A Modified Neo-Malthusian Population Growth Model for Predicting Demographic Processes in Nigeria

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ABSTRACT

This study develops a Modified Neo-Malthusian Population Growth Model tailored to Nigeria's demographic realities. Utilizing both classical and novel modeling frameworks, including exponential, logistic, Gompertz, von Bertalanffy, and Richards models, four new allometry-parameter-based models are introduced to better account for nonlinear growth dynamics. Parameters were estimated using Marquardt's Compromise Method and model performance assessed via R^2, RMSE, AIC, and BIC. Nigeria Population data as published on https://www.macrotrends.net/global-metrics/countries/nga/nigeria/population provided the basis for calibration and validation. Results indicate that Proposed Model 3 offers the most accurate predictions, while Proposed Model 1 and 2 demonstrated strong performance in Nigeria Population projections. This model suite provides enhanced tools for demographic forecasting and development planning in Nigeria.

Keywords: Growth Models, Nigeria Population Data, Hyperbolic Tanh, Allometry Parameter

INTRODUCTION

Population modeling is a cornerstone for demographic planning and policy development. Nigeria, Africa's most populous country, is projected to grow from 233 million in 2024 to over 380 million by 2050 (World Population Prospects, 2024). However, traditional population models—exponential, logistic, and their derivatives—often fail to incorporate the economic, environmental, and regional complexities influencing demographic trends.

The Neo-Malthusian perspective acknowledges resource limitations and seeks to model population growth through a flexible, nonlinear framework that accounts for real-world demographic transitions. This study aims to develop and validate modified Neo-Malthusian models that better reflect Nigeria's dynamic population structure.

Nigeria's rapid population growth demands a sophisticated model to guide sustainable development. Existing models oversimplify the complex relationship between population size, resource availability, and economic growth, leading to inconsistent policies that oscillate between overpopulation fears and economic stagnation concerns.

Population projections vary across Nigeria due to regional differences in fertility rates, urbanization, and economic conditions. Traditional models fail to capture the dynamic, bidirectional link between population growth and per capita income, limiting effective policy design tailored to regional needs.

Therefore, there is a need to develop a nonlinear growth model that incorporates an allometry parameter, such a model can provide adaptable policy insights to address disparities, manage resource demands, and ensure sustainable development.

Several growth models have attempted to describe population trends with varying degrees of success. Malthus's exponential model was among the earliest, but it assumes infinite resources and growth. Logistic models (Bennett & Dore, 2018) introduced carrying capacity constraints but still assume symmetric growth.

The Richards model introduces shape parameters (Volkan et al., 2022), while the Gompertz and von Bertalanffy models modify assumptions about growth rates and asymptotes. Recent studies (Rai & Garg, 2024) highlight the economic-demographic interplay, while machine learning approaches (Şahinarslan et al., 2021) offer data-driven alternatives.

However, most models still assume constant parameters or fail to address data contamination and nonlinear responses to policy or socioeconomic shocks. This study introduces an allometry-based modification to address such limitations.

METHODOLOGY

Parameters are quantities, which are known to be modeler before the model is constructed. Often they are constants, although it is possible for a parameter to change over time. In the Malthusian model the variable is the population and the parameter is the population growth rate. The differential equation describing exponential growth is given by;

$$\frac{dN}{dt} = rN \quad (1)$$

This can be integrated directly

$$\int_{N_0}^{N} \frac{dN}{N} = \int_0^t r\,dt \quad (2)$$

to give;

$$\ln\left(\frac{N}{N_0}\right) = rt \quad (3)$$

where, $N_0 = N(t = 0)$. Exponentiating,

$$N(t) = N_0 e^{rt} \quad (4)$$

Estimation of Parameters

We consider subsequently a method of optimization, due to Marquardt, often referred to as Marquardt's Compromise. When θ is not near the minimum it is possible for $Z'Z$ to be negative definite, thereby making the iterative process to diverge. Consequently, to combat these two extremes, the algorithm

$$\theta_{r+1} = \theta_r + [Z'Z + \alpha diag(Z'Z)]^{-1} Z'U \quad (5)$$

due to Marquardt (1963) was proposed where α is a scalar which may be adjusted to control the sequence of iterations and is such that $Z'Z + \alpha diag Z'Z$ is positive definite. When $\alpha \to \infty$ Equation (5) becomes

$$\theta_{r+1} = \theta_r + \lambda^* Z'u \quad (6)$$

which is essentially steepest descent, and when $\alpha \to 0$. In essence therefore, Marquardt's algorithm is a compromise between the Gauss-Newton algorithm and the method of steepest descent. The compromise works more efficiently when both the limitations of Taylor's series linearization and those of steepest descent are violated.

