Seasonal Analysis of Average Monthly Exchange Rate of Central Bank of Nigeria (CBN) when Trend Cycle Component is Quadratic

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Abstract: **seasonal analysis of average monthly exchange rate of Central Bank of Nigeria (CBN) when trend cycle component is quadratic is discussed in this study. Empirical example was taken from average monthly exchange rate of Central Bank of Nigeria over the period January, 2006 to December, 2017. This work is to investigate the trend pattern and suitable model for decomposition. Results indicate that the yearly standard deviations are stable, while the seasonal deviations differ, suggesting that the seasonal indices may be additive model**

Keywords: **Descriptive Time Series, Decomposition Model, Quadratic Trend Cycle Component, Suitable Model, Buys-Ballot Table**

I. INTRODUCTION

Descriptive time series analysis involves the investigation of the variations due to the components. This is normally known as the decomposition of the time series into four basic components, namely; the trend, the seasonal variation, the cyclical variation and irregular variation. Trend shows the presence of factors that persist for considerable durations of time. Examples of these factors include; population changes, technological improvements, price level fluctuations and various conditions that are peculiar to individual establishment. Sudden changes in trend may be caused by introduction of new element into or elimination of an old factor from forces confronting the series Alder and Roessler [1]. The cyclical variation on other hand is defined as the long term oscillation or swing about the general direction of the time series data. These cycle may or may not follow similar patterns after equal interval of time. However, if short period of time series data are involved, the cyclical component is jointly estimated into the trend [2] and the observed time series $\left(X_t,\ t = 1,\ 2,\ ...,\ n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular/residual component $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model

$$X_t = M_t + S_t + e_t \tag{1}$$

Multiplicative Model

$$X_t = M_t \times S_t \times e_t \tag{2}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t. \tag{3}$$

Oladugba, *et al*, [3] observed that when the seasonal fluctuation exhibits constant amplitude with respect to the trend, then the seasonal effect is said to be additive model and Equation (1) may be used. If the seasonal fluctuation is a function of the trend, then it is said to be multiplicative.

The two-dimensional arrangement of the time series data into *m* rows and *s* columns is called the Buys-Ballot table. The Buys-Ballot procedure has the advantage of jointly determining the model structure; additive or multiplicative or mixed model and checking the presence or absence of trend parameters and seasonal indices. Inspire of its advantages, the procedure has some limitations. Notable among the demerits is that, the estimation procedure of the Buys-Ballot table does not take into consideration cycles, outliers and missing observation that might be available in the series.

Iwueze and Ohakwe [4] presented detailed discussion of Buys-Ballot method to the case in which the trend cycle component is quadratic. They observed that the estimation of the slope of the curve is as in Iwueze and Nwogu [5]. The difference in method lies in the computation of *c* which is easily computed from differences in the periodic averages.

For a series arranged in Buys-Ballot table, Dozie [6] worked on estimation of the parameters of linear trend cycle and seasonal components with regards to the periodic, seasonal and overall averages for the mixed model in time series decomposition. His work discussed the linear trend cycle and seasonal component when there is no trend and b = 0.

II. MATERIALS AND METHODS

This study adopted Buys-Ballot method for time series. For detailed discussion of this method see Wei [7], Iwueze and Ohakwe [4], Dozie, *et al*, [8], Dozie and Ihekuna [9]

Table 1: Buys - Ballot Tabular Arrangement of Time Series Data

Rows (i)	Columns j								
	1	2	$\cdots$	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	$\cdots$	X_j	$\cdots$	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	$\cdots$	X_{s+j}	$\cdots$	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	$\cdots$	X_{2s+j}	$\cdots$	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	$\cdots$	$X_{(i-1)s+j}$	$\cdots$	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	$\cdots$	$X_{(m-1)s+j}$	$\cdots$	X_{ms}	$T_{m.}$	$\bar{X}_{m}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	$\cdots$	$T_{.j}$	$\cdots$	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\cdots$	$\bar{X}_{.j}$	$\cdots$	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	$\cdots$	$\hat{\sigma}_{.j}$	$\cdots$	$\hat{\sigma}_{.s}$			$\hat{\sigma}_x$

