Analytical Expression of Surface Wave Elevation Using Homotopy Analysis Method

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***Abstract*---This work focuses on the study of water wave equations which were solved by means of an analytic technique, namely the Homotopy Analysis Method (HAM). HAM is a capable and straight forward analytic tool for solving nonlinear differential equations and does not require small/large parameters in the governing equations unlike other well-known analytic approach such as the perturbation method. Using HAM, we obtain an approximate solution to the governing Rogue wave equations. The free surface displacement $\eta(x,t)$ and velocity potential, $\phi(x,z,t)$ obtained are compared with similar results using higher order Stokes approximations.**

Key words: rogue wave, surface displacement, velocity potential, Homotopy Analysis Method

I. INTRODUCTION

1. Rogue Wave Event

Rogue waves (also known as freak waves, giant waves, killer waves, monster waves, and extreme waves) are large water waves with large amplitude that often surprisingly appear and disappear from nowhere, on the sea surface sometime unexpected. These large waves are often associated with ocean wave current, energy focusing [1]. These giant waves are thus, threat to ships, ocean liners and onshore engineering structures. As stated in [2], the rogue waves have been noticeable part of marine problem for centuries.

[3-5] showed that the second order analytical models for the prediction of extreme event can also be derived by mean of the theory of quasi-determinism. The authors began from the general second order Stokes expansion (using perturbation method) of the surface displacement for long-crested waves, and should that the crest of the non-linear crest (trough) depends upon the initial crest (trough) amplitude.

[6] showed that the number of rogue waves and their cause differs spatially and note that each location is likely to have its own unique sensitivities which increase in the coastal seas and they concluded that forecast able predictors of rogue wave occurrence will need to be location specific, reflecting their cause.

[7] investigated the steady condition for the nonlinear interaction of two trains of propagating wave in deep water and obtained the solution for both resonant and non-resonant cases. By means of the analytical method called homotopy analysis method (HAM) developed in [7-11] a powerful analytic method for highly nonlinear problems. [12] considered the Hirota equation with fractional and integer-order time derivatives respectively, and derived their exact or approximate analytic rogue wave solution using homotopy analysis method (HAM).

In this work, we apply the homotopy analysis method (HAM) to the basic equations governing the dynamics of water waves to derive the mechanism for rogue wave generation.

II.BASIC IDEA OF HOMOTOPY ANALYSIS METHOD (HAM)

In this work, we use the homotopy analysis method (HAM) to solve the nonlinear partial differential equations. This method was proposed by a Chinese mathematician [Liao 1992]. We apply Liao's basic ideas to the nonlinear differential equation. Here, the nonlinear boundary-value problem governed by the PDEs (1-4) is solved by means of the homotopy analysis method.

To overcome the restrictions of perturbation methods and some additional non-perturbation techniques [7-11] developed an analytic technique for highly nonlinear problem, namely the homotopy analysis method (HAM). HAM gives us great freedom in the choice of the initial guess, the equation type of linear sub problems and basic functions of solution.

III. THE MATHEMATICAL DESCRIPTION

3.1 The Basic Equations Governing The Dynamics Of Water Waves

We first introduce the basic equations governing the dynamics of water waves. If we consider the nonlinear interaction of a train of progressive gravity waves of finite depth and assume that the fluid is inviscid and incompressible, the flow is irrotational and the surface tension is neglected. The x-axis positive in the direction of wave propagation and the z-axis points vertically upward from the still- water level, the problem is steady and is periodic in the x- variable. Let the vertical free surface displacement be $\eta(x,t)$ and the velocity potential $\phi(x,z,t)$. Both the velocity potential $\phi(x,z,t)$ and free surface displacement $\eta(x,t)$ have to satisfy the Laplace equation.

$$\nabla^2\phi(x,z,t)=0 \quad z=\eta(x,t) \tag{1}$$

The velocity potential ϕ is subject to the unknown $\eta(x,t)$ free-surface boundary conditions.

