# Nonlinear Dynamic Control of Autonomous Vehicle under Slip Using Improved Back Propagation Algorithm

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***Abstract*: This paper presents the nonlinear dynamic control of autonomous vehicle under slip using improved back propagation algorithm. The aim is to address the issue of nonlinearity experienced in autonomous vehicle under translational state due to slip force. This was achieved developing a nonlinear vehicle dynamic model under slip, improved model of the training algorithm, slip dataset, feature extraction model and the control model. The work was implemented using simulink and tested using the necessary simulation parameters. The result was evaluated using a regression analysis with predictive accuracy of 99.5%, and control response time of 0.005sec.**

***Keywords*: back-propagation, autonomous vehicle, slip, regression, control response**

## I. INTRODUCTION

According to [1] Autonomous vehicle (AV) is a vehicle capable of navigating District roadways and interpreting traffic-control devices without a driver actively operating any of the vehicle's control systems. They are vehicle designed with automated systems to provide electronic blind-spot assistance, crash avoidance, emergency braking, parking assistance, adaptive cruise control, lane-keep assistance, lane-departure warning, or traffic-jam and queuing assistance [1]. These features are embedded in the system (AV) to ensure controllability, reliability, confidence, accuracy and safety during motion. However certain factors like road frictional coefficients slip force, poor automatic brake system response, poor controller response, aggressive nature of servomotors, mechanical faults, rigid power steering among others, stand in the way of realizing this goals completely and as a result causes non linear vehicle dynamics during translation.

In recent times various methodologies have been adopted to ensure that these challenges can be eradicated, such includes the implementation of intelligent tires using accelerometers [2], smart systems for vital signs and vehicle stress condition monitoring [3], the use of intelligent slip estimation controllers [3] [1], among other techniques to mention a few. However despite the success they achieved, one will ask why is it that a complete control of AV is yet to be achieved?

According to [4] the major challenges of autonomous cruise control is that some factors that affects its stability like slip and road coefficients are inevitable. Once a vehicle is in motion, slip force will continue to act on the tires at a rate depending on the frictional coefficient of the road. In order words, the rate of slip force is dependent on whether wet or dry road surface. Due to this effect, controlling this slip parameter is very difficult.

Various works proposed in the past were able to combat these challenges using the methodologies aforementioned, yet there is still need for a complete control of slip in real time.

The major challenges hindering the success of the existing system, is that the time of slip force acting on the vehicle is less than the response time of the controllers, as a result the slip gets accumulated and reflects on the steering dynamics thus causing nonlinearity in the vehicle position. The author believes that if this slip can be predicted before it actually occurs, they can be controlled spontaneously in real time.

This paper therefore presents the use of artificial neural network to develop a controller with the ability to predict the slightest slip parameters and active control mechanism before it affects the vehicle dynamics. The prediction rate will be improved using an enhanced back-propagation algorithm for the training of the nonlinear autoregressive slip model.

## II. LITERATURE REVIEW

[2] Presented a research paper on "accelerometer based method for tire load and slip angle estimation". The work was able to estimate the rate of slip force acting on a tire during motion using the slip angle and magic formula. However they never proposed any means to control the effect in real time.

In 2016, [3] presented a research paper on " slip angle estimation using neural network for wheel vehicle", the wok was able to estimate the degree of slip angle on a vehicle. However this slip estimation was based on a road coefficient of 1 (dry road), but the wet road condition cannot be neglected because they induce more slip when vehicle is moving. Hence there is need for a model which considers various road condition in estimating slip.

In [5] they presented a research on adaptive critic anti-slip control of autonomous robot. According to them, when a wheeled autonomous robot drives with wheel slips, the velocity and posture control becomes difficult. They proposed the use of adaptive controllers for the stability response. However the response time of the fuzzy adaptive controller is more than the time of slip occurrence.

According to [6], presenting an integrated driver and active steering control for vision based lane keeping system. A nested PID (proportional–integral–derivative) steering control for autonomous vehicles equipped with artificial vision systems was designed so that the driver can override the automatic lane-keeping action and obtain complete control of the vehicle lateral dynamics without any switching strategy. However in a situation where the AV is on a high velocity and slip rate, the PID can get aggressive.

