Numerical Solutions for Free Convection Flow past a Vertical Cone using Alternating Direction Implicit (ADI) Technique

A. Kaushik

Department of Mathematics, MAIT, Delhi, India

***Abstract*- In the present study, an unsteady flow of a viscous incompressible fluid past a vertical cone with variable viscosity and thermal conductivity is studied. The dimensionless form of the governing equations for the flow is taken for the study. Alternating –Direction-Implicit (ADI) Technique is used to obtain numerical solutions of the non-linear differential equations governing flow and heat transfer. Flow parameters are obtained and are presented graphically. It is observed that velocity and temperature of fluid vary considerably with viscosity parameter and thermal conductivity parameter**

***Keywords*- vertical cone, variable viscosity, thermal conductivity, Alternating Direction Implicit technique.**

I. INTRODUCTION

For past many decades, the study of free convection flows has been receiving the attention of researchers due to its wide applications in engineering and science. Extensive analytical and experimental studies are conducted for the problem of free convection under varied conditions along inclined plane as well as along vertical cone ([1]-[5]). All these studies were conducted with the assumption that the viscosity as well as the thermal conductivity of the fluid is constant throughout the flow. However, various researches are available to study the flow behavior with variations of the fluid viscosity and thermal conductivity with temperature. Elbashbeshy [6] used shooting method to study a flow of a viscous incompressible fluid along a heated vertical plate, with variations of the viscosity and thermal diffusivity with temperature in the presence of a magnetic field. Seddeek [7] investigated a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the presence of radiation and variable viscosity. Hassanien et al. [8] studied the effect of variable viscosity and thermal conductivity on combined heat and mass transfer in mixed convection over a UHF/UMF wedge in porous media. Abo-Eldabah [9] studied a free convective steady laminar boundary layer flow in the presence of radiation with variations in temperature-dependent density, viscosity and thermal conductivity. Soundalgekar et al. [10] studied the effect of different parameters on an incompressible viscous fluid flow past a continuously moving semi-infinite plate with variable viscosity and variable temperature. Seddeek & Salem[11] presented similarity solutions for laminar mixed convection adjacent to vertical continuously stretching sheets, in the presence of variable viscosity and variable thermal diffusivity.

Assuming thermal diffusivity as a linear function of temperature, Seddeek and Abdelmeguid [12] studied the effects of radiation and thermal diffusivity on heat transfer over a stretching surface with a variable heat flux. Mahmoud [13] studied the flow and heat transfer of an incompressible viscous electrically conducting fluid over a continuously moving vertical infinite plate, with uniform suction and heat flux in the presence of radiation, taking into account the effects of variable viscosity. Saleh M. Al-Harbi [14] analyzed the effect of variable viscosity and thermal conductivity on the flow and heat transfer of electrically conducting viscous fluid on a continuously stretching surface. He assumed thermal conductivity and viscosity to vary as linearly and inverse linear functions of temperature, respectively. Elgazery[15] used Chebyshev pseudospectral method to study magneto-micropolar fluid flow, heat and mass transfer with suction and blowing through a porous medium in the presence of chemical reaction, Hall, ion-slip currents, variable viscosity and variable thermal diffusivity. Assuming fluid viscosity and thermal diffusivity as linear functions of temperature, Mukhopadhyay [16] studied the unsteady boundary layer flow and heat transfer of a fluid towards a porous stretching sheet. Using shooting method numerical solutions were obtained for the problem.

Palani and Kim [17] used Crank- Nicholson scheme to study free convection over an isothermal vertical plate immersed in a fluid with variable viscosity and thermal conductivity. Husnain et al [18] analyzed an unsteady boundary-layer flow with heat and mass transfer characteristics of a viscous fluid through porous media in the presence of fluid suction or blowing taking place at the surface. Animasaun [19] studied the effects of thermophoresis, Dufour, temperature dependent thermal conductivity and viscosity of an incompressible electrically conducting Casson fluid flow along a vertical porous plate in the presence of viscous dissipation, nth order chemical reaction and suction. Free convection effects on a vertical cone with variable viscosity and thermal conductivity were studied by Palani et al [20] using Crank- Nicholson scheme.

