On the Connectedness of A Set of Twenty-Three (23) Optimal Incomplete Block Designs of Size (T=9, B=9, K=3, R=3) Based on Circuits

***Anthony Ekpo[1], and Abimbola V. Oladugba[2]**

[1]Department of Statistics, Federal University of Agriculture, Makurdi, Nigeria

[2]Department of Statistics, University of Nigeria, Nsukka

***Corresponding Author**

DOI: https://doi.org/10.51584/IJRIAS.2025.100800045

Received: 25 July 2025; Accepted: 04 August 2025; Published: 05 September 2025

ABSTRACT

In this article, six (6) Incomplete Block Designs (IBDs) namely; A, B, C, D, E and F of size $(t=9, b=9, k=3, r=3)$ due to Nguyen, (1994) were extended by the construction of additional seventeen (17) new IBDs namely; G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V and W using all the initial blocks in Nguyen's via the cyclic method and JAVA codes. All the 23 IBDs were enumerated for "robustness" based on number of circuits; through shorter path-lengths. Our newly constructed design Q, turned out to be better than Nguyen's designs A-F with 28 shorter paths which is closely followed by designs G, K, N, O, P, S, V and W with 27 shorter-paths each. Design Q turned out to have the least number (8) of longer paths which makes it the best design for experimentation in terms of circuits. Incidentally, Nguyen's design A turns out to be the least connected design among all the 23 IBDs with only one longest path. In conclusion, we discovered that the overall best design is design Q based on connectedness. Therefore, design Q from the newly constructed designs becomes the most robust in terms of design connectedness and hence recommended for use with their experimental layouts to experimenters, investigators or evaluators who seek for the best optimal design of size $(t=9, b=9, k=3, r=3)$ in any field experiment.

Keywords: Concurrence, Connectedness, Canonical Efficiency Factors, Eigenvalues, Shorter Path-length and Circuits.

INTRODUCTION

It is important that any well-designed experiment should be able to give useful answers to scientific questions that are clear, concise and efficient at the initial instance. The analysis may require more than a few basic plots with evidence of responses as a result of transformation, depending on the available factors under consideration. Hence, for any well-designed experiment to be good enough, it is expected that the variance and the covariance of the expected parameters of interest will turn out to be small enough (Atkinson, 2014). Therefore, a properly designed experiment for particular research with specific objectives, is the basis of all successful experimentations (Sewenet, 2019). Its flexibility; is in allowing for each block to receive only some and not all the treatments to be compared (Toutenburg and Shalabh, 2009). In other instances, it is possible that the blocks available can only handle a limited number of treatments due to several reasons. For example, in the task of testing the effect of twenty types of ready-to-use-therapeutic feeds (RUTF) from different companies meant to be administered for the treatment of malnutrition among a group of under-5 malnourished children in an IDP camp, it may be difficult to get enough children having the same malnutrition status at the same time to conduct a complete block experiment. In such situation, a possible solution is to have less than twenty patients in each block, and not all the twenty feeds can be administered in all the blocks. Instead, few feeds are administered to the patients in one block and the remaining to the patients in other blocks (Hinkelmann and Kempthrone, 2007). In cases where the number of treatments to be compared exceeds the

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue VIII August 2025

number of patients in each block, the block size is reduced and the incomplete block designs can be used (Toutenburg and Shalabh, 2009. The incomplete block designs need a smaller number of observations in a block than a complete block design in testing necessary hypothesis without losing the efficiency of desired design of the planned experiment (Toutenburg and Shalabh, 2009). In such situation, a possible solution is to have less than twenty patients in each block. Not all the twenty feeds can be administered in all of the blocks. Instead a few feeds are administered to the patients in one block and the remaining to the patients in other blocks (Hinkelmann and Kempthrone, 2007). An incomplete-block design refers to a block design in which the size of each block is smaller than the number of treatments (Chigbu, 1998). Usually, the total number of plots in an incomplete-block design having r_i replications while administering t_i treatment in b_j block and of size k_j such that:

$$\sum_i r_{i.} = \sum_j k_j \quad (1.0)$$

is structurally represented:

$$y_{ijk} = a_0 + t_i + b_j + e_{ijk}, \;\; k = 1,2,...n_{ij} \quad (2.0)$$

where y_{ijk} is the k^{th} response in the $(ij)^{th}$ plot, a_0 is a constant, t_i and b_j are the i^{th} treatment and the j^{th} block effects, respectively, e_{ijk} are the random error, r_i and k_j are the number of replications of t_i and block b_j respectively, n_{ij} is the number of plots in the j^{th} block that receives the i^{th} treatment (Onukogu and Chigbu, 2002). In most experiments, especially the designed experiments in Agriculture, Industries, Pharmaceuticals, Medicine, Nutrition, Hydrology etc. the efficiency of Fisher's very popular Randomized Block Designs (RBD's) and Latin Square Designs (LSD) had been found to lose its efficiency when the number of plots per block or row and column increases to say, ten and above (Bose and Nair, 1939).

For instance, when 21 replications would be required for a design, a more robust design that can accommodate this situation by helping to reduce the number of replications, so that the pair of treatments can be arranged in different plots such that the difference between treatment effects of a pair for all the pairs in a block, can be estimated with the same precision without losing its connectedness is the Balanced Incomplete Block Design (BIBD). It is called "balanced" if there is an integer λ such that:

$$\lambda_{ij} = \lambda \quad (3.0)$$

This is because, some pair of treatment have same efficiency (Hinkelmann and Kempthrone, 2007). An incomplete block design of size (v, k, r), where v is treatment and set out in b- blocks of size k(< v) where the layout is in such a way that the each treatment is replicated r-times (Cheng and Bailey, 1991). The assumption here is that no treatment occurs more than once in a block. The information matrix for the adjusted treatment effects of the IBD of size (v, k, r) is as given below:

$$\underline{L} = rI - (\frac{1}{K})NN' \quad (4.0)$$

where NN' is the treatment concurrence matrix with a correspondence treatment concurrence graph (Nguyen, 1994). A common criterion being used by experimenters to compare incomplete block designs of the same size and class is the efficiency factor (Nguyen, 1994; Godolphin and Warren, 2014). Other design classification, profiling, enumeration and categorization strategies can be achieved by using the optimality criteria such as the $A-, D-$ and $E-Optimality$ etc. as well as ascertaining their connectedness in terms of shortest circuits and adjacency matrix (Lindsay, 1983; Chigbu and Ekpo, 2009; Chigbu and Ekpo, 2010). The search for a more robust optimal experimental treatment plan/layout that would provide an optimal combination of experimental units in experimentation, using an incomplete block design of size $t = 9, b = 9, k = 3, r = 3$; where nine (9)

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue VIII August 2025

is the number of treatments to be administered in nine (9) blocks, arranged in three (3) plots and replicated three (3) times necessitated this study. Therefore, the discovery and need to extend the work of (Nguyen, 1994) to capture more combinatorial properties such as the D- and E- optimality criteria, provide concurrence graphs, determine the extent of connectedness in terms of circuits and adjacency matrices with the construction of seventeen additional IBDs to be categorized alongside those of Nguyen becomes necessary. The aim of this article is to improve upon the work of Nguyen (1994) by extending its combinatorial properties while constructing additional sets of Incomplete Block Designs (IBDs) of the same size and provide a more robust optimal IBD layouts for experimentation requiring the same size of design. The specific objectives are to:

Construct additional Incomplete Block Design (IBD) of size (9, 9, 3, 3) using all the initial block in Nguyen (1994) by the cyclic method via Java codes;

Ascertain the extent of connectedness of all the designs based on their circuits via Design Graphs;

Enumerate the best designs for experimentation based on (i and ii) above.

