Application of Alternate Direction Implicit Technique to an Unsteady MHD Flow over a Semi-Infinite Vertical Plate with Viscous Dissipation

A. Kaushik

Department of Mathematics, MAIT, Delhi, India

Abstract- **An unsteady MHD flow of a viscous incompressible fluid over a semi-infinite plate with variable surface temperature in the presence of heat source is studied. The governing equations of the flow are converted into dimensionless form and the resulting non-linear differential equations are solved numerically using Alternating –Direction-Implicit (ADI) Technique. Flow parameters are obtained and are presented graphically. It was observed that the fluid velocity decreases with increase in magnetic field.**

Keywords: **Magnetohydodynamic fluid, semi-infinite vertical plate, Alternating Direction Implicit technique, variable surface temperature.**

I. INTRODUCTION

In engineering and technology, there are numerous applications of the study of transient natural convection flows over vertical plates. These studies are frequently used in heat transfer around different types of electronic and electrical devices, nuclear reactors etc. Extensive research is done for the study of free convection flows over vertical plates under variable conditions. Various techniques such as integral method [1], finite difference scheme [2],[3]. Crank Nicholson implicit scheme [4] etc are employed for these studies of the convection flows.

Alternate Direction Implicit scheme is an effective tool for the solution of problems expressed by elliptic and parabolic partial differential equations. In recent years, many researchers have employed this scheme for the solution of variety of problems. Cheng and Wang[5] applied this method to study forced convection in micropolar fluid flow over a wavy surface. Wang and Cheng [6] studied the flow through a sinusoidally curved converging–diverging channel and analyzed the skin-friction and Nusselt number for the flow for variable wavy geometry, Reynolds number and Prandtl number. In the presence of a transverse magnetic field, Wang and Chen[7] studied mixed convection boundary layer flow past an inclined wavy plate and presented numerical solution for the flow for different values of magnetic field, buoyancy, wavy geometry and material parameters. Navarro et al [8] presented numerical simulations of two dimensional incompressible fluid flows under the influence of a magnetic field at low magnetic Reynolds number. Hakeem et al [9] employed alternating direction implicit method to study natural convection cooling of thermally active plates kept at the center of an air filled cavity, taking two different boundary conditions applied on the cavity walls.

Nejad et al [10] studied mixed convection flow of electrically conducting power law fluids along a vertical wavy surface under the influence of a transverse magnetic field. They have discussed the effects of flow structure and dominant convection mode on the overall parameters of flow and heat transfer and investigated the alterations in boundary layers with magnetic field. Kiyasatfar and Pourmahmoud [11] studied viscous dissipation and joule heating effects in the presence of transverse magnetic field for electrically conducting non-Newtonian fluids through square microchannels. For different values of flow index and dimensionless shear rate parameter of modified power-law fluids, they obtained velocity, temperature profiles, product of friction factor–Reynolds number and Nusselt number. Majee and Shit [12] investigated unsteady flow of blood, treating it as Newtonian fluid and performed heat transfer to have better insight of blood flow through arteries under stenotic condition. Liu et al [13] studied two dimensional multi-term time fractional mixed diffusion and diffusion-wave equations and proved numerical stability and convergence of the alternating direction implicit (ADI) spectral method. Kaushik [14] used alternating direction implicit technique to numerically study an unsteady flow past a semi- infinite plate with temperature oscillations.

In this paper, we employ Alternating Direction Implicit scheme to study the combined effect of magnetic field and viscous dissipation past a semi-infinite vertical plate subjected to a variable surface temperature, when the fluid is incompressible, viscous and electrically conducting. For the analysis the dimensionless form of the governing boundary layer equations is used. The paper has six sections. Section I contains the introduction of the problem. The formulation of the problem along with the initial and boundary conditions is done in Section II. Section III discusses the Alternating Direction technique and its application to the nonlinear differential equations resulting from the MHD flow over a semi-infinite plate with variable surface temperature. Section IV consists of the stability analysis of the problem. Section V contains the results obtained and discussion of these results. Section VI concludes the research work.