From these studies, it is clear that adequate studies are not available to study the effect of variable viscosity and thermal conductivity on a free convection flow of a viscous incompressible fluid along an isothermal vertical cone. This has motivated the present study. In the present paper, the effect of variations of viscosity and thermal conductivity with temperature on velocity and temperature profiles is analyzed. The fluid viscosity is taken as exponential function and the thermal conductivity is taken as a linear function of temperature. The dimensionless form of the governing boundary layer equations is used and the resulting system of equations is then solved by Alternating- direction-implicit technique.

II. PROBLEM FORMULATION

Consider an unsteady, 2D viscous incompressible fluid flow past a vertical cone with variable viscosity and thermal diffusivity. The surface of the cone makes an angle θ with the horizontal. The local radius of the cone is r' . The X- axis is measured along the surface of the cone from the apex $(x'=0)$ and the Y-axis is measured normally from the cone to the fluid. We assume that at time $t' \le 0$, the cone and the fluid are at the same temperature. The ambient fluid temperature is T'_∞ and at $t' > 0$, the temperature of the cone is $T'_w > T'_\infty$ where T'_w is the temperature of the cone surface. The gravitational acceleration is acting downward. In our analysis, the fluid properties are assumed to be constant. The effect of viscous dissipation is assumed to be negligible.

Under these assumptions and with application of the Boussinesq approximation, the governing conservations equations are given by:

$$\frac{\partial(r'u')}{\partial x'} + \frac{\partial(r'v')}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = g\beta\sin\theta(T'-T'_\infty) + \frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right) \tag{2}$$

$$\frac{\partial T'}{\partial t'} + u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \frac{1}{\rho C_P}\frac{\partial}{\partial y'}\left(k\frac{\partial T'}{\partial y'}\right) \tag{3}$$

The initial and boundary conditions are:

$$\begin{aligned}
& t' \le 0 \ : \\
& \quad u' = 0 \ , v' = 0, \ T' = T'_\infty \\
& t' > 0 \ : \\
& \quad u' = 0 \ , v' = 0, T' = T'_w \qquad at \ \ y' = 0 \\
& \quad u' \to 0 \ , \ T' \to T'_\infty \qquad at \ \ y' \to \infty \\
& \quad u' = 0 \ , \ T' = T'_\infty \qquad at \ \ x' = 0
\end{aligned} \tag{4}$$

where u' and v' are the velocity components in the x' and y' directions, respectively; ρ is fluid density, g is acceleration due to gravity , t' is time and T' is temperature of the fluid in the boundary layer, β is volumetric coefficient of thermal expansion, μ is the variable dynamic coefficient of viscosity . T'_w is the temperature far away from the cone surface, C_P is the specific heat and k is the variable thermal conductivity of the fluid.

The fluid viscosity and thermal conductivity is written in terms of dimensionless temperature T as

$$\mu = \mu_0 e^{-\lambda T} \tag{5}$$

$$k = k_0(1+\gamma T) \tag{6}$$

where λ and γ are the viscosity parameter and thermal conductivity parameter respectively and μ_0 and k_0 are the dynamic viscosity and thermal conductivity respectively at temperature $T_w{}'$

Introducing the non-dimensional quantities

$$x = \frac{x'}{L} \ , \ y = \frac{y'}{L}Gr^{1/4} \ , u = \frac{u'L}{\upsilon}Gr^{-1/2} \ ,$$
$$v = \frac{v'L}{\upsilon}Gr^{-1/4} , t = \frac{\upsilon t'}{L^2}Gr^{1/2} \ , r = \frac{r'}{L}, \ T = \frac{(T'-T'_\infty)}{(T'_w-T'_\infty)} \ ,$$
$$Gr = \frac{g\beta L^3(T'_w-T'_\infty)\sin\theta}{\upsilon^2} \ , \upsilon = \frac{\mu_0}{\rho}, \ \Pr = \frac{\mu_0 C_p}{k_0} \tag{7}$$