[7] Presented a work on time delay sliding mode control of non holonomic wheeled mobile robot, their endeavor employs a hybrid control strategy for composite path tracking control of a holonomic wheeled mobile robotic (WMR) system under parametric and nonparametric uncertainties. However despite the success achieved in their hybrid methodology we believe the neural network will achieve better result an at a faster response and prediction time.

## III. METHODOLOGY

The system will be developed using the proposed system diagram in figure 1; the proposed system uses the nonlinear greybox model of the autonomous vehicle, which represents the nonlinear vehicle dynamics during translational motion consisting of slip angle and magic formula as the slip parameters. This features will be extracted and trained using an improved training algorithm. The prediction response of slip will trigger the automatic brake and steering system to control the vehicle speed and position.

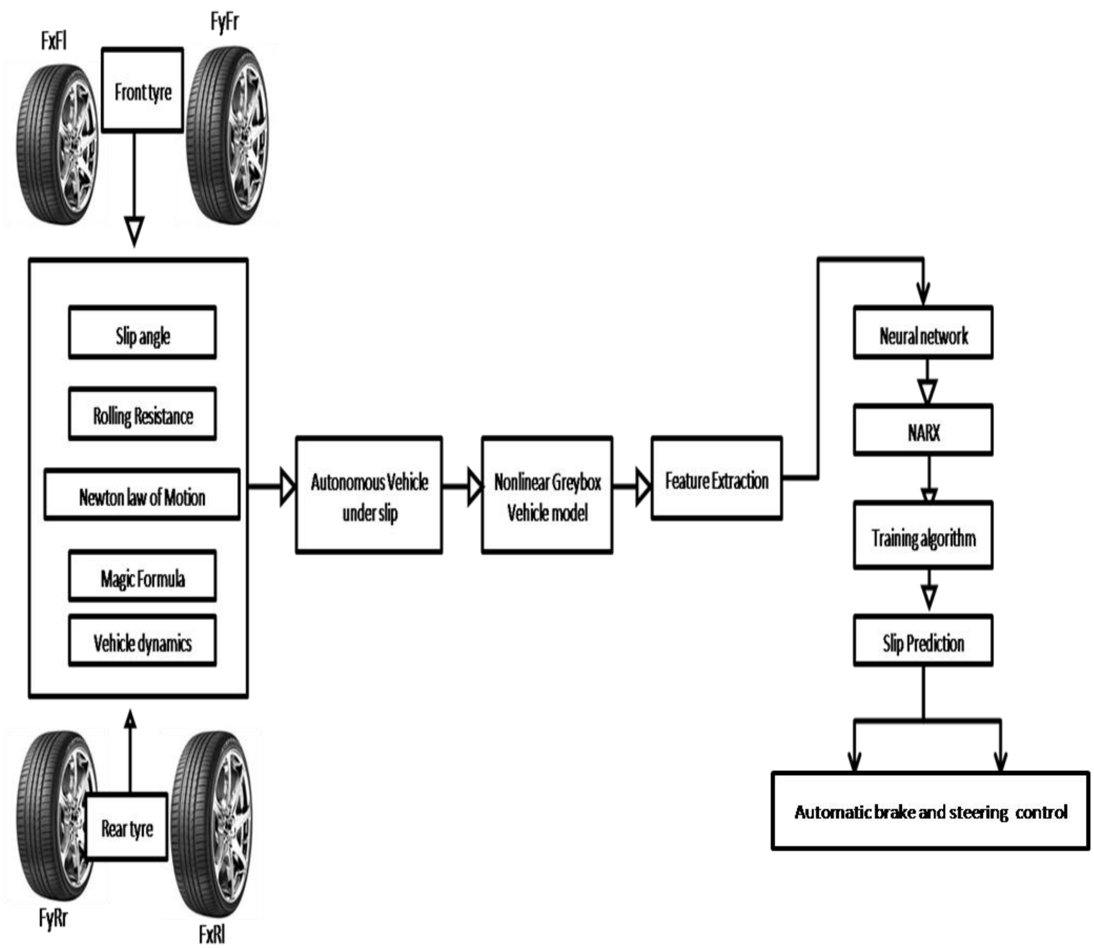


Figure 1: the proposed system

The proposed system will guide the system design starting with the vehicle model, slip model, neural network model, improved training algorithm, prediction model and the control model.

