# Remarks on Commutativity Results for Alternative Rings With $[x(x^2y^2), x] = 0$

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***Abstract:*** **In this article, it is shown that the commutativity of alternative ring satisfying the following properties:**

$(p_1)$ $[x(x^2y^2), x] = 0$

$(p_2)$ $[x(xy), x] = 0$

***Keywords:*** **Alternative ring, assosymetric ring, commutator, prime rings**.

## I. INTRODUCTION

Throughout $R$ represents an alternative ring, $C(R)$ the commutator, $A(R)$ the assosymetric ring. $N(R)$ the set of nilpotent element. An alternative ring R is a ring in which $(xx)y = x(xy)$, $y(xx) = (yx)x$ for all x, y in $R$, these equations are known as left and right alternative laws respectively. An assosymetric ring A(R) is one in which $(x, y, z) = (p(x), p(y), p(z))$, where p is any permutation of $x, y, z \in R$. An associator $(x, y, z)$ we mean by $(x, y, z) = (xy)z - x(yz)$ for all $x, y, z \in$ R. A ring $R$ is called a prime if whenever A and B are ideals of $R$ such that $AB = \{0\}$ then either $A = \{0\}$ $or$ $B = \{0\}$. If in a ring $R$, the identity $(x, y, x) = 0$ i.e. $(xy)x = x(yx)$ for all $x, y$ $in$ $R$ holds then $R$ is called flexible. A ring $R$ is said to be m-torsion tree if $mx = 0$ implies $x = 0, m$ is any positive number for all $x \in R$.A non-associative rings R is an additive abelian group in which multiplication is defined, which is distributive over addition on left as well as on right $[(x + y)z = xz + yz$, $z(x + y) = zx + zy, \forall x, y, z \in R]$.

Abujabal and Khan [1] proved the commutativity of associative ring satisfies the identity $(xy)^2 = xy^2x$. Gupta [2] established that a division ring $R$ is commutative if and only if $[xy, yx] = 0$. In addition, Madana and Reddy [3] have established the commutativity of non-associative ring satisfying the identities $(xy)^2 = x^2y^2$ and $(xy)^2 \in Z(R) \forall x, y \in R$. Further, Madana Mohana Reddy and Shobha latha.[4] established the commutativity of non-associative primitive rings satisfying the identities:$(x(x^2 + y^2)$+$(x^2 + y^2)x \in Z(R)$and $x(xy)^2 - (xy)^2x \in Z(R)$ ,Motivated by these observation it is natural to look commutativity of alternative rings satisfies:$(p_1)$ & $(p_2)$.

In the present paper we consider the following theorems.

## II. THE MAIN THEOREMS

*Theorem 2.1* Let $R$ be a 2,3-torsion free alternative ring with unity satisfy $(p_1)$, Then $R$ is commutative.

Now, we begin with the proof of our theorems.

*Proof of Theorem 2.1*

From the hypothesis $(p_1)$

(1) $x[x(x^2y^2)] = [x(x^2y^2)]x$ , for all x, y ∈ R.

Substitute $x = (1 + x)$ in (1), apply 2,3 torsion free and use (1) we get

(2) $y^2x = xy^2$ , for all x, y ∈ R.

Substitute $y = (y + 1)$ in (2)

(3) $xy^2 + 2xy + x = y^2x + 2yx + x$ , for all x, y ∈ R.

Use (3) and Apply 2-torsion free

This implies $xy = yx$ and R is commutative.

Since $R$ is a commutative ring and satisfies the identities either $(xx)y = x(xy)$ or

$y(xx) = (yx)x$, so that $R$ is an alternative ring. Hence an alternative ring $R$ with identity together with commutativity yields $(x, x, y) = 0 = (y, x, x)$, which completes the proof.

*Theorem 2.2* If $R$ is an alternative ring with unity satisfy $(p_2)$ then $R$ is commutative.

*Proof of Theorem 2.2*

From the hypothesis $(p_2)$, (4) $x[x(xy)] = [x(xy)]x$

Replace $x$ $by$ $(x + 1)$ in (4) and apply 2-torsion free we have

(5) $xy + x(xy) + y = yx + x(xy) + y$, for all x, y ∈ R.

Collect like terms in (5).

This implies $xy = yx$ and R is commutative.

Now using the same argument as in last paragraph of the proof of the theorem 2.1

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