Where L is the reference length, υ is the kinematic viscosity, Gr is the Grashof number, $\Pr$ is Prandtl number and $r' = x'\sin\theta$

We can write Eqn. (1)-(3) in non-dimensional form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{u}{x} = 0 \tag{8}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = T + e^{-\lambda T}\frac{\partial^2 u}{\partial y^2} - \lambda e^{-\lambda T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y} \tag{9}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1+\gamma T}{\Pr}\frac{\partial^2 T}{\partial y^2} + \frac{\gamma}{\Pr}\left(\frac{\partial T}{\partial y}\right)^2 \tag{10}$$

and the initial and boundary conditions (4) as

$t \le 0$:

$u = 0 \ , v = 0, \ T = 0$

$t > 0$:

$$u = 0 \ , v = 0, T = 1 \qquad at \ \ y = 0$$
$$u \to 0 \ , \ T \to 0 \qquad at \ \ y \to \infty$$
$$u = 0 \ , \ T = 0 \qquad at \ \ x = 0 \qquad (11)$$

III. NUMERICAL TECHNIQUE

Using Alternating-Direction-Implicit technique, the two dimensional, unsteady and non-linear partial differential equations given by (8)-(10) under the initial and boundary conditions (11) are solved. This scheme consists of two steps which splits an unsteady two dimensional problem into two separate one-dimensional problems. In first step the difference equations are made implicit in x at an intermediate time level $n+\frac{1}{2}$ and the unknowns associated with the x-derivatives are evaluated. The implicit difference equations at the time level $n+\frac{1}{2}$ are written as

$$\frac{u_{i+1,j}^{n+\frac{1}{2}} - u_{i-1,j}^{n+\frac{1}{2}}}{2\Delta x} + \frac{v_{i,j+1}^{n} - v_{i,j-1}^{n}}{2\Delta y} + \frac{u_{i,j}^{n}}{\Delta x} = 0 \qquad (12)$$

$$\frac{u_{i,j}^{n+\frac{1}{2}} - u_{i,j}^{n}}{\Delta t/2} + u_{i,j}^{n}\left(\frac{u_{i+1,j}^{n+\frac{1}{2}} - u_{i-1,j}^{n+\frac{1}{2}}}{2\Delta x}\right) + v_{i,j}^{n}\left(\frac{u_{i,j+1}^{n} - u_{i,j-1}^{n}}{2\Delta y}\right)$$
$$= T_{i,j}^{n} + e^{-\lambda T_{i,j}^{n}}\left(\frac{u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}}{(\Delta y)^2}\right) - \lambda e^{-\lambda T_{i,j}^{n}}\left(\frac{T_{i,j+1}^{n} - T_{i,j-1}^{n}}{2\Delta y}\right)\left(\frac{u_{i,j+1}^{n} - u_{i,j-1}^{n}}{2\Delta y}\right) \qquad (13)$$

$$\frac{T_{i,j}^{n+\frac{1}{2}} - T_{i,j}^{n}}{\Delta t/2} + u_{i,j}^{n}\left(\frac{T_{i+1,j}^{n+\frac{1}{2}} - T_{i-1,j}^{n+\frac{1}{2}}}{2\Delta x}\right) + v_{i,j}^{n}\left(\frac{T_{i,j+1}^{n} - T_{i,j-1}^{n}}{2\Delta y}\right)$$
$$= \frac{1+\gamma T_{i,j}^{n}}{\Pr}\left(\frac{T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{(\Delta y)^2}\right) + \frac{\gamma}{\Pr}\left(\frac{T_{i,j+1}^{n} - T_{i,j-1}^{n}}{2\Delta y}\right)^2 \qquad (14)$$