*Vehicle translational Model*

To develop the vehicle model under motion, force is applied on the vehicle longitudinal acceleration (x), lateral velocity (y), steering angle (t) and yaw rate (φ) to calculate the translational motion of the body fixed coordinates frame. This force applied is the resultant relationship between the longitudinal force Fx, lateral force Fy and yaw rate Fz to estimate the rotational force $F_B$ acting on the tyres. Relating this vehicle body whose origin is fixed coordinated in the center of gravity. The mass of the vehicle body m is assumed constant, where the applied force $[Fx, Fy, Fz]^T$ in the body fixed frame and velocity V is defined as;

$$F_B = \begin{bmatrix} Fx \\ Fy \\ Fz \end{bmatrix} = m\,(V + \omega \times V) \qquad \text{equation 1}$$

Where $M_b = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = I\omega + \omega \times (I\omega)$ and

$$I = \begin{bmatrix} Ixx & -Ixy & -Ixz \\ -Iyx & Iyy & -Iyz \\ -Izx & -Izy & Izz \end{bmatrix}$$

Where $M_b$ is the mass between the front and the rear tyres, I is the vehicle moment body of inertia along x,y and z axes.

Now to derive the relation between the fixed angular velocity vector of the body $[p\ q\ r]^T$, and the rate of Euler angle variation $[\varphi\ \Theta\ \psi]^{T,}$ this is designed furnishing the fixed body frame with the Euler rate as presented in the structure below;

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \varphi \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & sin\varphi \\ 0 & -sin\varphi & cos\varphi \end{bmatrix} \begin{bmatrix} 0 \\ \theta \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & sin\varphi \\ 0 & -sin\varphi & cos\varphi \end{bmatrix} \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \psi \end{bmatrix} = J^{-1} \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix}$$

equation 2

Where: φ, Θ, and ψ are the rotation of the vehicle fixed frames about the earth fixed x roll, y pitch and z yaw axes respectively. Inverting the function of J presents the desired Euler rate vector relationship of the system as;

$$\begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} = J \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & (sin\varphi\ tan\theta) & (cos\varphi\ tan\theta) \\ 0 & cos\varphi & -sin\varphi \\ 0 & \frac{sin\varphi}{cos\theta} & \frac{cos\varphi}{cos\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

equation 3

The applied force $F_b$ moment $M_b$ of the system are the defined considering the relationship between the force of gravity Fg, drag force $F_d$, moment $M_b$, suspension forces and the four tyres as shown in the model below;

$$F_b = \begin{bmatrix} Fx \\ Fy \\ Fz \end{bmatrix} = \begin{bmatrix} Fdx \\ Fdy \\ Fdz \end{bmatrix} + \begin{bmatrix} Fgx \\ Fgy \\ Fgz \end{bmatrix} + \begin{bmatrix} Fexzx \\ Fexzy \\ Fexzz \end{bmatrix} + \begin{bmatrix} Fflx \\ Ffly \\ Fflz \end{bmatrix} + \begin{bmatrix} Frlx \\ Frly \\ Frlz \end{bmatrix} + \begin{bmatrix} Frrx \\ Frry \\ Frrz \end{bmatrix} \qquad \text{equation 4}$$

$$M_b = \begin{bmatrix} Mx \\ My \\ Mz \end{bmatrix} = \begin{bmatrix} Mdx \\ Mdy \\ Mdz \end{bmatrix} + \begin{bmatrix} Mgx \\ Mgy \\ Mgz \end{bmatrix} + \begin{bmatrix} Mexzx \\ Mexzy \\ Mexzz \end{bmatrix} + \begin{bmatrix} Mflx \\ Mfly \\ Mflz \end{bmatrix} + \begin{bmatrix} Mrlx \\ Mrly \\ Mrlz \end{bmatrix} + \begin{bmatrix} Mrrx \\ Mrry \\ Mrrz \end{bmatrix} + M_F \qquad \text{equation 5}$$

Where; $F_{fl}$, $M_{fl}$ — Front left, $F_{fr}$, $M_{fr}$ — Front right and $F_{rl}$, Mrl — Rear left, Frr, Mrr — Rear right

