

# Primitive of Faltung Type Volterra Integro-Differential Equation of First Kind Using Elzaki Transform

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**Abstract:** Volterra integro-differential equations appear in different branches of engineering, physics, astronomy, biology, radiology and having many useful applications such as process of glass forming, diffusion process, heat and mass transfer, growth of cells and describing the motion of satellite. In this paper, authors determine the primitive of faltung type Volterra integro-differential equation of first kind using Elzaki transform. Four numerical problems have been considered and solved using Elzaki transform for explaining the applicability of present method. Results of numerical problems show that Elzaki transform is very effective integral transform for determining the primitive of faltung type Volterra integro-differential equation of first kind.

**Keywords:** Volterra integro-differential equation; Elzaki transform; Faltung; Inverse Elzaki transform.

## I. INTRODUCTION

Nowadays, integral transformations are one of the mostly used mathematical techniques to determine the answers of advance problems of space, science, technology and engineering. The most important feature of these transformations is providing the exact (analytical) solution of the problem without large calculation work. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations. In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations. Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of advance problems of population growth and decay by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31]. Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace

transformations of error function with applications. The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transformations and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations. Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51-52] used Elzaki and Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [53-54] by using Aboodh and Sadik transformations respectively. Aggarwal and Sharma [55] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [56]. Aggarwal and Bhatnagar [57] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [58]. Aggarwal [59] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [60] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [61] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [62]. Mishra et al. [63] defined the relationship between Sumudu and some efficient integral transforms.

The main aim of this paper is to determine the primitive of faltung type Volterra integro-differential equation of first kind with the help of Elzaki transform.

## II. DEFINITION OF ELZAKI TRANSFORM

The Elzaki transform of the function  $G(t)$  for all  $t \geq 0$  is defined as [64]

$$E\{G(t)\} = p \int_0^{\infty} G(t) e^{-\left(\frac{t}{p}\right)} dt = g(p), k_1 \leq p \leq k_2, \text{ where } E \text{ is Elzaki transform operator.}$$

Table 1 Fundamental Property of Elzaki Transforms [28]

S.N.	Name of Property	Mathematical Form
1.	Linearity	$E\{aG_1(t) + bG_2(t)\} = aE\{G_1(t)\} + bE\{G_2(t)\}$
2.	Change of Scale	$E\{G(at)\} = \left(\frac{1}{a^2}\right)g(ap)$
3.	Shifting	$E\{e^{at}G(t)\} = (1 - ap)g\left(\frac{p}{1 - ap}\right)$
4.	First Derivative	$E\{G'(t)\} = \frac{1}{p}g(p) - pG(0)$
5.	Second Derivative	$E\{G''(t)\} = \frac{1}{p^2}g(p) - G(0) - pG'(0)$
6.	nth Derivative	$E\{G^{(n)}(t)\} = \frac{1}{p^n}g(p) - \frac{1}{p^{n-2}}G(0) - \frac{1}{p^{n-3}}G'(0) \dots \dots - pG^{(n-1)}(0)$
7.	Faltung Theorem	$E\{G_1(t) * G_2(t)\} = \frac{1}{p}E\{G_1(t)\}E\{G_2(t)\}$

Table 2 Elzaki Transforms of Frequently Encountered Functions [5, 28]

S.N.	$G(t)$	$E\{G(t)\} = g(p)$
1.	1	$p^2$
2.	$t$	$p^3$
3.	$t^2$	$2!p^4$
4.	$t^n, n \in \mathbb{N}$	$n!p^{n+2}$
5.	$t^n, n > -1$	$\Gamma(n + 1)p^{n+2}$
6.	$e^{at}$	$\frac{p^2}{1 - ap}$
7.	$\sin at$	$\frac{ap^3}{1 + a^2p^2}$
8.	$\cos at$	$\frac{p^2}{1 + a^2p^2}$
9.	$\sinh at$	$\frac{ap^3}{1 - a^2p^2}$
10.	$\cosh at$	$\frac{p^2}{1 - a^2p^2}$

III. DUALITY BETWEEN ELZAKI AND LAPLACE TRANSFORMS [26]

If Elzaki and Laplace transformations of  $G(t)$  are  $g(p)$  and  $h(p)$  respectively then

$$g(p) = ph\left(\frac{1}{p}\right) \text{ and } h(p) = pg\left(\frac{1}{p}\right),$$

Where  $h(p) = \int_0^\infty G(t)e^{-pt}dt = L\{G(t)\}$  and  $L$  is the Laplace transform operator.

