Laplace-Carson Transform for the Primitive of Convolution Type Volterra Integro-Differential Equation of First Kind

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Abstract: **Volterra integro-differential equations appear in many branches of engineering, physics, biology, astronomy, radiology and having many interesting applications such as process of glass forming, diffusion process, heat and mass transfer, growth of cells and describing the motion of satellite. In this paper, authors present Laplace-Carson transform for the primitive of convolution type Volterra integro-differential equation of first kind. Four numerical problems have been considered and solved using Laplace-Carson transform for explaining the applicability of present transform. Results of numerical problems show that the Laplace-Carson transform is very effective integral transform for determining the primitive of convolution type Volterra integro-differential equation of first kind.**

Keywords: **Volterra integro-differential equation; Laplace-Carson transform; Convolution; Inverse Laplace-Carson transform.**

I. INTRODUCTION

Nowadays, integral transformations are one of the mostly used mathematical techniques to determine the answers of advance problems of space, science, technology and engineering. The most important feature of these transformations is providing the exact (analytical) solution of the problem without large calculation work. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations. In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations. Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of advance problems of population growth and decay by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31]. Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace transformations of error function with applications. The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transformations and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations. Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51-52] used Elzaki and Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [53-54] by using Aboodh and Sadik transformations respectively. Aggarwal and Sharma [55] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [56]. Aggarwal and Bhatnagar [57] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [58]. Aggarwal [59] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [60] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [61] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [62]. Mishra et al. [63] defined the relationship between Sumudu and some efficient integral transforms.

The main aim of this paper is to determine the primitive of convolution type Volterra integro-differential equation of first kind with the help of Laplace-Carson transform.

II. DEFINITION OF LAPLACE-CARSON TRANSFORM

The Laplace-Carson transform of the function $G(t)$ for all $t \geq 0$ is defined as [23]:

$L\{G(t)\} = p\int_0^{\infty} G(t)e^{-pt}dt = g(p)$, where L is Laplace-Carson transform operator.

TABLE 1 FUNDAMENTAL PROPERTIES OF LAPLACE-CARSON (MAHGOUB) TRANSFORMS [38, 20]

S.N.	Name of Property/Theorem	Mathematical Form
1	Linearity	$\begin{bmatrix} L\{aG_1(t)+bG_2(t)\} \\ = aL\{G_1(t)\}+bL\{G_2(t)\} \end{bmatrix}$
2	Change of Scale	$L\{G(at)\} = g\left(\frac{p}{a}\right)$
3	Shifting	$L\{e^{at}G(t)\} = \left(\frac{p}{p-a}\right)g(p-a)$
4	First Derivative	$\begin{bmatrix} L\{G^{'}(t)\} \\ = pg(p)-pG(0) \end{bmatrix}$
5	Second Derivative	$\begin{bmatrix} L\{G^{''}(t)\} = p^2g(p) \\ -p^2G(0)-pG^{'}(0) \end{bmatrix}$
6	nth Derivative	$\begin{bmatrix} L\{G^{(n)}(t)\} = p^ng(p) \\ -p^nG(0) \\ -p^{n-1}G^{'}(0) \\ -\cdots\ldots \\ -pG^{(n-1)}(0) \end{bmatrix}$
7	Convolution	$\begin{bmatrix} L\{G_1(t)*G_2(t)\} \\ = \frac{1}{p}L\{G_1(t)\}L\{G_2(t)\} \end{bmatrix}$

TABLE 2 LAPLACE-CARSON (MAHGOUB) TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [23]

S.N.	$G(t)$	$L\{G(t)\} = g(p)$
1.	1	1
2.	t	$\frac{1}{p}$
3.	t^2	$\frac{2!}{p^2}$
4.	$t^n, n \in N$	$\frac{n!}{p^n}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{p^n}$
6.	e^{at}	$\frac{p}{p-a}$
7.	$sinat$	$\frac{ap}{p^2+a^2}$
8.	$cosat$	$\frac{p^2}{p^2+a^2}$
9.	$sinhat$	$\frac{ap}{p^2-a^2}$
10.	$coshat$	$\frac{p^2}{p^2-a^2}$

III. DUALITY BETWEEN LAPLACE-CARSON AND LAPLACE TRANSFORMS [23]

If Laplace-Carson and Laplace transforms of $G(t)$ are $g(p)$ and $h(p)$ respectively then

$g(p) = ph(p)$ and $h(p) = \frac{1}{p}g(p)$,

where $h(p) = \int_0^{\infty} G(t)e^{-(pt)}dt = \ddot{L}\{G(t)\}$ and $\ddot{L}$ is Laplace transform operator.