*Slip Data model*

The data model will describe the vehicle slip data which will be used to train the neural network. This slip dataset will be modeled using the entity relationship diagram considering the magic input and road parameters as shown below;

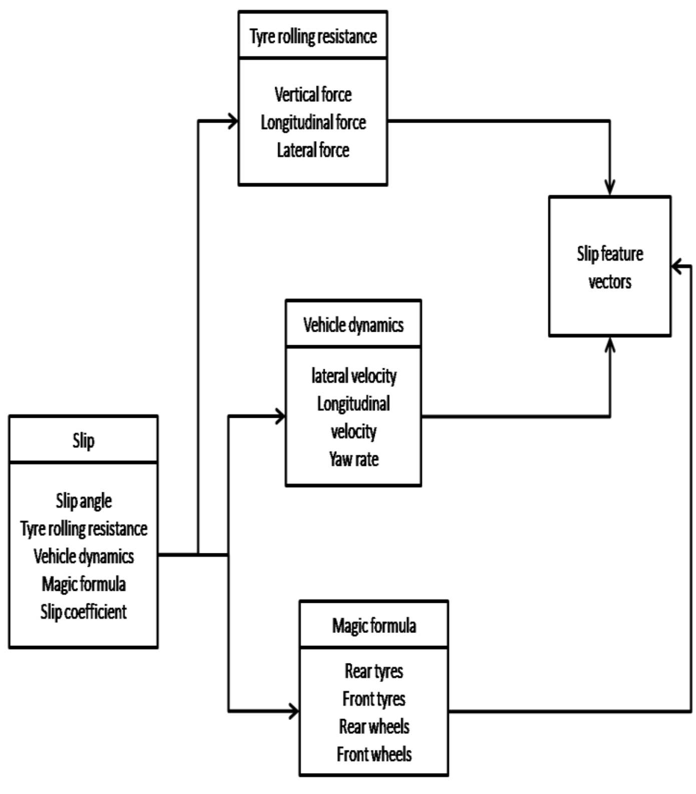


Figure 2: slip data model

*Feature extraction*

This is the extraction of statistics values from the slip data model as a compact feature vector for training purposes; this process is done using statistical method of feature extraction.

*Artificial neural network*

The extracted features will be feed to the neural network as a nonlinear auto regressive model as shown in the model below;

$$y(k+d)=N(y(k),y(k-1),\ldots,y(k-n+1),u(k),u(k-1),\ldots\ldots u(k-n+1)) \quad \text{equation 6}$$

Where u(k) is the feature vectors inputs, N is the non linear slip force, and y(k) is the system output. This identified slip model is trained using an improved back propagation algorithm.

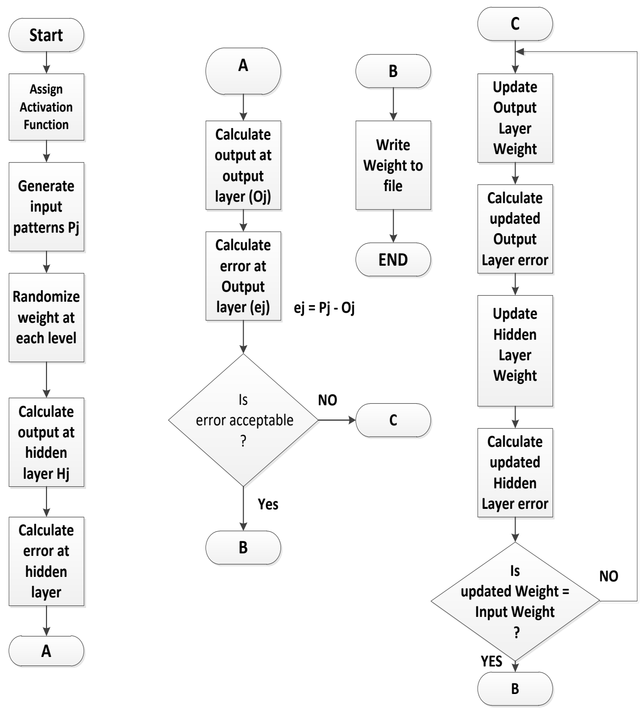


Figure 3: flow chart for the existing back propagation algorithm

*Improving the training model*

The model of the back-propagation algorithm produced a constant learning rate, hence only precised when the setting parameters are in order. However, when the training parameters are set too high or low, the algorithm becomes unstable and oscillates, while when the setting is too small, the algorithm converges. This is a serious problem because as the training process is initiated, most times the optimal learning rate varies. Hence there is need for a system which allows the learning rate to change as a result of the training process dynamics. To achieve this, there is need for a model which keeps the learning step size as large as possible and at the same time keeping the learning stable. This is done by developing a model which identifies the output of the initial algorithm in figure 3 and adjusting the bias variables according to the gradient descent as shown below; thus increasing the learning rate.