The most connected design provides a ready-to-use experimental design plan for experimenters who are seeking for a more robust optimal and well-connected design layouts of an incomplete block design of size $(9, 9, 3, 3)$. Specifically, it would provide an experimental layout for experiments, assessments and evaluations where nine treatments are required to be applied in block size of nine, with three replications and in three plots, hence saving scarce financial resources, valuable time and wastage of experimental materials. The two(2) best optimal experimental plans/layouts would be provided and enumerated as the best for experimentation which could help proffer solution to experimental problems in many fields of study, particularly it could be useful to those experimenters who are in need of optimal experimental layouts for use in experiments requiring nine (9) treatments in block sizes of three (3) and to be replicated three (3) time, especially those with limited resources in terms of funding, experimental material, time etc. The best design, if efficiently deployed, could also help to provide solution to the situation of the current scarcity and uncertainty surrounding the procurement and deployment of the ready-to-use-therapeutic food (Igoe, 2020) and the inability to achieve optimal results from the existing treatment plans of cases of severe acute malnutrition (SAM) among the under-5 children (Lindsey et al., 2016; Ekpo et al., 2019 and Ekpo et al., 2021) especially in some parts of Africa and Asia (Abate et al., 2020 and Kabalo et al., 2017) despite the multiple and sector-specific feeding programs of the ready-to-use-therapeutic food (RUTF) and the outpatient therapeutic feeding (OTF) mostly in the crises prone countries in Africa and beyond (Yebyo et al., 2013). Furthermore, this study would provide additional combinatorial properties to the six optimal incomplete block designs due to Nguyen (1994), such as: the $A-, D-$ and $E-Optimality$ criteria values through their canonical efficiency factors, connectedness through the provision of treatment concurrence graphs and circuits; and enumeration of adjacency levels of the designs through their adjacency matrices and consequently adding new literature to the body of knowledge in the area of Optimal design of experiments. The optimality criteria considered in this research are only the $A-, D-$ and $E-Optimality$ which is widely used in enumerating, comparing and categorizing designs (Jin, 2004; Chigbu, 1998; and John, 1987) obtained through a Python coded algorithm. The circuits showing longest and shortest paths together with the adjacency matrices of the twenty-three designs were enumerated. Therefore, in this study, we constructed seventeen additional optimal incomplete block designs to the already existing six IBDs constructed by Nguyen, (1994) and tested all the twenty-three designs for optimality using the $A-, D-$ and $E-$optimality criteria via their canonical efficiency factors, checked for connectedness via the provision of their treatment concurrence graphs and then enumerated them in a hierarchical order of bestness in terms of circuits. Although, the resultant optimal designs which this research enumerated satisfies all necessary and fundamental statistical, and theoretical combinatorial principles, however, putting it into practical use would require adequate resources, judicious deployment of expertise, adequate time devoted and series of field repetitive trials. However, the enumerated experimental layouts can only be deployed into full experimentation in the field with adequate funding, close monitoring and effective deployment of experimental material.

MATERIALS AND METHODS

Since this article dwells more on the connectedness of the incomplete block designs, especially based on circuits and concurrences of design, we laid credence to relevant literature relating to the historical and conceptual perspectives of the optimal designs in this work, especially the works of Mahanta (2018), Nguyen (1994), Li and Coster (2015), Morgan (2007), Habtamu (2019), Rink (2016), Atkinson (2014), Cheng and Bailey (1991), Harman and Filova (2014), Smucker et al. (2017), Uto and Bailey (2020), Safarkhani and Moerbeek (2015), Tommasi and Lopez-Fidalgo (2010), Godolphin and Warren (2014), Candel and Breukelen (2010), Evangelaras (2021), Sera (2015), John (1987), Sambo et al. (2014), Yin and Huang (2021), Ipinyomi and John (1985), Johnson et al. (2011). Chang-Yun and Po (2019), Bailey and Chigbu (1997) and Chigbu (2003), Gutman et al. (2014), Ekpo et al. (2021, Jones et al. (2020). Garcia-Rodenas et al. (2020), Bailey (2009), Eccleston and Hedayat (1974), Mba et al. (2021)**,** Lindsay (1983), Cameron (2006), Lin and Phoa (2016), Bhar (2014), Bailey et al. (2017), Filipiak and Markiewicz (2012), Bailey et al. (2013), Lee et al. (2018), Imhof et al. (2002) among others. The method employed in the article is to construct additional Incomplete Block Designs (IBDs) of size (9, 9, 3, 3) using all the initial blocks in Nguyen (1994) by the cyclic method via Java codes. Apparently, prior to this research, a set of six existing optimal incomplete block designs due to Nguyen (1994) namely; designs A, B, C, D, E and F each of size $(9,9,3,3)$, which were constructed by computer were identified as the existing set of similar designs. This existing sets of optimal IBD constructed were found to be optimal but not necessarily $A-, D-$and $E-Optimal$. The design construction, utilized codes in JAVA via the cyclic method of design construction, with all initial blocks in Nguyen (1994) and seventeen (17) more incomplete block designs of the same class and size were constructed; these newly constructed designs were named; designs G - W. Thus, the emerging complete sets of twenty-three IBDs were presented for enumeration of bestness for field experimentation. In most practical experimentation involving design works, it is usually required that there must be precision, and that the number of treatments (t) be known.

There are several ways of constructing a BIBD, and they should be used if the parameters are having large values. Only designs for small t and k can be constructed by trial and error without exorbitant effort (Bose, 2011). Furthermore, the method of constructing the BIBD is based on the concept of symmetrically repeated differences. By the use of this concept, it is possible to construct the entire design with the help of the initial blocks. The discovery of the initial blocks is very much facilitated by the use of the properties of the primitive roots of the binomial equations in the Galois field, but the use of these properties is not essentially here (Mahanta, 2018). The construction of a cyclic balanced incomplete block design requires the use of the initial block. However, the choice of the initial block is quite arbitrary, in that, it would lead to the construction of an appropriate design by a method of cyclical substitution. For convenience, the initial block will be taken to be the block with the lowest numerical values. Consider an IBD of size $(t=7, b=7, r=3, k=3, \lambda=1)$ for example, and using block $(0,1,3)$ as the initial block will give the following seven incomplete blocks: (0 1 3), (1 2 4), (2 3 5), (3 4 6), (4 5 0), (5 6 1), (6 0 2).

Here, treatment is replicated 3 times and every pair of treatments occurs together just ones in a block. The cyclic method of construction is such that a block is obtained just by adding one to each element in the previous block and reducing modulo 7 when necessary. The following theorems illustrates the construction of the IBDs using the cyclic method: Suppose an initial block $\{i_1, i_2, ..., i_k\}$ is given after labeling the treatment by the modulo t. The other blocks of the cyclic design are $\{i_1+1, i_2+1, ..., i_k+1\}$, and so on, with all arithmetic done in modulo t. This cyclic construction method works if b = t. Every choice of an initial block has a difference table as the one constructed below:

$$\begin{array}{c|cccc} & i_1 & i_2 & \cdots & i_k \\ \hline i_1 & 0 & i_2-i_1 & \cdots & i_k-i_1 \\ i_2 & i_1-i_2 & 0 & \cdots & i_k-i_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ i_k & i_1-i_k & i_2-i_k & \cdots & 0 \end{array} \quad (15.0)$$

The cyclic method of construction of designs is such that a block is obtained by adding one to each element in the previous block and reducing to appropriate modulo when necessary (John, 1987; Bailey and Cameron, 2019). The cyclic method of constructing incomplete block designs is used here in the construction of the seventeen new IBDs of the same size (9, 9, 3, 3) as those of Nguyen (1994), i.e., designs G, H, I, J, K ,L, M, N, O, P, Q, R, S, T, U, V and W. Design G is constructed using Nguyen's initial block (3 1 7) while design H was also constructed using Nguyen's initial block (3 1 9), design I with initial block (9 8 4), design J with initial block (7 8 4), design K with initial block (5 2 6), design L with initial block (3 2 7), design M with initial block (9 2 7), design N with initial block (1 2 7), design O with initial block (5 9 8), design P with initial block (5 3 8), design Q with initial block (6 1 4), design R with initial block (5 9 4), design S with initial block (2 8 3), design T with initial block (2 8 9), design U with initial block (2 8 1), design V with initial block (7 1 6) and design W with initial block (7 3 6). The newly constructed incomplete block designs are as given in Appendix 2.

In any connected design, all the treatment contrast within the block are estimable and pair-wise comparison of estimators have similar variances. Given two treatment effects τ_{i1} and τ_{i2} it is possible to have treatment effects in the chains: τ_{i1} , τ_{1j} , τ_{2j} , ..., τ_{nj} , τ_{i2} such that two adjoining treatments, in this chain occurs in the same block. If the blocks of a design are given in such a way that any treatment can be reached from any treatment, then such a design is connected (Toutenburg and Shalabh, 2009). Further insight into the concept of connectedness can be obtained by generating the incidence matrix:

$$N = (n_{ij}) \quad (16.0)$$

Here, N is a $(t \times b)$matrix where n_{ij} is the number of times that the i^{th} treatment appears in the j^{th} block of the design. The equality will hold if and only if the design is connected especially in a binary design in which each element in the incidence matrix N is either 0 or 1 (Onukogu and Chigbu, 2002).

To achieve this objective, we made use of the theory of graph as a collection of points and lines joining together pairs of these points. The term path and circuit are used here, bearing in mind that: a path joining the distinct points i and j is a route from $i - j$ traversing other lines in the graph; and a circuit at a point i is a route joining i to itself. For instance, in a graph with a path $0 - 2 - 3$, a path joining points 0 and 2, and another path $0 - 3 - 4 - 5 - 0$ is a circuit at point 0 (John, 1987; Bailey and Cameron, 2011). The treatment concurrence graph of the IBD have been defined here to be in routes between treatment in blocks of specific sizes (Lindsay, 1983).