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Hyperbolic Tangent function (tanh x)

We shall now look at the hyperbolic function tanh x. In speech, this function is pronounced as “Tanh”, or sometimes as “than”. The function is defined by the formula;

$$\tanh x = \frac{\sinh x}{\cosh x} \quad (7)$$

tanh x can be obtained in terms of exponential functions;

$$\tanh x = \frac{e^x - e^{-x}}{2} \div \frac{e^x + e^{-x}}{2} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (8)$$

We can use what we know about sinh x and cosh x to sketch the graph of tanh x. We first

take $x = 0$. We know that sinh 0 = 0 and cosh 0 = 1, so;

$$\tanh 0 = \frac{\sinh 0}{\cosh 0} = \frac{0}{1} = 0 \quad (9)$$

So as x gets large, $\sinh x \approx \cosh x$, so tanh x gets close to 1:

$$\tanh x \approx 1 \, For\, large\, x \quad (10)$$

But sinh x is always less than cosh x, so tanh x is always slightly less than 1. It gets close to 1 as x gets very large, but never reaches it. As x gets large and negative, $\sinh x \approx -\cosh x$, so tanh x gets close to -1:

$$\tanh x \approx 1 \, For\, large\, negative\, x \quad (11)$$

But $\sinh x$ is always greater than $-cosh(x)$, so $\tanh x$ is always slightly greater than -1. It gets close to 1 as x gets very large and negative, but never reaches it.

We can now sketch the graph of $\tanh x$ in Figure as shown below. Note that $\tanh(-x) = -tanh(x)$.

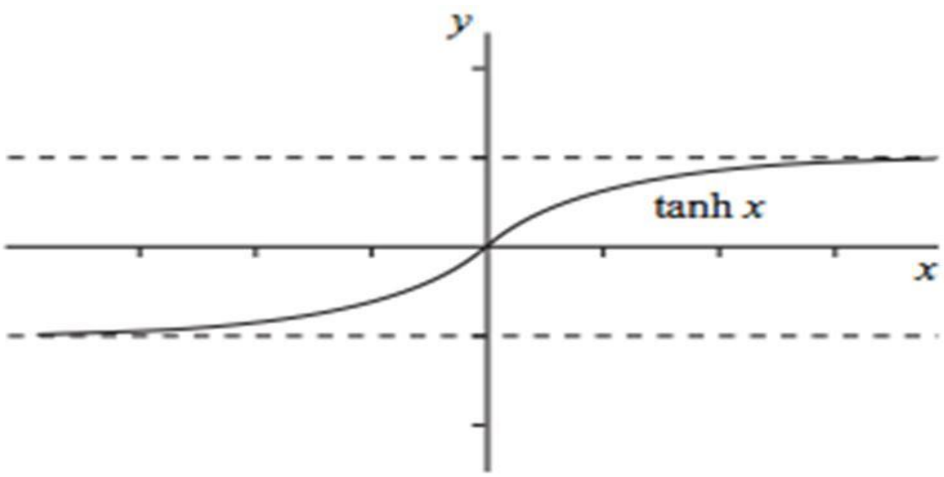

Figure 1: Graph of the $\tanh x$ functions

Proposed Models

The application of allometric scaling to demographic processes builds on fundamental work in biological scaling (West et al., 1997; Brown et al., 2004) and its extension to social systems (Bettencourt et al., 2007). In demographic contexts, allometric relationships can capture how population growth efficiency changes with population size, economic development, and social complexity.

