Statistical Analysis of Time Series Model: Additive Case

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***Abstract*: The purpose of this paper is to examine the condition(s) under which the linear trend cycle component with emphasis on the additive time series model is the most appropriate model in time series analysis. This paper is to identify the series that admits additive model using the Buys-Ballot procedure. Also, to estimate missing observations by the method of mean imputation. Table 1 show that, the seasonal variance of Buys-Ballot table for additive model, is a function of trend parameters only but do not contain seasonal indices.**

***Keywords:* Descriptive Time Series, Trend-Cycle component, Additive Model, Choice of Model, Buys-Ballot Estimates.**

I. INTRODUCTION

One of the aims of descriptive time series analysis is to isolate the four time series components available in the series. That is, to de-compose an observed time series $\left(X_t,\ t = 1, 2, ..., n\right)$ into components, representing the trend $\left(T_t\right)$, the seasonal $\left(S_t\right)$, cyclical $\left(C_t\right)$ and irregular $\left(e_t\right)$(Kendal and Ord [1], Chatfield [2]. Seasonal component refers to the regular periodic movements in time series data associated with the time of the year. Such movements happen due to recurring events which take place annually. Many time series, such as sales figures and temperature readings, displays a variation which is annual in period. Davey and Flores [3] provided a method which adds statistical test of seasonal indexes for the multiplicative model that helps to identify seasonality with greater confidence. Kendall and Ord [1] also provided test for seasonality. Apart from seasonal effects, many time series displays variation at a fixed periods or at periods that are not fixed but which are predictable. The differences between a seasonal component and a cyclical component is that former occurs at regular seasonal interval or fixed periods, while cyclical components have normally a longer duration of time that varies from cycle to cycle. For short duration of data, cyclical component is superimposed into the trend [2] and the observed time series $\left(X_t,\ t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular/residual component $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \qquad (1)$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \qquad (2)$$

and Mixed Model

$$X_t = M_t \times S_t + e_t . \qquad (3)$$

It is always assumed that the seasonal effect, when it exists, has period *s*, that is, it repeats after *s* time periods.

$$S_{t+s} = S_t \text{, for all t} \qquad (4)$$

For additive model given in equation (1), we make assumption that the sum of the seasonal components over a complete period is zero, ie ,

$$\sum_{j=1}^{s} S_{t+j} = 0 . \qquad (5)$$

Also, for multiplicative and mixed models given in equations (2) and (3), we equally make assumptions that the sum of the seasonal components over a complete period is *s*.

$$\sum_{j=1}^{s} S_{t+j} = s . \qquad (6)$$

This study considers additive model which assumes that the effect of the trend, the season, the cycles and the residual are equal in absolute terms throughout the period of time. This assumption is usually true when short periods are involved or where the rate of growth or decline in the trend is small and transformation is not needed.

Chatfield [2] suggested how to use run sequence plot (time plot) for choice of model in time series decomposition. Puerto and Rivera [4] also, provided the use of the coefficients of variation of seasonal differences (CV (d)) and seasonal quotients (CV(c)) for choice of model. Linde [5] presented the difference in choice of model between additive and multiplicative. For additive model, the seasonal variation is independent of the absolute level of the time series and its amplitude is relatively close while in the multiplicative model, the amplitude of the seasonal factor varies with the level of the time series

II. MATERIALS AND METHODS

The method adopted in this paper is the Buys-Ballot procedure for time series decomposition. For more details of Buys-Ballot table/procedure, see Wei [6], Iwueze and Nwogu [7,8 and 9] Dozie [10], Dozie, *et al* [11], Dozie and Ijomal [12]

For the additive model, the row, column and overall averages and variances obtained by Iwueze and Nwogu [9] and given in Table 1. It is observed from Table 1 that the seasonal variance of the Buys-Ballot Table is a function of trend parameters only and do not contain seasonal indices Iwueze and Nwogu [9] also, provided a test for constant variance as basis for choice between additive and multiplicative model.