II. MATHEMATICAL ANALYSIS

An unsteady, 2D electrically conducing, viscous incompressible fluid flow over a semi-infinite vertical plate having variable surface temperature is considered. We are making following assumptions

1. The X- axis $(y'=0)$ is measured along the plate vertically upward and the Y-axis $(x'=0)$ is measured normally to the plate.
2. The fluid is at rest and at temperature T'_∞ . The variable temperature of the plate is $T'_w > T'_\infty$ and is taken as $T'_w = T'_\infty + Ax^n$.
3. The acceleration due to gravity is acting downward.
4. All fluid properties are constant except for effect of density variation with temperature, which is taken in the body force term.
5. The energy equation includes the effect of viscous dissipation.
6. A uniform magnetic field B_0 is applied along the Y-axis.

With these assumptions and application of the Boussinesq approximation, the governing conservations equations are given by:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \qquad (1)$$

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = g\beta(T'-T'_\infty) + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \qquad (2)$$

$$\frac{\partial T'}{\partial t'} + u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho C_P}\frac{\partial^2 T'}{\partial y'^2} - Q_0\left(T'-T'_\infty\right) + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \qquad (3)$$

where u' and v' are the velocity components in the x' and y' directions, respectively; ρ is fluid density, g is gravitational acceleration , t' is time and T' is temperature of the fluid in the boundary layer, β is volumetric coefficient of thermal expansion, μ is coefficient of viscosity . B_0 is the magnetic field induction, σ is electrical conductivity, Q_0 is heat generation/absorption, C_P is the specific heat and k is the thermal conductivity of the fluid.

The initial and boundary conditions are:

$$\begin{aligned}
&t' \le 0 : \quad u'=0 \ , v'=0,\ T'=T'_\infty && \forall\ x', y' \\
&t' > 0 : \quad u'=0 \ , v'=0,\ T'=T'_\infty + Ax^n && at\ \ y'=0 \\
&\qquad\quad u' \to 0\ ,\ T' \to T'_\infty && at\ \ y' \to \infty \\
&\qquad\quad u'=0\ ,\ T'=T'_\infty && at\ \ x'=0
\end{aligned} \qquad (4)$$

We define the following non-dimensional quantities

$$x = \frac{x'}{L}\ ,\ y = \frac{y'}{L}Gr^{1/4}\ ,\ u = \frac{u'L}{\upsilon}Gr^{-1/2}$$

$$v = \frac{v'L}{\upsilon}Gr^{-1/4}\ ,\ t = \frac{\upsilon t'}{L^2}Gr^{1/2}\ ,\ T = \frac{(T'-T'_\infty)}{(T'_w-T'_\infty)},$$

$$Gr = \frac{g\beta L^3(T'_w-T'_\infty)}{\upsilon^2}\ ,\ \upsilon = \frac{\mu}{\rho},\ \Pr = \frac{\mu C_p}{k},$$

$$\varphi = \frac{Q_0 L^2 Gr^{-1/2}}{\rho C_p \upsilon}, M = \frac{\sigma B_0^{\,2} L^2 Gr^{-1/2}}{\rho\upsilon},$$

$$\varepsilon = \frac{1}{C_p L^2}\frac{\upsilon^2 Gr^2}{(T'_w-T'_\infty)} \qquad (5)$$

where L is the reference length, υ is the kinematic viscosity, Gr is the Grashof number, $\Pr$ is Prandtl number, M is the magnetic field parameter, φ is the heat source parameter and ε is viscous dissipation parameter. Eqn. (1)-(3) reduces to non-dimensional form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = T + \frac{\partial^2 u}{\partial y^2} - Mu \qquad (7)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + \varepsilon\left(\frac{\partial u}{\partial y}\right)^2 - \varphi T \qquad (8)$$

and the initial and boundary conditions (4) as

$$\begin{aligned}
&t \le 0 : u=0 \ , v=0,\ T=0 && \forall\ x', y' \\
&t > 0 : u=0 \ , v=0,\ T=x^n && at\ \ y=0 \\
&\qquad u \to 0\ ,\ T \to 0 && at\ \ y \to \infty \\
&\qquad u=0\ ,\ T=0 && at\ \ x=0
\end{aligned} \qquad (9)$$