In a treatment concurrence graph (TCG), there are specific numbers of points, lines joining those pairs which occur such as i and j. Each path gives rise to an unbiased estimators of treatment differences with the shorter ones implying estimates with lower or weaker variances. The length of the path therefore is the number of lines in it even though other paths also contribute to the overall estimate (Bailey and Cameron, 2013; Lindsay, 1983; Onukogu and Chigbu, 2002).

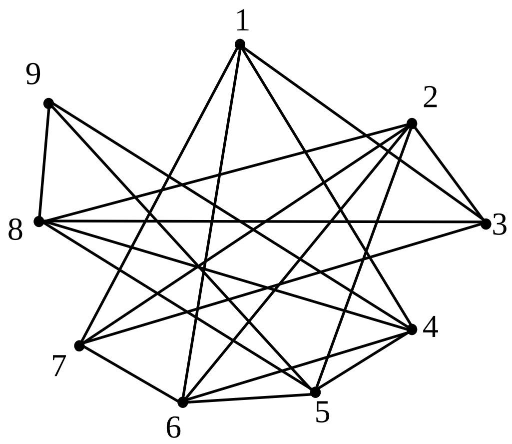
In general, when more shorter lengths in $(i - j)$ paths are noticed in a treatment concurrence graph, more information is available on the difference existing between treatments i and j , hence, a good design should have few circuits of short length. In other words, a path of length n which does not go over any line twice gives an estimator having: $2n\sigma^2$= variance. Such that designs with more estimators having higher number of small variances are considered to be well connected and good enough for experimentation (Lindsay,1983; Onukogu and Chigbu, 2002; Chigbu and Ekpo; 2008, 2009, 2010; and Bailey and Cameron, 2011). Bailey (2009) investigated how to allocate actual treatments to the abstract ones in the design under the assumption that the design is equireplicate. Particular interested was in the difference between the effects of treatments i and j, while ensuring that the variance of the estimator of this difference is small enough. Thus, the primary interest is therefore, to explore when properties of the variances can be deduced from an examination of the graph itself without necessarily calculating the generalized inverse of a matrix. In particular, when it is true, the variance decreases as concurrence increases or that variance increases with distance. Terminology and three instructive examples were further introduced and it was showed that, for a certain class of block designs, a simple statement linking variance to concurrence or distance holds for all incomplete-block designs in that

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue VIII August 2025

class. By letting Δ be an incomplete-block design for $v(\Delta)$points (treatments) and denoting it by $\tau(\Delta)$ the set of treatments of Δ, that Δ is a family of $b(\Delta)$ subsets of $\tau(\Delta)$, called blocks, each of size $k(\Delta)$. Each treatment occurs in $\tau(\Delta)$ blocks; the number $r(\Delta)$ is called the replication. The concurrence λ_{ij} (Δ) of treatments i and j becomes the number of blocks in which i and j both occurred: in particular, λ_{ij} $(\Delta) = r(\Delta)$ for all i in $r(\Delta)$.The concurrence matrix $\phi(\Delta)$ becomes the v × v matrix with entries λ_{ij} (Δ) with $G(\Delta)$ as the concurrence graph of Δ thereby concluding that the vertex set of $G(\Delta)$ is $T(\Delta)$, and that the number of edges between vertices i and j is λ_{ij} , if i = j: there are no loops. It was finally concluded that the block design is said to be connected if $G(\Delta)$ is a connected graph.

RESULT

In this section, our focus is on the presentation of result as obtained from the technique employed to achieve objective one (1). Specifically, in the design construction, we used the cyclic method of design construction, with all initial blocks in Nguyen (1994) via codes in JAVA and to construct seventeen (17) more incomplete block designs of the same class and size as shown in appendix 2; these newly constructed designs are named; designs G - W. Thus, together with the six (6) due to Nguyen (1994), a complete set of twenty-three (23) IBDs are presented for enumeration for bestness in field experimentation, and thereafter documented as new body of knowledge in related literature.



Treatment Concurrence Tables and Graphs for Designs A-F

Treatment Pair	Shortest Path	Length	Variance
1,2	1-3-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-4-5	2	$4\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-4-8	2	$4\sigma^2$
1,9	1-4-9	2	$4\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-5-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-8-9	2	$4\sigma^2$
3,4	3-1-4	2	$4\sigma^2$
3,5	3-8-5	2	$4\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-8-9	2	$4\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-1-6	2	$4\sigma^2$
4,7	4-1-7	2	$4\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-2-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-5-8	2	$4\sigma^2$
6,9	6-5-9	2	$4\sigma^2$
7,8	7-2-8	2	$4\sigma^2$
7,9	7-2-8-9	3	$6\sigma^2$
8,9	8-9	1	$4\sigma^2$

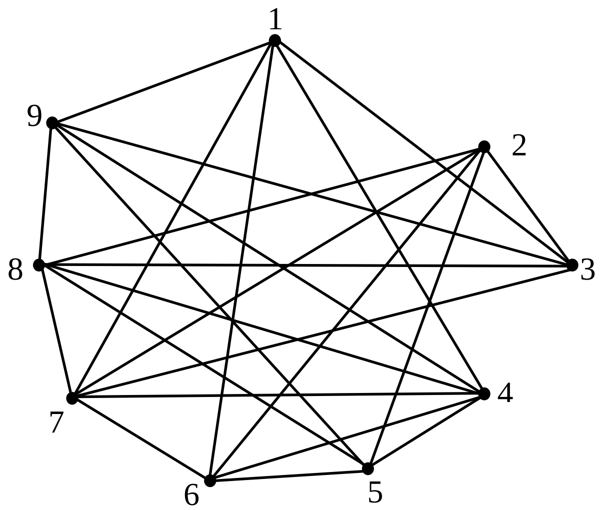
Concurrence Graph for Design A

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue VIII August 2025

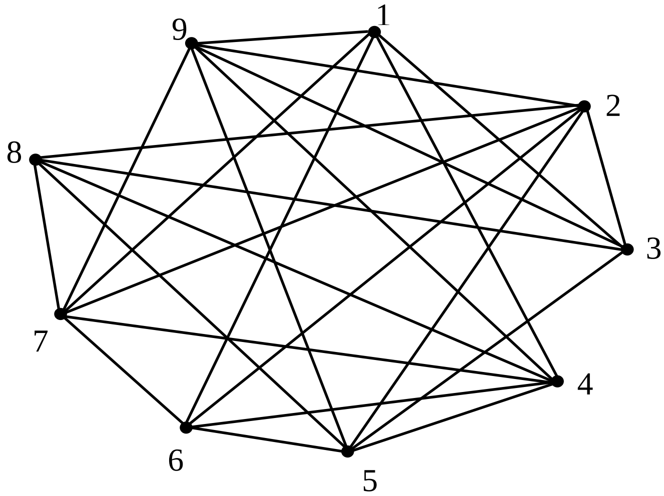
Treatment Concurrence Table for Design A

Treatment Pair	Shortest Path	Length	Variance
1,2	1-3-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-1	1	$2\sigma^2$
1,5	1-4-5	2	$4\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-7-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-5-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-8-9	2	$4\sigma^2$
3,4	3-8-4	2	$4\sigma^2$
3,5	3-8-5	2	$4\sigma^2$
3,6	3-1-6	2	$4\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	1	$2\sigma^2$
6,9	6-1-9	2	$4\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Treatment Concurrence Table for Design B



Concurrence Graph for Design B



ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue VIII August 2025

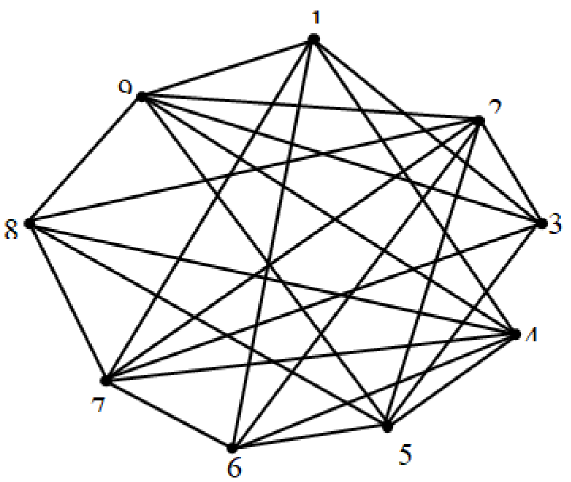
Treatment Pair	Shortest Path	Length	Variance
1,2	1-3-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-4-5	2	$4\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-4-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-9-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-8-4	2	$4\sigma^2$
3,5	3-9-5	2	$4\sigma^2$
3,6	3-1-6	2	$4\sigma^2$
3,7	3-2-7	2	$4\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-1-9	2	$4\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-2-9	2	$4\sigma^2$

Concurrence Graph for Design C

Treatment Concurrence Table for Design C

Treatment Pair	Shortest Path	Length	Variance
1,2	1-3-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-9-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-8-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-7-4	2	$4\sigma^2$
3,5	3-9-5	2	$4\sigma^2$
3,6	3-7-6	2	$4\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-1-9	2	$4\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design D