Where, m = number of years, s = length of periodic interval and n = length of the series

2.1 Quadratic Trend Cycle and Seasonal Components

The expression of the quadratic trend is given by

$$\bar{X}_i = a + bt + ct^2 \tag{13}$$

Iwueze and Nwogu [5] discussed the expression of the estimation of the trend and seasonal indices for an additive model when trend-cycle component is quadratic as;

$$\hat{a} = a^| + \left(\frac{s-1}{2}\right)\hat{b} - \left(\frac{(s-1)(2s-1)}{6}\right)\hat{c} \tag{14}$$

$$\hat{b} = \frac{b^|}{s} + \hat{c}(s-1) \tag{15}$$

$$\hat{c} = \frac{c^|}{s^2} \tag{16}$$

$$\hat{S}_j = \bar{X}_{.j} - d_j \tag{17}$$

$$d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \hat{b} + \hat{c}(n-s)j + \hat{c}\, j^2 \tag{18}$$

III. EMPIRICAL EXAMPLE

The empirical example is based on short series for which the trend cycle component is jointly estimated. The data is on average monthly exchange rate of Central Bank of Nigeria (CBN) for the period of eleven (11) years. The series, which is presented as monthly data is shown in Appendix A with its row, column and overall means and standard deviations. The corresponding graphs are shown in Figures 3.1, 3.2 and 3.3. As Figures 3.1 and 3.2 and Appendix A indicate, that the data is seasonal with evidence of upward or downward trend. The yearly standard deviations are clearly more stable than the seasonal standard deviations, suggesting that the seasonal indices may be additive.

3.1 Quadratic Trend Cycle and Seasonal Components

$$\bar{X}_i = 4.9018 - 0.0349i + 0.0080i^2 \tag{19}$$

Using (14), (15) and (16) we obtain

$$\hat{c} = \frac{0.0084}{144} = 0.0001$$

$$\hat{b} = \frac{-0.0349}{12} + 0.0001(12-1)$$

$$= -0.0018$$

$$\hat{a} = 4.9018 + \left(\frac{12-1}{2}\right)(-0.0018) - \left(\frac{(12-1)(24-1)}{6}\right)0.0001$$

$$= 4.8877$$

The analysis in this study is performed using the Buys-Ballot approach. Trend analysis by the Buys-Ballot estimates gave the following estimates $\hat{a} =$ 4.8877, $\hat{b} =$ - 0.0018 and $\hat{c} =$ 0.0001. Therefore,

$$\hat{M}_t = 4.8877 - 0.0018t + 0.0001t^2$$

The seasonal effects are derived by averaging the difference $X_t - \hat{M}_t$ at each season. The associated seasonal analysis procedure estimated the seasonal effects as $\hat{S}_1 =$ 0.048,

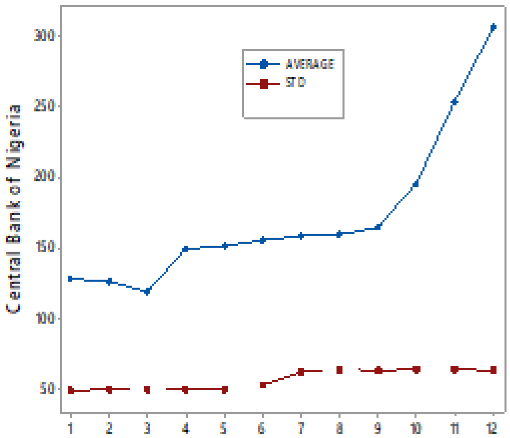

Fig.3.1: Means and standard deviations of Central Bank of Nigeria

Using (17) we have

$$\hat{S}_j = \bar{X}_{.j} - \left(5.3743 + 0.0132j + 0.001j^2\right)$$

Table 2: Estimates of Seasonal Effect

j	$\bar{X}_j$	S_j
1	5.0798	0.0480
2	5.0860	0.0407
3	5.0862	0.0272
4	5.0864	0.0135
5	5.0872	0.0002
6	5.1041	0.0028
7	5.1222	0.0209
8	5.1281	-0.0024
9	5.1284	-0.017
10	5.1259	-0.0346
11	5.1253	-0.0501
12	5.1421	-0.0492
$\sum_{j=1}^{s} \hat{S}_j$		0.0000