III. NUMERICAL TECHNIQUE

The dimensionless partial differential equations governing the flow given by (6)-(8) under the initial and boundary conditions (9) are solved using Alternating-Direction-Implicit technique. The scheme consists of two steps. The first step comprise of writing PDEs into difference equations which are implicit in x at an intermediate time level $m+\frac{1}{2}$. The unknowns associated with the x-derivatives are evaluated in this step. The implicit difference equations at the time level $m+\frac{1}{2}$ are written as

$$\frac{u_{i+1,j}^{m+\frac{1}{2}}-u_{i-1,j}^{m+\frac{1}{2}}}{2\Delta x}+\frac{v_{i,j+1}^{m}-v_{i,j-1}^{m}}{2\Delta y}=0 \quad (10)$$

$$\frac{u_{i,j}^{m+\frac{1}{2}}-u_{i,j}^{m}}{\Delta t/2}+u_{i,j}^{m}\left(\frac{u_{i+1,j}^{m+\frac{1}{2}}-u_{i-1,j}^{m+\frac{1}{2}}}{2\Delta x}\right)+v_{i,j}^{m}\left(\frac{u_{i,j+1}^{m}-u_{i,j-1}^{m}}{2\Delta y}\right)=T_{i,j}^{m}+\left(\frac{u_{i,j+1}^{m}-2u_{i,j}^{m}+u_{i,j-1}^{m}}{(\Delta y)^{2}}\right)-Mu_{i,j}^{m} \quad (11)$$

$$\frac{T_{i,j}^{m+\frac{1}{2}}-T_{i,j}^{m}}{\Delta t/2}+u_{i,j}^{m}\left(\frac{T_{i+1,j}^{m+\frac{1}{2}}-T_{i-1,j}^{m+\frac{1}{2}}}{2\Delta x}\right)+v_{i,j}^{m}\left(\frac{T_{i,j+1}^{m}-T_{i,j-1}^{m}}{2\Delta y}\right)=\frac{1}{\Pr}\left(\frac{T_{i,j+1}^{m}-2T_{i,j}^{m}+T_{i,j-1}^{m}}{(\Delta y)^{2}}\right)+\varepsilon\left(\frac{u_{i,j+1}^{m}-u_{i,j-1}^{m}}{2\Delta y}\right)^{2}-\varphi T_{i,j}^{m} \quad (12)$$

The eqns. (10) - (12) reduces into tri-diagonal form. Thomas Algorithm is used for solving this tri-diagonal form to obtain solutions for $u_{i,j}^{m+\frac{1}{2}}$ and $T_{i,j}^{m+\frac{1}{2}}$ for all i, keeping j fixed. This step is repeated for next value $j+1$ and so on. The values of $u_{i,j}^{m+\frac{1}{2}}$ and $T_{i,j}^{m+\frac{1}{2}}$ at intermediate time level $m+\frac{1}{2}$ is known for all $(i, j,)$ in the end of this step.

In the next step, we write difference equations which are implicit in y at time level n and obtain unknowns associated with the y-derivatives. The implicit difference equations at the time level n are written as

$$\frac{u_{i+1,j}^{m+\frac{1}{2}}-u_{i-1,j}^{m+\frac{1}{2}}}{2\Delta x}+\frac{v_{i,j}^{m+1}-v_{i,j-1}^{m+1}}{\Delta y}=0 \quad (13)$$

$$\frac{u_{i,j}^{m+1}-u_{i,j}^{m+\frac{1}{2}}}{\Delta t}+u_{i,j}^{m}\left(\frac{u_{i+1,j}^{m+\frac{1}{2}}-u_{i-1,j}^{m+\frac{1}{2}}}{2\Delta x}\right)+v_{i,j}^{m}\left(\frac{u_{i,j+1}^{m+1}-u_{i,j-1}^{m+1}}{2\Delta y}\right)=T_{i,j}^{m+\frac{1}{2}}+\left(\frac{u_{i,j+1}^{m+1}-2u_{i,j}^{m+1}+u_{i,j-1}^{m+1}}{(\Delta y)^{2}}\right)-Mu_{i,j}^{m+\frac{1}{2}} \quad (14)$$