The allometric equations for population growth were formulated as an ordinary differential equations and expressed as:

1. $\frac{dP}{dt} = a^{\theta}P$

2. $\frac{dP}{dt} = r^{\frac{1}{\theta}}P$

3. $\frac{dP}{dt} = P\left[r + \frac{\theta}{\sqrt{1+t^2}}\right]$

4. $\frac{dP}{dt} = rt^{\theta}$

Conditioning the allometry parameter on intrinsic rate of growth. The first –order differential equation for proposed model 1 is:

$$\frac{dP}{dt} = r^{\theta}P$$

Separating varibles and integrating gives:

$P = Ce^{r^{\theta}t}$ for proposed model 1

while it gave,

$P = Ce^{\alpha t}$ for proposed model 2,

where θ is the allometric exponent that captures scaling effects. When θ = 1, this reduces to exponential growth; θ < 1 indicates sublinear scaling (growth rate decreases with size), while θ > 1 indicates superlinear scaling (growth rate increases with size). Also, $\alpha = \frac{r}{\theta}$ and C is the constant of integration.

Proposed model 3 is where an alternative trigonometric function was introduced to the intrinsic rate of increase in other to further improve the solution of the given partial differential equation,

$$\frac{dp}{dt} = p\left[r + \frac{\theta}{\sqrt{1+t^2}}\right] \quad (12)$$

Solving equation 11 gave;

$$P = P_0 e^{rt+\sin(\arctan(t))} \quad (13)$$

Proposed model 4,

$$\frac{dP}{dt} = rt^{\theta} \quad (14)$$

Was solved and the resulting solution gave,

$$P = \frac{r}{\theta+1}t^{\theta+1} + C \ (15)$$

To determine the value of the constant of integration C, we considered the approach below,

Consider that there is no specific initial condition given, hence, C is left as a constant that can take any value. In this context, we assume $P(0) = P_0$, where P_0 is a known value, with this we can substitute t=0 and $P = P_0$ into the equation (15)

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$P_0 = \frac{r}{\theta+1}(0)^{\theta+1} + C$ (16)

$$P_0 = 0 + C$$

Therefore, the constant C is equal to P_0. So, the specific solution with the initial condition $P(0) = P_0$ is:

$P = \frac{r}{\theta+1}t^{\theta+1} + P_0$ (17)

The solutions obtained in proposed models 1 – 4 were applied to Nigeria population data as published on https://www.macrotrends.net/global-metrics/countries/nga/nigeria/population while the existing models comparison was based on R^2, MSE, MAE, AIC and BIC.

DISCUSION OF RESULTS

Growth Model Evaluation and Statistical Analysis

The study employed nine competing mathematical models, as stated in table 1, to analyze Nigeria's population growth patterns: exponential, logistic, Gompertz, Richards, von Bertalanffy, and four proposed models (labeled proposed1-4). Each model represents different mathematical approaches to population growth dynamics, with varying assumptions about growth patterns, carrying capacity, and demographic transition stages. The statistical evaluation of these models using goodness-of-fit metrics (R^2, AIC, RMSE) revealed that all models captured the overall growth trajectory reasonably well as stated in figure 2, with R^2 values ranging from 0.9922 (von Bertalanffy) to 0.9998 (proposed3).

Table 1: Model Performance Evaluation based on the Nigeria Population Data

	R2	AIC	BIC	RMSE
exponential	0.99952584	2356.06203	2363.05423	1254184.19
logistic	0.99952584	2358.06203	2367.38497	1254184.23
gompertz	0.99916535	2401.03739	2410.36032	1663989.33
richards	0.99744454	2488.0796	2499.73327	2911610
von_bertalar	0.99223437	2570.55184	2579.87478	5075599.29
proposed1	0.99964423	2336.22955	2345.55249	1086379.49
proposed2	0.99964423	2336.22955	2345.55249	1086379.49
proposed3	0.99980329	2291.19785	2300.52079	807824.39
proposed4	0.99930096	2387.56219	2396.88512	1522822.2

The proposed3 model emerged as the superior framework, demonstrating the highest coefficient of determination (R^2=0.9998), lowest Akaike Information Criterion (AIC=2291.2), and lowest Root Mean Square Error (RMSE=807,824.4). This statistical superiority indicates that the proposed3 model best balances mathematical complexity with predictive accuracy, capturing nuances in Nigeria's population dynamics that simpler models might miss. Conversely, the von Bertalanffy model showed the poorest fit with the highest RMSE (5,075,599) and AIC (2570.55), suggesting its assumptions about growth dynamics may be less applicable to Nigeria's demographic context.