2.1 Estimation of Trend Parameters

Row and overall means are used to estimate parameters of the trend line. We assume that the length of periodic interval is s For additive model, using the expression in table 1, we obtain

$$\bar{X}_i = a - \frac{b}{2}(s-1) + (bs)i \quad (7)$$

$$\equiv \alpha + \beta_i \quad (8)$$

where $\alpha = a - \frac{b}{2}(s-1),\ \beta = bs$

Table 1: Estimates of means and variances for additive model

Measure	Linear trend-cycle component: $M_t = a + bt$, $t = 1, 2, ..., n = ms$
	Additive model
$\bar{X}_{i.}$	$a - \frac{b}{2}(s-1) + (bs)i$
$\bar{X}_{.j}$	$a + \frac{b}{2}(n-s) + bj + S_j$
$\bar{X}_{..}$	$a + \frac{b(n+1)}{2}$
$\hat{\sigma}^2_{i.}$	$b^2\left(\frac{s(s+1)}{12}\right) + \left(\frac{2b}{s-1}\right)\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2$
$\hat{\sigma}^2_{.j}$	$b^2\left(\frac{n(n+s)}{12}\right)$
$\hat{\sigma}^2_{x}$	$b^2\left(\frac{n(n+1)}{12}\right) + \frac{1}{n-1}\left\{2bm\sum_{j=1}^{s} jS_j + m\sum_{j=1}^{s} S_j^2\right\}$

Source: Iwueze and Nwogu (2014)

$$\therefore \hat{a} = \alpha + \frac{b}{2}(s-1) \quad (9)$$

$$\hat{b} = \frac{\beta}{s} \quad (10)$$

Estimates of $S_j,\ j = 1, 2, ..., 5$

The seasonal and grand means are used to estimate the seasonal indices. Again, we assume that the length of periodic interval is s. Using the expression in Table 1 we obtain, additive model

$$\bar{X}_{.j} = a + \frac{b}{2}(n-s) + b_j + S_j \quad (11)$$

$$\equiv \left[\alpha + \beta_j\right] + S_j \quad (12)$$

where $\alpha = a - \frac{b}{2}(n-s)$

$$\beta = b \quad (13)$$

$$\therefore \hat{S}_j = \bar{X}_j - \left(a + \frac{b}{2}(n-s) + b_j\right) \quad (14)$$

Table 2**:** Estimates of parameters for linear trending curve and seasonal indices

Parameter	Additive model
a	$\alpha + \frac{\hat{b}}{2}(s-1)$
b	$\frac{\beta}{s}$
S_j	$\bar{X}_{.j} - \left(a + \frac{b}{2}(n-s) + b_j\right)$

Note: α and β are estimates obtained from the regression equations of row means on row

2.2 Method of Estimating Missing Value:

Mean imputation (MI) is one of the methods of replacing missing observations. This method replaces the missing observations with mean of the values before the missing position. This is achieved by taking the summation of the values and dividing by the number of observation before the missing position.

$$MI = \widehat{X}_{(i-1)s+j} = \frac{1}{(i-1)s+j-1}\left[X_1 + X_2 + X_3 + ... + X_{(i-1)s+j-1}\right] \quad (15)$$

$$MI = \frac{1}{n^*}\sum_{t=1}^{n^*} X_t$$

Where

$n^* = (i-1)s + j - 1$ Is the number of observations preceding the missing value.

2.3 Test for Constant Variance

Levene's test statistic for the null hypothesis

$$H_0 : \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$$

against the alternative

$$H_1 : \sigma_1^2 \neq \sigma_j^2 \text{ for at least one pair } (ij)$$

is defined as

$$W = \frac{(N-K)}{K-1} \cdot \frac{\sum_{j=1}^{K} N_i (Z_{\cdot j} - Z_{\cdot\cdot})^2}{\sum_{i=1}^{K}\sum_{j=1}^{N} (Z_{ij} - Z_{\cdot j})^2} \qquad (16)$$

Where K is the number of different groups to which the sampled cases belong.

Ni Is the number of cases in the j[th] group

N Is the total number of cases in all groups.