$$\frac{\partial^2\phi(x,z,t)}{\partial t^2}+g\frac{\partial\phi(x,z,t)}{\partial z}+\frac{1}{2}\frac{\partial}{\partial t}\left[\left(\frac{\partial\phi(x,z,t)}{\partial x}\right)^2+\left(\frac{\partial\phi(x,z,t)}{\partial z}\right)^2\right]-g\left(\frac{\partial\phi(x,z,t)}{\partial x}\right)\left(\frac{\partial\eta(x,t)}{\partial x}\right)=0 \tag{2}$$

$$g\eta(x,t)+\frac{\partial\phi(x,z,t)}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\phi(x,z,t)}{\partial x}\right)^2+\left(\frac{\partial\phi(x,z,t)}{\partial z}\right)^2\right]=0 \tag{3}$$

The solid boundary condition at the horizontal bottom is given as

$$\frac{\partial\phi(x,z,t)}{\partial z}=0 \text{ at } z=-d \tag{4}$$

where g is the acceleration of gravity. Although the governing equation (1) and the bottom boundary condition (4) are linear, the two nonlinear boundary conditions (2) and (3) are satisfied on the unknown free surface $\eta(x,t)$. Such nonlinear partial differential equations (PDEs) are difficult to solve in general, often with rather complicated solutions. In this work we apply the homotopy analysis method (HAM) to solve this boundary-value problem with the nonlinear conditions with an unknown free surface displacement $\eta(x,t)$.

3.3 Continuous Deformation

To solve the nonlinear boundary-value problem governed by the PDEs equations (1-4) by means of HAM, we start with the following initial approximations. Let $\phi_0(x,z,t)$, $\eta_0(x,t)$ denote the initial guesses of the velocity potential $\phi(x,z,t)$ and free surface displacement $\eta(x,t)$ respectively. Let $p\in[0,1]$ denote an embedding parameter and let h be the so-called convergence-control parameter. Here, both p and h are auxiliary parameter without physical meaning. Instead of solving the nonlinear PDEs in equation (1-4) directly, we first construct a family (with respect to p) of PDEs about two continuous deformations $\phi(x,z,t)$ and $\eta(x,t)$ governed by the so-called zero-order deformation equations,

$$\nabla^2\varphi(x,z\ t;p)=0 \qquad -d<z<\eta(x,t;p) \tag{5}$$

subject to the two boundary conditions on the unknown free surface $z=\eta(x,t:p)$

$$(1-p)L[\varphi(x,z,t;p)-\phi_0 x(x,z,t)]=phN[\varphi(x,z,t;p)] \tag{6}$$

$$(1-p)\eta(x,t;p)=ph\Re[\eta(x,t;p),\varphi(x,z,t;p)] \tag{7}$$

And the bottom condition

$$\frac{\partial\varphi(x,z,t;p)}{\partial z}=0 \text{ at } z=-d, \tag{8}$$

Where h is an auxiliary parameter and it is important to know that one has great freedom to choose auxiliary parameter h, L is an auxiliary linear operator with the property $L(0)=0$, N *and* $\Re$ are nonlinear differential operators. If we choose our nonlinear operator to be

$$N(\varphi(x,z,t;p))=\frac{\partial^2\varphi(x,z,t;p)}{\partial t^2}+g\frac{\partial\varphi(x,z,t;p)}{\partial z}+$$
$$\frac{1}{2}\frac{\partial}{\partial t}\left[\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)^2+\left(\frac{\partial\varphi(x,z,t;p)}{\partial z}\right)^2\right]-g\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)\frac{\partial\eta(x,t;p)}{\partial x} \tag{9}$$

and

$$\Re[\varphi(x,z,t;p)]=\frac{1}{g}\left(\frac{\partial\varphi(x,z,t;p)}{\partial t}-\frac{1}{2}\left[\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)^2+\left(\frac{\partial\varphi(x,z,t,p)}{\partial z}\right)^2\right]\right) \tag{10}$$

We choose the auxiliary linear operations as

$$L[\varphi(x,z,t;p)]=\frac{\partial^2\varphi(x,z,t;p)}{\partial t^2}+g\frac{\partial\varphi(x,z,t;p)}{\partial z} \tag{11}$$

Note that, the definitions of $N,\Re$ and L are based on the two boundary conditions equations (2) and (3) respectively.

