.0126	0.0171	1.3669
	0.0106	**0.0102**	0.0144	0.0104	0.0143	1.1231

Table 27: Performance of estimation methods via Variance for $\alpha = 3.5, \beta = 2$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.3522	0.4096	0.4237	0.4892	0.4219	6.3196
	0.2376	0.28	0.2994	0.3214	0.2979	4.7076
$\alpha = 3.5$	**0.1513**	0.1695	0.1994	0.1936	0.1985	3.537
	0.0853	0.0946	0.1157	0.1049	0.115	2.8486
	0.0442	0.0456	0.0584	0.049	0.058	2.1536
	0.0278	0.0289	0.0388	0.0304	0.0386	1.5705
$\beta = 2$	**0.0213**	0.0225	0.0292	0.0235	0.0291	1.1947
	0.017	0.018	0.0235	0.0186	0.0234	1.0151
	0.0143	0.0149	0.0194	0.0154	0.0193	0.8237

Table 28: Performance of estimation methods via Variance for $\alpha = 4.5, \beta = 4$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	1.5306	**1.5084**	1.7759	1.9243	1.7701	17.4788
	1.0143	1.0352	1.2473	1.2609	1.2425	13.6203
$\alpha = 4.5$	**0.6602**	1.1659	0.8196	0.7786	0.8171	10.5997
	0.3815	0.3833	0.4719	0.4339	0.4699	8.1549
	0.1875	0.3327	0.2337	0.2055	0.2329	5.8457
	0.1186	0.1197	0.1529	0.1271	0.1525	4.1995
$\beta = 4$	**0.0907**	0.0925	0.1142	0.0972	0.1139	3.1916
	0.0738	0.0747	0.0923	0.0779	0.0921	2.7093
	0.0615	0.0621	0.0761	0.0645	0.0758	2.1761

Table 29: Performance of estimation methods via Variance for $\alpha = 3.8, \beta = 3.5$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	1.1483	1.161	1.3802	1.4729	1.3745	19.3036
	0.7688	0.7943	0.9656	0.961	0.9607	15.1375
$\alpha = 3.8$	**0.4902**	1.1684	0.638	0.5935	0.635	12.2603
	0.2822	0.2918	0.3687	0.3299	0.3663	10.0684
	0.1386	0.6293	0.1835	0.1558	0.1824	7.6778
	0.0883	0.0906	0.1204	0.096	0.1197	5.6112
$\beta = 3.5$	**0.0689**	0.0702	0.0901	0.0738	0.0897	4.2689
	0.0558	0.0568	0.0726	0.0591	0.0723	3.6297
	0.0465	0.0471	0.0598	0.0489	0.0596	2.9443

Table 30: Performance of estimation methods via Variance for $\alpha = 3.5, \beta = 4.8$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	2.1586	**2.0924**	2.4167	2.6692	2.4112	16.1514
	1.4706		1.7063	1.7591	1.702	12.4668
$\alpha = 3.5$	**0.954**	1.2877	1.1147	1.0849	1.1129	9.2953
	0.5428	**0.5343**	0.6391	0.6052	0.6378	6.6622
	0.2634	0.2655	0.3156	0.2879	0.3152	4.443
	0.0423	0.0425	0.0517	0.0492	0.0516	2.0734
$\beta = 4.8$	**0.1274**	0.1294	0.1539	0.136	0.1538	2.3754
	0.1033	0.1044	0.1253	0.1087	0.1252	2.0152
	0.0862	0.087	0.1029	0.0901	0.1028	1.6012

Table 31: Performance of estimation methods via Variance for $\alpha = 3.5, \beta = 3.5$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	0.9806	1.0043	1.1796	1.2667	1.1747	16.58
	0.6583	0.687	0.8259	0.8269	0.8217	12.9496
$\alpha = 3.5$	**0.4186**	0.9477	0.546	0.5095	0.5434	10.4304
	0.2405	0.2504	0.3157	0.2828	0.3136	8.5539
	0.1181	0.3172	0.1573	0.1327	0.1563	6.519
	0.0752	0.0776	0.1033	0.0823	0.1027	4.7636
$\beta = 3.5$	**0.0587**	0.0602	0.0773	0.0632	0.077	3.624
	0.0476	0.0487	0.0623	0.0506	0.0621	3.0812
	0.0398	0.0404	0.0514	0.0419	0.0512	2.4995