IV. INVERSE LAPLACE-CARSON TRANSFORM

If $L\{G(t)\} = g(p)$ then $G(t)$ is called the inverse Laplace-Carson transform of $g(p)$ and mathematically it is defined as

$G(t) = L^{-1}\{g(p)\}$, where L^{-1} is the inverse Laplace-Carson transform operator.

TABLE 3 INVERSE LAPLACE-CARSON TRANSFORMS OF FREQUENTLY ENCOUNTERED FUNCTIONS

S.N.	$g(p)$	$G(t) = L^{-1}\{g(p)\}$
1.	1	1
2.	$\frac{1}{p}$	t
3.	$\frac{1}{p^2}$	$\frac{t^2}{2!}$
4.	$\frac{1}{p^n}, n\epsilon N$	$\frac{t^n}{n!}$
5.	$\frac{1}{p^n}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{p}{p-a}$	e^{at}
7.	$\frac{p}{p^2+a^2}$	$\frac{sinat}{a}$
8.	$\frac{p^2}{p^2+a^2}$	$cosat$
9.	$\frac{p}{p^2-a^2}$	$\frac{sinhat}{a}$
10.	$\frac{p^2}{p^2-a^2}$	$coshat$

V. LAPLACE-CARSON TRANSFORM FOR THE PRIMITIVE OF CONVOLUTION TYPE VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF FIRST KIND

In this part of the paper, authors used Laplace-Carson transform for determining the primitive of convolution type Volterra integro-differential equation of first kind.

Convolution type Volterra integro-differential equation of first kind is given by

$$\left.\begin{aligned} &\int_0^t K_1(t-u)\,\omega(u)du + \\ &\int_0^t K_2(t-u)\,\omega^{(n)}(u)du \\ &= F(t), K_2(t-u) \neq 0 \end{aligned}\right\} \quad (1)$$

$$\text{with} \quad \left.\begin{aligned} &\omega(0) = \delta_0, \omega^{'}(0) = \delta_1, \\ &\omega^{''}(0) = \delta_2, \ldots\ldots, \\ &\omega^{(n-1)}(0) = \delta_{n-1} \end{aligned}\right\} \quad (2)$$

$$\text{where} \left.\begin{matrix} \begin{bmatrix} K_1(t-u), K_2(t-u) \\ = faltung\ type\ kernels \\ of\ integral\ equation \end{bmatrix} \\ \begin{bmatrix} \omega(t) = unknown \\ function \end{bmatrix} \\ \begin{bmatrix} \omega^{(n)}(t) = nth\ derivative \\ of unknown\ function \end{bmatrix} \\ \begin{bmatrix} F(t) = known \\ function \end{bmatrix} \\ \begin{bmatrix} \delta_0, \delta_1, \delta_2, \dots, \delta_{n-1} \\ = real\ numbers \end{bmatrix} \end{matrix}\right\}$$

Taking Laplace-Carson transform of both sides of (1), we have

$$\begin{bmatrix} L\left\{\int_0^t K_1(t-u)\,\omega(u)du\right\} \\ +L\left\{\int_0^t K_2(t-u)\,\omega^{(n)}(u)du\right\} \\ = L\{F(t)\} \end{bmatrix} \quad (3)$$

Applying convolution theorem of Laplace-Carson transform on (3), we have

$$\begin{bmatrix} \frac{1}{p}L\{K_1(t)\}L\{\omega(t)\} \\ +\frac{1}{p}L\{K_2(t)\}L\{\omega^{(n)}(t)\} = L\{F(t)\} \end{bmatrix} \quad (4)$$

Applying the property "Laplace-Carson transform of derivative of functions" on (4), we get

$$\begin{bmatrix} \frac{1}{p}L\{K_1(t)\}L\{\omega(t)\} \\ +\frac{1}{p}L\{K_2(t)\}\begin{bmatrix} p^n L\{\omega(t)\} \\ -p^n\omega(0) \\ -p^{n-1}\omega'(0) \\ -p^{n-2}\omega''(0) \\ -\cdots\dots \\ -p\omega^{(n-1)}(0) \end{bmatrix} = L\{F(t)\} \end{bmatrix} \quad (5)$$