$$dx = Ir * \frac{De^k}{dx} \quad \text{equation 7}$$

Where ($De^k$) is the back propagation derivative performance with respect to the weight and bias variables (x), where Ir is the learning rate. Now, at each training epoch, if the performance decreases towards the training goal, the learning rate (Ir) automatically increases by a factor of Ir_inc (which is equal to 0.5, epoch), on the other hand if the training performance increases which might result to over fitting, the learning rate automatically decreases the epoch factor (Ir_dc) by 0.7. In training the network, the aim is to achieve optimal number of hidden layer neurons and also the learning parameter. So, through training of different combination of hidden layer neurons and the learning parameter, the optimal number of hidden layer neurons and the learning parameter were obtained. This is summarized using the flow chart in figure 4.

Table 1: Neural Network Parameters

| | |
|---|---|
| Training epochs | 10 |
| Size of hidden layers | 10 |
| Controller training segments | 30 |
| No. delayed reference input | 2 |
| Maximum feature output | 3.1 |
| Maximum feature input | 15 |
| Number of non hidden layers | 2 |
| Maximum interval per sec | 2 |
| No. delayed output | 1 |
| No. delayed feature output | 2 |
| Minimum reference value | -0.7 |
| Maximum reference value | 0.7 |

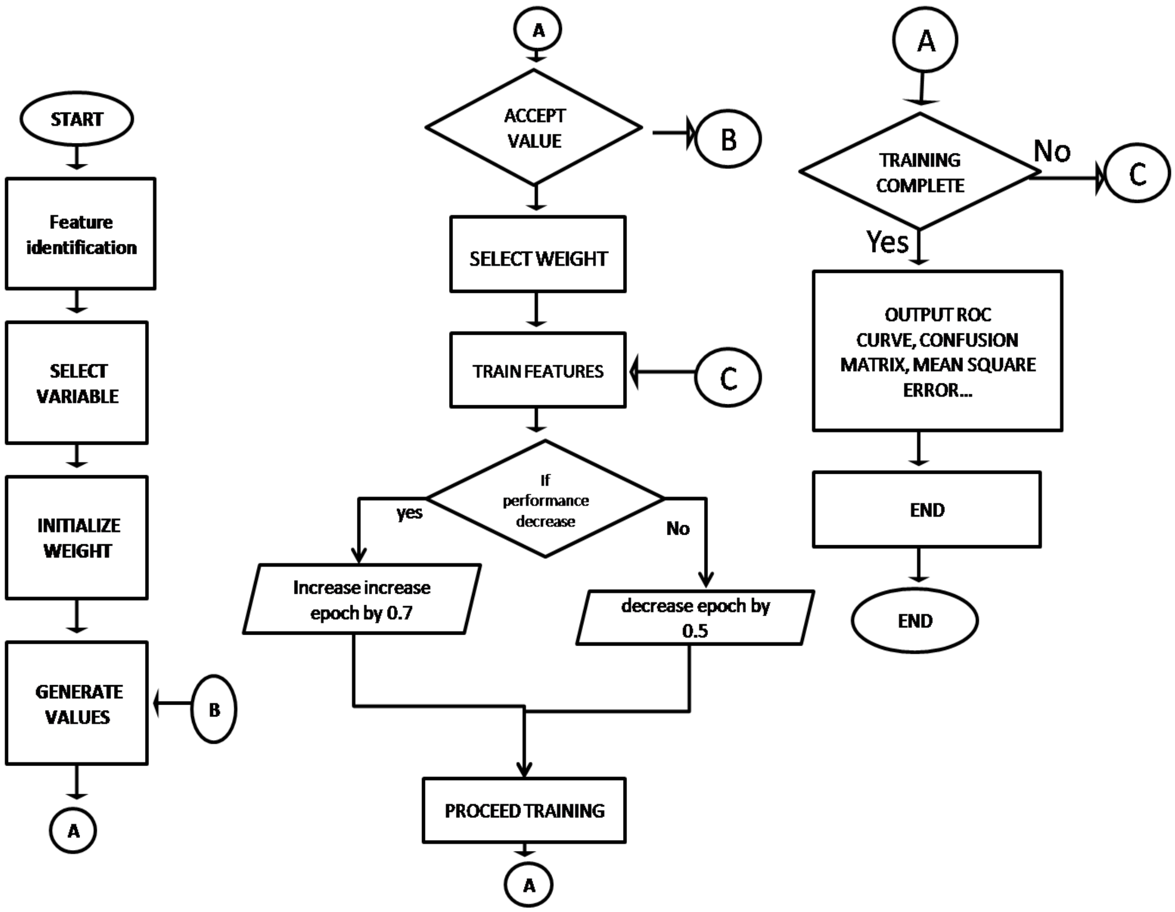


Figure 4: flow chart of the improved training algorithm

The resultant effect of the training algorithm produces the approximate slip model as shown in equation 8;

$$\hat{y}(k+d)=f(y(k),y(k-1),\ldots,y(k-n+1),u(k-1),\ldots,u(k-m+1))+g(y(k),y(k-1),\ldots,y(k-n+1),u(k-1),\ldots,u(k-m+1))\cdot u(k)$$

equation 8

This approximate model is classified with the reference slip model obtained from the training of the slip dataset (see figure 5) as shown in the structure;

$$u(k+1) = \frac{yr(k+d) - f[y(k),\ldots,y(k-n+1),u(k),\ldots,u(k-n+1)]}{g[y(k),\ldots,y(k-n+1),u(k),\ldots,u(k-n+1)]}$$

equation 9

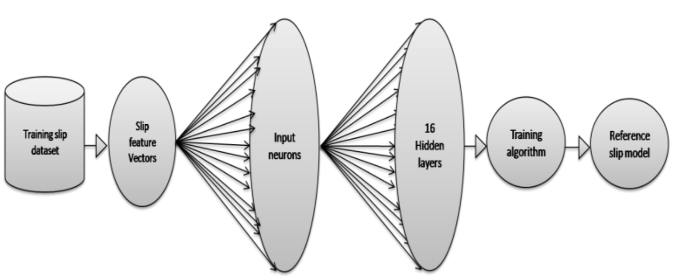


Figure 5: neural network slip training architecture

This reference model is classified with the slip trained slip model to predict the slip force using the prediction model below;