Machine Learning Application and Data Partitioning

The implementation of machine learning methodologies through data partitioning (70% training, 20% testing, 10% validation) as stated in figure 2, provided robust evaluation of model performance beyond simple curve fitting. This cross-validation approach enabled assessment of each model's generalizability to unseen data, a critical factor for reliable population projections. The visualization of model predictions against actual values across all three data partitions demonstrated remarkable consistency in performance, with most models showing similar accuracy across training, testing, and validation sets.

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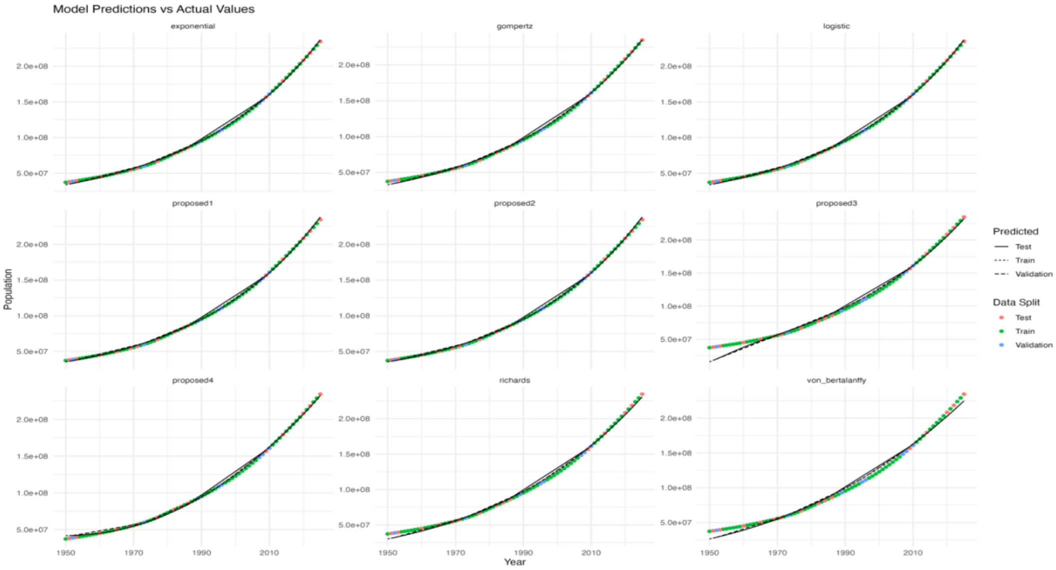

Figure 2: Model Predictions versus actual values based on 3 stage data calibration

This consistency across partitions suggests minimal overfitting, indicating that the models genuinely captured underlying population dynamics rather than simply memorizing the training data. The tight clustering of data points around prediction lines for top-performing models (particularly proposed1, proposed2, and proposed3) across all partitions reinforces their reliability for population projections. The machine learning framework thus provides stronger evidence for model selection than traditional curve fitting alone would offer, enhancing confidence in the selected growth models for policy applications.

Residual Analysis and Model Limitations

The residual analysis in figure 3 and 4 below, revealed important patterns and limitations across all models. Most notably, all models exhibited a wave-like pattern in their residuals over time, suggesting systematic errors that none of the mathematical formulations fully addressed. Models typically underestimated population in early years (1950-1970), overestimated during middle periods (1970-2000), and then underestimated again in recent years (2000-2010). This consistent pattern across different mathematical approaches indicates that key demographic factors might be missing from all models, potentially including cyclical economic factors, policy changes, or demographic transition elements not captured by continuous growth functions.