IV. INVERSE ELZAKI TRANSFORM

If  $E\{G(t)\} = g(p)$  then  $G(t)$  is called the inverse Elzaki transform of  $g(p)$  and mathematically it is defined as

$G(t) = E^{-1}\{g(p)\}$ , where  $E^{-1}$  is the inverse Elzaki transform operator.

Table 3 Inverse Elzaki Transforms Of Frequently Encountered Functions

S.N.	$g(p)$	$G(t) = E^{-1}\{g(p)\}$
1.	$p^2$	1
2.	$p^3$	$t$
3.	$p^4$	$\frac{t^2}{2!}$
4.	$p^{n+2}, n \in \mathbb{N}$	$\frac{t^n}{n!}$
5.	$p^{n+2}, n > -1$	$\frac{t^n}{\Gamma(n + 1)}$
6.	$\frac{p^2}{1 - ap}$	$e^{at}$
7.	$\frac{p^3}{1 + a^2p^2}$	$\frac{\sin at}{a}$
8.	$\frac{p^2}{1 + a^2p^2}$	$\cos at$
9.	$\frac{p^3}{1 - a^2p^2}$	$\frac{\sinh at}{a}$
10.	$\frac{p^2}{1 - a^2p^2}$	$\cosh at$

V. PRIMITIVE OF FALTUNG TYPE VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF FIRST KIND USING ELZAKI TRANSFORM

In this part of the paper, authors determine the primitive of faltung type Volterra integro-differential equation of first kind with the help of Elzaki transform.

Faltung type Volterra integro-differential equation of first kind is given by

$$\left. \begin{aligned} &\int_0^t K_1(t - u) \omega(u) du + \\ &\int_0^t K_2(t - u) \omega^{(n)}(u) du \end{aligned} \right\} = F(t), K_2(t - u) \neq 0 \tag{1}$$

$$\text{With } \left. \begin{aligned} &\omega(0) = \delta_0, \omega'(0) = \delta_1, \\ &\omega''(0) = \delta_2, \dots, \\ &\omega^{(n-1)}(0) = \delta_{n-1} \end{aligned} \right\} \tag{2}$$

$$\left. \begin{aligned} &\left[ \begin{aligned} &K_1(t - u), K_2(t - u) \\ &= \text{faltung type kernels} \\ &\text{of integral equation} \end{aligned} \right] \\ &\left[ \begin{aligned} &\omega(t) = \text{unknown} \\ &\text{function} \end{aligned} \right] \\ \text{where } &\left[ \begin{aligned} &\omega^{(n)}(t) = \text{nth derivative} \\ &\text{of unknown function} \end{aligned} \right] \\ &\left[ \begin{aligned} &F(t) = \text{known} \\ &\text{function} \end{aligned} \right] \\ &\left[ \begin{aligned} &\delta_0, \delta_1, \delta_2, \dots, \delta_{n-1} \\ &= \text{real numbers} \end{aligned} \right] \end{aligned} \right\}$$

Taking Elzaki transform of both sides of (1), we have

$$\begin{bmatrix} E \left\{ \int_0^t K_1(t-u) \omega(u) du \right\} \\ + E \left\{ \int_0^t K_2(t-u) \omega^{(n)}(u) du \right\} \\ = E\{F(t)\} \end{bmatrix} \quad (3)$$

Applying faltung theorem of Elzaki transform on (3), we have

$$\begin{bmatrix} \frac{1}{p} E\{K_1(t)\} E\{\omega(t)\} \\ + \frac{1}{p} E\{K_2(t)\} E\{\omega^{(n)}(t)\} = E\{F(t)\} \end{bmatrix} \quad (4)$$