So when $p=0$, the zeroth-order deformation equations (5-8) have the solution

$$\varphi(x,z,t;0)=\phi_0(x,z,t) \tag{12}$$

$$\eta(x,t;0)=0=\eta_0(x,t) \tag{13}$$

And when $p=1$, the zeroth–order deformation equations (5-8) are equivalent to the original PDEs

$$\varphi(x,z,t;1)=\phi(x,z,t) \tag{14}$$

$$\eta(x,t;1)=\eta(x,t) \tag{15}$$

Therefore, as the embedding parameter $p\in(0,1)$ increases from 0 to 1, $\varphi(x,z,t;p)$ and $\eta(x,t;p)$ very continuously from their initial guess solution $\phi_0(x,z,t)$ and $\eta_0(x,t)=0$ to the exact velocity potential $\phi(x,z,t)$ and the free surface displacement $\eta(x,t)$ respectively. So, the zero-order deformation equations (5-8) indeed construct two continuous deformation are called homotopy, expressed by

$$\varphi(x,z,t;p):\phi_0(x,z,t)\to\phi(x,z,t) \tag{16}$$

$$\eta(x,z,t;p):\eta_0(x,t)\to\eta(x,t) \tag{17}$$

But the above two continuous deformation are also dependent upon the convergence-control parameter h, which has no physical meaning but provides a convenient way to guarantee the convergence of approximations. It is important to know that one has great freedom to choose auxiliary parameter h. Infact, it is the so-called convergence- control parameter h that differentiates, the HAM from all other analytic techniques as pointed by (Liao 2012).

Using Taylors theorem in equations (12) and (13), we expand $\varphi(x,z,t;p)$ and $\eta(x,z,t;p)$ in the power series of p as follows

$$\varphi(x,z,t;p)=\phi_0(x,z,t)+\sum_{n=1}^{+\infty}\frac{\phi_0^{(n)}(x,z,t)}{n!}p^n \tag{18}$$

$$\eta(x,t;p)=\eta_0(x,t)+\sum_{n=1}^{+\infty}\frac{\eta_0^{(n)}(x,t)}{n!}p^n \tag{19}$$

Where

$$\phi_0^{(n)}(x,z,t)=\frac{\partial^n\phi(x,z,t;p)}{\partial p^n}\Big|_{p=0} \tag{20}$$

$$\eta_0^n(x,t)=\frac{\partial^n\eta(x,t;p)}{\partial p^n}\Big|_{p=0} \tag{21}$$

Using equation (14) and (15) and assuming that the convergence-control parameter h is properly chosen so that the above series are convergent at $p=1$, from (14) and (15) we have

$$\phi(x,z,t)=\phi_0(x,z,t)+\sum_{n=1}^{+\infty}\frac{\phi_0^n(x,z,t)}{n!} \tag{22}$$

$$\eta(x,t)=\eta_0(x,t)+\sum_{n=1}^{+\infty}\frac{\eta_o^n(x,t)}{n!} \tag{23}$$

The unknown term $\phi_o^{(n)}(x,z,t)$ is governed by a linear PDE, and it is straight forward to obtain $\eta_0^{(n)}(x,t)$ as long as $\phi_0^{(n-1)}(x,z,t)$ is known. In this way, the original nonlinear PDEs (2-3) are transformed into an infinite number of linear PDEs. However, unlike perturbation techniques, such transformation in the context of the HAM does not need any small physical parameters.