Table 32: Performance of estimation methods via Variance for $\alpha = 5, \beta = 5$

Parameters	GMLE	MLE	PWM	MPS	LMOM	MOM
	1.7809	**1.7398**	1.975	2.2002	1.971	12.5218
	1.2174		1.3968	1.4521	1.3937	9.5562
$\alpha = 5$	**0.7918**	0.9967	0.9117	0.8938	0.9107	6.9827
	0.4472	**0.44**	0.5223	0.4978	0.5215	4.9244
	0.2176	0.2185	0.258	0.2369	0.2579	3.234
	0.1359	0.1386	0.1688	0.147	0.1688	2.2607
$\beta = 5$	**0.1044**	0.1065	0.1259	0.1119	0.1259	1.7217
	0.0847	0.0859	0.1026	0.0893	0.1025	1.4605
	0.0707	0.0715	0.0842	0.0742	0.0843	1.1583

For the above level of parameters, the GMLE has the least value of variance followed by the MLE implying that in terms of variance, **GMLE** is the best estimator.

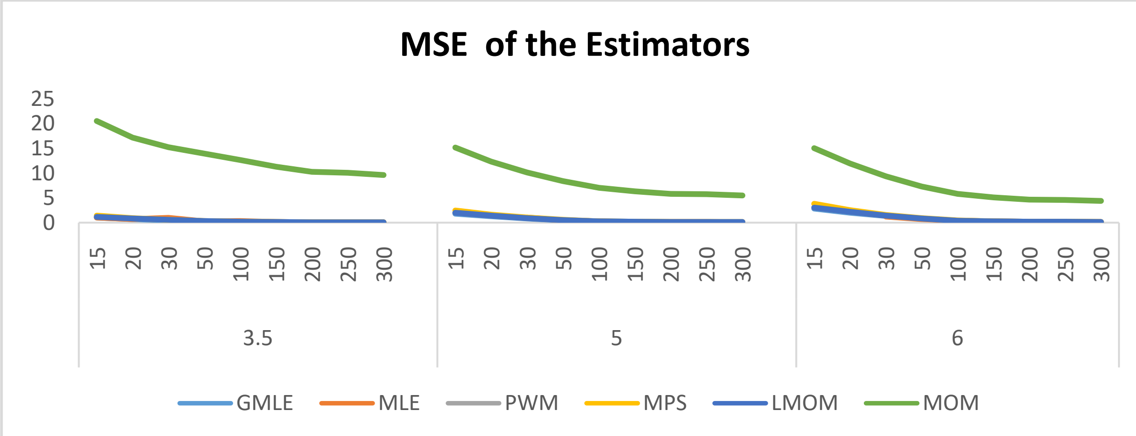

Figure 1: MSE of the estimates of α = β = 3.5, α = β = 5 and α = β = 6 for N = 1000 simulated samples, considering different values of **n** using the following estimation methods (GMLE, MLE, PWM, MPS, LMOM and MOM)