Applying the property ‘‘Elzaki transform of derivative of functions’’ on (4), we get

$$\begin{bmatrix} \frac{1}{p} E\{K_1(t)\} E\{\omega(t)\} \\ + \frac{1}{p} E\{K_2(t)\} \begin{bmatrix} \frac{1}{p^n} E\{\omega(t)\} \\ - \frac{1}{p^{n-2}} \omega(0) \\ - \frac{1}{p^{n-3}} \omega'(0) \\ - \frac{1}{p^{n-4}} \omega''(0) \\ \dots \\ - p \omega^{(n-1)}(0) \end{bmatrix} = E\{F(t)\} \end{bmatrix} \quad (5)$$

Now using (2) in (5), we have

$$\begin{bmatrix} \frac{1}{p} E\{K_1(t)\} E\{\omega(t)\} \\ + \frac{1}{p} E\{K_2(t)\} \begin{bmatrix} \frac{1}{p^n} E\{\omega(t)\} \\ - \frac{1}{p^{n-2}} \delta_0 \\ - \frac{1}{p^{n-3}} \delta_1 \\ - \frac{1}{p^{n-4}} \delta_2 \\ \dots \\ - p \delta_{n-1} \end{bmatrix} = E\{F(t)\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \begin{bmatrix} \frac{1}{p} E\{K_1(t)\} \\ + \frac{1}{p^{n+1}} E\{K_2(t)\} \end{bmatrix} E\{\omega(t)\} \\ E\{F(t)\} \\ \begin{bmatrix} \frac{1}{p^{n-1}} \delta_0 \\ + \frac{1}{p^{n-2}} \delta_1 \\ + \frac{1}{p^{n-3}} \delta_2 \\ + \dots \\ + \delta_{n-1} \end{bmatrix} \end{bmatrix}$$

$$\Rightarrow E\{\omega(t)\} = \frac{\begin{bmatrix} E\{F(t)\} \\ + E\{K_2(t)\} \begin{bmatrix} \frac{1}{p^{n-1}} \delta_0 \\ + \frac{1}{p^{n-2}} \delta_1 \\ + \frac{1}{p^{n-3}} \delta_2 \\ + \dots \\ + \delta_{n-1} \end{bmatrix} \end{bmatrix}}{\begin{bmatrix} \frac{1}{p} E\{K_1(t)\} \\ + \frac{1}{p^{n+1}} E\{K_2(t)\} \end{bmatrix}}, \quad (6)$$

$$\begin{bmatrix} \frac{1}{p} E\{K_1(t)\} \\ + \frac{1}{p^{n+1}} E\{K_2(t)\} \end{bmatrix} \neq 0$$

The inverse Elzaki transform of both sides of (6) gives the required primitive of faltung type Volterra integro-differential equation of first kind which is given by (1) with (2).

### VI. NUMERICAL PROBLEMS

In this part of the paper, some numerical problems have been considered for explaining the complete methodology.

*Problem: 1* Consider the following faltung type Volterra integro-differential equation of first kind

$$\begin{bmatrix} \int_0^t (t-u) \omega(u) du \\ + \int_0^t (t-u)^2 \omega'(u) du \\ = 3t - 3sint \end{bmatrix} \quad (7)$$

$$\text{With } \omega(0) = 0 \quad (8)$$

Taking Elzaki transform of both sides of (7), we have

$$\begin{bmatrix} E \left\{ \int_0^t (t-u) \omega(u) du \right\} \\ + E \left\{ \int_0^t (t-u)^2 \omega'(u) du \right\} \\ = E\{3t - 3sint\} \end{bmatrix} \quad (9)$$

Applying faltung theorem of Elzaki transform on (9), we have

$$\begin{bmatrix} \frac{1}{p} E\{t\} E\{\omega(t)\} + \frac{1}{p} E\{t^2\} E\{\omega'(t)\} \\ = E\{3t - 3sint\} \\ = 3E\{t\} - 3E\{sint\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p^2 E\{\omega(t)\} + 2p^3 E\{\omega'(t)\} \\ = 3p^3 - \frac{3p^3}{1+p^2} \end{bmatrix} \quad (10)$$