Reducing the eqns. (12) - (14) into tri-diagonal form, we obtain solution for $u_{i,j}^{n+\frac{1}{2}}$ and $T_{i,j}^{n+\frac{1}{2}}$ for all i, keeping j fixed, using Thomas Algorithm. This step is repeated for next value $j+1$ and so on. In the end of this step, the values of $u_{i,j}^{n+\frac{1}{2}}$ and $T_{i,j}^{n+\frac{1}{2}}$ at intermediate time level $n+\frac{1}{2}$ is known for all (i, j).

In the next step, difference equations are made implicit in y at time level n and the unknowns associated with the y-derivatives are evaluated. The implicit difference equations at the time level n are written as

$$\frac{u_{i+1,j}^{n+\frac{1}{2}} - u_{i-1,j}^{n+\frac{1}{2}}}{2\Delta x} + \frac{v_{i,j}^{n+1} - v_{i,j-1}^{n+1}}{\Delta y} + \frac{u_{i,j}^{n}}{\Delta x} = 0 \qquad (15)$$

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n+\frac{1}{2}}}{\Delta t} + u_{i,j}^{n}\left(\frac{u_{i+1,j}^{n+\frac{1}{2}} - u_{i-1,j}^{n+\frac{1}{2}}}{2\Delta x}\right) + v_{i,j}^{n}\left(\frac{u_{i,j+1}^{n+1} - u_{i,j-1}^{n+1}}{2\Delta y}\right) = T_{i,j}^{n+\frac{1}{2}} +$$
$$e^{-\lambda T_{i,j}^{n}}\left(\frac{u_{i,j+1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j-1}^{n+1}}{(\Delta y)^2}\right) - \lambda e^{-\lambda T_{i,j}^{n}}\left(\frac{T_{i,j}^{n} - T_{i,j-1}^{n}}{\Delta y}\right)\left(\frac{u_{i,j}^{n+1} - u_{i,j-1}^{n+1}}{\Delta y}\right) \qquad (16)$$

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n+\frac{1}{2}}}{\Delta t} + u_{i,j}^{n}\left(\frac{T_{i+1,j}^{n+\frac{1}{2}} - T_{i-1,j}^{n+\frac{1}{2}}}{2\Delta x}\right) + v_{i,j}^{n}\left(\frac{T_{i,j+1}^{n+1} - T_{i,j-1}^{n+1}}{2\Delta y}\right)$$
$$= \frac{\gamma}{\Pr}\left(\frac{T_{i,j}^{n} - T_{i,j-1}^{n}}{\Delta y}\right)^2 + \frac{1+\gamma T_{i,j}^{n}}{\Pr}\left(\frac{T_{i,j+1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j-1}^{n+1}}{(\Delta y)^2}\right) \qquad (17)$$

Reducing eqns. (15)-(17) to tri-diagonal form we yield solution for $v_{i,j}^{n}$, $u_{i,j}^{n}$ and $T_{i,j}^{n}$ for all j, keeping i fixed, using Thomas Algorithm. The calculations are repeated for all values of i . The values of $v_{i,j}^{n}$, $u_{i,j}^{n}$ and $T_{i,j}^{n}$ at next time level n is known for all (i, j) at the end of this step. Here, the subscript i in $u_{i,j}^{n}$, $v_{i,j}^{n}$, $T_{i,j}^{n}$ represents the grid node along the x- direction and j subscript represents the grid node along the y- direction.

The stability of Alternating Differencing Implicit scheme can be easily established by employing Von-Neumann Technique as done in [21].

The domain of integration is assumed to be a rectangular region with sides

$$x = 0 \ , \ x = 1$$
$$y = 0 \ , y = 14$$

where the boundary condition $y = 14$ corresponds to conditions at infinity. The mesh size is taken as $\Delta x = 0.05$, $\Delta y = 0.25$ with the time step as $\Delta t = 0.01$. Computations are performed to make the absolute difference between values of both u and T at two consecutive time steps as negligible $(\approx 10^{-5})$.