$$\frac{T_{i,j}^{m+1}-T_{i,j}^{m+\frac{1}{2}}}{\Delta t}+u_{i,j}^{m}\left(\frac{T_{i+1,j}^{m+\frac{1}{2}}-T_{i-1,j}^{m+\frac{1}{2}}}{2\Delta x}\right)+v_{i,j}^{m}\left(\frac{T_{i,j+1}^{m+1}-T_{i,j-1}^{m+1}}{2\Delta y}\right)=\frac{1}{\Pr}\left(\frac{T_{i,j+1}^{m+1}-2T_{i,j}^{m+1}+T_{i,j-1}^{m+1}}{(\Delta y)^{2}}\right)+\varepsilon\left(\frac{u_{i,j+1}^{m}-u_{i,j-1}^{m}}{2\Delta y}\right)^{2}-\varphi T_{i,j}^{m+\frac{1}{2}} \quad (15)$$

We are reducing eqns. (13)-(15) to tri-diagonal form and yield solutions for $v_{i,j}^{m}$, $u_{i,j}^{m}$ and $T_{i,j}^{m}$ for all j, keeping i fixed, again using Thomas Algorithm. The calculations are repeated for all values of i . The values of $v_{i,j}^{m}$, $u_{i,j}^{m}$ and $T_{i,j}^{m}$ at next time level m is known for all (i, j) at the end of this step. Note that, here, the subscript i in $u_{i,j}^{m}$, $v_{i,j}^{m}$, $T_{i,j}^{m}$ represents the grid node along the x- direction and j subscript represents the grid node along the y- direction.

Assuming the domain of integration as a rectangular region

$$x=0\ ,\ x=1;\ \ y=0\ ,y=14,$$

with sides where $y=14$ corresponds to conditions at infinity, computations are performed. The mesh size is taken as $\Delta x=0.05$, $\Delta y=0.25$ with the time step as $\Delta t=0.01$.

IV. STABILITY ANALYSIS

The stability of differencing scheme is investigated using Von-Neumann Technique. The general term of the Fourier expansion for u and T at an arbitrary time $t=0$ is assumed to be of the form $e^{iax}e^{iby}$ where $i=\sqrt{-1}$. At any time t, these can be written as

$$u=F(t)e^{iax}e^{iby}$$
$$T=G(t)e^{iax}e^{iby} \quad (16)$$

Substituting these in Eqns. (14) and (15) and taking

$$u^{m+1}=F'(t)\ \ ,\ u^{m+\frac{1}{2}}=F(t)$$
$$T^{m+1}=G'(t)\ \ ,T^{m+\frac{1}{2}}=G(t)$$

Eqn. (14) reduces to

$$F'\left[\frac{1}{\Delta t}+\frac{2}{(\Delta y)^{2}}\left(1-\cos b\Delta y\right)+i\frac{v}{2\Delta y}\sin\left(b\Delta y\right)\right]=F\left(\frac{1}{\Delta t}-M-i\frac{u}{\Delta x}\sin\left(a\Delta x\right)\right)+G \quad (17)$$

and Eqn. (15) reduces to

$$G'\left(\frac{1}{\Delta t}+\frac{2}{(\Delta y)^2}\frac{1}{\Pr}\left(1-\cos b\Delta y\right)+i\frac{v}{\Delta y}\sin(b\Delta y)\right)$$
$$=G\left(\frac{1}{\Delta t}-\varphi-i\frac{u\sin(a\Delta x)}{\Delta x}\right)+\varepsilon\left(\frac{\Delta u}{\Delta y}\right)^2 \quad (18)$$