Applying the property ‘‘Elzaki transform of derivative of functions’’ on (10), we get

$$\begin{bmatrix} p^2 E\{\omega(t)\} \\ + 2p^3 \left[ \frac{1}{p} E\{\omega(t)\} - p \omega(0) \right] \\ = 3p^3 - \frac{3p^3}{1+p^2} \end{bmatrix} \quad (11)$$

Now using (8) in (11), we have

$$\begin{bmatrix} 3p^2 E\{\omega(t)\} = 3p^3 - \frac{3p^3}{1+p^2} \end{bmatrix}$$

$$\Rightarrow \left[ \begin{array}{c} E\{\omega(t)\} \\ = p - \frac{p}{(p^2+1)} = \frac{p^3}{(p^2+1)} \end{array} \right] \quad (12)$$

Taking inverse Elzaki transform of both sides of (12), we get the required solution of (7) with (8) as

$$\left[ \omega(t) = E^{-1} \left\{ \frac{p^3}{(p^2+1)} \right\} = sint \right].$$

*Problem: 2* Consider the following faltung type Volterra integro-differential equation of first kind

$$\left[ \begin{array}{c} \int_0^t sin(t-u) \omega(u) du \\ - \frac{1}{2} \int_0^t (t-u) \omega''(u) du \\ = \frac{t}{2} - \frac{tcost}{2} \end{array} \right] \quad (13)$$

$$\text{with } [\omega(0) = 0, \omega'(0) = 1] \quad (14)$$

Taking Elzaki transform of both sides of (13), we have

$$\left[ \begin{array}{c} E \left\{ \int_0^t sin(t-u) \omega(u) du \right\} \\ - \frac{1}{2} E \left\{ \int_0^t (t-u) \omega''(u) du \right\} \\ = E \left\{ \frac{t}{2} - \frac{tcost}{2} \right\} \end{array} \right] \quad (15)$$

Applying faltung theorem of Elzaki transform on (15), we have

$$\begin{aligned} & \left[ \begin{array}{c} \frac{1}{p} E\{sint\} E\{\omega(t)\} \\ - \frac{1}{2} \left(\frac{1}{p}\right) E\{t\} E\{\omega''(t)\} \\ = \frac{1}{2} E\{t\} - \frac{1}{2} E\{tcost\} \end{array} \right] \\ & \Rightarrow \left[ \begin{array}{c} \left(\frac{1}{p}\right) \frac{p^3}{(p^2+1)} E\{\omega(t)\} \\ - \frac{1}{2} \left(\frac{1}{p}\right) (p^3) E\{\omega''(t)\} \\ = \frac{1}{2} (p^3) - \frac{1}{2} \left(\frac{p^3(1-p^2)}{(p^2+1)^2}\right) \end{array} \right] \quad (16) \end{aligned}$$

Applying the property “Elzaki transform of derivative of functions” on (16), we get

$$\left[ \begin{array}{c} \frac{p^2}{(p^2+1)} E\{\omega(t)\} \\ - \frac{1}{2} (p^2) \left[ \begin{array}{c} \frac{1}{p^2} E\{\omega(t)\} \\ - \omega(0) \\ - p\omega'(0) \end{array} \right] \\ = \frac{1}{2} (p^3) - \frac{1}{2} \left(\frac{p^3(1-p^2)}{(p^2+1)^2}\right) \end{array} \right] \quad (17)$$

Now using (14) in (17), we have

$$\begin{aligned} & \left[ \begin{array}{c} \frac{p^2}{(p^2+1)} E\{\omega(t)\} \\ - \frac{1}{2} (p^2) \left[ \begin{array}{c} \frac{1}{p^2} E\{\omega(t)\} - p \end{array} \right] \\ = \frac{1}{2} (p^3) - \frac{1}{2} \left(\frac{p^3(1-p^2)}{(p^2+1)^2}\right) \end{array} \right] \\ & \Rightarrow E\{\omega(t)\} = \frac{p^3}{(p^2+1)} \quad (18) \end{aligned}$$