Y_i Is the value of the measured variable for the j[th] case from the group.

$$Z_{ij} = \begin{vmatrix} Y_{ij} - \bar{Y}_{i.} \\ Y_{ij} - \hat{Y}_i \end{vmatrix}$$

$$\bar{Y}_{i.} \textit{ is a mean of the } i^{th} \textit{ group.} \qquad (17)$$

$$\hat{Y}_i \textit{ is a median of the } i^{th} \textit{ group}$$

$Z_i = \frac{1}{Ni}\sum_{j=1}^{N_i} Z_{ij}$ Is the mean of the Z_{ij} for group i. (18)

$Z_{\cdot\cdot} = \frac{1}{N}\sum_{i=1}^{K}\sum_{j=1}^{N_j} Z_{ij}$ Is the mean of all Z_{ij} (19)

The test statistic W is approximately as F-distribution with *k–I* and $N-k$ degrees of freedom. We re-write the Levene's test statistic and modify it to suit the Buys-Ballot procedure. Using the parameters of the Buys-Ballot table $N = ms,\ k = s,\ N_i = m$ the statistic in (16) is given as

$$W = \frac{(ms - S)}{s-1}\left|\frac{\sum_{j=1}^{S} m(Z_{\cdot j} - Z_{\cdot\cdot})^2}{\sum_{i=1}^{m}\sum_{j=1}^{s}(Z_{ij} - Z_{\cdot j})}\right| \qquad (20)$$

$$W = \frac{S(m-1)m}{S-1} \cdot \frac{\sum_{j=1}^{S} m(Z_{\cdot j} - Z_{\cdot\cdot})^2}{\sum_{i=1}^{m}\sum_{j=1}^{s}(Z_{ij} - Z_{\cdot j})} \qquad (21)$$

$$Z_{ij} = \left| y_{ij} - y_{i\cdot} \right| \qquad (22)$$

$$y_{i\cdot} = \textit{column mean}(\textit{median})$$

$$Z_{\cdot j} = \frac{1}{m}\sum_{i=1}^{m} Z_{ij} \qquad (23)$$

$$Z_{\cdot\cdot} = \frac{1}{s}\sum_{j=1}^{s} Z_{\cdot j} \qquad (24)$$

III. ANALYSIS

The data analysis is based on monthly time series data on number of registered baptism at Assumpta Cathedral Owerri, Imo State, Nigeria for a period 2009 to 2019. One hundred and fourteen (114) registered baptisms were considered from January 2009 to December 2018 in which six (6) observations were not registered shown in Appendix A. Therefore, the process was repeated with the missing data estimated and replaced using the method of Mean Imputation. The seasonal (monthly) variances are shown in Table 4. The first step is to determine whether the time series data admits additive model. The levene's test statistic given (21) is applied. The null hypothesis that the data admits additive model is rejected, if W is greater than the tabulated value, for which $F_{\alpha(k-1)(N-k)}$ level of significance, or do not reject null hypothesis otherwise

Table 4: Difference between observed values and seasonal means ($X_{ij} - \bar{X}_{\cdot j}$)

S/N	$\bar{X}_{\cdot j}$	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	10.4	5.6	15.6	6.7	-3.6	4.5	8	-3.5	7.4	-3.1	-2	18.9	10.4
2	18.4	5.6	5.6	5.7	-0.6	-5.5	10	5.5	-1.6	5.9	0	7.9	6.4
3	14.3	-2.4	-0.4	-3.3	-1.6	7.5	-2.0	6.5	1.6	4.9	6	4.9	-6.6
4	15.6	3.6	-4.4	3.7	6.4	-4.5	-1.0	8.5	-5.6	2.9	3	3.9	3.4
5	18.5	6.6	2.6	5.7	7.4	-2.5	-5.0	9.5	2.4	-2.1	-1	-15.1	4.4
6	16.0	-2.4	-7.4	-0.3	7.4	13.5	6	-4.5	0.4	8.9	0	-1.1	-1.6
7	17.5	-5.4	2.6	-5.3	-5.6	-0.5	-1.0	-8.5	0.4	1.9	-3	-5.1	-0.6
8	19.6	-1.4	-0.4	-3.3	-3.6	-6.5	-3.0	-7.5	7.4	-7.1	5	-2.1	-2.6
9	16.1	-4.4	-8.4	-4.3	-4.6	-0.5	-6.0	-6.5	-9.6	-5.1	-5	-9.1	-6.6
10	13.0	-5.4	-5.4	-5.3	-1.6	-5.5	-8	0.5	0.4	-7.1	-3	-3.1	-6.6
11	20.1												
12	16.6												
	Total	0	0	0	0	0	-2	0	0	0	0	0	0