Treatment Concurrence Table for Design D

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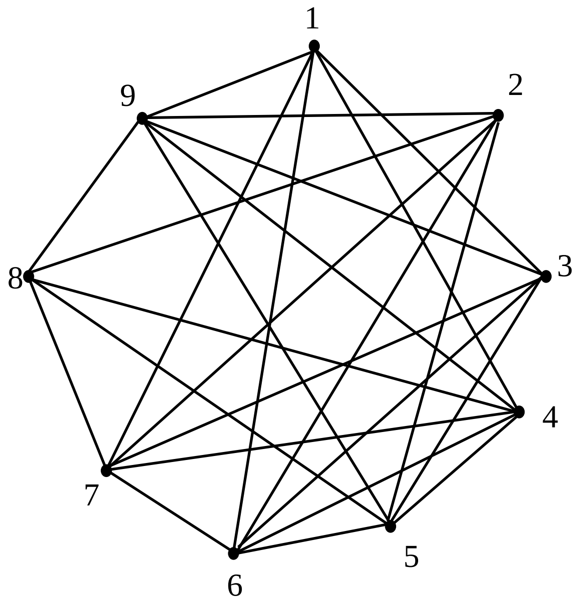
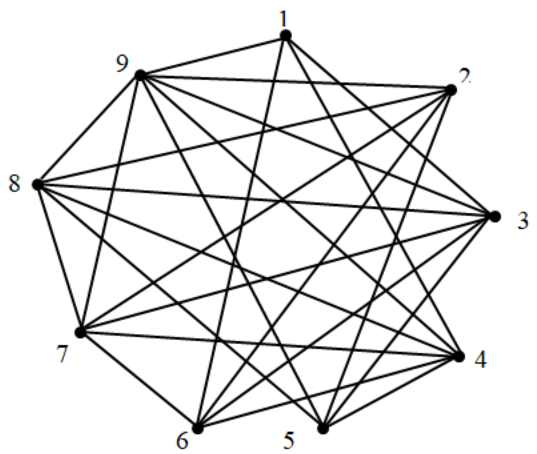


Treatment Concurrence Table for Design E

Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-4-5	2	$4\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-9-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-5-3	2	$4\sigma^2$
2,4	2-5-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-5-4	2	$4\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-1-9	2	$4\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design E



Treatment Concurrence Table for Design F

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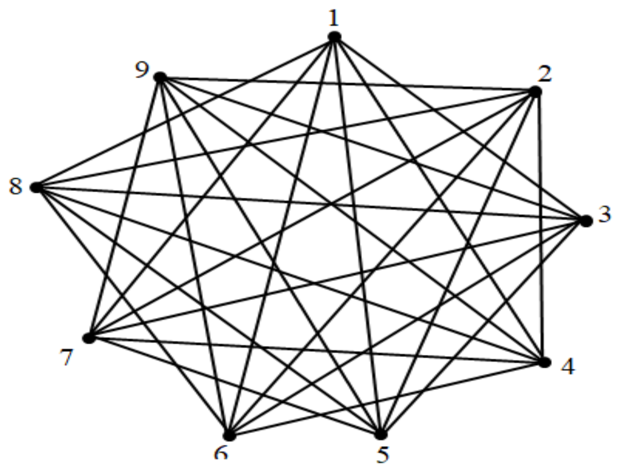
Treatment Pair	Shortest Path	Length	Variance
1,2	1-6-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-4-5	2	$4\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-6-7	2	$4\sigma^2$
1,8	1-9-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-5-3	2	$4\sigma^2$
2,4	2-5-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-5-4	2	$4\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-4-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	5-7	1	$2\sigma^2$
6,8	6-2-8	2	$4\sigma^2$
6,9	6-1-9	2	$4\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design F



Treatment Concurrence Tables and Graphs for Designs G-W

Concurrence Graph for Design G

Treatment Pair	Shortest Path	Length	Variance
1,2	1-3-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-8	1	$2\sigma^2$
1,9	1-3-9	2	$4\sigma^2$
2,3	2-6-3	2	$4\sigma^2$
2,4	2-4	1	$2\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-6-4	2	$4\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-7-5	2	$4\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-2-6	2	$4\sigma^2$
5,7	5-7	1	$2\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-2-7	2	$4\sigma^2$
6,8	6-8	1	$2\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-1-8	2	$4\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-2-9	2	$4\sigma^2$

Treatment Concurrence Table for Design G

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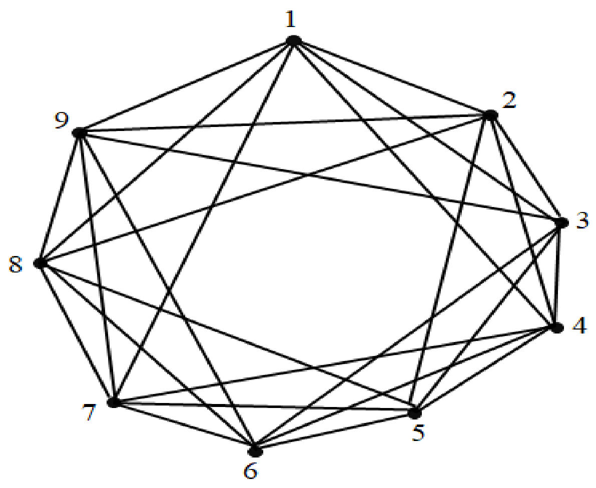
Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-4-5	2	$4\sigma^2$
1,6	1-3-6	2	$4\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-8	1	$2\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-3-6	2	$4\sigma^2$
2,7	2-8-7	2	$4\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-6-7	2	$4\sigma^2$
3,8	3-9-8	2	$4\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-7-8	2	$4\sigma^2$
4,9	4-3-9	2	$4\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-7	1	$2\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-8-9	2	$4\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-8	1	$2\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-2-9	2	$2\sigma^2$

Treatment Concurrence Table for Design H

Concurrence Graph for Design H



Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-5-4	2	$4\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-6-7	2	$4\sigma^2$
1,8	1-9-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-6-5	2	$4\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-7-8	2	$4\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-7-6	2	$4\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-8-9	2	$4\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-8-7	2	$4\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-9-8	2	$4\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-1-9	2	$4\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

INTERNATIONAL JOURNAL OF RESEARCH AND INNOVATION IN APPLIED SCIENCE (IJRIAS)
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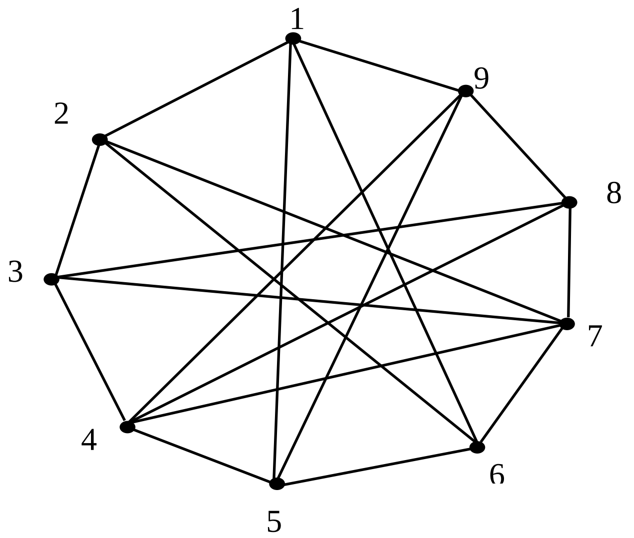
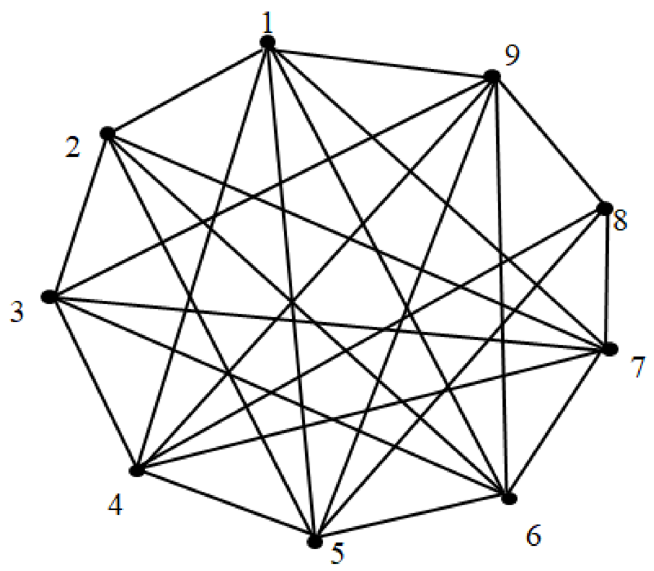


Treatment Concurrence Table for Design I

Concurrence Graph for Design I



Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-7-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-7-8	2	$4\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-7-8	2	$4\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design J