Now using (2) in (5), we have

$$\begin{bmatrix} \frac{1}{p}L\{K_1(t)\}L\{\omega(t)\} \\ +\frac{1}{p}L\{K_2(t)\}\begin{bmatrix} p^n L\{\omega(t)\} \\ -p^n\delta_0 \\ -p^{n-1}\delta_1 \\ -p^{n-2}\delta_2 \\ -\cdots\dots \\ -p\delta_{n-1} \end{bmatrix} = L\{F(t)\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \begin{bmatrix} \frac{1}{p}L\{K_1(t)\} \\ +p^{n-1}L\{K_2(t)\} \end{bmatrix} L\{\omega(t)\} \\ = \begin{bmatrix} L\{F(t)\} \\ +L\{K_2(t)\}\begin{pmatrix} p^{n-1}\delta_0 \\ +p^{n-2}\delta_1 \\ +p^{n-3}\delta_2 \\ +\cdots\dots \\ +\delta_{n-1} \end{pmatrix} \end{bmatrix} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} L\{\omega(t)\} = \dfrac{\begin{bmatrix} L\{F(t)\}+ \\ L\{K_2(t)\}\begin{pmatrix} p^{n-1}\delta_0 \\ +p^{n-2}\delta_1 \\ +p^{n-3}\delta_2 \\ +\cdots\dots \\ +\delta_{n-1} \end{pmatrix} \end{bmatrix}}{\begin{bmatrix} \frac{1}{p}L\{K_1(t)\} \\ +p^{n-1}L\{K_2(t)\} \end{bmatrix}}, \\ \begin{bmatrix} \frac{1}{p}L\{K_1(t)\} \\ +p^{n-1}L\{K_2(t)\} \end{bmatrix} \neq 0 \end{bmatrix} \quad (6)$$

The inverse Laplace-Carson transform of both sides of (6) gives the required primitive of convolution type Volterra integro-differential equation of first kind which is given by (1) with (2).

VI. NUMERICAL PROBLEMS

In this part of the paper, some numerical problems have been considered for explaining the complete methodology.

Problem: 1 Consider the following convolution type Volterra integro-differential equation of first kind

$$\begin{bmatrix} \int_0^t (t-u)\,\omega(u)du \\ +\int_0^t (t-u)^2\,\omega'(u)du \\ = 3t - 3sint \end{bmatrix} \quad (7)$$

with $\omega(0) = 0$ (8)

Taking Laplace-Carson transform of both sides of (7), we have

$$\begin{bmatrix} L\left\{\int_0^t (t-u)\,\omega(u)du\right\} \\ +L\left\{\int_0^t (t-u)^2\,\omega'(u)du\right\} \\ = L\{3t - 3sint\} \end{bmatrix} \quad (9)$$

Applying convolution theorem of Laplace-Carson transform on (9), we have

$$\begin{bmatrix} \frac{1}{p}L\{t\}L\{\omega(t)\} + \frac{1}{p}L\{t^2\}L\{\omega'(t)\} \\ = L\{3t - 3sint\} \\ = 3L\{t\} - 3L\{sint\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{p^2}L\{\omega(t)\} + \frac{2}{p^3}L\{\omega'(t)\} \\ = \frac{3}{p} - \frac{3p}{(p^2+1)} \end{bmatrix} \quad (10)$$

Applying the property "Laplace-Carson transform of derivative of functions" on (10), we get

$$\left[\begin{array}{c} \frac{1}{p^2}L\{\omega(t)\} \\ +\frac{2}{p^3}[pL\{\omega(t)\}-p\omega(0)] \\ =\frac{3}{p}-\frac{3p}{(p^2+1)} \end{array}\right] \quad (11)$$

Now using (8) in (11), we have

$$\left[\frac{3}{p^2}L\{\omega(t)\}=\frac{3}{p}-\frac{3p}{(p^2+1)}\right]$$

$$\Rightarrow\left[\begin{array}{c} L\{\omega(t)\} \\ =p-\frac{p^3}{(p^2+1)}=\frac{p}{(p^2+1)} \end{array}\right] \quad (12)$$

Taking inverse Laplace-Carson transform of both sides of (12), we get the required primitive of (7) with (8) as

$$\left[\omega(t)=L^{-1}\left\{\frac{p}{(p^2+1)}\right\}=sint\right].$$

Problem: 2 Consider the following convolution type Volterra integro-differential equation of first kind

$$\left[\begin{array}{c} \int_0^t sin(t-u)\,\omega(u)du \\ -\frac{1}{2}\int_0^t (t-u)\,\omega''(u)du \\ =\frac{t}{2}-\frac{tcost}{2} \end{array}\right] \quad (13)$$

with $[\omega(0)=0, \omega'(0)=1]$ (14)