3.5 High Order Deformation Equation

Differentiating the zero-order deformation equations (5-8) m times with respect to p, them dividing them by m! and setting $p=0$ we have the mth-order deformation equation

$$\nabla^2\phi_m=0 \qquad -d<z<0 \tag{24}$$

Subject to the two boundary condition at $z=0$

$$\frac{\partial^m}{\partial p}\frac{1}{m!}[(1-p)L(\varphi(x,z,t;p)-\phi_0(x,z,t))=X_n phN[\varphi(x,z,t;p)]] \tag{25}$$

and

$$\frac{\partial^m}{\partial p}\frac{1}{m!}[(1-p)\eta(x,t;p)]=X_n ph\Re[\eta(x,t;p),\ \varphi(x,z,t;p)] \tag{26}$$

and the bottom condition

$$\frac{\partial\phi_0^n(x,z,t)}{\partial z}=0 \qquad z=-d \tag{27}$$

Where $X_n=\begin{cases}0 & when\quad n\le 1\\ 1 & when\quad n>1\end{cases}$ (28)

Moreover, differentiating equations (6) and (7) m times with respect to the embedding parameter at $p=0$ we obtain the respective free-surface boundary conditions defined at $z=\eta_0(x,t)$ as

$$\frac{D^nL(\varphi(x,z,t;p))}{Dp^n}\Big|_{p=0}=n\left\{X_n\frac{D^{n-1}L(\varphi(x,z,t;p))}{Dp^{n-1}}+h\frac{D^{n-1}N[\varphi(x,z,t;p)]}{Dp^{n-1}}\right\}\Big|_{p=0} \tag{29}$$

and

$$\eta_0^{(n)}(x,t)=n\left\{(X_n+h)\eta_0^{n-1}(x,t)-h\frac{D^{n-1}\Re[\varphi(x,z,t;p)]}{Dp^{n-1}}\Big|_{p=0}\right\} \tag{30}$$

where

$$\frac{D^{n-1}L[\varphi(x,z,t;p)]}{Dp^{n-1}}=\frac{D^{n-1}\left(\frac{\partial^2\varphi^n(x,z,t;p)}{\partial t^2}\right)}{Dp^{n-1}}\Big|_{p=0}+g\frac{D^{n-1}\left(\frac{\partial\varphi^n(x,z,t;p)}{\partial z}\right)}{Dp^{n-1}}\Big|_{p=0} \tag{31}$$

$$\frac{D^{n-1}N[\varphi(x,z,t;p)]}{Dp^{n-1}}=\frac{D^{n-1}\left(\frac{\partial^2\varphi^n(x,z,t;p)}{\partial t^2}\right)}{Dp^{n-1}}\Big|_{p=0}+g\frac{D^{n-1}\left(\frac{\partial\varphi^n(x,z,t;p)}{\partial z}\right)}{Dp^{n-1}}\Big|_{p=0}+$$
$$\frac{D^{n-1}}{Dp^{n-1}}\left[\frac{1}{2}\frac{\partial}{\partial t}\left[\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)^2+\left(\frac{\partial\varphi(x,z,t;p)}{\partial z}\right)^2\right]-g\left(\frac{\partial\varphi(x,z,t;p)}{\partial xbn}\right)\frac{\partial\eta(x,t;p)}{\partial x}\right] \tag{32}$$

and

$$\frac{D^{n-1}\Re(\varphi(x,z,t;p))}{Dp^{n-1}}=\frac{D^{n-1}}{Dp^{n-1}}\left[\frac{1}{g}\frac{\partial\varphi(x,z,t;p)}{\partial t}-\frac{1}{2}\left[\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)^2+\left(\frac{\partial\varphi(x,z,t;p)}{\partial z}\right)^2\right]\right] \tag{33}$$