VI. STABILITY ANALYSIS

We examine the stability of differencing scheme by employing Von-Neumann Technique. The general term of the Fourier expansion for u and T at an arbitrary time $t=0$ is assumed to be of the form $e^{iax}e^{iby}$ where $i=\sqrt{-1}$. At any time t, these can be written as

$$u = F(t)e^{iax}e^{iby}$$
$$T = G(t)e^{iax}e^{iby}$$

We substitute these in Eqns. (16) and (17). Let us take

$$u^{n+1} = F'(t) \quad u^{n+\frac{1}{2}} = F(t)$$
$$T^{n+1} = G'(t) \quad T^{n+\frac{1}{2}} = G(t) \tag{18}$$

Eqn. (16) on simplification gives

$$F'\left(\frac{1}{\Delta t}+\frac{4}{(\Delta y)^2}e^{-\lambda T}\sin^2\left(\frac{b\Delta y}{2}\right)+\lambda e^{-\lambda T}\frac{\Delta T}{\Delta y}\left(1-e^{-ib\Delta y}\right)\right) + iF'\frac{v}{\Delta y}\sin(b\Delta y) = F\left(\frac{1}{\Delta t}-i\frac{u\sin(a\Delta x)}{\Delta x}\right)+G \tag{19}$$

Eqn. (17) on simplification gives

$$G'\left(\frac{1}{\Delta t}+\frac{2}{(\Delta y)^2}\frac{1+\gamma T}{\Pr}\left(1-\cos b\Delta y\right)+i\frac{v}{\Delta y}\sin(b\Delta y)\right) = G\left(\frac{1}{\Delta t}-i\frac{u\sin(a\Delta x)}{\Delta x}\right)+\frac{\gamma}{\Pr}\left(\frac{\Delta T}{\Delta y}\right)^2 \tag{20}$$

Taking

$$\frac{1}{\Delta t}+e^{-\lambda T}\left[\frac{4}{(\Delta y)^2}\sin^2\left(\frac{b\Delta y}{2}\right)+\lambda\frac{\Delta T}{\Delta y}\left(1-e^{-ib\Delta y}\right)\right]+i\frac{v}{\Delta y}\sin(b\Delta y)=A$$
$$\frac{1}{\Delta t}+\frac{2}{(\Delta y)^2}\frac{1+\gamma T}{\Pr}\left(1-\cos b\Delta y\right)+i\frac{v}{\Delta y}\sin(b\Delta y)=B$$
$$\frac{1}{\Delta t}-i\frac{u\sin(a\Delta x)}{\Delta x}=C$$

Ignoring last term in Equation (20), Equations (19) and (20) in matrix form can be written as

$$\begin{bmatrix}F'\\G'\end{bmatrix}=\begin{bmatrix}\frac{C}{A} & \frac{1}{A}\\ 0 & \frac{C}{B}\end{bmatrix}\begin{bmatrix}F\\G\end{bmatrix} \tag{21}$$

The stability of the differencing scheme can be established if the modulus of each Eigen value of the matrix does not exceed unity. The Eigen values for (21) are (C/A) and (C/B). In first Eigen value C/A, clearly, the real part of A is always greater than real part of C. Therefore,

$$\left|\frac{C}{A}\right|\le 1$$

Similarly, we can prove that

$$\left|\frac{C}{B}\right|\le 1 \tag{18}$$

Hence, the differencing scheme is unconditionally stable.

V. RESULT AND DISCUSSION

Numerical computations were carried out for different values of viscosity parameter λ and thermal conductivity parameter γ, taking Pr=0.7(air) and 7.0 (water) and applying the Alternating direction implicit (ADI) technique discussed in section 3. For calculations we take $\Delta x = 0.05$, $\Delta y = 0.25;$ $\Delta t = 0.01$. The ADI algorithm has been implemented in MATLAB programming language. The accuracy of numerical results is compared with the previous studies available in literature.