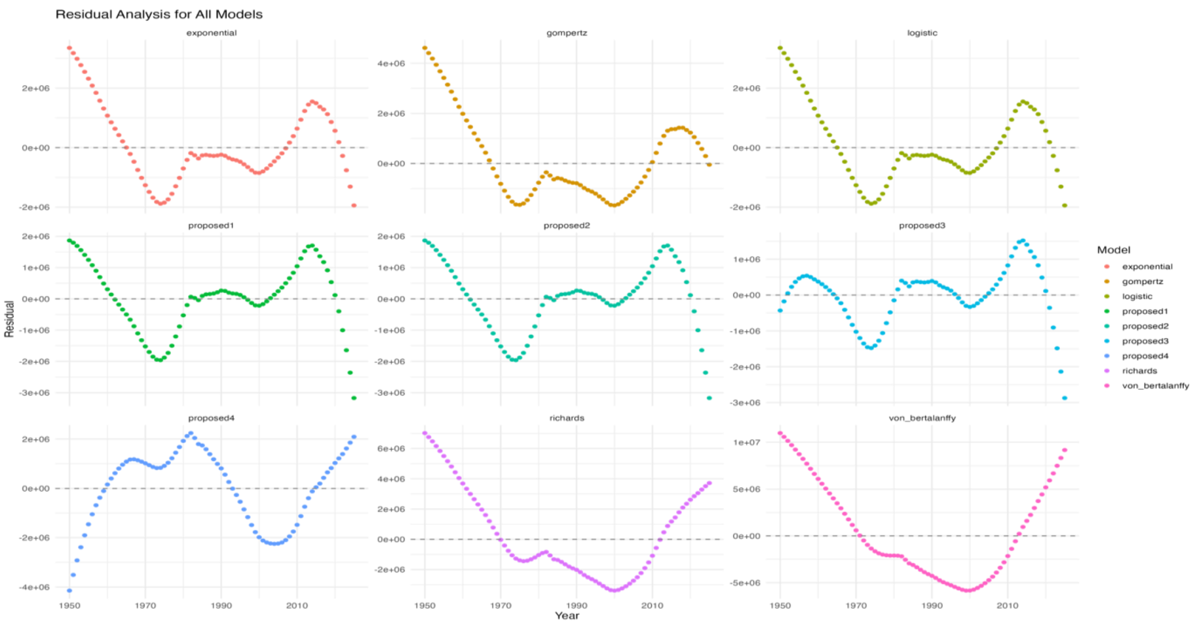

Figure 3: Residual Analysis based on the 9 competing models

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The comparison of residuals between models further highlighted their relative strengths and weaknesses. The von Bertalanffy and Richards models showed the most pronounced systematic deviations, with larger residual magnitudes and stronger patterns. Even the best-performing proposed3 model exhibited the same general pattern of residuals as other models, albeit with smaller magnitude, suggesting fundamental limitations to purely mathematical approaches to population modeling. These systematic residual patterns indicate potential areas for model improvement, perhaps through incorporation of additional socioeconomic variables or hybrid modeling approaches.