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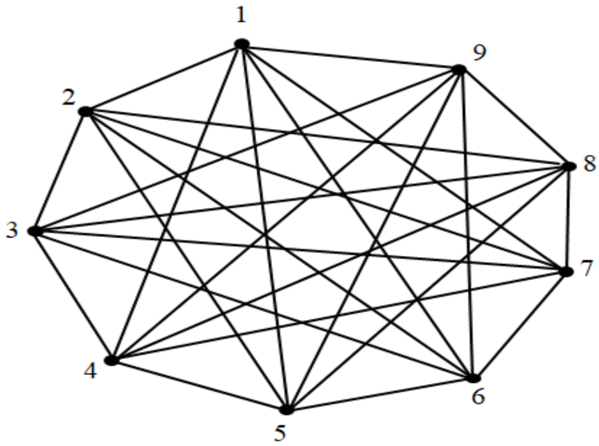
Treatment Concurrence Graph for Design J



Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-7-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design K

Treatment Concurrence Table for Design K

Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-5-4	2	$4\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-2-7	2	$4\sigma^2$
1,8	1-9-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-6-5	2	$4\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-7-8	2	$4\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-7-6	2	$4\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-8-9	2	$4\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-8-7	2	$4\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-9-8	2	$4\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-1-9	2	$4\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Treatment Concurrence Table for Design L

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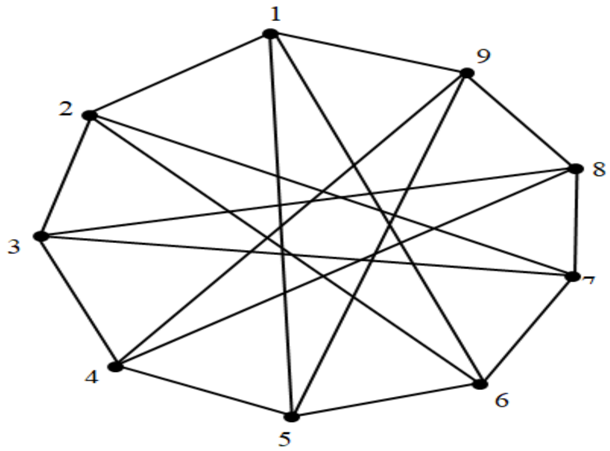
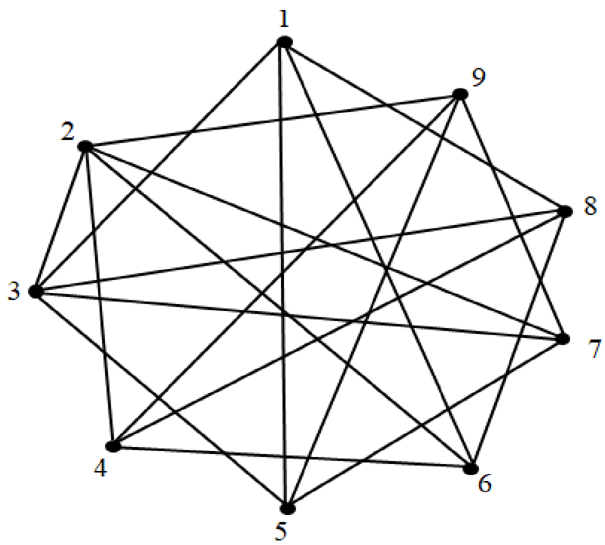
Concurrence Graph for Design L



Treatment Pair	Shortest Path	Length	Variance
1,2	1-6-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-6-4	2	$4\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-5-7	2	$4\sigma^2$
1,8	1-8	1	$2\sigma^2$
1,9	1-5-9	2	$4\sigma^2$
2,3	2-7-3	2	$4\sigma^2$
2,4	2-4	1	$2\sigma^2$
2,5	2-9-5	2	$4\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-6-8	2	$4\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-8-4	2	$4\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-1-6	2	$4\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-5-9	2	$4\sigma^2$
4,5	4-9-5	2	$4\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-9-7	2	$4\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-1-6	2	$4\sigma^2$
5,7	5-7	1	$2\sigma^2$
5,8	5-1-8	2	$4\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-2-7	2	$4\sigma^2$
6,8	6-8	1	$2\sigma^2$
6,9	6-2-9	2	$4\sigma^2$
7,8	7-3-8	2	$4\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-4-9	2	$4\sigma^2$

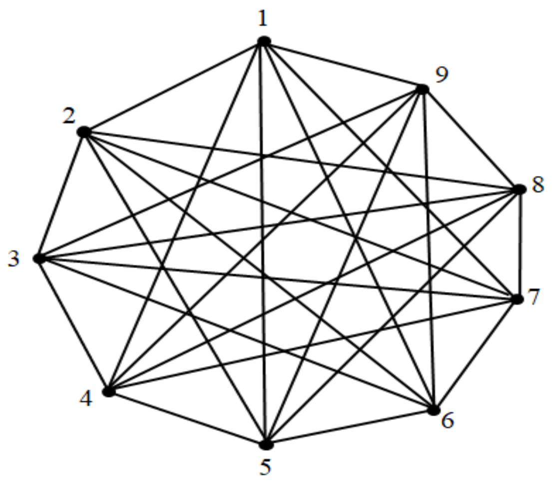
Treatment Concurrence Table for Design M



Concurrence Graph for Design M

Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-7-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design N



Treatment Concurrence Table for Design N

Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-7-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

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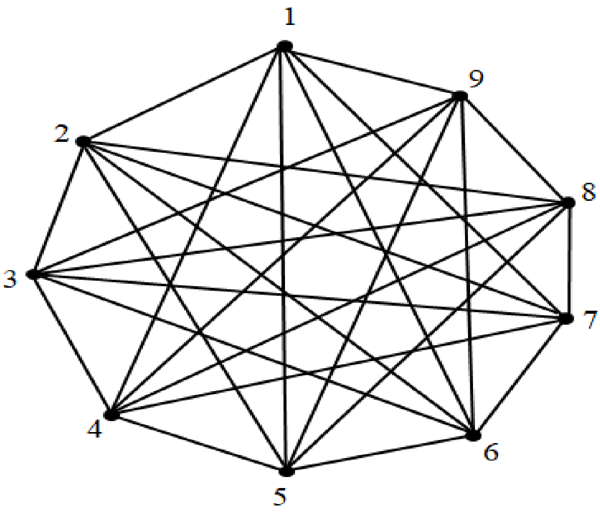
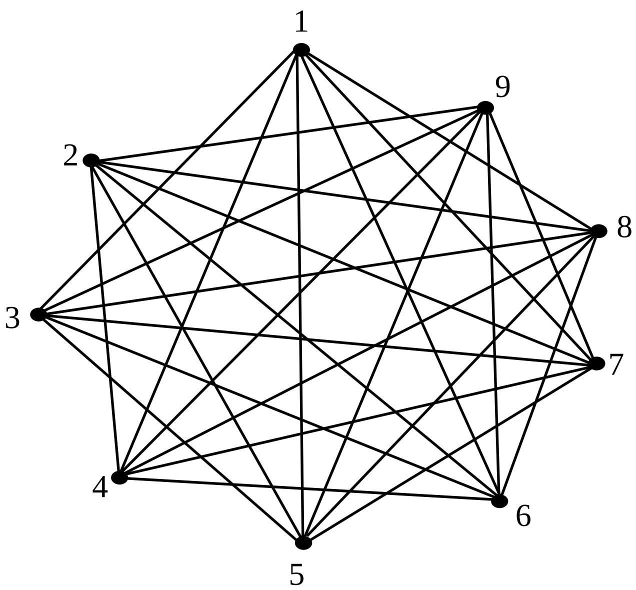
Concurrence Graph for Design O



Treatment Concurrence Table for Design O

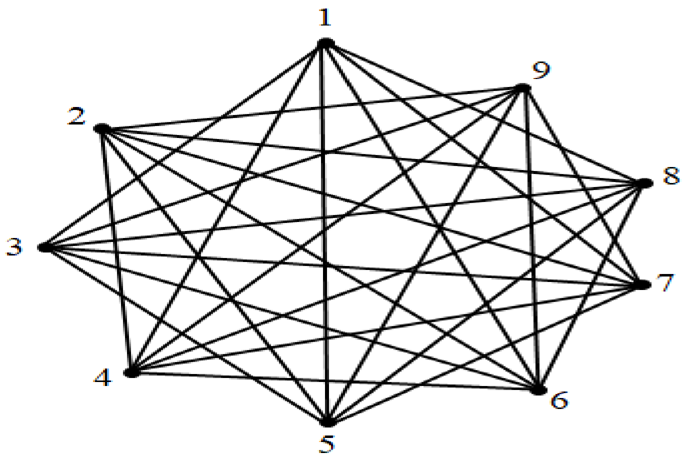
Treatment Pair	Shortest Path	Length	Variance
1,2	1-8-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-8	1	$2\sigma^2$
1,9	1-6-9	2	$4\sigma^2$
2,3	2-7-3	2	$4\sigma^2$
2,4	2-4	1	$2\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-6-4	2	$4\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-9-5	2	$4\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-8-6	2	$4\sigma^2$
5,7	5-7	1	$2\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-9-7	2	$4\sigma^2$
6,8	6-8	1	$2\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-1-8	2	$4\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-5-9	2	$4\sigma^2$