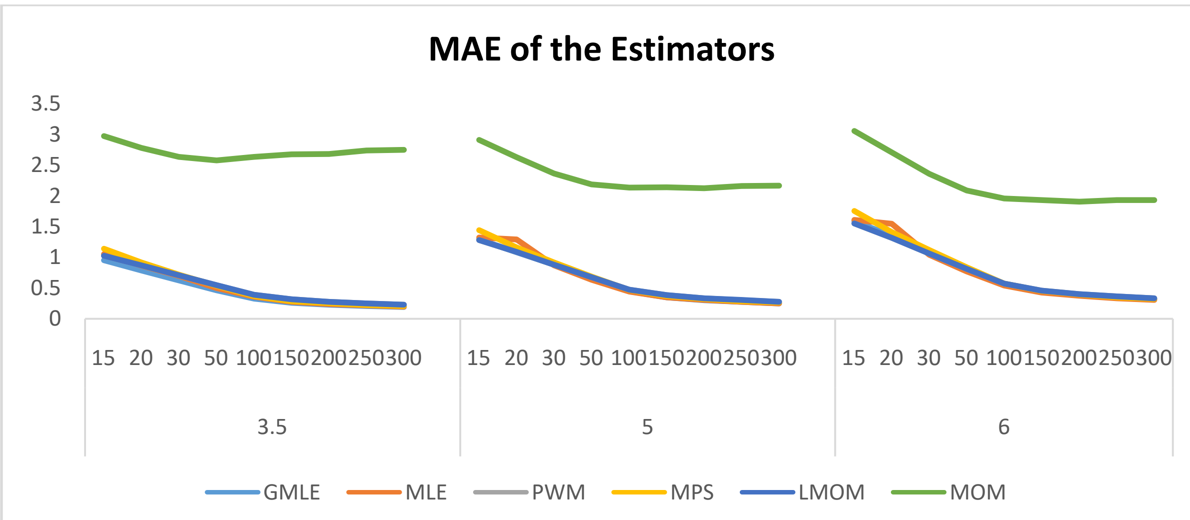

Figure 2: MAE of the estimates of α = β = 3.5, α = β = 5 and α = β = 6 for N = 1000 simulated samples, considering different values of n using the following estimation methods (GMLE, MLE, PWM, MPS, LMOM and MOM)

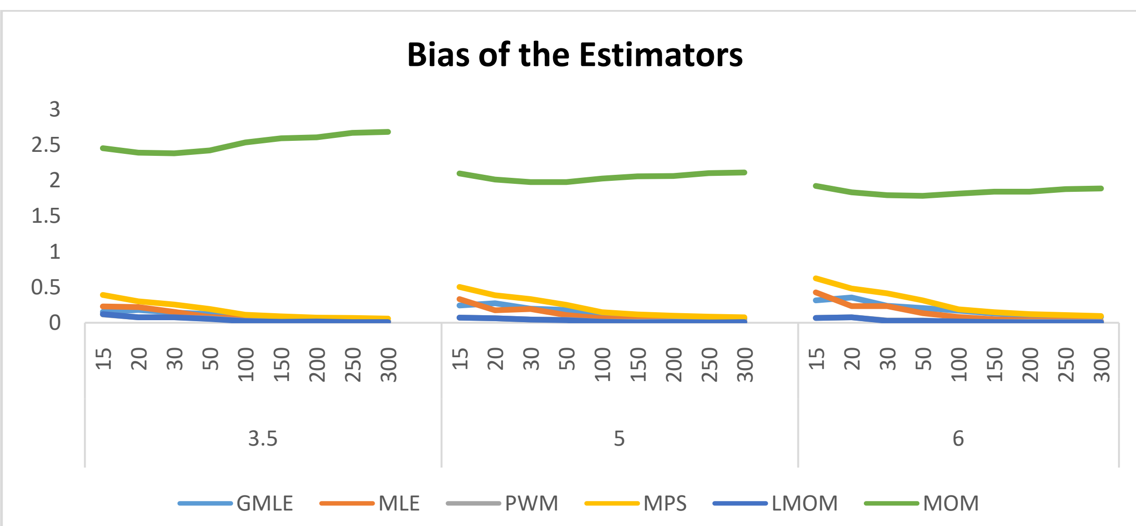

Figure 3: Bias of the estimates of α = β = 3.5, α = β = 5 and α = β = 6 for N = 1000 simulated samples, considering different values of n using the following estimation methods (GMLE, MLE, PWM, MPS, LMOM and MOM)