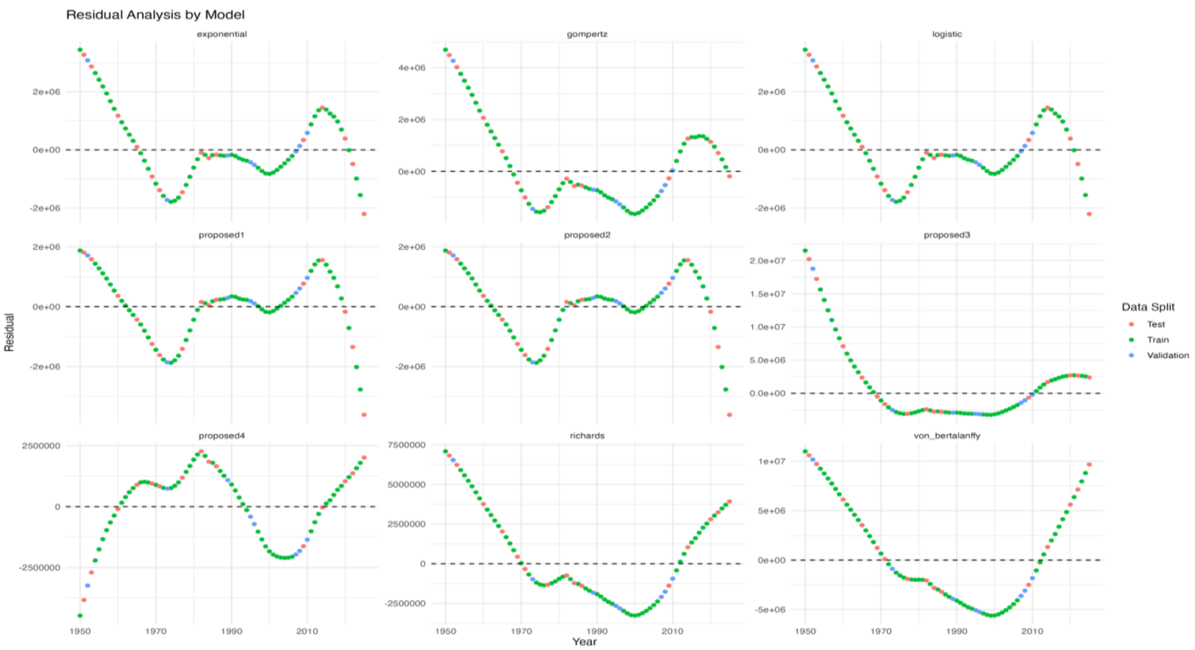

Figure 4: Residual Analysis based on the 3 Stage Data Calibration

Based on the prediction and residual analysis graphs in figure 4, the following models demonstrate superior performance:

Proposed3 Model: Clearly the top performer with the smallest and most evenly distributed residuals across all data partitions. The prediction graph shows excellent alignment between predicted values and actual data points in training, testing, and validation sets. The residuals hover closer to zero compared to other models, with fewer extreme values.

Proposed1 and Proposed2 Models: These appear nearly identical in performance (likely variants of the same model with minor differences) and show strong prediction capability with well-controlled residuals. Their residual patterns are very similar and show relatively balanced distribution around zero.

Exponential and Logistic Models: Despite being simpler models, they perform surprisingly well, with prediction curves closely matching the data points across all partitions. Their residual patterns are nearly identical, suggesting they capture similar aspects of the population growth dynamics.

Comparative Model Performance and Selection

When all models were plotted together on a single graph, their fitted curves showed surprising similarity as shown in figure 5, despite statistical differences in their performance metrics. Models diverged most noticeably in earlier years (1950-1970) and in future projections beyond 2010, while converging significantly during the middle period (1970-2000). This visual comparison reveals that while statistical metrics indicate clear differences between models, the practical implications for population estimates during most periods may be less dramatic than the metrics alone suggest.