Treatment Concurrence Table for Design P

Treatment Pair	Shortest Path	Length	Variance
1,2	1-8-2	2	$4\sigma^2$
1,3	1-3	1	$2\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-8	1	$2\sigma^2$
1,9	1-7-9	2	$4\sigma^2$
2,3	2-9-3	2	$4\sigma^2$
2,4	2-4	1	$2\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-7-4	2	$4\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-7-5	2	$4\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-8-6	2	$4\sigma^2$
5,7	5-7	1	$2\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-9-7	2	$4\sigma^2$
6,8	6-8	1	$2\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-1-8	2	$4\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design P



Concurrence Graph for Design Q

Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-5-4	2	$4\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-6-7	2	$4\sigma^2$
1,8	1-9-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-6-5	2	$4\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-7-8	2	$4\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-2-6	2	$4\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-8-9	2	$4\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-8-7	2	$4\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-9-8	2	$4\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-5-9	2	$4\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-9	1	$2\sigma^2$

Treatment Concurrence Table for Design Q

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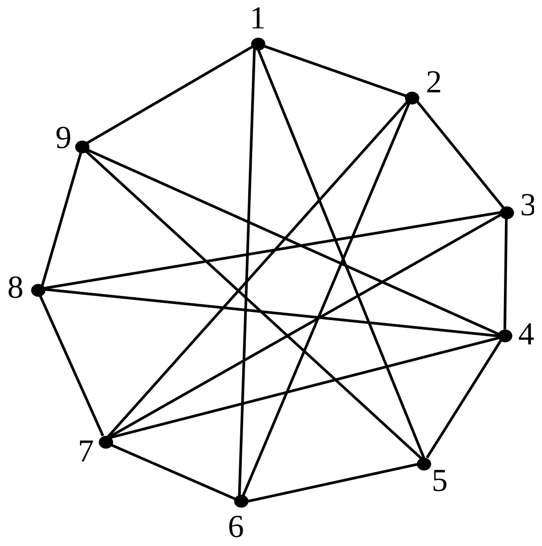
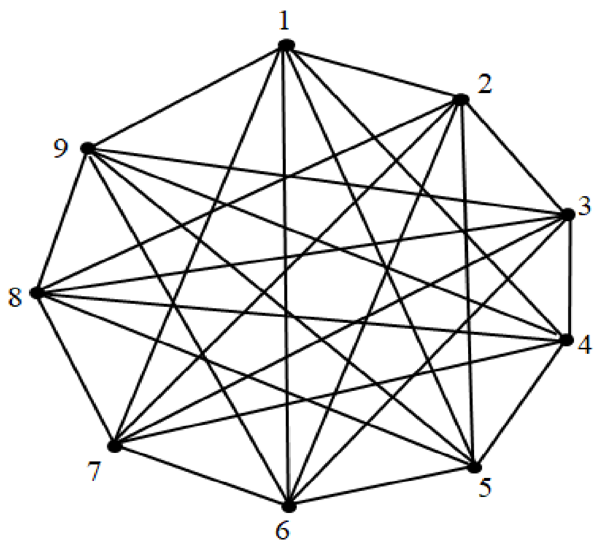


Treatment Concurrence Table for Design R

Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-9-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-8-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-2-6	2	$4\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$4\sigma^2$

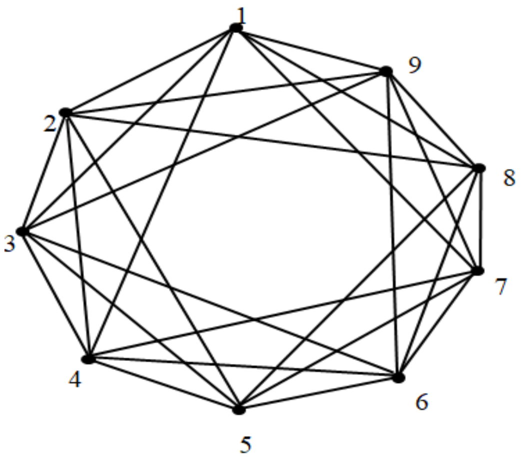


Treatment Concurrence Table for Design S

Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-2-5	2	$4\sigma^2$
1,6	1-9-6	2	$4\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-8	1	$2\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-5-6	2	$4\sigma^2$
2,7	2-9-7	2	$4\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-9	1	$2\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-6-7	2	$4\sigma^2$
3,8	3-6-8	2	$4\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-7-8	2	$4\sigma^2$
4,9	4-3-9	2	$4\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-7	1	$2\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-8-9	2	$4\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-8	1	$2\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design T

Treatment Concurrence Table for Design T



Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-4-5	2	$4\sigma^2$
1,6	1-8-6	2	$4\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-8	1	$2\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-4	1	$2\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-5-6	2	$4\sigma^2$
2,7	2-8-7	2	$4\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-6-4	2	$4\sigma^2$
3,5	3-5	1	$2\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-6-7	2	$4\sigma^2$
3,8	3-2-8	2	$4\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-6	1	$2\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-8-9	2	$4\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-7	1	$2\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-8	1	$2\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-9	1	$2\sigma^2$
8,9	8-9	1	$2\sigma^2$

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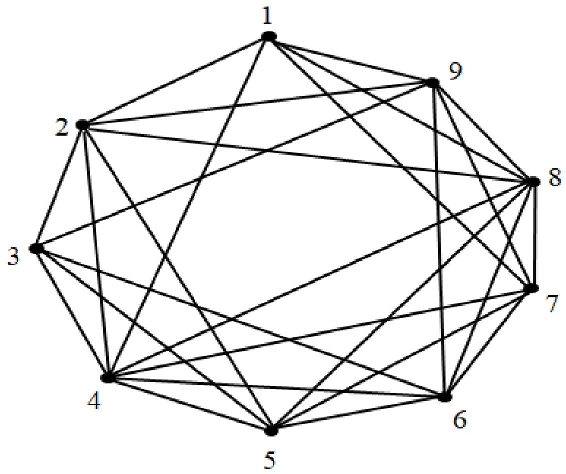
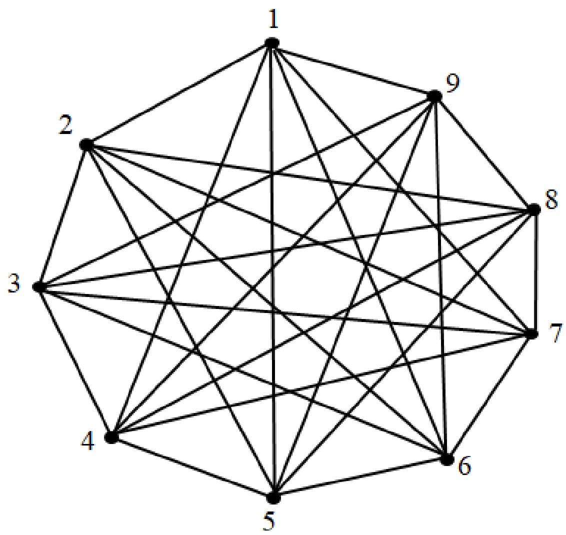


Concurrence Graph for Design U

Treatment Concurrence Table for Design U

Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-7-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-5-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-6-5	2	$4\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Treatment Concurrence Table for Design V