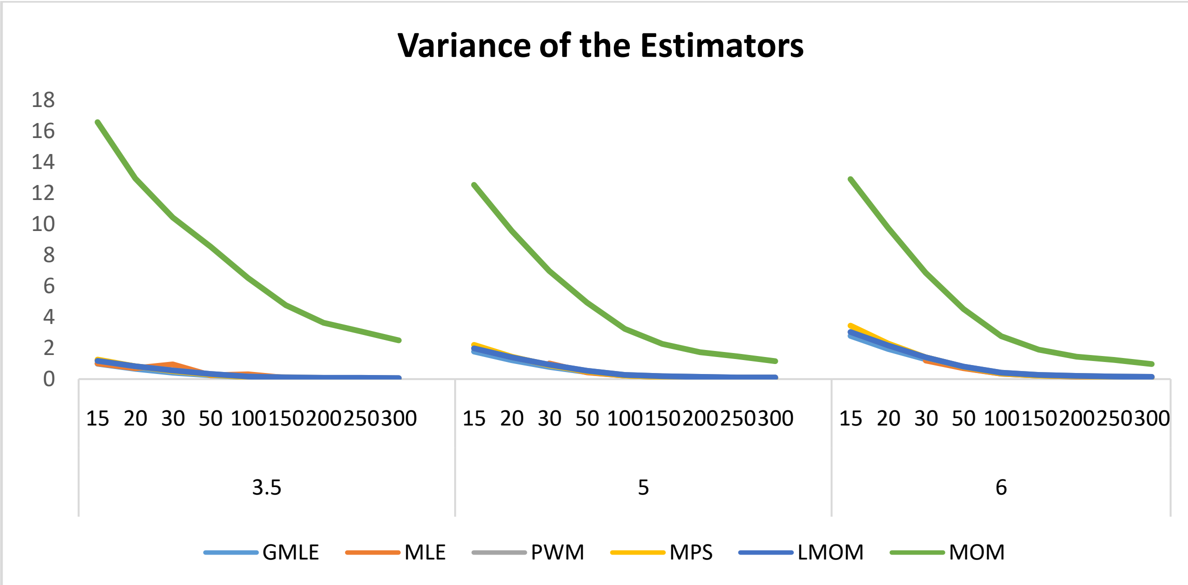

Figure 4: Variance of the estimates of α = β = 3.5, α = β = 5 and α = β = 6 for N = 1000 simulated samples, considering different values of n using the following estimation methods (GMLE, MLE, PWM, MPS, LMOM and MOM)

Performance of the estimators

Table 33: Frequency of Estimators performance

	Count					
Criterion	**GMLE**	**MLE**	**PWM**	**MPS**	**LMOM**	**MOM**
Bias	4	19	17	0	**42**	0
MSE	**50**	31	0	0	0	0
MAE	**55**	22	0	0	4	0
VAR	**61**	20	0	0	0	0

The best method of parameter estimation was based on the modal class for each of the four performance measures considered (Bias, M.S.E, M.A.E. and Variance). We therefore discovered that, Generalized Maximum Likelihood Estimators [GMLE] is the best method of estimation for all the performance measures for Frechet distribution considered except for the bias performance measure where L-moment outperformed other estimators.

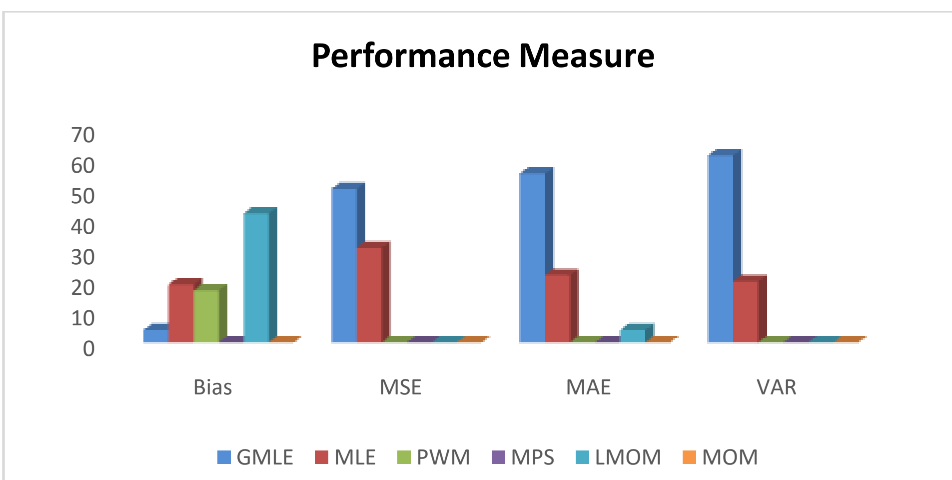

Figure 5: Histogram Plot

The Histogram plot above depicts the performance measure graphically. The conclusion made above is also the same as the one on the plot above. From the Histogram plot, the generalized maximum likelihood estimator outperformed the other estimation methods (it has the highest peak) in terms of MSE, MAE and Variance while, L-MOM is the best estimator in terms or bias (it has the highest peak).