Taking Laplace-Carson transform of both sides of (13), we have

$$\left[\begin{array}{c} L\left\{\int_0^t sin(t-u)\,\omega(u)du\right\} \\ -\frac{1}{2}L\left\{\int_0^t (t-u)\,\omega''(u)du\right\} \\ =L\left\{\frac{t}{2}-\frac{tcost}{2}\right\} \end{array}\right] \quad (15)$$

Applying convolution theorem of Laplace-Carson transform on (15), we have

$$\left[\begin{array}{c} \frac{1}{p}L\{sint\}L\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p}\right)L\{t\}L\{\omega''(t)\} \\ =\frac{1}{2}L\{t\}-\frac{1}{2}L\{tcost\} \end{array}\right]$$

$$\Rightarrow\left[\begin{array}{c} \frac{1}{(p^2+1)}L\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^2}\right)L\{\omega''(t)\} \\ =\frac{1}{2}\left(\frac{1}{p}\right)-\frac{p}{2}\left(\frac{p^2-1}{(p^2+1)^2}\right) \end{array}\right] \quad (16)$$

Applying the property "Laplace-Carson transform of derivative of functions" on (16), we get

$$\left[\begin{array}{c} \frac{1}{(p^2+1)}L\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^2}\right)\left[\begin{array}{c} p^2L\{\omega(t)\} \\ -p^2\omega(0) \\ -p\omega'(0) \end{array}\right] \\ =\frac{1}{2}\left(\frac{1}{p}\right)-\frac{p}{2}\left(\frac{p^2-1}{(p^2+1)^2}\right) \end{array}\right] \quad (17)$$

Now using (14) in (17), we have

$$\left[\begin{array}{c} \frac{1}{(p^2+1)}L\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^2}\right)[p^2L\{\omega(t)\}-p\,] \\ =\frac{1}{2}\left(\frac{1}{p}\right)-\frac{p}{2}\left(\frac{p^2-1}{(p^2+1)^2}\right) \end{array}\right]$$

$$\Rightarrow L\{\omega(t)\}=\frac{p}{(p^2+1)} \quad (18)$$

Taking inverse Laplace-Carson transform of both sides of (18), we get the required primitive of (13) with (14) as

$$\left[\omega(t)=L^{-1}\left\{\frac{p}{(p^2+1)}\right\}=sint\right].$$

Problem: 3 Consider the following convolution type Volterra integro-differential equation of first kind

$$\left[\begin{array}{c} \int_0^t cos(t-u)\,\omega(u)du \\ +\int_0^t sin(t-u)\,\omega'''(u)du \\ =1+sint-cost \end{array}\right] \quad (19)$$

$$\text{with}\left[\begin{array}{c} \omega(0)=1, \\ \omega'(0)=1, \omega''(0)=-1 \end{array}\right] \quad (20)$$

Taking Laplace-Carson transform of both sides of (19), we have

$$\left[\begin{array}{c} L\left\{\int_0^t cos(t-u)\,\omega(u)du\right\} \\ +L\left\{\int_0^t sin(t-u)\,\omega'''(u)du\right\} \\ =L\{1+sint-cost\,\} \end{array}\right] \quad (21)$$

Applying convolution theorem of Laplace-Carson transform on (21), we have

$$\left[\begin{array}{c} \frac{1}{p}L\{cost\}L\{\omega(t)\} \\ +\frac{1}{p}L\{sint\}L\{\omega'''(t)\} \\ =L\{1\}+L\{sint\}-L\{cost\} \end{array}\right]$$

$$\Rightarrow\left[\begin{array}{c} \frac{p}{(p^2+1)}L\{\omega(t)\} \\ +\frac{1}{(p^2+1)}L\{\omega'''(t)\} \\ =1+\frac{p}{(p^2+1)}-\frac{p^2}{(p^2+1)} \end{array}\right] \quad (22)$$

Applying the property "Laplace-Carson transform of derivative of functions" on (22), we get

$$\begin{bmatrix} \frac{p}{(p^2+1)} L\{\omega(t)\} \\ +\frac{1}{(p^2+1)} \begin{bmatrix} p^3 L\{\omega(t)\} \\ -p^3\omega(0) \\ -p^2\omega'(0) \\ -p\omega''(0) \end{bmatrix} \\ = 1+\frac{p}{(p^2+1)}-\frac{p^2}{(p^2+1)} \end{bmatrix} \quad (23)$$