Concurrence Graph for Design V

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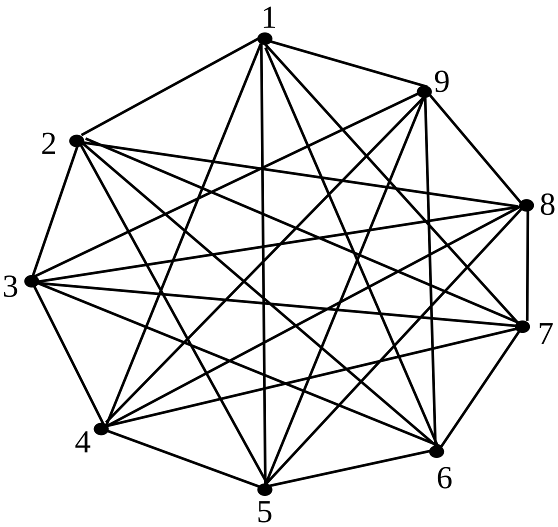
Treatment Pair	Shortest Path	Length	Variance
1,2	1-2	1	$2\sigma^2$
1,3	1-2-3	2	$4\sigma^2$
1,4	1-4	1	$2\sigma^2$
1,5	1-5	1	$2\sigma^2$
1,6	1-6	1	$2\sigma^2$
1,7	1-7	1	$2\sigma^2$
1,8	1-7-8	2	$4\sigma^2$
1,9	1-9	1	$2\sigma^2$
2,3	2-3	1	$2\sigma^2$
2,4	2-3-4	2	$4\sigma^2$
2,5	2-5	1	$2\sigma^2$
2,6	2-6	1	$2\sigma^2$
2,7	2-7	1	$2\sigma^2$
2,8	2-8	1	$2\sigma^2$
2,9	2-1-9	2	$4\sigma^2$
3,4	3-4	1	$2\sigma^2$
3,5	3-4-5	2	$4\sigma^2$
3,6	3-6	1	$2\sigma^2$
3,7	3-7	1	$2\sigma^2$
3,8	3-8	1	$2\sigma^2$
3,9	3-9	1	$2\sigma^2$
4,5	4-5	1	$2\sigma^2$
4,6	4-5-6	2	$4\sigma^2$
4,7	4-7	1	$2\sigma^2$
4,8	4-8	1	$2\sigma^2$
4,9	4-9	1	$2\sigma^2$
5,6	5-6	1	$2\sigma^2$
5,7	5-6-7	2	$4\sigma^2$
5,8	5-8	1	$2\sigma^2$
5,9	5-9	1	$2\sigma^2$
6,7	6-7	1	$2\sigma^2$
6,8	6-7-8	2	$4\sigma^2$
6,9	6-9	1	$2\sigma^2$
7,8	7-8	1	$2\sigma^2$
7,9	7-8-9	2	$4\sigma^2$
8,9	8-9	1	$2\sigma^2$

Concurrence Graph for Design S



Treatment Concurrence Table for Design W

Concurrence Graph for Design W

Table 1.0: Result Based on Circuits and Shortest Path-Length

Designs	$2\sigma^2$ **(1 Path)**	$4\sigma^2$ **(2 Paths)**	$6\sigma^2$ **(3 Paths)**
A	20	15	1
B	25	11	0
C	24	12	0
D	26	10	0

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E	27	9	0
F	26	10	0
G	27	9	0
H	26	10	0
I	18	18	0
J	25	11	0
K	27	9	0
L	18	18	0
M	18	18	0
N	27	9	0
O	27	9	0
P	27	9	0
Q	**28**	**8**	**0**
R	20	16	0
S	27	9	0
T	25	11	0
U	26	10	0
V	27	9	0
W	27	9	0

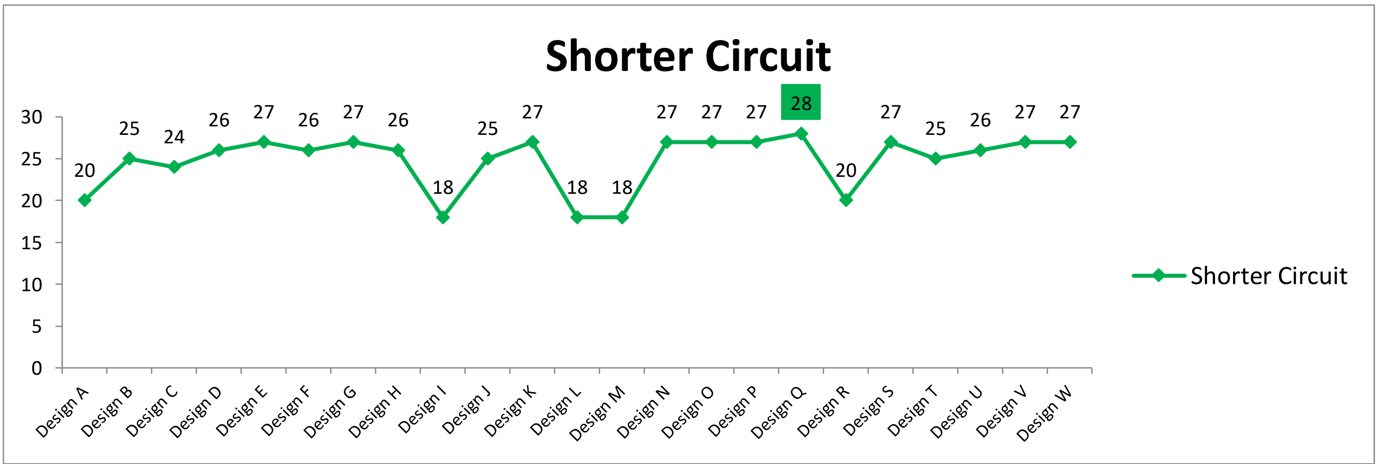

Figure 1.0: Graph Showing the Best of the Twenty-Three Optimal IBDs Based on Shorter Circuits

DISCUSSION

In terms of shorter-paths which implies minimum variance as shown in table 1.0 and figure 1.0, design Q, one of the newly constructed designs turned out to be better than Nguyen's designs A-F with 28 shorter paths (minimum variance of 1 path), this is closely followed by designs E, G, K, N, O, P, S, V and W with 27

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shorter-paths. Design Q has the least number eight (8) of longer paths with zero (0) longest path, hence the best design for experimentation in terms of circuits. However, Nguyen's design A turns out to be the least connected design among all the twenty-three (23) IBDs with only 1 longest path (maximum variance - 3 paths). This result was collaborated by Bailey (2009) where it was stated that for the difference between the effects of treatments i and j, one should ensure that the variance of the estimator of this difference is small enough. Thus, the primary interest is therefore, to explore when properties of the variances can be deduced from an examination of the graph itself without necessarily calculating the generalized inverse of a matrix; and that the variance decreases as concurrence increases or that variance increases with distance as shown in the concurrence graphs and tables. Again, this finding was collaborated by the findings of Eccleston and Hedayat (1974) which asserted that connectedness is an important property which every block design must possess if it is to provide an unbiased estimator for all elementary treatment contrasts under the usual linear additive model. Similarly, finding by Lindsay (1983) further supports the result obtained here, with the assertion which described the connection between the efficiency of incomplete block designs and numbers of circuits of different lengths in their variety concurrence graphs stating that numbers of circuits offer in their own right an intuitively cogent way of assessing designs; to show that they provide reasonable approximations to the conventional harmonic mean efficiency factor with emphasis on statistical motivation and on the capacity of graph theory to simplify; while algebraic details were kept to a minimum.

Application

Clearly, design Q happens to be better than all other designs including Nguyen's (A-F) in terms of maximum number of shorter circuit (path-length), hence it becomes the best design to be used for experimentation when connectedness is the focus. For instance, researchers in the health sector seeking to apply 9 treatment combinations, on 9 patients, using 3 hospital wards, with the process repeated in 3 different hospitals during clinical trials can implement the process using design Q layout.

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Appendix 1: Existing Incomplete Block Designs Due to Nguyen (1994)

3	1	7
9	8	4
5	2	6
3	2	7
5	9	8
6	1	4
5	9	4
2	8	3
7	1	6

(*A*)

3	1	9
7	8	4
5	2	6
3	2	7
5	9	8
6	1	4
5	9	4
2	8	3
7	1	6

(*B*)

3	1	9
7	8	4
5	2	6
9	2	7
5	3	8
6	1	4
5	9	4
2	8	3
7	1	6

(*C*)

3	1	9
7	8	4
5	2	6
3	2	7
5	3	8
6	1	4
5	9	4
2	8	9
7	1	6

(*D*)

3	1	9
7	8	4
5	2	6
1	2	7
5	3	8
6	1	4
5	9	4
2	8	9
7	3	6

(*E*)

3	1	9
7	8	4
5	2	6
9	2	7
5	3	8
6	1	4
5	9	4
2	8	1
7	3	6

(*F*)

Note: (A) - (F) signifies name of designs, e.g., (A) is Design A

Appendix 2: Java Output of the Newly Constructed IBDs with Nguyen's Initial Blocks

BLOCK G	BLOCK H	BLOCK I	BLOCK J	BLOCK K	BLOCK L	BLOCK M	BLOCK N	BLOCK O	BLOCK P	BLOCK Q	BLOCK R	BLOCK S	BLOCK T	BLOCK U	BLOCK V	BLOCK W
317	319	984	784	526	327	927	127	598	538	614	594	283	289	281	716	736
428	421	195	895	637	438	138	238	619	649	725	615	394	391	392	827	847
539	532	216	916	748	549	249	349	721	751	836	726	415	412	413	938	958
641	643	327	127	859	651	351	451	832	862	947	837	526	523	524	149	169
752	754	438	238	961	762	462	562	943	973	158	948	637	634	635	251	271
863	865	549	349	172	873	573	673	154	184	269	159	748	745	746	362	382
974	976	651	451	283	984	684	784	265	295	371	261	859	856	857	473	493
185	187	762	562	394	195	795	895	376	316	482	372	961	967	968	584	514
296	298	873	673	415	216	816	916	487	427	593	483	172	178	179	695	625