Taking inverse Elzaki transform of both sides of (18), we get the required solution of (13) with (14) as

$$\left[ \omega(t) = E^{-1} \left\{ \frac{p^3}{(p^2+1)} \right\} = sint \right].$$

*Problem: 3* Consider the following faltung type Volterra integro-differential equation of first kind

$$\left[ \begin{array}{c} \int_0^t cos(t-u) \omega(u) du \\ + \int_0^t sin(t-u) \omega'''(u) du \\ = 1 + sint - cost \end{array} \right] \quad (19)$$

$$\text{with } \left[ \begin{array}{c} \omega(0) = 1, \\ \omega'(0) = 1, \omega''(0) = -1 \end{array} \right] \quad (20)$$

Taking Elzaki transform of both sides of (19), we have

$$\left[ \begin{array}{c} E \left\{ \int_0^t cos(t-u) \omega(u) du \right\} \\ + E \left\{ \int_0^t sin(t-u) \omega'''(u) du \right\} \\ = E\{1 + sint - cost\} \end{array} \right] \quad (21)$$

Applying faltung theorem of Elzaki transform on (21), we have

$$\begin{aligned} & \left[ \begin{array}{c} \frac{1}{p} E\{cost\} E\{\omega(t)\} \\ + \frac{1}{p} E\{sint\} E\{\omega'''(t)\} \\ = E\{1\} + E\{sint\} - E\{cost\} \end{array} \right] \\ & \Rightarrow \left[ \begin{array}{c} \left(\frac{1}{p}\right) \left(\frac{p^2}{(p^2+1)}\right) E\{\omega(t)\} \\ + \left(\frac{1}{p}\right) \left(\frac{p^3}{(p^2+1)}\right) E\{\omega'''(t)\} \\ = p^2 + \left(\frac{p^3}{(p^2+1)}\right) - \left(\frac{p^2}{(p^2+1)}\right) \end{array} \right] \quad (22) \end{aligned}$$

Applying the property “Elzaki transform of derivative of functions” on (22), we get

$$\left[ \begin{array}{c} \left(\frac{p}{(p^2+1)}\right) E\{\omega(t)\} \\ + \left(\frac{p^2}{(p^2+1)}\right) \left[ \begin{array}{c} \frac{1}{p^3} E\{\omega(t)\} \\ - \frac{1}{p} \omega(0) \\ - \omega'(0) \\ - p\omega''(0) \end{array} \right] \\ = p^2 + \left(\frac{p^3}{(p^2+1)}\right) - \left(\frac{p^2}{(p^2+1)}\right) \end{array} \right] \quad (23)$$

Now using (20) in (23), we have

$$\begin{aligned} & \left[ \begin{array}{c} \left(\frac{p}{p^2+1}\right) E\{\omega(t)\} \\ + \left(\frac{p^2}{(p^2+1)}\right) \left[\frac{1}{p^3} E\{\omega(t)\} - \frac{1}{p} - 1 + p\right] \\ = p^2 + \left(\frac{p^3}{(p^2+1)}\right) - \left(\frac{p^2}{(p^2+1)}\right) \end{array} \right] \\ \Rightarrow & \left[ E\{\omega(t)\} = p^3 + \left(\frac{p^2}{(p^2+1)}\right) \right] \end{aligned} \quad (24)$$

Taking inverse Elzaki transform of both sides of (24), we get the required solution of (19) with (20) as

$$\begin{aligned} & \left[ \begin{array}{c} \omega(t) = E^{-1} \left\{ p^3 + \frac{p^2}{(p^2+1)} \right\} \\ = E^{-1}\{p^3\} + E^{-1} \left\{ \frac{p^2}{(p^2+1)} \right\} \end{array} \right] \\ \Rightarrow & \omega(t) = t + cost. \end{aligned}$$