From Appendix B and Table 4

$Z_{ij} = \left| X_{ij} - \bar{X}_{\cdot j} \right| = \left| -2 \right| = 2$, $Z_{\cdot j} = \frac{1}{10} \times 2 = \frac{2}{10} = 0.2$, $Z_{\cdot\cdot} = \frac{1}{12} \times 0.2 = \frac{0.2}{12} = 0.017$, S = 12, M =10. Hence, $W = \frac{108}{11} \times \frac{0.33489}{3.24}$ = 1.0. $F_{\alpha(k-1)\,(N-k)} = F_{0.05\,(12-1)(120-12)}$ = 1.83. When compared with the critical value (1.83), W is less than, indicating that the data admits additive time series model. However, the study shows that data evaluation requires logarithm transformation to meet the constant variance and normality assumptions in the distribution. When the seasonal variances of the logarithm transformed time series data obtained in Table 5 are subjected to test for constant variance, the calculated levene's test statistic (1.92) is greater than the tabulated (1.83) at $F_{\alpha(k-1)\,(N-k)}$ level significant. This shows that the variance is not constant and the transformed series does not admit additive model. This paper further confirms that the appropriate model of actual time series data is additive.

Table 5: Difference between observed values (transformed) and seasonal means ($X_{ij} - \bar{X}_{\cdot j}$)

S/N	$\bar{X}_{\cdot j}$	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	2.24	0.53	0.68	0.57	-0.22	0.27	0.47	-0.15	0.57	-0.06	0.13	0.78	0.62
2	2.85	0.53	0.33	-0.17	0.01	-0.31	-0.55	0.35	0.16	0.45	0.03	0.45	0.46
3	2.47	-0.16	0.04	-0.07	-0.06	0.39	0.07	0.39	0.16	0.4	0.41	0.34	-0.38
4	2.70	0.4	-0.21	0.42	0.39	-0.23	0	0.47	-0.09	0.3	0.24	0.3	-0.38
5	2.87	0.59	0.19	-0.17	0.44	-0.1	-0.31	0.51	0.36	0	-0.05	-1.27	0.36
6	2.71	-0.16	-0.45	0.17	0.44	0.6	0.38	-0.23	-0.43	0.56	0.03	0.06	0.03
7	2.79	-0.63	0.19	-0.27	-0.4	0.02	0	-0.59	-0.43	-0.56	-0.23	-0.17	0.09
8	2.73	-0.04	0.04	-0.07	-0.22	-0.39	-0.15	-0.49	0.57	-0.44	0.36	0.01	-0.04
9	2.64	-0.45	-0.55	-0.17	-0.3	0.02	-0.23	-0.39	-0.43	-0.24	-0.45	-0.48	-0.38
10	2.53	-0.63	-0.29	-0.27	-0.06	-0.31	-0.63	0.1	-0.43	-0.44	-0.23	-0.05	-0.38
11	2.88												
12	2.68												
	Total	-0.02	-0.03	-0.03	0.02	-0.04	0.01	-0.03	0.01	-0.03	-0.02	-0.03	0

From Appendix D and Table 5

$$Z_{ij} = -0.02 - 0.03 - 0.03 + 0.02 - 0.04 + 0.01 - 0.03 + 0.01 - 0.03 - 0.02 - 0.03 = -0.19$$

$$|-0.19| = 0.19$$

$$Z_{\cdot j} = \frac{1}{10} \times 0.19 = \frac{0.19}{10} = 0.019,$$

$$Z_{\cdot\cdot} = \frac{1}{12} \times 0.019 = \frac{0.019}{12} = 0.0016$$

$$W = \frac{12(10-1)}{12-1} \times \frac{10(0.019 - 0.0016)^2}{(0.19 - 0.019)}$$

$$W = \frac{108}{11} \times \frac{0.0057276}{0.029241} = 1.92$$

IV. CONCLUSION

This paper has examined the statistical analysis of time series model with emphasis on the additive time series model in descriptive time series analysis. This study estimated missing observations using mean imputation method. Also, the study indicated that the appropriate time series model that best describe the pattern in the transformed series is multiplicative. This showed that the appropriate model of actual time series data is additive.

REFERENCES

[1] Kendal, M. G. & Ord, J. K. (1990). *Time Series* (3rd ed.). Charles Griffin, London.