Therefore substituting equations (31) and (32) into (29) we have

$$\left(\frac{\partial^2\phi^n(x,z,t)}{\partial t^2}\right)+g\left(\frac{\partial\phi^n(x,z,t)}{\partial z}\right)=$$
$$\left[\begin{array}{l} nx_n\left(\frac{\partial^2\phi^{(n-1)}(x,z,t)}{\partial t^2}+g\frac{\partial\phi^{(n-1)}(x,z,t)}{\partial z}\right)- \\ -h\left[\begin{array}{l}\frac{\partial^2\phi^{(n-1)}(x,z,t)}{\partial t^2}+g\frac{\partial\phi^{(n-1)}(x,z,t)}{\partial z}\\ +\frac{1}{2}\frac{\partial}{\partial t}\left[\left(\frac{\partial\phi^{n-1}(x,z,t)}{\partial x}\right)^2+\left(\frac{\partial\phi^{n-1}(x,z,t)}{\partial z}\right)^2\right]-g\frac{\partial\phi^{n-1}(x,z,t)}{\partial x}\frac{\partial\eta^{n-1}(x,t)}{\partial x}\end{array}\right]\end{array}\right] \tag{34}$$

$$\eta^n(x,t)=n\left\{(X_n+h)\eta_0^{n-1}-\frac{h}{g}\left[\frac{\partial\phi^{n-1}(x,z,t)}{\partial t}-\frac{1}{2}\left[\left(\frac{\partial\phi^{n-1}(x,z,t)}{\partial x}\right)^2+\left(\frac{\partial\phi^{n-1}(x,z,t)}{\partial z}\right)^2\right]\right]\right\} \tag{35}$$

The boundary value problem at the m[th]-order approximation is defined by the governing equation (24) and the boundary conditions (27), (34) and (35). So $\phi^n(x,z,t)$ velocity potential and $\eta^n(x,t)$ free surface displacement can be easily symbolically solved by the computer software MATHEMATICA and directly calculated from equations (34) and (35). From equation (35) we now successfully obtain the equations of $\eta^n(x,t)$ free surface displacement from zero order to higher order as

$$\eta^0 = 0 \tag{36}$$

$$\eta_1 = \frac{1}{2}A_1 h(A_1 k + 2\cos[tw-kx]) \quad \text{(where } A_1 = a\text{)} \tag{37}$$

$$\eta_2 = A_2 h\left(2(1+h-w^2)\cos[tw-kx] + k(1+h-w^4)\cos[2tw-2kx]A_2\right)$$

where $A_2 = -\frac{A_1^2 h}{4w}$ (38)

$$\eta_3 = 3A_2 h\begin{pmatrix} 2kw\cos[2tw-2kx] + 2k^3w^2A_3 \\ +(1+h)\left(2(1+h-w^2)\cos[tw-kx]A_3 + k(1+h-w^4)\cos[2tw-2kx]A_3\right) \end{pmatrix} \tag{39}$$

Where $A_3 = \frac{6h^2(2+h)A_2^2 - 6h + 8}{13w}$

$$\eta_4 = 2h\begin{pmatrix} 4kw\cos[2tw-2kx]A_3 + 8kw^2\cos[4tw-4kx]A_4 + \\ 4k^4w^2(\cos[2tw-2kx]A_3 + 2\cos[4tw-4kx]A_4)^2 + 4k^4\sin[2tw-2kx]^2(A_3 + 3w\cos[2tw-2kx]A_4)^2 + \\ +6(1+h)A_4\left(2kw\cos[2tw-2kx] + 2k^3w^2A_4 + (1+h)\begin{pmatrix} 2(1+h-w^2)\cos[tw-kx] \\ +k(1+h-w^4\cos[2tw-2kx]A_4) \end{pmatrix}\right) + \end{pmatrix} \tag{40}$$