$\hat{S}_2 = 0.0407$, $\hat{S}_3 =$ 0.0272, $\hat{S}_4 =$ 0.0135, $\hat{S}_5 = 0.0002$, $\hat{S}_6 =$ 0.0028, $\hat{S}_7 =$ 0.0209, $\hat{S}_8 =$ -0.0024, $\hat{S}_9 =$ -0.017, $\hat{S}_{10} =$ -0.0346, $\hat{S}_{11} =$ -0.0501, $\hat{S}_{12} =$ -0.0492 shown in Table 3 for additive model. The next step is to discuss the error component of our time series data derived by subtracting estimates of trend cycle component and seasonal effect from the time series data at each time period. The adjusted residual mean was obtained as zero, while the variance is 0.0870. Therefore, the residual for the model indicates that the fitted model is inadequate. Hence, the fitted model becomes

$$\hat{X}_t = 4.8877 - 0.0018t + 0.0001t^2 + \hat{S}_t$$

Table 3: Buys-Ballot Estimates of Quadratic trend parameters and seasonal Indices.

parameter	Quadratic trend and seasonal indices
$\hat{a}$	4.8877
$\hat{b}$	-0.0018
$\hat{c}$	0.0001
$\hat{S}_1$	0.0480
$\hat{S}_2$	0.0407
$\hat{S}_3$	0.0272
$\hat{S}_4$	0.0135
$\hat{S}_5$	0.0002
$\hat{S}_6$	0.0028
$\hat{S}_7$	0.0209
$\hat{S}_8$	-0.0024
$\hat{S}_9$	-0.0170
$\hat{S}_{10}$	-0.0346
$\hat{S}_{11}$	-0.0501
$\hat{S}_{12}$	-0.0492

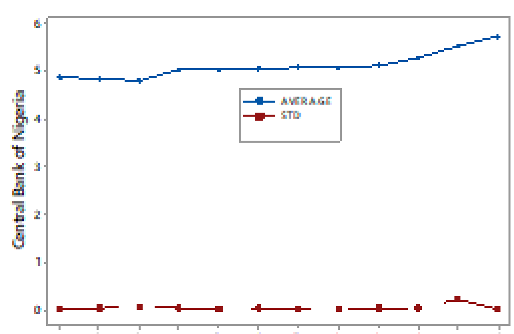

Fig. 3.2: Means and standard deviations of transformed series of CBN

Table 4: Estimates of Trend, Seasonal and Residual Values

Year	t	Y_t	$\hat{T}_t$	$\hat{S}_t$	$\hat{Y}_t = \hat{T}_t + \hat{S}_t$	$\hat{R}_t = Y_t - \hat{Y}_t$	$Adj\ \hat{R}_t$
2006	1	4.8573	4.8860	0.0480	4.9340	-0.0767	-0.3038
2007	2	4.8339	4.8845	0.0407	4.9252	-0.0913	-0.3184
2008	3	4.7784	4.8832	0.0272	4.9104	-0.1320	-0.3591
2009	4	5.0093	4.8821	0.0135	4.8956	0.1137	-0.1134
2010	5	5.0179	4.8812	0.0002	4.8814	0.1365	-0.0906
2011	6	5.0489	4.8805	0.0028	4.8833	0.1656	-0.0615
2012	7	5.0673	4.8800	0.0209	4.9009	0.1664	-0.0607
2013	8	5.0719	4.8797	-0.0024	4.8773	0.1946	-0.0325
2014	9	5.1047	4.8796	-0.0170	4.8626	0.2421	0.0154
2015	10	5.2754	4.8797	-0.0346	4.8451	0.4303	0.2032
2016	11	5.5139	4.8800	-0.0501	4.8299	0.6840	0.4569
2017	12	5.7229	4.8805	-0.0492	4.8313	0.8916	0.6645