Taking

$$\frac{1}{\Delta t}+\frac{4}{(\Delta y)^2}\sin^2\left(\frac{b\Delta y}{2}\right)+i\frac{v}{2\Delta y}\sin(b\Delta y)=A$$

$$\frac{1}{\Delta t}+\frac{2}{(\Delta y)^2}\frac{1}{\Pr}\left(1-\cos b\Delta y\right)+i\frac{v}{\Delta y}\sin(b\Delta y)=B$$

$$\frac{1}{\Delta t}-M-i\frac{u\sin(a\Delta x)}{\Delta x}=C$$

$$\frac{1}{\Delta t}-\varphi-i\frac{u\sin(a\Delta x)}{\Delta x}=D$$

Ignoring $\varepsilon\left(\frac{\Delta u}{\Delta y}\right)^2$ in eqn. (18), equations (17) and (18) in matrix form can be written as

$$\begin{bmatrix}F'\\G'\end{bmatrix}=\begin{bmatrix}\frac{C}{A} & \frac{1}{A}\\ 0 & \frac{D}{B}\end{bmatrix}\begin{bmatrix}F\\G\end{bmatrix} \quad (19)$$

If the modulus of each Eigen value of the matrix is less than unity, the stability of the differencing scheme can be established. Since coefficient matrix in Eqn. (19) is triangular, its Eigen values are the diagonal elements i.e. $\frac{C}{A}$ and $\frac{D}{B}$. We have to check if $\left|\frac{C}{A}\right|\le 1$

$$\left|\frac{C}{A}\right|=\left|\frac{\frac{1}{\Delta t}-M-i\frac{u\sin(a\Delta x)}{\Delta x}}{\frac{1}{\Delta t}+\frac{4}{(\Delta y)^2}\sin^2\left(\frac{b\Delta y}{2}\right)+i\frac{v}{2\Delta y}\sin(b\Delta y)}\right|$$

Clearly, the real part of A is always greater than or equal to the real part of C. Therefore, $\left|\frac{C}{A}\right|\le 1$

Similarly, we can prove that $\left|\frac{D}{B}\right|\le 1$. This proves that the differencing scheme is unconditionally stable.

V. RESULT AND DISCUSSION

Numerical computations are carried out at different time intervals and for different values of $\Pr = 0.7, 7.0$; $M = 0.0, 0.5, 1.0$; $\varphi = 0.0, 0.5, 1.0$ $\varepsilon = 0,1,2$; $n = 0.5, 1.0$ unknown variables $u-$velocity and temperature T are obtained. Taking $\Delta x = 0.05$, $\Delta y = 0.25$; $\Delta t = 0.01$, the ADI algorithm has been implemented in MATLAB programming language. The unknown quantities are obtained at all node points and the accuracy of these numerical results is compared to precedi ng literature studies

In Fig.1 and Fig. 2, the transient velocity and temperature profiles are plotted for two values (Pr=0.71 (air), 7.0 (water)) of the Prandtl number of the fluid and three different values ($M = 0.0,0.5,1.0$) of magnetic field parameter. It can be seen that fluid velocity increases with time and then decreases subsequently. An increase in magnetic field results in decrease in the fluid velocity. This result holds because with increase in magnetic field, the Lorentz forces increases, which oppose the flow. This results in decrease in velocity of the flow.

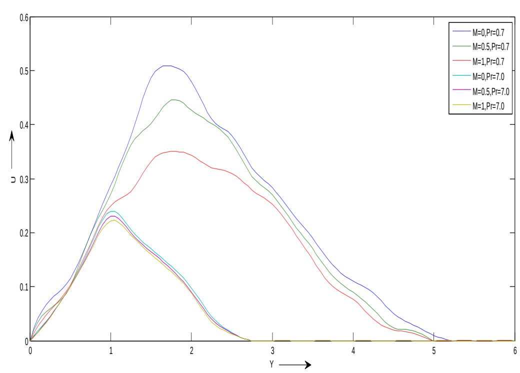

Fig. 1: Velocity profile at x=1.0 for $n = 0.5, = \varphi = 0.5, \varepsilon = 1.0$ at t =1.5

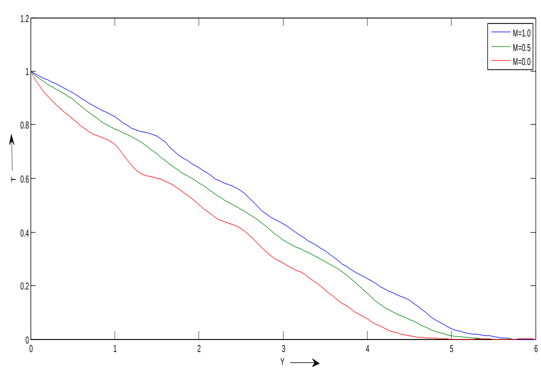

Fig.2: Temperature profile at x=1.0 for $n = 0.5, = \varphi = 0.5, \varepsilon = 1.0$ at t =1.5