where $A_4 = \frac{4h\left(12(2h+3h^2+h^3-kw)A_2^3 - 14\right)}{(9+4h)w}$

$$\eta_5 = 5\begin{pmatrix} 2h(h+1)\begin{pmatrix} 4kw\cos[2tw-2kx]A_4 + 8kw^2\cos[4tw-4kx]A_5 + 4k^4w^2(\cos[2tw-2kx]A_4 + 2\cos[4tw-4kx]A_5)^2 + \\ 4k^4\sin[2tw-2kx]^2(A_4 + 3w\cos[2tw-2kx]A_5)^2 \\ +6(1+h)A_5\begin{pmatrix} 2kw\cos[2tw-2kx] + 2k^3w^2A_5 + \\ (1+h)(2(1+h-w^2)\cos[tw-kw] + k(1+h-w^4\cos[2tw-2kx]A_5)) \end{pmatrix} - \\ h\begin{pmatrix} -2kw\cos[2tw-2kx]A_3 - 4kw^2\cos[4tw-4kx]A_4 - 4kw^2\cos[4tw-4kx]A_3A_4 - 8kw^3\cos[8tw-8kx]A_5 \\ +\frac{1}{2}\begin{pmatrix} -4k^4(\cos[2tw-2kx]A_3 + 2w\cos[4tw-4kx](1+A_2)A_4 + 4w^2\cos[8tw-8kx]A_5)^2 \\ +k^4(2\sin[2tw-2kx]A_3(1+3w\cos[2tw-2kx]A_4) + w\sin[4tw-4kx](3A_4 + 8w\cos[4tw-4kx]A_5))^2 \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \tag{41}$$

Where $A_5 = \frac{5h\left(12(2h+3h^2+h^3-kw)A_3^2 - 14(8h^2+32h^3-kw)A_4\right)}{(15+4h)w}$

Therefore the summation free surface displacement i.e.

$$\eta^n(x,t) = \eta^0 + \eta^1 + \eta^2 + \eta^3 + \eta^4 + \eta^5 + \ldots$$

$$= \frac{1}{2}A_1h(A_1k + 2\cos[tw-kx]) + A_2h\left(2(1+h-w^2)\cos[tw-kx] + k(1+h-w^4)\cos[2tw-2kx]A_2\right) +$$

$$3A_2h\begin{pmatrix} 2kw\cos[2tw-2kx] + 2k^3w^2A_3 \\ +(1+h)\left(2(1+h-w^2)\cos[tw-kx]A_3 + k(1+h-w^4)\cos[2tw-2kx]A_3\right) \end{pmatrix} +$$

$$2h\begin{pmatrix} 4kw\cos[2tw-2kx]A_3 + 8kw^2\cos[4tw-4kx]A_4 + \\ 4k^4w^2(\cos[2tw-2kx]A_3 + 2\cos[4tw-4kx]A_4)^2 + 4k^4\sin[2tw-2kx]^2(A_3 + 3w\cos[2tw-2kx]A_4)^2 + \\ +6(1+h)A_4\left(2kw\cos[2tw-2kx] + 2k^3w^2A_4 + (1+h)\begin{pmatrix} 2(1+h-w^2)\cos[tw-kx] \\ +k(1+h-w^4\cos[2tw-2kx]A_4) \end{pmatrix}\right) + \end{pmatrix} +$$

$$5\begin{pmatrix} 2h(h+1)\begin{pmatrix} 4kw\cos[2tw-2kx]A_4 + 8kw^2\cos[4tw-4kx]A_5 + 4k^4w^2(\cos[2tw-2kx]A_4 + 2\cos[4tw-4kx]A_5)^2 + \\ 4k^4\sin[2tw-2kx]^2(A_4 + 3w\cos[2tw-2kx]A_5)^2 \\ +6(1+h)A_5\begin{pmatrix} 2kw\cos[2tw-2kx] + 2k^3w^2A_5 + \\ (1+h)(2(1+h-w^2)\cos[tw-kw] + k(1+h-w^4\cos[2tw-2kx]A_5)) \end{pmatrix} - \\ h\begin{pmatrix} -2kw\cos[2tw-2kx]A_3 - 4kw^2\cos[4tw-4kx]A_4 - 4kw^2\cos[4tw-4kx]A_3A_4 - 8kw^3\cos[8tw-8kx]A_5 \\ +\frac{1}{2}\begin{pmatrix} -4k^4(\cos[2tw-2kx]A_3 + 2w\cos[4tw-4kx](1+A_2)A_4 + 4w^2\cos[8tw-8kx]A_5)^2 \\ +k^4(2\sin[2tw-2kx]A_3(1+3w\cos[2tw-2kx]A_4) + w\sin[4tw-4kx](3A_4 + 8w\cos[4tw-4kx]A_5))^2 \end{pmatrix} \end{pmatrix} \end{pmatrix} \end{pmatrix} \tag{42}$$

Where k is the wave number and in deep water $k = \frac{\omega^2}{g}$, ω is the frequency. The coefficient a is amplitude of the wave component.We should remember that both $\phi(x, z, t)$ velocity potential and $\eta(x,t)$ free surface displacement are dependent of $\hbar$ an auxiliary parameter which no physical meaning but it is convergence-control parameter.

