Solutions of Population Growth and Decay Problems Using Sumudu Transform

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Abstract: **In recent years, many scholars have paid attention to determine the solution of advance problems of engineering and sciences by using integral transforms method. In this paper, authors determine the solutions of population growth and decay problems with the help of Sumudu transform. These problems have much importance in the field of economics, chemistry, biology, physics, social science and zoology. Authors have considered some numerical applications to demonstrate the effectiveness of Sumudu transform for determining the solutions of population growth and decay problems. Results of numerical applications show that Sumudu transform is a very useful integral transform for determining the solutions of population growth and decay problems.**

Keywords: **Sumudu transform, Inverse Sumudu transform, Population growth problem, Decay problem, Half-life.**

I. INTRODUCTION

Nowadays, integral transformations are one of the mostly used mathematical techniques to determine the answers of advance problems of space, science, technology and engineering. The most important feature of these transformations is providing the exact (analytical) solution of the problem without large calculation work. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations.

In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations. Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of advance problems of population growth and decay by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31].

Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace transformations of error function with applications. The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transformations and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations.

Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51-52] used Elzaki and Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [53-54] by using Aboodh and Sadik transformations respectively. Aggarwal and Sharma [55] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [56].

Aggarwal and Bhatnagar [57] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [58]. Aggarwal [59] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [60] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [61] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [62]. Mishra et al. [63] defined the relationship between Sumudu and some efficient integral transforms.

The main aim of this paper is to determine the solutions of population growth and decay problems with the help of Sumudu transform.

II. DEFINITION OF SUMUDU TRANSFORM

Sumudu transform of the function $F(t), t \geq 0$ was proposed by Watugula, G.K. [64] as:

$$S\{F(t)\} = \frac{1}{v}\int_0^\infty F(t)e^{-\frac{t}{v}}dt$$

$= T(v), 0 < k_1 \leq v \leq k_2$, where operator S is called the Sumudu transform operator.

Problem of Growth

The growth of the population (a plant, or a cell, or an organ, or a species) is mathematically expressed in terms of a first order ordinary linear differential equation [14-20, 53-54] as

$\frac{dQ}{dt} = KQ$ (1)

with initial condition $Q(t_0) = Q_0$ (2)

where K is a positive real number, Q is the amount of population at time t and Q_0 is the initial population at time $t = t_0$.

Equation (1) is known as the Malthus law of population growth.

Problem of Decay

The decay problem of the substance is defined mathematically by the following first order ordinary linear differential equation [14-20, 53-54] as

$\frac{dQ}{dt} = -KQ$ (3)

with initial condition $Q(t_0) = Q_0$ (4)

where Q is the amount of substance at time t, K is a positive real number and Q_0 is the initial amount of the substance at time $t = t_0$.

In equation (3), the negative sign in the R.H.S. is taken because the mass of the substance is decreasing with time and so the derivative $\frac{dQ}{dt}$ must be negative.

Linearity Property of Sumudu Transforms [29, 35]

If $S\{F(t)\} = T_1(v)$ and $S\{G(t)\} = T_2(v)$ then

$S\{aF(t) + bG(t)\} = aS\{F(t)\} + bS\{G(t)\}$

$\Rightarrow S\{aF(t) + bG(t)\} = aT_1(v) + bT_2(v)$, where a, b are arbitrary constants.

Table 1 SUMUDU TRANSFORM OF SOME USEFUL FUNCTIONS [29, 35]

S.N.	$F(t)$	$S\{F(t)\} = T(v)$
1.	1	1
2.	t	v
3.	t^2	$2!\,v^2$
4.	$t^n, n \in N$	$n!\,v^n$
5.	$t^n, n > -1$	$\Gamma(n+1)v^n$
6.	e^{at}	$\frac{1}{1-av}$
7.	$sinat$	$\frac{av}{1+a^2v^2}$
8.	$cosat$	$\frac{1}{1+a^2v^2}$
9.	$sinhat$	$\frac{av}{1-a^2v^2}$
10.	$coshat$	$\frac{1}{1-a^2v^2}$

III. INVERSE SUMUDU TRANSFORM [29]

If $S\{F(t)\} = T(v)$ then $F(t)$ is called the inverse Sumudu transform of $T(v)$ and mathematically it is defined as $F(t) = S^{-1}\{T(v)\}$, where the operator S^{-1} is called the inverse Sumudu transform operator.