Now using (20) in (23), we have

$$\begin{bmatrix} \frac{p}{(p^2+1)} L\{\omega(t)\} \\ +\frac{1}{(p^2+1)} \begin{bmatrix} p^3 L\{\omega(t)\} \\ -p^3-p^2+p \end{bmatrix} \\ = 1+\frac{p}{(p^2+1)}-\frac{p^2}{(p^2+1)} \end{bmatrix}$$

$$\Rightarrow \left[L\{\omega(t)\} = \frac{1}{p}+\frac{p^2}{(p^2+1)}\right] \quad (24)$$

Taking inverse Laplace-Carson transform of both sides of (24), we get the required primitive of (19) with (20) as

$$\begin{bmatrix} \omega(t) = L^{-1}\left\{\frac{1}{p}+\frac{p^2}{(p^2+1)}\right\} \\ = L^{-1}\left\{\frac{1}{p}\right\} + L^{-1}\left\{\frac{p^2}{(p^2+1)}\right\} \end{bmatrix}$$

$$\Rightarrow \omega(t) = t + cost.$$

Problem: 4 Consider the following convolution type Volterra integro-differential equation of first kind

$$\begin{bmatrix} \int_0^t (t-u)^2\,\omega(u)du \\ -\frac{1}{12}\int_0^t (t-u)^3\,\omega'''(u)du = \frac{t^4}{12} \end{bmatrix} \quad (25)$$

with $\begin{bmatrix} \omega(0)=0, \\ \omega'(0)=3, \omega''(0)=0 \end{bmatrix}$ (26)

Taking Laplace-Carson transform of both sides of (25), we have

$$\begin{bmatrix} L\left\{\int_0^t (t-u)^2\,\omega(u)du\right\} \\ -\frac{1}{12} L\left\{\int_0^t (t-u)^3\,\omega'''(u)du\right\} \\ = \frac{1}{12} L\{t^4\} \end{bmatrix} \quad (27)$$

Applying convolution theorem of Laplace-Carson transform on (27), we have

$$\begin{bmatrix} \frac{1}{p} L\{t^2\} L\{\omega(t)\} \\ -\frac{1}{12}\left(\frac{1}{p}\right) L\{t^3\} L\{\omega'''(t)\} = \frac{2}{p^4} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{2}{p^3} L\{\omega(t)\} \\ -\frac{1}{12}\left(\frac{6}{p^4}\right) L\{\omega'''(t)\} = \frac{2}{p^4} \end{bmatrix} \quad (28)$$

Applying the property "Laplace-Carson transform of derivative of functions" on (28), we get

$$\begin{bmatrix} \frac{2}{p^3} L\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^4}\right) \begin{bmatrix} p^3 L\{\omega(t)\} \\ -p^3\omega(0) \\ -p^2\omega'(0) - p\omega''(0) \end{bmatrix} = \frac{2}{p^4} \end{bmatrix} \quad (29)$$

Now using (26) in (29), we have

$$\begin{bmatrix} \frac{2}{p^3} L\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^4}\right)[p^3 L\{\omega(t)\} - 3p^2] = \frac{2}{p^4} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left[\frac{2}{p^3}-\frac{1}{2}\left(\frac{1}{p}\right)\right] L\{\omega(t)\} \\ = \frac{2}{p^4}-\frac{3}{2}\left(\frac{1}{p^2}\right) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left[\frac{4-p^2}{2p^3}\right] L\{\omega(t)\} \\ = \frac{2}{p^4}-\frac{3}{2}\left(\frac{1}{p^2}\right) = \left[\frac{4-3p^2}{2p^4}\right] \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} L\{\omega(t)\} = \left[\frac{1}{p}-\frac{2p}{(4-p^2)}\right] \\ = \left[\frac{1}{p}+\frac{2p}{(p^2-4)}\right] \end{bmatrix} \quad (30)$$

Taking inverse Laplace-Carson transform of both sides of (30), we get the required primitive of (25) with (26) as

$$\begin{bmatrix} \omega(t) = L^{-1}\left\{\frac{1}{p}+\frac{2p}{(p^2-4)}\right\} \\ = L^{-1}\left\{\frac{1}{p}\right\} + 2L^{-1}\left\{\frac{p}{(p^2-4)}\right\} \end{bmatrix}$$

$$\Rightarrow \omega(t) = t + sinh2t.$$

VII. CONCLUSIONS

In this paper, authors successfully discussed the Laplace-Carson transform for the primitive of convolution type Volterra integro-differential equation of first kind and complete methodology explained by giving four numerical problems. The results of numerical problems show that the Laplace-Carson transform is very useful integral transformation for determining the primitive of convolution type Volterra integro-differential equation of first kind. In future, Laplace-Carson transform can be used for solving system of convolution type Volterra integro-differential equations of first kind.