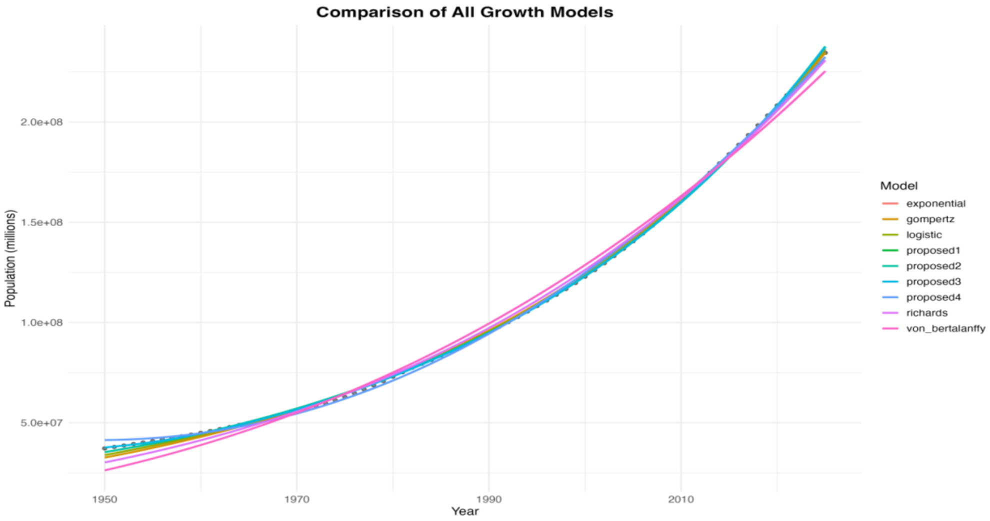

Figure 5: Prediction of the Nigeria Population data by the 9 competing models

The direct comparison of models without data partitioning further emphasized that all models struggle with the same time periods, particularly early years and recent years. The consistent residual patterns suggest a potential ceiling effect in model improvement using these mathematical formulations alone. Despite these shared limitations, the proposed3 model consistently demonstrated superior performance with smaller overall residuals, validating its selection as the preferred model for Nigeria's population dynamics. Simpler models like exponential and logistic performed surprisingly well given their mathematical simplicity, suggesting they may remain valuable for quick approximations or when data limitations prevent more complex modeling.

The application of machine learning approaches through systematic data partitioning and validation as shown in figure 6, enhances the methodological rigor of demographic modeling, moving beyond simple curve fitting to assess genuine predictive capability, the detailed residual analysis as shown in figure 3, identifies systematic patterns in model errors that point toward specific periods where all models struggle, suggesting opportunities for future research incorporating additional variables or alternative modeling approaches.

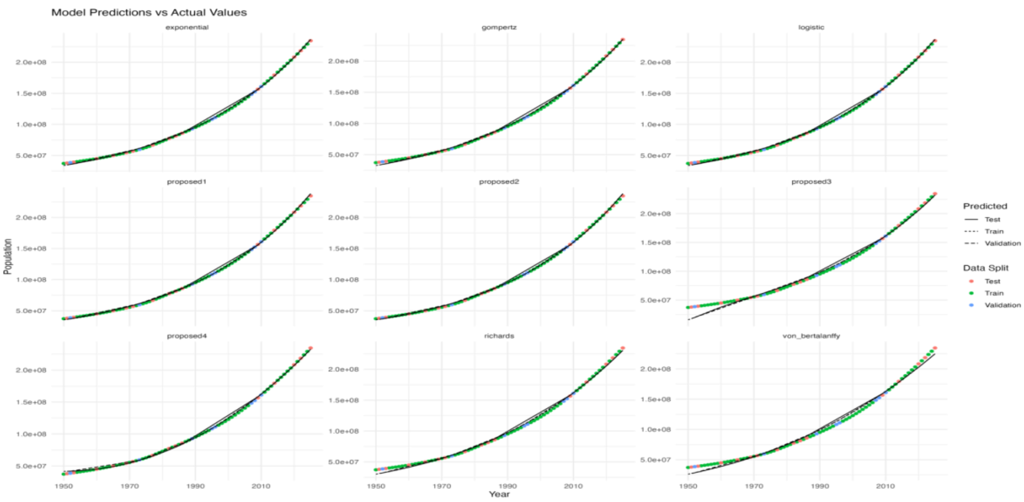

Figure 6: Model performance based on Data Partitioning of the Nigeria Population Data

Perhaps most significantly, this analysis reveals that despite the mathematical sophistication of various growth models, all share similar limitations in capturing certain aspects of Nigeria's population dynamics. This finding suggests that future advances in demographic modeling may require interdisciplinary approaches that integrate mathematical modeling with deeper understanding of socioeconomic factors, policy impacts, and migration patterns. By identifying both the capabilities and limitations of current modeling approaches, this study provides a foundation for more nuanced population projections essential for Nigeria's sustainable development planning.

CONCLUSIONS

The analysis rigorously compares multiple population growth models to find the best fit for Nigeria's population data. The combination of visual representations and statistical metrics provides a detailed picture of population trends and the effectiveness of different modeling approaches. The choice of the best model was not based only on statistical fit (AIC, BIC, R-squared, RMSE) but also its interpretability and potential for generalization to future periods. The combined analysis confirms that proposed3 is the best model overall, but also reveals that the practical differences between the top models (proposed1-3, exponential, logistic) are less dramatic than statistical metrics alone would suggest. The consistent residual patterns across all models indicate that further improvements might require incorporating additional factors beyond mathematical growth functions - perhaps including socioeconomic, policy, migration, or other demographic transition factors that could explain the cyclical pattern in prediction errors that persist across all models.

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