Real Life Data Analysis Result

Fitting Generalized extreme value distribution type II (GEV II) Model to Nigeria annual rainfall (2010-2015) and testing the data for goodness of fit for GEV II distribution, using the Kolmogorov Smirnov test on easyfit software, the results of the analysis shows that the data is a good fit for GEV II distribution.

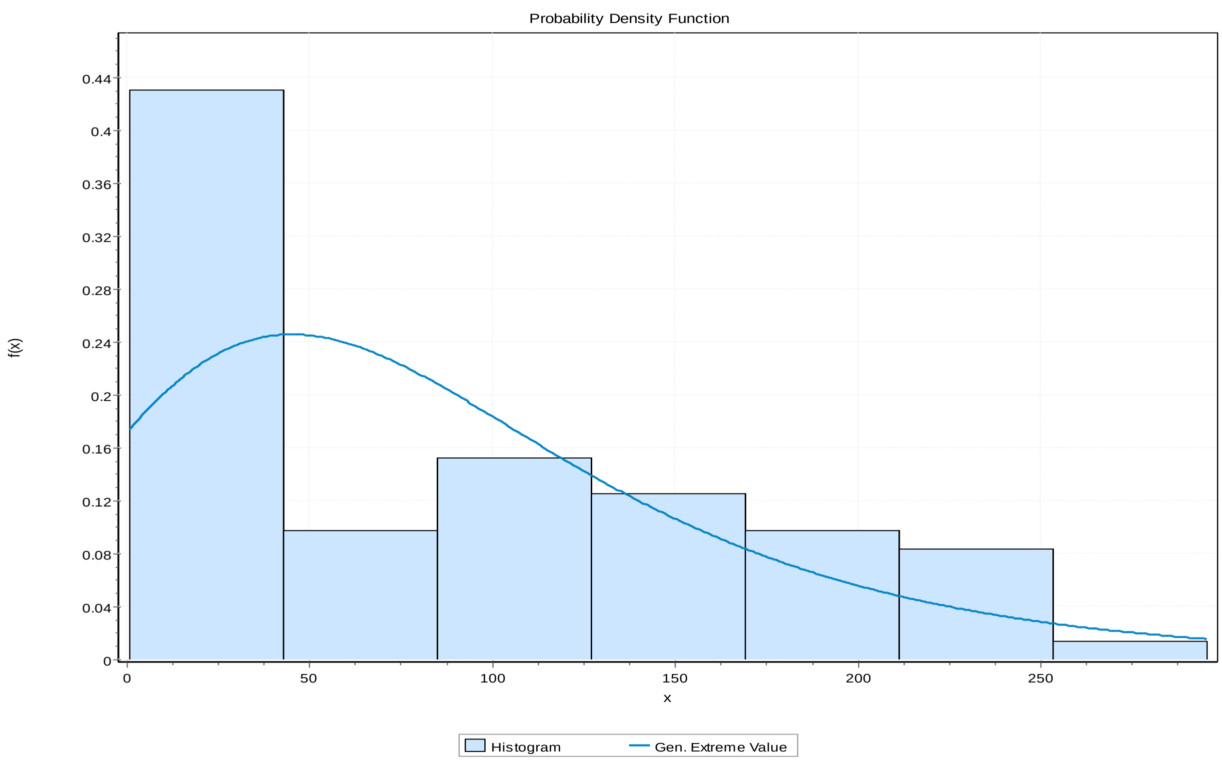

Figure 1: Probability density function plot showing the shape of the GEV II distribution.

Parameter estimates.

The criteria for judgment shall be standard error. The method with the minimum standard error is crowned the best estimation method.

Table 34: The top 3 estimators with minimum standard errors

Method	Parameters	Estimate	Standard Error
GMLE	shape (α)	0.5225	0.6593
	scale (β)	42.4419	20.6769
MLE	shape (α)	1.3077	0.2559
	scale (β)	29.8308	6.8269
MPS	shape (α)	1.2162	**0.16112**
	scale (β)	32.2082	**6.48129**

Table 34 above shows the top 3 estimators with minimum standard error, the MPS estimator has the minimum standard error for both the shape and scale parameters, we then conclude that, for the real life data, the MPS is the best estimator.