REFERENCES

[1]. Aggarwal, S., Chauhan, R., & Sharma, N. (2018). A new application of Mahgoub transform for solving linear Volterra integral equations. *Asian Resonance*, *7*(2), 46-48.

[2]. Aggarwal, S., Sharma, N., & Chauhan, R. (2018). Application of Mahgoub transform for solving linear Volterra integral equations of first kind. *Global Journal of Engineering Science and Researches*, *5*(9), 154-161.

[3]. Aggarwal, S., Sharma, N., & Chauhan, R. (2018). A new application of Aboodh transform for solving linear Volterra integral equations. *Asian Resonance*, *7*(3), 156-158.

[4]. Aggarwal, S., Gupta, A. R., & Sharma, S. D. (2019). A new application of Shehu transform for handling Volterra integral

equations of first kind. *International Journal of Research in Advent Technology*, *7*(4), 439-445.

[5]. Aggarwal, S., Chauhan, R., & Sharma, N. (2018). Application of Elzaki transform for solving linear Volterra integral equations of first kind. *International Journal of Research in Advent Technology*, *6*(12), 3687-3692.

[6]. Aggarwal, S., Sharma, N., & Chauhan, R. (2018). Application of Aboodh transform for solving linear Volterra integral equations of first kind. *International Journal of Research in Advent Technology*, *6*(12), 3745-3753.

[7]. Aggarwal, S., Sharma, N., & Chauhan, R. (2018). Solution of linear Volterra integral equations of second kind using Mohand transform. *International Journal of Research in Advent Technology*, *6*(11), 3098-3102.

[8]. Aggarwal, S., Chauhan, R., & Sharma, N. (2018). A new application of Kamal transform for solving linear Volterra integral equations. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *7*(4), 138-140.

[9]. Aggarwal, S., Sharma, N., & Chauhan, R. (2018). Solution of linear Volterra integro-differential equations of second kind using Mahgoub transform. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *7*(5), 173-176.

[10]. Aggarwal, S., & Gupta, A. R. (2019). Solution of linear Volterra integro-differential equations of second kind using Kamal transform. *Journal of Emerging Technologies and Innovative Research*, *6*(1), 741-747.

[11]. Aggarwal, S., Sharma, N., & Chauhan, R. (2018). Application of Aboodh transform for solving linear Volterra integro-differential equations of second kind. *International Journal of Research in Advent Technology*, *6*(6), 1186-1190.

[12]. Chauhan, R., & Aggarwal, S. (2018). Solution of linear partial integro-differential equations using Mahgoub transform. *Periodic Research*, *7*(1), 28-31.

[13]. Gupta, A. R., Aggarwal, S., & Agrawal, D. (2018). Solution of linear partial integro-differential equations using Kamal transform. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *7*(7), 88-91.

[14]. Singh, G. P., & Aggarwal, S. (2019). Sawi transform for population growth and decay problems. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *8*(8), 157-162.

[15]. Aggarwal, S., Sharma, N., & Chauhan, R. (2018). Solution of population growth and decay problems by using Mohand transform. *International Journal of Research in Advent Technology*, *6*(11), 3277-3282.

[16]. Aggarwal, S., Gupta, A. R., Asthana, N., & Singh, D. P. (2018). Application of Kamal transform for solving population growth and decay problems. *Global Journal of Engineering Science and Researches*, *5*(9), 254-260.

[17]. Aggarwal, S., Sharma, S. D., & Gupta, A. R. (2019). Application of Shehu transform for handling growth and decay problems. *Global Journal of Engineering Science and Researches*, *6*(4), 190-198.

[18]. Aggarwal, S., Singh, D. P., Asthana, N., & Gupta, A. R. (2018). Application of Elzaki transform for solving population growth and decay problems. *Journal of Emerging Technologies and Innovative Research*, *5*(9), 281-284.

[19]. Aggarwal, S., Gupta, A. R., Singh, D. P., Asthana, N., & Kumar, N. (2018). Application of Laplace transform for solving population growth and decay problems. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *7*(9), 141-145.

[20]. Aggarwal, S., Pandey, M., Asthana, N., Singh, D. P., & Kumar, A. (2018). Application of Mahgoub transform for solving population growth and decay problems. *Journal of Computer and Mathematical Sciences*, *9*(10), 1490-1496.

