Effects of Variable Pressure Gradient on Magnetohydrodynamic Flow between Parallel Plates considering Variable Transverse Magnetic Fields

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***Abstract-* Analysis of the effects of applying variable pressure gradient to a Magnetohydrodynamic fluid flowing between two parallel plates under the influence of variable transverse magnetic fields are investigated. The study involves a steady, incompressible hydromagnetic fluid flowing through parallel plates. The upper plate is considered porous moving in the opposite direction to the fluid flow while the lower plate remains stationary. The results obtained shows that velocity profiles decreases whenever Reynold number, magnetic number or suction parameter is increased. As the pressure gradient is increased, velocity of the flow increased but temperature decreased. Also, increase in suction number yields to increase in temperature profile.**

***Index Terms-* Magnetohydrodynamic flow, porous parallel plates, variable pressure gradient, variable transverse magnetic fields.**

I. INTRODUCTION

Liquids and gases are commonly referred to as fluids. Incompressible fluid flow between parallel plates normal to them is referred to as Hartmann. Hartmann number is defined to as the ratio of electromagnetic force to the viscous force experienced by fluid flow through magnetic fields. Under magnetic fields, the flow of an electrically conducting fluid induces electric currents and therefore Lorentz force is developed.

Manyonge, *et al* (2012) investigated two dimensional Magnetohydrodynamic poiseuille flow of incompressible steady fluid. The fluid was flowing between porous channel influenced by slanting magnetic field and uniform pressure gradient. The analysis obtained showed that velocity distribution decreased as magnetic field strength increased. Unsteady Magnetohydrodynamic couette flow between infinite porous plates where the lower plate was considered porous past sloped magnetic field with heat transmission was analyzed by Joseph, *et al* (2014). They found out that energy losses were reduced by high magnetic field. Singh (2014) studied steady laminar flow of viscous incompressible fluid between two parallel infinite plates with constant pressure gradient. The results showed that increase of inclination of magnetic field decreased velocity profile.

Kiema, *et al* (2015) studied MHD fluid steadily flowing with constant pressure gradient between two limitless parallel pervious sheets under the action of steady magnetic field. The fluid entered through the lower sheet and exited through the top sheet. The results showed that velocity was lowered whenever Hartmann number was increased. Mbugua, *et al* (2016) investigated electrically conducting flow of incompressible unsteady Newtonian fluid flowing through porous non conducting sheets in the presence of variable transverse magnetic field. The top sheet moved contrary to the fluid flow as the bottom sheet remained stationary. Pressure gradient was taken to be uniform throughout. They concluded that reduced suction parameter increased the velocity distribution and decreased the temperature profiles. They also deduced that suction stabilized the boundary layer growth.

MHD fluid through parallel plates subject to an inclined magnetic field under uniform pressure gradient was studied by Mburu & Kwanza (2016). They found out that increase in Hartmann number lowered velocity and pressure gradient was directly proportional to velocity. Dash & Ojha (2018) studied viscoelastic hydromagnetic flow between two permeable sheets in the presence of sinusoidal pressure gradient in the presence of magnetic field and porous matrix. They found out that flowing back of fluid could be prevented by having pressure gradient oscillation at low frequency. Using finite difference method, MHD flow of two parallel plates influenced by sliding magnetic field was analyzed by Aruna and Dubewar (2019). They confirmed that as the slant angle increased velocity decreased.

The aim of this paper is to analyze the effects of variable pressure gradient applied to a MHD fluid when the variable magnetic fields are transverse to the upper plate.

II. FORMULATION OF THE PROBLEM

We consider effects of various thermos-physical parameters on velocity and temperature profiles due to a variable pressure gradient on 2- dimensional Magnetohydrodynamic fluid flow between parallel plates. The fluid is considered to be steady, incompressible and viscous. When the fluid is at rest, at t=0, both plates are stationary and at time greater than zero (t>0), the lower plate is immobile as the top plate move in the opposite direction to the fluid flow. The two plates are considered to be of infinite length in the x- and z- directions and transverse magnetic fields act on the y- axis.