The necessary condition for the series to be convergent is $|1+h|<1$ i.e., $-2 \le h < 0$

It is interesting that the convergence region of the solution series depends upon the value of h. The closer the value of h $(-2 \le h < 0)$ to zero, the larger the convergence region of the series.

Physically, we found that, the higher order deformation equations of wave elevation satisfied the definition and characteristic of Rogue wave, for a fully developed Rogue wave system.

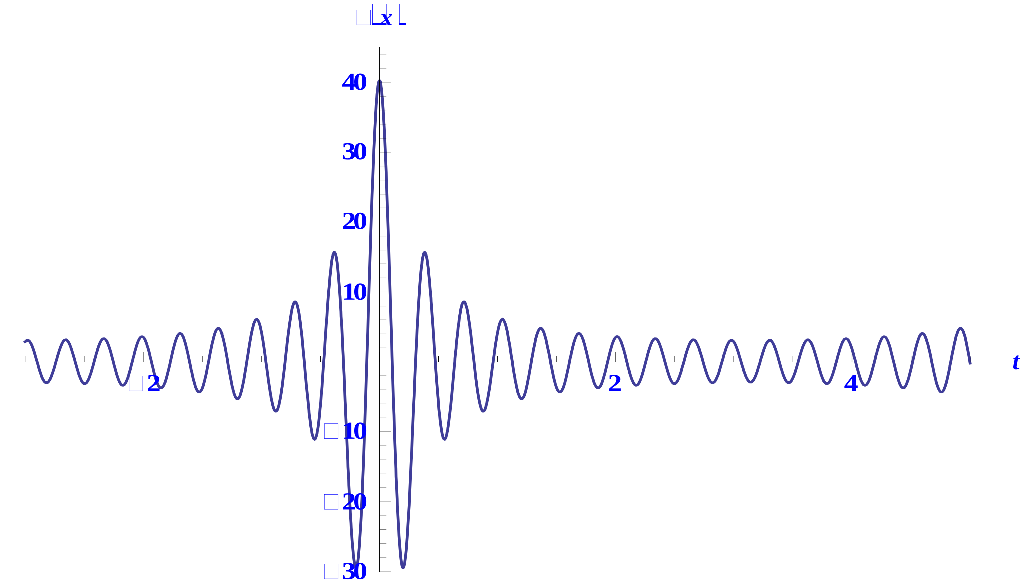

Fig 1 surface wave elevation plot at $h=-1$

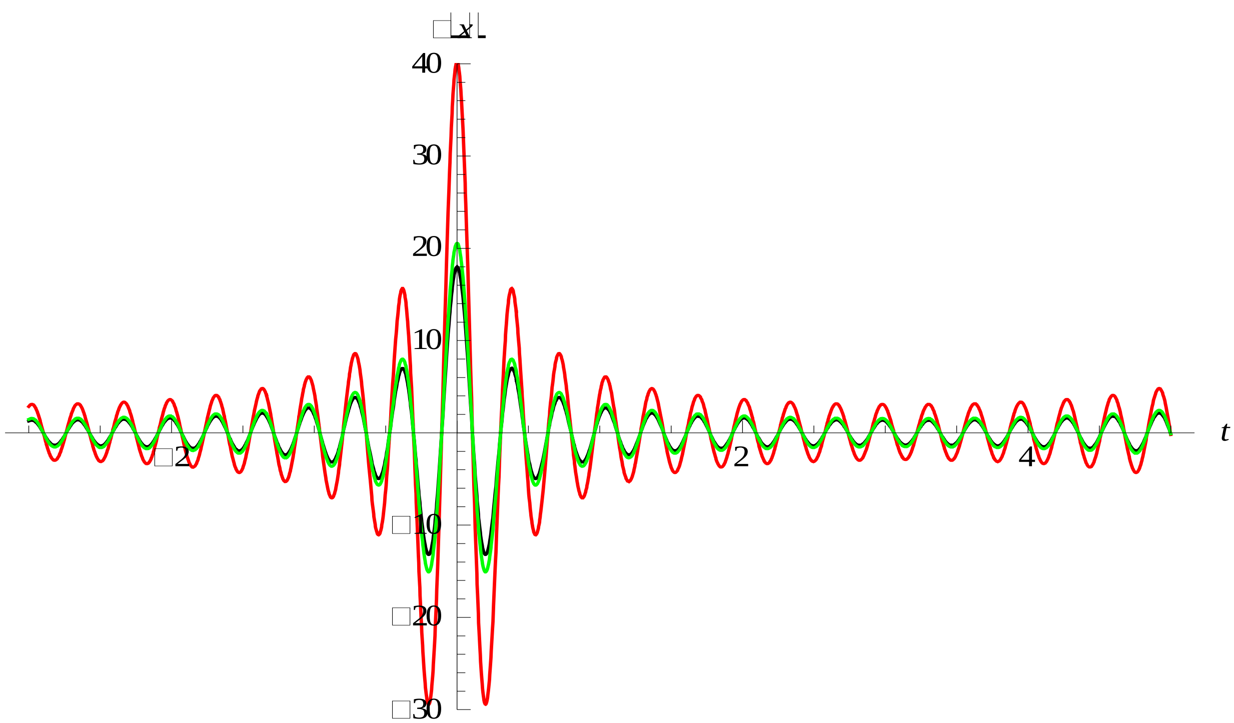

Fig 2 Comparison of the surface wave elevation

Figure 2 is the Comparison between the analytic method, numerical result based on second order QD and the laboratory generated New Year Wave (Draupner wave). Red colour; analytic method, Green colour; Numerical result and Black colour; New Year Wave.

The space-time evolution of the analytic surface wave has been studied and compared with the Draupner wave generated in a wave tank and second-order predictions of the QD (quasi determinism) theory [13]. It has been shown in fig.1 that the higher order deformation equation in equation (42) of wave profile predicts a Rogue wave, showing a wall of water appearing from nowhere. The analytic solution of the nonlinear wave group in time domain at different point close to x=0 has been carried out using equation (42). It shows that x=0 designates the location where the largest wave occurs, and symmetrical profile is obtained. Fig 2 demonstrates the comparison between the analytic methods, laboratory generated Draupner New Year Wave and second order free surface QD theory (Petrova et al. 2011). From the plot of the surface wave we concluded that the real Draupner wave cannot be the largest wave during the actual sea. We have shown that the analytic method can be used in describing the wave groups either in the space domain at any fixed time or in the time domain at any fixed point. The illustrative examples suggest that HAM is an efficient and exact method for nonlinear problems in many areas of science and engineering.

V. CONCLUSION

Our interest in this investigation is to extend this expansion to higher order and observe the behaviour of surface profile. Firstly we start with zero order term of this solution of expression to the high order using HAM. Our aim is to study it's effects in relation to observed wave height for rogue wave event. The deduction of higher order terms are not the interesting part of it, but it's geophysical impart it will made on the development of rogue wave phenomenon.

The usual appearance of waves with extreme high crest and deep trough can be initiated by the local intercrossing of a large number of quasi-monochromatic wave group with differing phases, wave numbers and the wave current interaction are effective mechanism capable of initiating rogue wave event. In this consideration and at the initial state of wave development in deep water short wave group are at the front of long wave group (wave packet). As time involves, the longer wave components with higher group velocity will overtake the shorter one. Consequently, the longer wave components will extract energy from the shorter components, thus it will grow in size. This study will eventually lead to significant understanding of the Evolution of this rogue wave event.

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