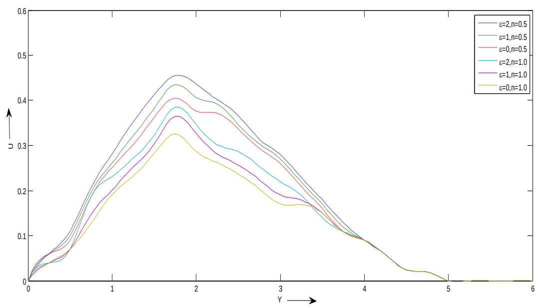

Fig. 3: Velocity profile at x=1.0 for $\mathrm{Pr} = 0.7, \mathrm{M} = 0.5, \varphi = 0.5$ at t =1.5

In Fig.3 and Fig. 4, the transient velocity and temperature profiles for Pr=0.7 are plotted for three different values of the viscous dissipation parameter (ε=0, 1, 2) of the fluid and two different values of exponent ($n = 0.5, 1$). We see that an increase in value of exponent n decreases the velocity of flow. Also, larger values of ε resultsin higher velocities. An increase in value of n, results in decrease in temperature, whereas temperature increases with increase in value of ε. This is agreeable because larger value of viscous dissipative heat increases the

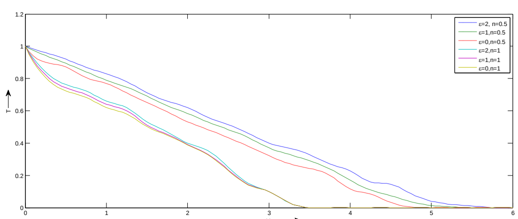

Fig. 4: Temperature profile at x=1.0 for $\mathrm{Pr} = 0.7, \mathrm{M} = 0.5, \varphi = 0.5$ at t =1.5

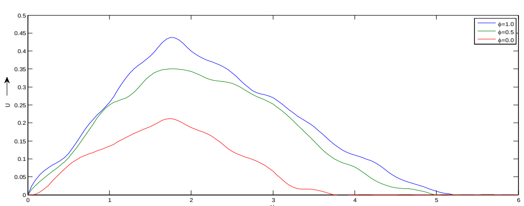

Fig. 5: Velocity profile at x=1.0 for $\mathrm{Pr} = 0.7, \mathrm{M} = 1.0, \varepsilon = 1.0, \mathrm{n} = 0.5$ at t =1.5

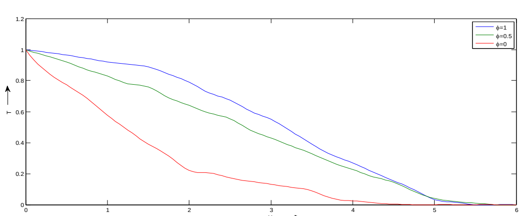

Fig. 6: Temperature profile at x=1.0 for $\mathrm{Pr} = 0.7, \mathrm{M} = 1.0, \varepsilon = 1.0, \mathrm{n} = 0.5$ at t =1.5

Temperature of the fluid. In Fig.5 and Fig. 6, the transient velocity and temperature profiles are plotted for three different values of the φ=0.0, 0.5, 1.0 for Pr=0.7. An increase in velocity of flow can be observed with increase in heat source parameter. Also, the presence of heat source produces energy, which results in higher fluid temperature. The same can be observed in Fig. 6.

VI. CONCLUSION

Numerical solutions for an unsteady flow past a semi- infinite vertical plate subjected to a variable surface temperature under the influence of magnetic field and viscous dissipation are obtained using Alternating –Direction-Implicit (ADI) Technique. Numerical computations were performed for velocity and temperature for different values of Prandtl number, magnetic field parameter, viscous dissipation parameter and heat source parameter. It was observed that the fluid velocity decreases with increase in magnetic field or with increase in value of exponent n, whereas increase in ε or heat source parameter results in increase in velocity of fluid. It was also observed that increase in n results in the decrease in temperature, whereas temperature increases with increase in value of ε. The findings obtained align well with previous studies[15] available.