[21]. Aggarwal, S., Sharma, N., & Chauhan, R. (2020). Duality relations of Kamal transform with Laplace, Laplace–Carson, Aboodh, Sumudu, Elzaki, Mohand and Sawi transforms. *SN Applied Sciences*, *2*(1), 135.

[22]. Aggarwal, S., & Bhatnagar, K. (2019). Dualities between Laplace transform and some useful integral transforms. *International Journal of Engineering and Advanced Technology*, *9*(1), 936-941.

[23]. Chauhan, R., Kumar, N., & Aggarwal, S. (2019). Dualities between Laplace-Carson transform and some useful integral transforms. *International Journal of Innovative Technology and Exploring Engineering*, *8*(12), 1654-1659.

[24]. Aggarwal, S., & Gupta, A. R. (2019). Dualities between Mohand transform and some useful integral transforms. *International Journal of Recent Technology and Engineering*, *8*(3), 843-847.

[25]. Aggarwal, S., & Gupta, A. R. (2019). Dualities between some useful integral transforms and Sawi transform. *International Journal of Recent Technology and Engineering*, *8*(3), 5978-5982.

[26]. Aggarwal, S., Bhatnagar, K., & Dua, A. (2019). Dualities between Elzaki transform and some useful integral transforms. *International Journal of Innovative Technology and Exploring Engineering*, *8*(12), 4312-4318.

[27]. Aggarwal, S., Sharma, N., Chaudhary, R., & Gupta, A. R. (2019). A comparative study of Mohand and Kamal transforms. *Global Journal of Engineering Science and Researches*, *6*(2), 113-123.

[28]. Aggarwal, S., Mishra, R., & Chaudhary, A. (2019). A comparative study of Mohand and Elzaki transforms. *Global Journal of Engineering Science and Researches*, *6*(2), 203-213.

[29]. Aggarwal, S., & Sharma, S. D. (2019). A comparative study of Mohand and Sumudu transforms. *Journal of Emerging Technologies and Innovative Research*, *6*(3), 145-153.

[30]. Aggarwal, S., & Chauhan, R. (2019). A comparative study of Mohand and Aboodh transforms. *International Journal of Research in Advent Technology*, *7*(1), 520-529.

[31]. Aggarwal, S., & Chaudhary, R. (2019). A comparative study of Mohand and Laplace transforms. *Journal of Emerging Technologies and Innovative Research*, *6*(2), 230-240.

[32]. Aggarwal, S., Gupta, A. R., & Kumar, A. (2019). Elzaki transform of error function. *Global Journal of Engineering Science and Researches*, *6*(5), 412-422.

[33]. Aggarwal, S., & Singh, G. P. (2019). Aboodh transform of error function. *Universal Review*, *10*(6), 137-150.

[34]. Aggarwal, S., & GP, S. (2019). Shehu Transform of Error Function (Probability Integral). *Int J Res Advent Technol*, *7*, 54-60.

[35]. Aggarwal, S., & Sharma, S. D. (2019). Sumudu transform of error function. *Journal of Applied Science and Computations*, *6*(6), 1222-1231.

[36]. Aggarwal, S., Gupta, A. R., & Kumar, D. (2019). Mohand transform of error function. *International Journal of Research in Advent Technology*, *7*(5), 224-231.

[37]. Aggarwal, S., & Singh, G. P. (2019). Kamal transform of error function. *Journal of Applied Science and Computations*, *6*(5), 2223-2235.

[38]. Aggarwal, S., Gupta, A. R., Sharma, S. D., Chauhan, R., & Sharma, N. (2019). Mahgoub transform (Laplace-Carson transform) of error function. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *8*(4), 92-98.

[39]. Aggarwal, S., Singh, A., Kumar, A., & Kumar, N. (2019). Application of Laplace transform for solving improper integrals whose integrand consisting error function. *Journal of Advanced Research in Applied Mathematics and Statistics*, *4*(2), 1-7.

[40]. Aggarwal, S., Sharma, N., Chauhan, R., Gupta, A. R., & Khandelwal, A. (2018). A new application of Mahgoub transform for solving linear ordinary differential equations with variable coefficients. *Journal of Computer and Mathematical Sciences*, *9*(6), 520-525.

[41]. Aggarwal, S., & Sharma, S. D. (2019). Application of Kamal transform for solving Abel's integral equation. *Global Journal of Engineering Science and Researches*, *6*(3), 82-90.