*Prediction model*

The prediction is done using the numerical optimization program to determine the training result over the specified horizon using the model below;

$$J = \sum_{j=N1}^{N2}\left(yr\ (t+1) - ym\ (t+j)\right)^2 + p\ \sum_{j=1}^{Nu}(u`(t+j - 2)\text{^}2$$

equation 10

Where $N_1$, $N_2$, and $N_u$ define the horizons over which the prediction error and the learning increments are evaluated. The $u'$ variable is the tentative feature values signal, $y_r$ is the desired trained response, and $y_m$ is the network model response. The ρ value determines the contribution that the sum of the squares of the learning rate has on the performance index. The optimization block determines the values of $u'$ that minimize $J$, and then the optimal $u$ is input to the training algorithm.

*Control model*

The effect of slip affects the positional dynamics of the vehicle, hence to control this effect the speed and position must be controlled. This will be achieved using the automatic brake and steer control mechanism.

*Brake model*

A disc brake converts brake cylinder pressure from the brake cylinder into force. The disc brake applies the force at the brake pad mean radius. The equation that the block uses to calculate brake torque, depends on the wheel speed, *Ω*, such that when Ω≠0,

$$T= \frac{\mu k P \pi Db\ ^2 * RmN}{4} \qquad \text{equation 11}$$

However when Ω=0, the torque applied by the brake is equal to the torque that is applied externally for wheel rotation. The maximum value of the torque that the brake can apply when Ω=0, is

$$T=\frac{\mu s P \pi Db\ ^2 * RmN}{4} \qquad \text{equation 12}$$

In any case, $Rm= \frac{Ro+Ri}{2}$

Table 2: equations parameters description

| | |
|---|---|
| $\mu_k$ is the disc pad-rotor coefficient of kinetic friction. | *T* is the brake torque. |
| $D_b$ is the brake actuator bore diameter. | *P* is the applied brake pressure. |
| $R_m$ is the mean radius of brake pad force application on brake rotor. | *Ω* is the wheel speed. |
| $R_o$ is the oukter radius of brake pad. | *N* is the number of brake pads in disc brake assembly. |
| $R_i$ is the inner radius of brake pad. | $\mu_s$ is the disc pad-rotor coefficient of static friction. |