Appendix 3: Java Program for Construction of the New Seventeen IBDs with Nguyen's Initial Blocks

```
/*
 * To change this license header, choose License Headers in Project Properties.
 * To change this template file, choose Tools | Templates
 * and open the template in the editor.
 */
package experimentaldesign;
```

```
import java.awt.Dimension;

import java.awt.Toolkit;

import javax.swing.JFrame;

import javax.swing.JOptionPane;

import javax.swing.JScrollPane;

import javax.swing.JTable;

/**

 *

 */

public class ExperimentalDesign {

/**

* @param args the command line arguments

*/

public static void main(String[] args) {

// TODO code application logic here

String str = JOptionPane.showInputDialog(null, "Enter the Number of Blocks Not More than 30");

if ((Integer.parseInt(str)) < 0 && str!="" && str.isEmpty() && (Integer.parseInt(str))>30) {

str = JOptionPane.showInputDialog(null, "Enter the Number of Blocks (Input a Non-Negative Value not Greater than 30)");

}

int numOfBlocks = Integer.parseInt(str);

String blocks[] = new String[numOfBlocks];

String b = "1";

for (int i = 0; i < numOfBlocks; i++) {

b = JOptionPane.showInputDialog(null, "Enter Block Number (Not More Than Three Digits): " + (i + 1));

if ((Integer.parseInt(b)) < 0 && b!="" && b.isEmpty() && b.length()>3) {

b = JOptionPane.showInputDialog(null, "Enter Block Number (Input a Non-Negative Value): " + (i + 1));

}

blocks[i] = b;

}
```

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```
int indigits[] = new int[3];

String data[][] = new String[9][numOfBlocks];

String[] initialColumns = new String[numOfBlocks];

String column[] = {"BLOCK G", "BLOCK H", "BLOCK I", "BLOCK J", "BLOCK K", "BLOCK L", "BLOCK M", "BLOCK N", "BLOCK O", "BLOCK P", "BLOCK Q", "BLOCK R", "BLOCK S", "BLOCK T", "BLOCK U", "BLOCK V", "BLOCK W", "BLOCK X", "BLOCK Y", "BLOCK Z", "BLOCK 21", "BLOCK 22", "BLOCK 23", "BLOCK 24", "BLOCK 25", "BLOCK 26", "BLOCK 27", "BLOCK 28", "BLOCK 29", "BLOCK 30"};

for (int i = 0; i < numOfBlocks; i++) {

initialColumns[i] = column[i];

}

// String column[] = {"BLOCK 1", "BLOCK 2", "BLOCK 3", "BLOCK 4", "BLOCK 5", "BLOCK 6", "BLOCK 7", "BLOCK 8"};

/*if (numOfBlocks == 1)

}

if (numOfBlocks == 2) {

}

if (numOfBlocks == 3) {

}

if (numOfBlocks == 4) {

}

if (numOfBlocks == 5) {

}

if (numOfBlocks == 6) {

}

if (numOfBlocks == 7) {

}

if (numOfBlocks == 8) {

}

if (numOfBlocks == 9) {

}

if (numOfBlocks == 10) {
```

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```
}
if (numOfBlocks == 11) {
}
if (numOfBlocks == 12) {
}
if (numOfBlocks == 13) {
}
if (numOfBlocks == 14) {
}
if (numOfBlocks == 15) {
}
if (numOfBlocks == 16) {
}
if (numOfBlocks == 17) {
}
if (numOfBlocks == 18) {
}
if (numOfBlocks == 19) {
}
if (numOfBlocks == 20) {
}
if (numOfBlocks == 21) {
}
if (numOfBlocks == 22) {
}
if (numOfBlocks == 23) {
}
if (numOfBlocks == 4) {
}
```

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```
if (numOfBlocks == 25) {
}
if (numOfBlocks == 26) {
}
if (numOfBlocks == 27) {
}
if (numOfBlocks == 28) {
}
if (numOfBlocks == 29) {
}
if (numOfBlocks == 30) {
}*/
//for(int i=0; i<numOfBlocks; i++){
String bla[] = new String[9];
String blb[] = new String[9];
String blc[] = new String[9];
String bld[] = new String[9];
String ble[] = new String[9];
String blf[] = new String[9];
String blg[] = new String[9];
String blh[] = new String[9];
String bli[] = new String[9];
String blj[] = new String[9];
String blk[] = new String[9];
String bll[] = new String[9];
//}
String temp = "1";
data[0] = blocks;
for (int i = 0; i < numOfBlocks; i++) {
```

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```
if (i == 0) {
System.out.print("BLOCK " + (i + 1));
} else {
System.out.print("       BLOCK " + (i + 1));
}
}
for (int i = 0; i < numOfBlocks; i++) {
System.out.println();
System.out.println("BLOCK " + (i + 1));
//System.out.print(blocks[i]);
//data[i][0] =blocks[i];
indigits[0] = Integer.parseInt(blocks[i].substring(0, 1));
indigits[1] = Integer.parseInt(blocks[i].substring(1, 2));
indigits[2] = Integer.parseInt(blocks[i].substring(2, 3));
int m = indigits[0];
int n = indigits[1];
int o = indigits[2];
//System.out.println(m + "M" );
//System.out.println(n + "N");
//System.out.println(o + "O");
if (i == 0) {
bla[i] = blocks[i];
}
if (i == 1) {
blb[i] = blocks[i];
}
if (i == 2) {
blc[i] = blocks[i];
}
```

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```
if (i == 3) {
bld[i] = blocks[i];
}
if (i == 4) {
ble[i] = blocks[i];
}
if (i == 5) {
blf[i] = blocks[i];
}
if (i == 6) {
blg[i] = blocks[i];
}
if (i == 7) {
blh[i] = blocks[i];
}
/*if (i == 8) {
bli[i] = Integer.parseInt(blocks[i]);
}
if (i == 9) {
blj[i] = Integer.parseInt(blocks[i]);
}
if (i == 10) {
blk[i] = Integer.parseInt(blocks[i]);
}
if (i == 11) {
bll[i] = Integer.parseInt(blocks[i]);
}*/
for (int k = 0; k < 8; k++) {
System.out.println();
```

```
if (m == 9) {
m = 0;
}
if (n == 9) {
n = 0;
}
if (o == 9) {
o = 0;
}
m += 1;
n += 1;
o += 1;
//System.out.print();
temp = m + "" + n + "" + o;
data[k + 1][i] = temp;
if (i == 0) {
bla[k + 1] = temp;
}
if (i == 1) {
blb[k + 1] = temp;
}
if (i == 2) {
blc[k + 1] = temp;
}
if (i == 3) {
bld[k + 1] = temp;
}
if (i == 4) {
ble[k + 1] = temp;
```

```
}
if (i == 5) {
blf[k + 1] = temp;
}
if (i == 6) {
blg[k + 1] = temp;
}
if (i == 7) {
blh[k + 1] = temp;
}
/* if (i == 8) {
bli[k + 1] = Integer.parseInt(temp);
}
if (i == 9) {
blj[k + 1] = Integer.parseInt(temp);
}
if (i == 10) {
blk[k + 1] = Integer.parseInt(temp);
}
if (i == 11) {
bll[k + 1] = Integer.parseInt(temp);
}*/
//System.out.println(m + "" + n + "" + o + "");
}
}
/*print blocks
for (int i = 0; i < numOfBlocks; i++) {
if (i == 0) {
System.out.print("BLOCK " + (i + 1));
```

```
} else {
if (i > 0) {
System.out.print("       BLOCK " + (i + 1));
}
}
System.out.println();
for (int k = 0; k < 9; k++) {
//data[i][k]=
if (i == 0) {
System.out.print(bla[k]);
} else {
System.out.print("        " + blb[k]);
if (blc.length != 0) {
System.out.print("        " + blc[k]);
}
if (bld.length != 0) {
System.out.print("        " + bld[k]);
}
if (ble.length != 0) {
System.out.print("        " + ble[k]);
}
}
//System.out.print(bla[k] + "" + blb[k] + blc[k] + bld[k] + ble[k]);
System.out.println();
}*/
Dimension screenSize = Toolkit.getDefaultToolkit().getScreenSize();
int screenHeight = screenSize.height;
int screenWidth = screenSize.width;
JFrame f = new JFrame();
```

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```
JTable jt = new JTable(data, initialColumns);

//jt.setBounds(30, 40, 200, 300);

JScrollPane sp = new JScrollPane(jt);

f.add(sp);

//this.setPreferredSize(this.getParent().getPreferredSize());

//this.pack();

f.setSize(screenWidth, screenHeight);

f.setVisible(true);

}

private static Object getParent() {

throw new UnsupportedOperationException("Not supported yet."); //To change body of generated methods, choose Tools | Templates.

}

private static void pack() {

throw new UnsupportedOperationException("Not supported yet."); //To change body of generated methods, choose Tools | Templates.

}

}
```