[42]. Aggarwal, S., & Gupta, A. R. (2019). Sumudu transform for the solution of Abel's integral equation. *Journal of Emerging Technologies and Innovative Research*, *6*(4), 423-431.

[43]. Aggarwal, S., Sharma, S. D., & Gupta, A. R. (2019). A new application of Mohand transform for handling Abel's integral

equation. *Journal of Emerging Technologies and Innovative Research*, *6*(3), 600-608.

[44]. Aggarwal, S., & Sharma, S. D. (2019). Solution of Abel's integral equation by Aboodh transform method. *Journal of Emerging Technologies and Innovative Research*, *6*(4), 317-325.

[45]. Aggarwal, S., & Gupta, A. R. (2019). Shehu Transform for Solving Abel's Integral Equation. *Journal of Emerging Technologies and Innovative Research*, *6*(5), 101-110.

[46]. Aggarwal, S., Chauhan, R., & Sharma, N. (2018). Mohand transform of Bessel's functions. *International Journal of Research in Advent Technology*, *6*(11), 3034-3038.

[47]. Aggarwal, S., Gupta, A. R., & Agrawal, D. (2018). Aboodh transform of Bessel's functions. *Journal of Advanced Research in Applied Mathematics and Statistics*, *3*(3), 1-5.

[48]. Aggarwal, S., Sharma, N., & Chauhan, R. (2018). Mahgoub transform of Bessel's functions. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *7*(8), 32-36.

[49]. Aggarwal, S. (2018). Elzaki transform of Bessel's functions. *Global Journal of Engineering Science and Researches*, *5*(8), 45-51.

[50]. Chaudhary, R., Sharma, S.D., Kumar, N. & Aggarwal, S. (2019). Connections between Aboodh transform and some useful integral transforms. *International Journal of Innovative Technology and Exploring Engineering, 9*(1), 1465-1470.

[51]. Aggarwal, S., Chauhan, R. & Sharma, N. (2018). Application of Elzaki transform for solving linear Volterra integral equations of first kind. *International Journal of Research in Advent Technology, 6*(12), 3687-3692.

[52]. Aggarwal, S., Sharma, N. & Chauhan, R. (2018). Application of Kamal transform for solving linear Volterra integral equations of first kind. *International Journal of Research in Advent Technology, 6*(8), 2081-2088.

[53]. Aggarwal, S., Asthana, N. & Singh, D.P. (2018). Solution of population growth and decay problems by using Aboodh transform method. International Journal of Research in Advent Technology, 6(10), 2706-2710.

[54]. Aggarwal, S. & Bhatnagar, K. (2019). Sadik transform for handling population growth and decay problems. *Journal of Applied Science and Computations, 6*(6), 1212-1221.

[55]. Aggarwal, S. & Sharma, S.D. (2019). Sadik transform of error function (probability integral). *Global Journal of Engineering Science and Researches, 6*(6), 125-135.

[56]. Aggarwal, S., Gupta, A.R. & Sharma, S.D. (2019). Application of Sadik transform for handling linear Volterra integro-differential equations of second kind. *Universal Review*, *10*(7), 177-187.

[57]. Aggarwal, S. & Bhatnagar, K. (2019). Solution of Abel's integral equation using Sadik transform. *Asian Resonance, 8*(2), (Part-1), 57-63.

[58]. Aggarwal, S. (2019). A comparative study of Mohand and Mahgoub transforms. *Journal of Advanced Research in Applied Mathematics and Statistics, 4*(1), 1-7.

[59]. Aggarwal, S. (2018). Kamal transform of Bessel's functions. *International Journal of Research and Innovation in Applied Science*, *3*(7), 1-4.

[60]. Chauhan, R. & Aggarwal, S. (2019). Laplace transform for convolution type linear Volterra integral equation of second kind. *Journal of Advanced Research in Applied Mathematics and Statistics, 4*(3&4), 1-7.

[61]. Sharma, N. & Aggarwal, S. (2019). Laplace transform for the solution of Abel's integral equation. *Journal of Advanced Research in Applied Mathematics and Statistics, 4*(3&4), 8-15.

[62]. Aggarwal, S. & Sharma, N. (2019). Laplace transform for the solution of first kind linear Volterra integral equation. *Journal of Advanced Research in Applied Mathematics and Statistics, 4*(3&4), 16-23.

[63]. Mishra, R., Aggarwal, S., Chaudhary, L., & Kumar, A. (2020). Relationship between Sumudu and some efficient integral transforms. *International Journal of Innovative Technology and Exploring Engineering, 9*(3), 153-159.