Table 2 INVERSE SUMUDU TRANSFORM OF SOME USEFUL FUNCTIONS [29]

S.N.	$T(v)$	$F(t) = S^{-1}\{T(v)\}$
1.	1	1
2.	v	t
3.	v^2	$\frac{t^2}{2!}$
4.	$v^n, n\epsilon N$	$\frac{t^n}{n!}$
5.	$v^n, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{1-av}$	e^{at}
7.	$\frac{v}{1+a^2v^2}$	$\frac{sinat}{a}$
8.	$\frac{1}{1+a^2v^2}$	$cosat$
9.	$\frac{v}{1-a^2v^2}$	$\frac{sinhat}{a}$
10.	$\frac{1}{1-a^2v^2}$	$coshat$

Sumudu Transform of the Derivatives of the Function $F(t)$ [29, 35]

If $S\{F(t)\} = T(v)$ then

a) $S\{F'(t)\} = \frac{T(v)}{v} - \frac{F(0)}{v}$

b) $S\{F''(t)\} = \frac{T(v)}{v^2} - \frac{F(0)}{v^2} - \frac{F'(0)}{v}$

c) $S\{F^{(n)}(t)\} = \frac{T(v)}{v^n} - \frac{F(0)}{v^n} - \frac{F'(0)}{v^{n-1}} - \cdots \ldots - \frac{F^{(n-1)}(0)}{v}$.

IV. SOLUTION OF POPULATION GROWTH PROBLEM USING SUMUDU TRANSFORMS

In this section, authors determine the solution of population growth problem given by (1) and (2) using Sumudu transform.

Taking Sumudu transform on both sides of (1), we have

$S\left\{\frac{dQ}{dt}\right\} = KS\{Q(t)\}$ (5)

Now applying the property, Sumudu transform of derivative of function, on (5), we have

$\frac{1}{v}S\{Q(t)\} - \frac{Q(0)}{v} = KS\{Q(t)\}$ (6)

Using (2) in (6) and on simplification, we have

$$\left(\frac{1}{v} - K\right) S\{Q(t)\} = \frac{Q_0}{v}$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0}{(1-Kv)} \quad (7)$$

Operating inverse Sumudu transform on both sides of (7), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0}{(1-Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{1}{(1-Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 e^{Kt} \quad (8)$$

which is the required amount of the population at time t.

V. SOLUTION OF DECAY PROBLEM USING SUMUDU TRANSFORMS

In this section, authors determine the solution of decay problem which is mathematically expressed in terms of (3) and (4) using Sumudu transform.

Applying the Sumudu transform on both sides of (3), we have

$$S\left\{\frac{dQ}{dt}\right\} = -KS\{Q(t)\} \quad (9)$$

Now applying the property, Sumudu transform of derivative of function, on (9), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{Q(0)}{v} = -KS\{Q(t)\} \quad (10)$$

Using (4) in (10) and on simplification, we have

$$\left(\frac{1}{v} + K\right) S\{Q(t)\} = \frac{Q_0}{v}$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0}{(1+Kv)} \quad (11)$$

Operating inverse Sumudu transform on both sides of (11), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0}{(1+Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{1}{(1+Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 e^{-Kt} \quad (12)$$

which is the required amount of substance at time t.

VI. APPLICATIONS

In this section, authors have considered some numerical applications to demonstrate the effectiveness of Sumudu transform for determining the solutions of population growth and decay problems.

Application: 1 The population of a city grows at a rate proportional to the number of people presently living in the city. If after four years, the population has tripled, and after five years the population is 50,000, estimate the number of people initially living in the city.

This problem can be written in mathematical form as:

$$\frac{dQ(t)}{dt} = KQ(t) \quad (13)$$

where Q denote the number of people living in the city at any time t and K is the constant of proportionality. Consider Q_0 is the number of people initially living in the city at $t = 0$.