*Steering model*

To develop the steering model, the ackerman model was adopted and presented below as (mathworks, 2018)

$$\cot(\delta_L) - \cot(\delta_R) = \frac{TW}{WB} \qquad \text{equation 13}$$

$$\delta_{vir} = \frac{\delta in}{y}$$

$$\delta_L = \tan^{-1}\left(\frac{WBtan\ (\delta vir)}{WB - 0.5TW \tan(\delta vir)}\right)$$

$$\delta_R = \tan^{-1}\left(\frac{WBtan\ (\delta vir)}{WB + 0.5TW \tan(\delta vir)}\right)$$

Table 3: The illustration and equations use these variables.

| | |
|---|---|
| $\delta_{in}$ | Steering angle |
| $\delta_L$ | Left wheel angle |
| $\delta_R$ | Right wheel angle |
| $\delta_{vir}$ | Virtual wheel angle |
| *TW* | Track width |
| *WB* | Wheel base |
| *Γ* | Steering ratio |

## IV. RESULTS

The models were implemented using simulink and simulated using the parameters in table 1. The results generated will be discussed considering the neural network training performance. This begins with the mean square error (MSE) performance of the training process. From the figure the aim is to achieve a similar correlated pattern between the training, test and validation training (multi dataset) process. However in a case where this three transfer training functions are non correlated in terms of pattern, this shows physically that the network under performs. Now blending this to the result obtained from the training process as shown in the MSE performance in figure 6, it was observed that the relationship between the multi dataset training patterns are correlated as expected. The implication of this MSE training performance is to show the best training epoch value, this epoch value is the epoch point at which the best training accuracy is achieved and then the training process stops. This is when the root mean square error performance is recorded as 5.8375e-10 root mean square error which is good. Showing that the training performance produces a précised regression value

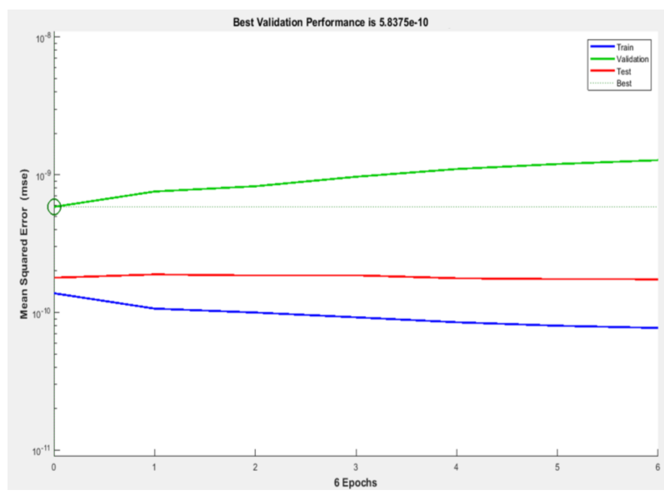


Figure 6: NN training MSE performance

The next result which will be reported in this paper is the regression analysis of the training performance. This work will be used to evaluate the training accuracy of the test, training and validation process as shown in figure 7 the aim of the training performance is to achieve a regression value equal or approximately one.