[2] Chatfield, C. (2004). *The analysis of time Series: An introduction*. Chapman and Hall,/CRC Press, Boca Raton.

[3] Davey A.M & Flores (1993). Identification of seasonality in time series: a note, maths. Comput. Modelling 18(6), 73-81

[4] Puerto, J. & Rivera, M. P. (2001). *Descriptive analysis of time series applied to housing prices in Spain*, Management Mathematics for European Schools 94342 – CP – 2001 – DE – COMENIUS – C21

[5] Linde, P.(2005). *Seasonal Adjustment*, Statistics Denmark. www.dst.dk/median/konrover/13-forecasting-org/seasonal/001pdf

[6] Wei, W. W. S (1989). *Time series analysis: Univariate and multivariate methods*, Addison-Wesley publishing Company Inc, Redwood City.

[7] Iwueze, I. S. & Nwogu E.C. (2004). *Buys-Ballot estimates for time series decomposition*, Global Journal of Mathematics, 3(2), 83-98.

[8] Iwueze, I. S. & Nwogu, E.C. (2005). *Buys-Ballot estimates for exponential and s-shaped curves, for time series*, Journal of the Nigerian Association of Mathematical Physics, 9, 357-366.

[9] Iwueze, I. S. & Nwogu, E.C. (2014). *Framework for choice of models and detection of seasonal effect in time series.* Far East Journal of Theoretical Statistics 48(1), 45– 66

[10] Dozie, K. C. N (2020). Buys-Ballot Estimates for Mixed Model in Descriptive Time Series. International Journal of Theoretical and Mathematical Physics 10(1), 22-27

[11] Dozie, K.C.N, Nwogu, E.C, Nwanya, J.C (2020). Buys-Ballot technique for analysis of time Series model. International Journal of Scientific Research and Innovative Technology 7(1), 63-78

[12] Dozie, K.C.N, Ijomah M.A (2020). A comparative study on additive and mixed models in descriptive time series. American Journal of Mathematical and Computer Modelling 5(1), 12-17

Appendix A: Buys-Ballot table for the original time series data with incomplete observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	16	34	21	12	23	24	14	27	13	11	39	27	21.75	80.93
2010	-	24	20	15	11	26	23	18	22	13	28	23	20.08	25.54
2011	8	18	11	-	26	14	24	18	21	19	25	10	17.33	36.24
2012	14	14	18	22	14	15	26	14	19	16	24	20	18.0	19.8
2013	17	21	20	23	16	11	27	22	14	12	5	21	17.42	37.72
2014	8	11	14	23	32	22	13	20	25	13	19	15	17.92	46.81
2015	5	21	9	-	18	15	9	-	18	10	15	16	13.83	25.95
2016	9	18	11	12	12	13	10	-	9	18	18	14	14.25	27.30
2017	6	10	10	11	18	12	11	10	11	8	11	10	10.67	7.88
2018	5	13	9	-	13	8	18	20	9	10	17	10	12.17	20.15
$\bar{X}_{.j}$	10.4	18.4	14.3	15.6	18.5	16.0	17.5	19.6	16.1	13.0	20.1	16.6	16.34	
$\sigma^2_{.j}$	23.38	51.38	24.46	26.04	50.78	35.56	47.39	27.16	32.22	13.11	90.1	35.6		41.37

Source: Assumpta Cathedral Parish Owerri (2009-2018)

Appendix B: Buys-Ballot table for the original time series data with complete observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	16	34	21	12	23	24	14	27	13	11	39	27	21.75	80.93
2010	16	24	20	15	11	26	23	18	22	13	28	23	20.08	25.54
2011	8	18	11	14	26	14	24	18	21	19	25	10	17.33	36.24
2012	14	14	18	22	14	15	26	14	19	16	24	20	18.0	19.8
2013	17	21	20	23	16	11	27	22	14	12	5	21	17.42	37.72
2014	8	11	14	23	32	22	13	20	25	13	19	15	17.92	46.81
2015	5	21	9	10	18	15	9	20	18	10	15	16	13.83	25.95
2016	9	18	11	12	12	13	10	27	9	18	18	14	14.25	27.30
2017	6	10	10	11	18	12	11	10	11	8	11	10	10.67	7.88
2018	5	13	9	14	13	8	18	20	9	10	17	10	12.17	20.15
$\bar{X}_{.j}$	10.4	18.4	14.3	15.6	18.5	16.0	17.5	19.6	16.1	13.0	20.1	16.6	16.34	
$\sigma^2_{.j}$	23.38	51.38	24.46	26.04	50.78	35.56	47.39	27.16	32.22	13.11	90.1	35.6		41.37