Applying the Sumudu transform on both sides of (13), we have

$$S\left\{\frac{dQ}{dt}\right\} = KS\{Q(t)\} \quad (14)$$

Now applying the property, Sumudu transform of derivative of function, on (14), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{Q(0)}{v} = KS\{Q(t)\} \quad (15)$$

Since at $t = 0, Q = Q_0$, so using this in (15), we have

$$\left(\frac{1}{v} - K\right) S\{Q(t)\} = \frac{Q_0}{v}$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0}{(1-Kv)} \quad (16)$$

Operating inverse Sumudu transform on both sides of (16), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0}{(1-Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{1}{(1-Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 e^{Kt} \quad (17)$$

Now at $t = 4$, $Q = 3Q_0$, so using this in (17), we have

$$3Q_0 = Q_0 e^{4K}$$

$$\Rightarrow e^{4K} = 3$$

$$\Rightarrow K = \frac{1}{4}log_e 3 = 0.275 \quad (18)$$

Now using the condition at $t = 5, Q = 50{,}000$, in (17), we have

$$50{,}000 = Q_0 e^{5K} \quad (19)$$

Putting the value of K from (18) in (19), we have

$$50{,}000 = Q_0 e^{5\times 0.275}$$

$$\Rightarrow 50{,}000 = 3.955Q_0$$

$$\Rightarrow Q_0 \simeq 12642 \quad (20)$$

which are the required number of people initially living in the city.

Application: 2 A radioactive substance is known to decay at a rate proportional to the amount present. If initially there is 100 milligrams of the radioactive substance present and after six hours it is observed that the radioactive substance has lost 30 percent of its original mass, find the half life of the radioactive substance.

This problem can be written in mathematical form as:

$$\frac{dQ(t)}{dt} = -KQ(t) \tag{21}$$

where Q denote the amount of radioactive substance at time t and K is the constant of proportionality. Consider Q_0 is the initial amount of the radioactive substance at time $t = 0$.

Applying the Sumudu transform on both sides of (21), we have

$$S\left\{\frac{dQ}{dt}\right\} = -KS\{Q(t)\} \tag{22}$$

Now applying the property, Sumudu transform of derivative of function, on (22), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{Q(0)}{v} = -KS\{Q(t)\} \tag{23}$$

Since at $t = 0, Q = Q_0 = 100$, so using this in (23), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{100}{v} = -KS\{Q(t)\}$$

$$\Rightarrow \left(\frac{1}{v} + K\right) S\{Q(t)\} = \frac{100}{v}$$

$$\Rightarrow\ S\{Q(t)\} = \frac{100}{(1+Kv)} \tag{24}$$

Operating inverse Sumudu transform on both sides of (24), we have

$$Q(t) = S^{-1}\left\{\frac{100}{(1+Kv)}\right\}$$

$$= 100S^{-1}\left\{\frac{1}{(1+Kv)}\right\}$$

$$\Rightarrow Q(t) = 100e^{-Kt} \tag{25}$$

Now at $t = 6$, the radioactive substance has lost 30 percent of its original mass 100 mg so $Q = 100 - 30 = 70$, using this in (25), we have

$$70 = 100e^{-6K}$$

$$\Rightarrow e^{-6K} = 0.70$$

$$\Rightarrow K = -\frac{1}{6}log_e 0.70 = 0.059 \tag{26}$$

We required t when $Q = \frac{Q_0}{2} = \frac{100}{2} = 50$ so from (25), we have

$$50 = 100e^{-Kt} \tag{27}$$

Putting the value of K from (26) in (27), we have

$$50 = 100e^{-0.059t}$$

$$\Rightarrow e^{-0.059t} = 0.50$$

$$\Rightarrow t = -\frac{1}{0.059}log_e 0.50$$

$$\Rightarrow t = 11.75 \text{ hours} \tag{28}$$

which is the required half-time of the radioactive substance.

VII. CONCLUSIONS

In this paper, authors successfully determined the solutions of population growth and decay problems using Sumudu transform and complete methodology explained by giving some numerical applications in application section. Results of numerical applications show that Sumudu transform is a very effective integral transform for determining the solutions of population growth and decay problems. The scheme defined in this paper can be applied for the continuous compound interest and heat conduction problems in fututre.

CONFLICT OF INTEREST

The authors confirm that this article contents have no conflict of interest.

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