REFERENCES

[1] Pohlhausen, E., Der Warmeaustausch zwischen festen korpern und Flussingkeiten mit kleiner Reibung und ˙kleiner Warmeleitung, ¨ Z. Angew. Math. Mech., 1921, vol. 1, pp. 115–121.

[2] Takhar, H.S., Ganesan, P., Ekambavannan, K., and Soundalgekar, V.M., Transient Free Convection Flow past a Semi-Infinite Vertical Plate with Variable Surface Temperature, Int. J. Num. Meth. Heat Fluid Flow, 1997, vol. 7, no. 4, pp. 280–296.

[3] Ganesan, P. and Palani, G., Transient Free Convection Flow of a Viscous Dissipative Fluid past a Semi-Infinite Inclined Plate, Int. J. Appl. Mech. Eng., 2003, vol. 8, no. 3, pp. 395–402.

[4] Palani, G., Viscous Dissipative Fluid past a Semi-Infinite Vertical Plate with Variable Surface Temperature, Int. J. Num. Meth. Fluids, 2007, vol. 55, pp. 1157–1169.

[5] Ching-Yang Cheng and Chi-Chang Wang (2000). Forced convection in micropolar fluid flow over a wavy surface. Numerical Heat Transfer, Part A: Applications, 37(3),

[6] Wang, C.C and Chen, C.K. (2002). Forced convection in a wavy-wall channel. International Journal of Heat and Mass Transfer, 45(12), 2587-2595.

[7] Chi-Chang Wang and Cha'o-Kuang Chen(2005). Mixed convection boundary layer flow on inclined wavy plates including the magnetic field effect. International Journal of Thermal Sciences, 44(6), 577-586.

[8] Hélio Aparecido Navarro Luben Cabezas-Gómez ,Renato Césarda Silva and Arlindo NetoMontagnoli (2007). A generalized alternating-direction implicit scheme for incompressible magnetohydrodynamic viscous flows at low magnetic Reynolds number**.** Applied Mathematics and Computation, 189(2), 1601-1613

[9] A.K.Abdul Hakeem, S.Saravanan and P.Kandaswamy (2011). Natural convection in a square cavity due to thermally active plates for different boundary conditions**.** Computers & Mathematics with Applications 62 (1), 491-496

[10] Mehrzad Mirzaei Nejad, K. Javaherdeh and M. Moslemi(2015). MHD mixed convection flow of power law non-Newtonian fluids over an isothermal vertical wavy plate**.** Journal of Magnetism and Magnetic Materials. 389, 66-72.

[11] Mehdi Kiyasatfar and Nader Pourmahmoud (2016). Laminar MHD flow and heat transfer of power-law fluids in square

microchannels. International Journal of Thermal Sciences, 99, 26-35

[12] Sreeparna Majee and G.C.Shit (2017). Numerical investigation of MHD flow of blood and heat transfer in a stenosed arterial segment. Journal of Magnetism and Magnetic Materials, 424, 137-147

[13] Zeting Liu, Fawang Liu and Fanhai Zeng(2019). An alternating direction implicit spectral method for solving two dimensional multi-term time fractional mixed diffusion and diffusion-wave equations. Applied Numerical Mathematics,136, 139-151

[14] Kaushik, A. Numerical Solutions for unsteady flow past a semi-infinite plate using Alternating –Direction-Implicit (ADI) Technique (2020). , International Journal of Scientific Research in Mathematical and Statistical Sciences, 7 (1)

[15] Palani, G., Srikanth, U. and Kwang-Yong Kim(2017)., Combined Effects of Viscous Dissipation and MHD on Free Convection Flow past a Semi-Infinite Vertical Plate with Variable Surface Temperature in the Presence of Heat Source, Journal of Engineering Thermophysics, 26(1). 1, 113–124.