Source: Assumpta Cathedral Parish Owerri (2009-2018)

Appendix C: Buys-Ballot table for the transformed time series data with incomplete observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	2.77	3.53	3.04	2.48	3.14	3.18	2.64	3.30	2.56	2.40	3.66	3.30	3.00	0.18
2010	-	3.18	2.30	2.71	2.56	3.26	3.14	2.89	3.09	2.56	3.33	3.14	2.91	0.11
2011	2.08	2.89	2.40	-	3.26	2.64	3.18	2.89	3.04	2.94	3.22	2.30	2.79	0.15
2012	2.64	2.64	2.89	3.09	2.64	2.71	3.26	2.64	2.94	2.77	3.18	2.30	2.81	0.08
2013	2.83	3.04	2.30	3.14	2.77	2.40	3.30	3.09	2.64	2.48	1.61	3.04	2.72	0.22
2014	2.08	2.40	2.64	3.14	3.47	3.09	2.56	2.30	3.22	2.56	2.94	2.71	2.76	0.17
2015	1.61	3.04	2.20	-	2.89	2.71	2.20	-	2.08	2.30	2.71	2.77	2.43	0.16
2016	2.20	2.89	2.40	2.48	2.48	2.56	2.30	-	2.20	2.89	2.89	2.64	2.60	0.11
2017	1.79	2.30	2.30	2.40	2.89	2.48	2.40	2.30	2.40	2.08	2.40	2.30	2.34	0.06
2018	1.61	2.56	2.20	-	2.56	2.08	2.89	2.30	2.20	2.30	2.83	2.30	2.37	0.12
$\bar{X}_{.j}$	2.24	2.85	2.47	2.70	2.87	2.71	2.79	2.73	2.64	2.53	2.88	2.68	2.67	
$\sigma^2_{.j}$	0.24	0.14	0.09	0.10	0.11	0.14	0.18	0.18	0.17	0.08	0.32	0.15		0.13

Source: Assumpta Cathedral Parish Owerri (2009-2018)

Appendix D: Buys-Ballot table for the transformed time series data with complete observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	2.77	3.53	3.04	2.48	3.14	3.18	2.64	3.30	2.56	2.40	3.66	3.30	3.00	0.18
2010	2.77	3.18	2.30	2.71	2.56	3.26	3.14	2.89	3.09	2.56	3.33	3.14	2.91	0.11
2011	2.08	2.89	2.40	2.64	3.26	2.64	3.18	2.89	3.04	2.94	3.22	2.30	2.79	0.15
2012	2.64	2.64	2.89	3.09	2.64	2.71	3.26	2.64	2.94	2.77	3.18	2.30	2.81	0.08
2013	2.83	3.04	2.30	3.14	2.77	2.40	3.30	3.09	2.64	2.48	1.61	3.04	2.72	0.22
2014	2.08	2.40	2.64	3.14	3.47	3.09	2.56	2.30	3.22	2.56	2.94	2.71	2.76	0.17
2015	1.61	3.04	2.20	2.30	2.89	2.71	2.20	2.30	2.08	2.30	2.71	2.77	2.43	0.16
2016	2.20	2.89	2.40	2.48	2.48	2.56	2.30	3.30	2.20	2.89	2.89	2.64	2.60	0.11
2017	1.79	2.30	2.30	2.40	2.89	2.48	2.40	2.30	2.40	2.08	2.40	2.30	2.34	0.06
2018	1.61	2.56	2.20	2.64	2.56	2.08	2.89	2.30	2.20	2.30	2.83	2.30	2.37	0.12
$\bar{X}_{.j}$	2.24	2.85	2.47	2.70	2.87	2.71	2.79	2.73	2.64	2.53	2.88	2.68	2.67	
$\sigma^2_{.j}$	0.24	0.14	0.09	0.10	0.11	0.14	0.18	0.18	0.17	0.08	0.32	0.15		0.13

Source: Assumpta Cathedral Parish Owerri (2009-2018)