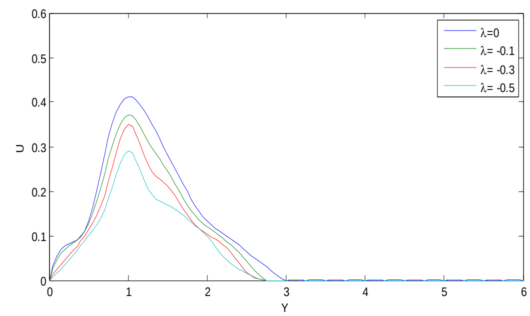

Fig. 1: Velocity profile at x=1.0 for Pr=0.7, γ=3 at t=1.5 for different values of λ

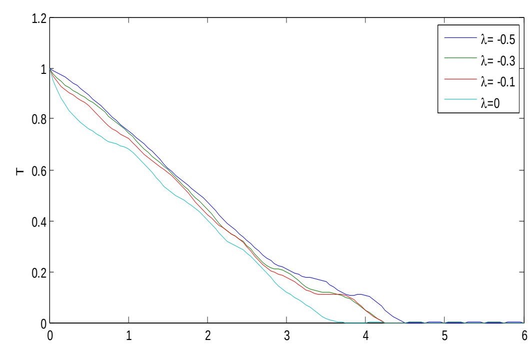

Fig. 2: Temperature profile at x=1.0 for Pr=0.7, γ=3 at t=1.5 for different values of λ

In Fig.1 and Fig. 2, the transient velocity profile and temperature profile are plotted for Prandtl number Pr=0.7 (air) of the fluid for γ=3 for different values of λ. In Fig. 1, it can be seen that velocity increases to a maximum and then start decreasing and finally reduces to zero.

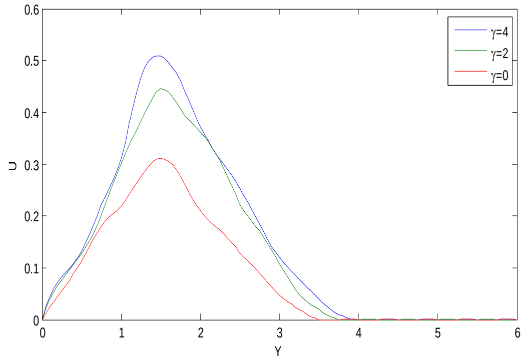

Fig. 3: Velocity profile at x=1.0 for Pr=0.7, λ = -0.5 at t=1.5 for different values of γ

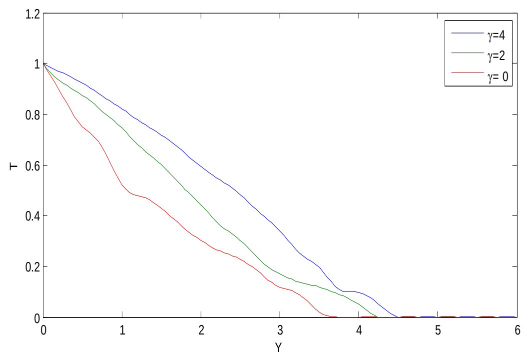

Fig. 4: Temperature profile at x=1.0 for Pr=0.7, λ = -0.5 at t=1.5 for different values of γ

In Fig. 3 and Fig. 4, the transient velocity profiles and temperature profiles are plotted for Prandtl number Pr=0.7 (air) of the fluid for λ = -0.5 for different values of γ. It is observed that an increase in γ results in increase in velocity as well as temperature