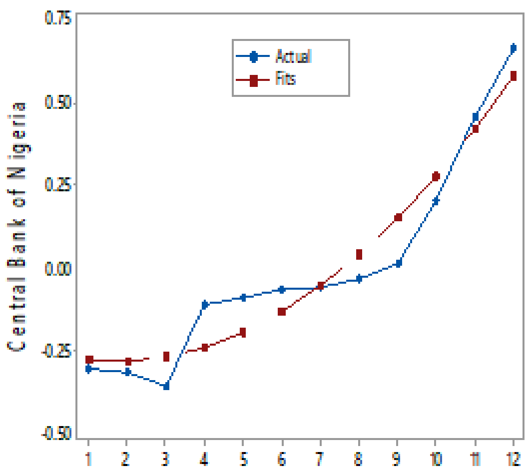

Fig3.3: Plot of residuals of CBN, between 2006 to 2017

IV. CONCLUDING REMARKS

This paper has presented seasonal analysis of average monthly exchange rate of Central Bank of Nigeria (CBN) when trend cycle component is quadratic. The study indicates that, the time series is seasonal with evidence of upward or downward trend. The periodic standard deviations are stable while the seasonal standard deviations differ, suggesting that the seasonal indices may be additive model. The adjusted error mean was obtained as zero, while the variance is 0.0870. Therefore, the fitted model becomes

$$\hat{X}_t = 4.8877 - 0.0018t + 0.0001t^2 + \hat{S}_t$$

REFERENCES

[1] Alder, H.L. and Roesslar, E.B. (1975): Introduction to Probability and Statistics, W.H.Freeman and Company, San Francisco

[2] Chatfield, C. (2004). *The analysis of time Series: An introduction*. Chapman and Hall,/CRC Press, Boca Raton

[3] Oladugba, A.V., Ukaegbu, E.C., Udom, A.U., Madukaife, M.S., Ugah, T.E. &Sanni, S.S., (2014). *Principles of Applied Statistic,* University of Nigeria Press Limited.

[4] Wei, W. W. S (1989). *Time series analysis: Univariate and multivariate methods*, Addison-Wesley publishing Company Inc, Redwood

[5] Iwueze, I. S. & Nwogu, E.C. (2014). *Framework for choice of models and detection of seasonal effect in time series.*Far East Journal of Theoretical Statistics 48(1), 45– 66

[6] Iwueze, I. S. and Ohakwe, J. (2004). *Buys-Ballot estimates when stochastic trend is quadratic*. Journal of the Nigerian Association of Mathematical Physics, 8, 311-318.

[7] Dozie, K. C. N (2020). Buys-Ballot estimates for mixed model in descriptive time series. International Journal of Theoretical and Mathematical Physics 10(1), 22-27

[8] Dozie, K.C.N, Ibebuogu, C.C, Mbachu, H.I & Raymond, M.C (2020). Buys-Ballot modeling of church marriages in Owerri, Imo State, Nigeria. American Journal of Mathematics and Statistics 10(1): 26-31

[9] Dozie, K.C.N and Ihekuna S.O (2020). Buys-Ballot estimates of quadratic trend component and seasonal indices and effect of incomplete data in time series. International Journal of science and healthcare research 5(2): 341-348