IV. CONCLUSION

We simulated samples from the generalized extreme value distribution type II (Frechet) for variety of combinations of sample sizes and parameter values. The dimension used throughout the simulation study was two (2), that is, the parameters of interest are two (shape $=\alpha$ and scale $=\beta$). For each combination, the simulation process was replicated 1000 times. For each combination, we also estimated the parameters in the six methods afore-mentioned. We then selected the best method on the basis of Bias, Mean Square Error (MSE), Mean Absolute Error (MAE), and Variance. We used Nigeria annual maximum rainfall (2010-2015) for real life data

The results of the analysis showed that Generalized Maximum Likelihood Estimation (GMLE) is the best estimation method for Frechet distribution parameters in terms of Mean Square Error (MSE), Mean Absolute Error (MAE) and Variance, though L-Moment performed better in terms of Bias, having frequency of 42 as summarized in table 33 above.

Table 34 above shows the top 3 estimators with minimum standard error. It is clear that the Maximum Product Spacing (MPS) estimator has the least standard errors 0.16112 and 6.48129 for both the shape (α) and scale parameters (β), we then conclude that, for the real life data, the Maximum Product Spacing (MPS) is the best estimation method for Frechet distribution parameters. Fitting Generalized extreme value distribution type II (GEV II) Model to Nigeria maximum annual rainfall (2010-2015) and testing the data for goodness of fit for GEV II distribution, using the Kolmogorov Smirnov test on easyfit software, the results of the analysis showed that the p-value 0.18903 was greater than the theoretical value 0.05 at 0.01 level of significance, that is, $p=0.18903 > 0.05$ at $\alpha= 0.01$. Hence, there was 99% confidence that the Nigeria annual maximum rainfall data is a good fit for Frechet distribution

REFERENCES

[1] Alkasasbeh, M.Z. and Raqab, M.Z. (2009). Estimation of the generalized logistic distribution parameters: Comparative study, Statistical Methodology 6, pp. 262–279.

[2] Gupta, R.D. and D. Kundu, D. (2001). Generalized exponential distribution: different method of estimations, Journal of Statistical Computation and Simulation 69, pp. 315–337.

[3] Harlow, D.G. (2002). Applications of the Frechet distribution function. International Journal of Material and product technology, 5(17):482-495.

[4] Hobert, J.P .and Casella, G. (1996). The effect of improper priors on gibbs sampling in hierarchical linear mixed models, Journal of the American Statistical Association 91, pp. 1461– 1473.

[5] Hosking, J.R. (1990). L-moments: analysis and estimation of distributions using linear combinations of order statistics, Journal of the Royal Statistical Society. Series B (Methodological), pp. 105–124.

[6] Katz, S. and Nadarajah, S. (2002). Extreme value distributions: theory and applications, World Scientific.

[7] Mubarak, M. (2011). Parameter estimation based on the Frechet progressive type II censored data with binomial removals. International Journal of Quality, Statistics and Reliability, 2012, Article ID 245910.

[8] Murphy, K.P. (2012). Machine learning: a probabilistic perspective, MIT press, R Core Team, R: A Language and Environment for Statistical Computing, R Foundation for Statistical Computing, Vienna, Austria (2017). Available at https://www. R-project.org/.

[9] Ramos, P. and Louzada, F. (2016). The generalized weighted lindley distribution: Properties, estimation and applications, Cogent Mathematics 3.

[10] Ramos, P., Nascimento, D. and Louzada, F. (2017). The long term Frechet distribution: Estimation, properties and its application, BiomBiostatInt J 6, p. 00170.

[11] Ramos, P.L., Louzadz, Francisco; Ramos, Eduardo and Dey, Sanku (2018). The Frechet distribution: Estimation and Application an overview. Ar Xiv: 1801.05327v1 [Stat.AP]

[12] Teimouri, M., Hoseini, S.M. and Nadarajah, S. (2013). Comparison of estimation methods for the weibull distribution, Statistics 47, pp. 93–109. climateportal@worldbank.org