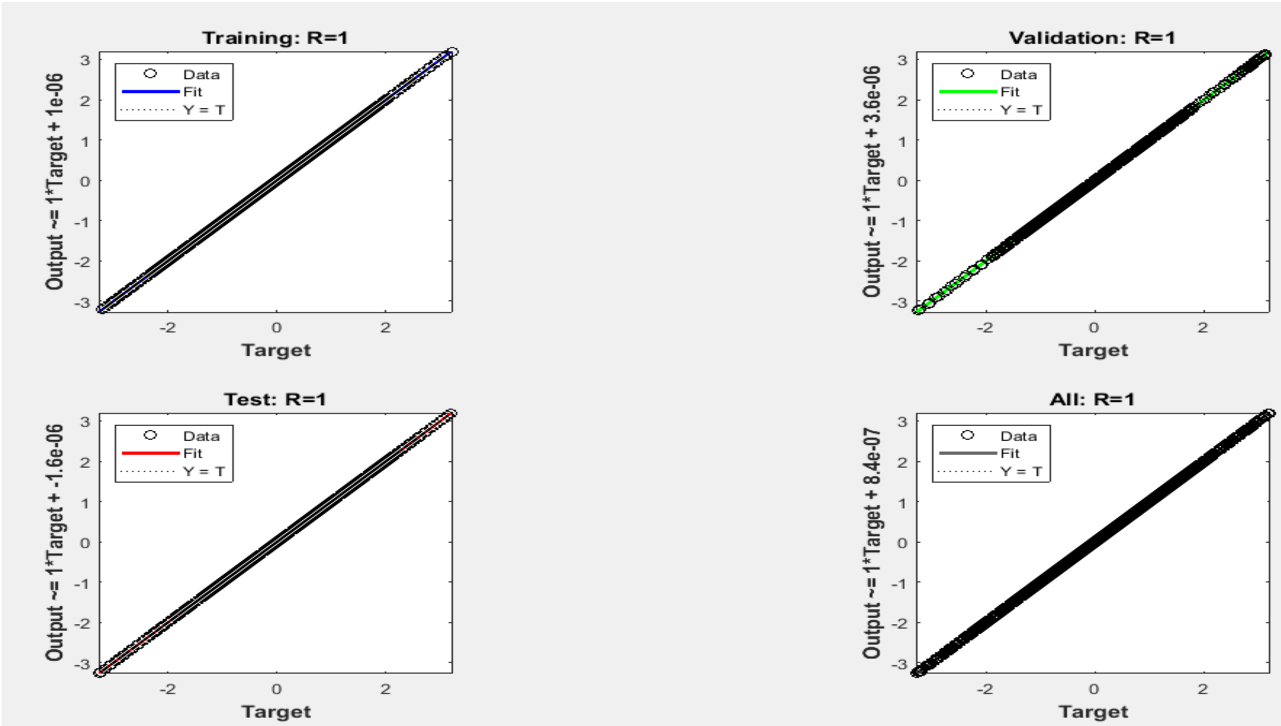


Figure 7: NN regression performance

From the four plots presented in figure 7, representing the training, test, validation and the summarized results, the dash lines in each result represents the perfect result-output= targets. The solid lines represent the best fit linear regression line between output and targets. The R value is an indication of the relationship between the output and targets. If R is zero then there is no linear relationship between the output and target, however as shown by our result, R = 1, showing the a précised prediction performance is achieved. In the next result the step response time of the euro controller will be evaluated. This is to know the time it takes for the control mechanisms modeled in equation 3.12 for speed control and equation 3.13 for position control to response to slip force causing the vehicle nonlinearity. This is shown in the figure 8

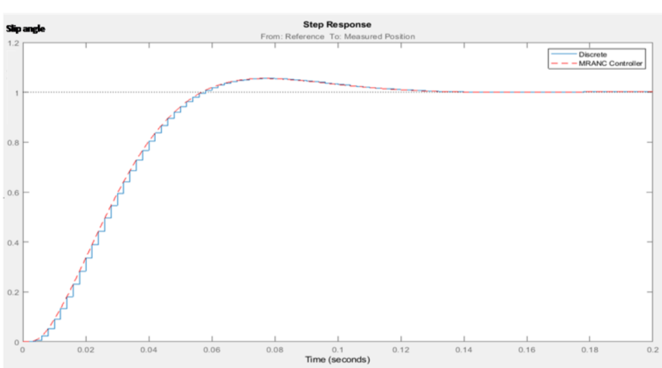


Figure 8: step response of the slip controller

From the result in figure 8, it was observed that due to the predictive nature of the controller, designed, it almost instantaneously at 0.005s detect the slip as shown in the result and hence prevent its effect on the vehicle. This is to say that within 0.005 seconds of slip detection, the control response is triggered for normalization.

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