Appendix A: Average Monthly Exchange Rate of Central Bank of Nigeria (2006-2017)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2006	129.93	129.33	128.68	128.58	128.57	128.50	128.43	128.43	128.39	128.42	128.42	128.39	1544.1	128.67	0.474
2007	128.37	128.33	128.25	128.25	127.62	127.50	127.26	126.57	125.74	123.56	119.45	118.11	1509.0	125.75	3.550
2008	117.72	117.50	116.79	117.47	117.79	117.74	117.71	117.69	117.62	117.72	117.88	134.33	1428.0	119.00	4.840
2009	146.59	149.12	149.12	149.10	149.00	148.54	149.88	155.23	153.25	150.22	151.03	149.80	1800.9	150.07	2.520
2010	150.33	150.97	150.08	150.38	151.49	151.27	150.27	150.70	152.62	151.78	150.55	152.63	1813.1	151.09	0.885
2011	152.57	152.75	155.21	154.60	156.17	155.66	152.62	153.36	156.70	159.82	158.76	162.27	1870.5	155.87	3.090
2012	161.31	158.59	157.72	157.44	157.46	162.33	161.33	158.97	157.78	157.24	157.58	157.33	1905.1	158.76	1.840
2013	159.56	157.52	158.63	158.20	158.02	160.02	161.12	161.15	161.96	159.83	158.79	159.05	1913.9	159.49	1.380
2014	160.23	163.62	164.61	162.19	161.89	162.82	162.25	161.99	162.93	164.64	171.10	180.33	1978.6	164.88	5.560
2015	181.78	194.48	197.07	197.00	197.00	196.92	196.97	197.00	197.00	196.99	196.99	196.99	2346.2	195.52	4.390
2016	197.00	197.00	197.00	197.00	197.00	231.76	294.57	309.73	305.23	305.21	303.18	305.22	3041.9	253.49	53.90
2017	305.20	305.31	306.40	306.05	305.54	305.72	305.86	305.67	305.89	305.62	305.90	306.31	3669.5	305.79	0.360
total	1990.59	2004.52	2009.56	2006.26	2007.52	2048.78	2108.27	2126.49	2125.11	2121.05	2121.63	2150.76			
$\bar{y}_{.j}$	165.88	167.04	167.46	167.19	167.29	170.73	175.69	177.21	177.09	176.75	176.80	179.23			
$\sigma_{.j}$	49.10	49.60	50.20	50.00	49.80	52.60	61.70	64.30	63.50	63.70	64.00	62.80			

Appendix B: Transformed Series of Average Monthly Exchange Rate of Central Bank of Nigeria (2006-2017)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2006	4.8670	4.8624	4.8573	4.8566	4.8565	4.8559	4.8554	4.8554	4.8551	4.8553	4.8553	4.8551	58.29	4.8573	0.0037
2007	4.8549	4.8546	4.8540	4.8540	4.8491	4.8481	4.8462	4.8408	4.8342	4.8167	4.7829	4.7716	58.01	4.8340	0.0287
2008	4.7683	4.7664	4.7604	4.7662	4.7689	4.7685	4.7682	4.7681	4.7675	4.7683	4.7697	4.9003	57.34	4.7784	0.0385
2009	4.9876	5.0048	4.9844	5.0046	5.0040	5.0009	5.0280	5.0328	5.0543	5.0741	5.0674	5.0893	60.11	5.0093	0.0168
2010	5.0128	5.0171	5.0112	5.0132	5.0205	5.0191	5.0124	5.0153	5.0280	5.0224	5.0143	5.0280	60.21	5.0179	0.0059
2011	5.0276	5.0288	5.0448	5.0408	5.0510	5.0477	4.6347	4.8363	5.0304	4.8978	4.7958	4.0073	60.59	5.0469	0.0197
2012	5.0833	5.0663	5.0608	5.0590	5.0592	5.0896	5.0835	5.0687	5.0612	5.0578	5.0599	5.0584	60.80	5.0673	1.0115
2013	5.0724	5.0596	5.0666	5.0639	5.0627	5.0753	5.0822	5.0822	5.0874	5.0741	5.0676	5.0692	60.86	5.0719	0.0087
2014	5.0766	5.0976	5.1036	5.0888	5.0867	5.0927	5.0891	5.0875	5.0933	5.1038	5.1423	5.1948	61.26	5.1047	0.0327
2015	5.2028	5.2703	5.2836	5.2832	5.2832	5.2828	5.2831	5.2832	5.2832	5.2832	5.2832	5.2832	63.30	5.2754	0.0232
2016	5.2832	5.2832	5.2832	5.2832	5.2832	5.4457	5.6855	5.7357	5.7211	5.7210	5.7209	5.7210	66.17	5.5139	0.2176
2017	5.7210	5.7213	5.7249	5.7238	5.7221	5.7227	5.7231	5.7225	5.7232	5.7223	5.7233	5.7246	68.67	5.7229	0.0012
total	60.958	61.032	61.034	61.037	61.047	61.249	61.467	61.537	61.541	61.511	61.504	61.705			
$\bar{y}_{.j}$	5.0798	5.0860	5.0862	5.0864	5.0872	5.0141	5.1222	5.128	5.128	5.126	5.125	5.142			
$\sigma_{.j}$	0.2479	0.2519	0.2553	0.2538	0.2532	0.2685	0.3037	0.3118	0.3094	0.3112	0.3146	0.3047			