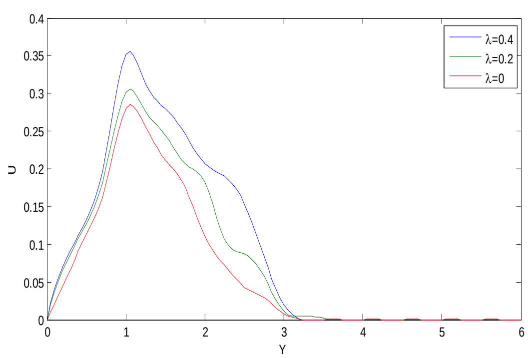

Fig. 5: Velocity profile at x=1.0 for Pr=7.0, γ=0.05 at t=2 for different values of λ

In Fig. 5 and Fig. 6, the transient velocity profiles and temperature profiles are plotted for Prandtl number Pr=7.0 (water) of the fluid for γ=0.05 for different values of λ. It can be seen that velocity increase with increase in λ whereas, temperature decreases with increase in λ.

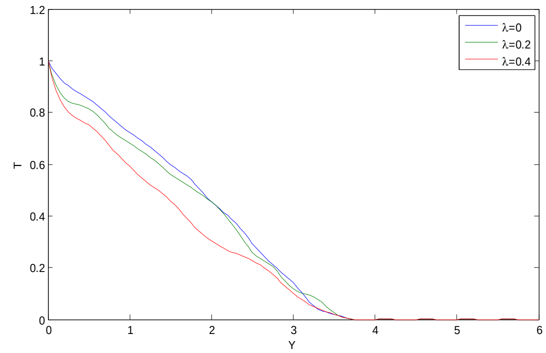

Fig. 6: Temperature profile at x=1.0 for Pr=7.0, γ=0.05 at t=2 for different values of λ

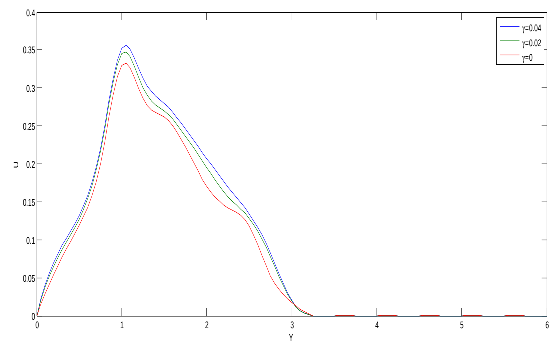

Fig. 7: Velocity profile at x=1.0 for Pr=7.0, λ = 0.5 at t=2 for different values of γ

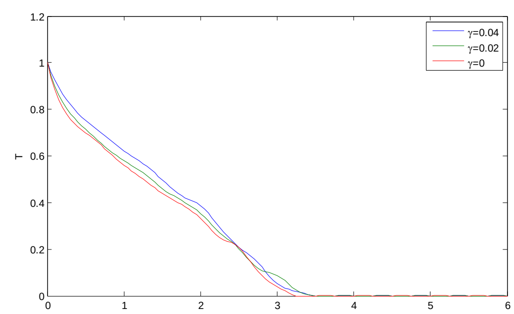

Fig. 8: Temperature profile at x=1.0 for Pr=7.0, λ = 0.5 at t=2 for different values of γ

In Fig. 7 and Fig. 8, the transient velocity profiles and temperature profiles are plotted for Prandtl number Pr=7.0 (water) of the fluid for λ = 0.5 for different values of γ. It is observed that for Pr=7.0, an increase in γ results in increase in velocity as well as temperature.

VI. CONCLUSION

An unsteady flow of a viscous incompressible fluid past a vertical cone with variable viscosity and thermal conductivity is studied in this paper. The dimensionless governing equations are solved numerically using ADI technique. The conclusions of the study are as follows:

1. An increase in γ results in increase in velocity of fluid for both values of Pr.
2. An increase in γ results in increase in temperature for both values of Pr.
3. The velocity of the fluid increase with increase in λ.
4. The temperature of fluid decreases with increase in λ.
5. The results obtained are in good agreement with the previous studies [20] available.

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