**Problem: 4** Consider the following faltung type Volterra integro-differential equation of first kind

$$\left[ \begin{array}{c} \int_0^t (t-u)^2 \omega(u) du \\ - \frac{1}{12} \int_0^t (t-u)^3 \omega'''(u) du = \frac{t^4}{12} \end{array} \right] \quad (25)$$

$$\text{with } \left[ \begin{array}{c} \omega(0) = 0, \\ \omega'(0) = 3, \omega''(0) = 0 \end{array} \right] \quad (26)$$

Taking Elzaki transform of both sides of (25), we have

$$\left[ \begin{array}{c} E \left\{ \int_0^t (t-u)^2 \omega(u) du \right\} \\ - \frac{1}{12} E \left\{ \int_0^t (t-u)^3 \omega'''(u) du \right\} \\ = \frac{1}{12} E\{t^4\} \end{array} \right] \quad (27)$$

Applying faltung theorem of Elzaki transform on (27), we have

$$\begin{aligned} & \left[ \begin{array}{c} \frac{1}{p} E\{t^2\} E\{\omega(t)\} \\ - \frac{1}{12} \left(\frac{1}{p}\right) E\{t^3\} E\{\omega'''(t)\} = 2p^5 \end{array} \right] \\ \Rightarrow & \left[ \begin{array}{c} \left(\frac{1}{p}\right) (2p^4) E\{\omega(t)\} \\ - \frac{1}{12} \left(\frac{1}{p}\right) (6p^5) E\{\omega'''(t)\} = 2p^6 \end{array} \right] \end{aligned} \quad (28)$$

Applying the property “Elzaki transform of derivative of functions” on (28), we get

$$\left[ \begin{array}{c} 2p^3 E\{\omega(t)\} \\ - \frac{1}{2} (p^4) \left[ \begin{array}{c} \frac{1}{p^3} E\{\omega(t)\} \\ - \frac{1}{p} \omega(0) \\ - \omega'(0) \\ - p\omega''(0) \end{array} \right] \end{array} \right] = 2p^6 \quad (29)$$

Now using (26) in (29), we have

$$\begin{aligned} & \left[ \begin{array}{c} 2p^3 E\{\omega(t)\} \\ - \frac{1}{2} (p^4) \left[ \frac{1}{p^3} E\{\omega(t)\} - 3 \right] = 2p^6 \end{array} \right] \\ \Rightarrow & \left[ \begin{array}{c} \left[ 2p^3 - \frac{1}{2} p \right] E\{\omega(t)\} \\ = 2p^6 - \frac{3}{2} p^4 \end{array} \right] \\ \Rightarrow & \left[ \begin{array}{c} \left[ \frac{4p^3 - p}{2} \right] E\{\omega(t)\} \\ = 2p^6 - \frac{3}{2} p^4 = \left[ \frac{4p^6 - 3p^4}{2} \right] \end{array} \right] \\ \Rightarrow & \left[ \begin{array}{c} E\{\omega(t)\} = \left[ p^3 - \frac{2p^3}{4p^2-1} \right] \\ = \left[ p^3 + \frac{2p^3}{1-4p^2} \right] \end{array} \right] \end{aligned} \quad (30)$$

Taking inverse Elzaki transform of both sides of (30), we get the required solution of (25) with (26) as

$$\begin{aligned} & \left[ \begin{array}{c} \omega(t) = E^{-1} \left\{ p^3 + \frac{2p^3}{1-4p^2} \right\} \\ = E^{-1}\{p^3\} + 2E^{-1} \left\{ \frac{p^3}{1-4p^2} \right\} \end{array} \right] \\ \Rightarrow & \omega(t) = t + \sinh 2t. \end{aligned}$$

## VII. CONCLUSIONS

In this paper, authors successfully determine the primitive of faltung type Volterra integro-differential equation of first kind with the help of Elzaki transform and complete methodology explained by giving four numerical problems. The results of numerical problems show that the Elzaki transform is very useful integral transform for determining the primitive of faltung type Volterra integro-differential equation of first kind. In future, this technique can be used for solving system of faltung type Volterra integro-differential equations of first kind.

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