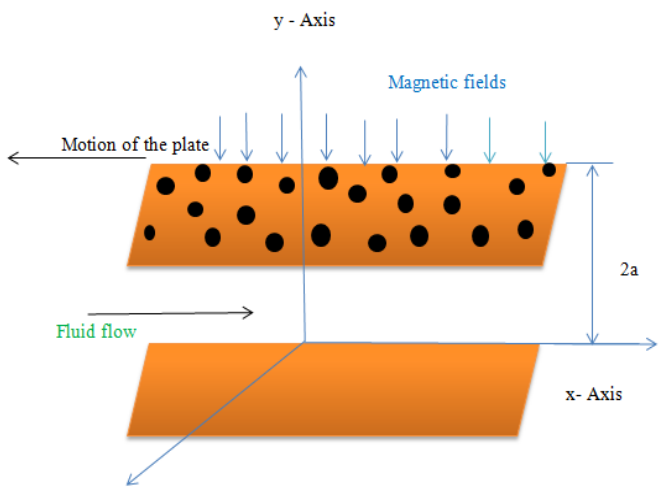

Figure 1: Flow configuration

The equations governing the flow are:

a) Equation of continuity

In tensor form, the equation of continuity is given by,

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0. \quad (1)$$

Where, $i = 1,2,3$ represent x, y, z directions respectively. Since the flow is steady and incompressible, the density of fluid does not change with time. In Cartesian coordinates equation (1) becomes,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (2)$$

The parallel plates are of immeasurable length in x- and z- directions hence the velocity components do not depend on x and z. Simplifying equation (2) we get

$$\frac{\partial v}{\partial y} = 0 \quad (3)$$

Resulting to

$$v = v_\circ \quad (4)$$

$v_\circ$ is the equivalent to suction velocity of the upper porous plate

b) Navier- Stokes equation

The general Navier- Stokes equation in tensor is expressed as,

$$\rho\left(\frac{\partial ui}{\partial t}\right) + u_j + \left(\frac{\partial u_i}{\partial x_j}\right) = - \frac{\partial P}{\partial xj} + \mu \nabla^2 ui + \rho F_i \quad (5)$$

In this research we consider magnetic force, electric force and shear stress while gravitational force is taken to be negligible. We consider a 2-dimensional flow in the x- direction. Velocity profile in the y-axis is zero.

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{F_x}{\rho} \quad (6)$$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{F_y}{\rho} \quad (7)$$

In equation (7), gravitational forces are considered negligible hence,

$$P = P(x) \quad (8)$$

The plates are of immeasurable length in x- and y- directions which simplifies equation (6) to

$$\frac{\partial u}{\partial t} + v_\circ\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{F_x}{\rho} \quad (9)$$

Considering Lorentz force, a particle of charge q moving with a velocity $\vec{V}$ in an electric field $\vec{E}$ and a magnetic field $\vec{B}$ experiences a force of,

$$\vec{F} = q\vec{E} + q\vec{V} \times \vec{B} \quad (10)$$

Considering Ohms law equation and assumption that forces due to electric field are negligible,

$$\vec{J} = \sigma(\vec{V} \times \vec{B}) \quad (11)$$

Also, the velocity vector of the fluid is given by

$$\vec{V} = \vec{V}(U, 0,0) \quad (12)$$

Then,

$$\vec{J} = \vec{V} \times \vec{B} = \sigma \begin{vmatrix} i & j & k \\ U & 0 & 0 \\ 0 & B_y & 0 \end{vmatrix} = \sigma U B_y k \quad (13)$$

Applying Lorentz force equation

$$\vec{J} \times \vec{B} = \begin{vmatrix} i & j & k \\ 0 & 0 & \sigma U B_y \\ 0 & B_y & 0 \end{vmatrix} = -\sigma B_y^2 U i. \quad (14)$$

Substituting magnetic permeability equation $B_y = \mu_e H_y$ to equation (14) we get

$$\vec{J} \times \vec{B} = -\sigma \mu_e^2 H_y^2 U i \quad (15)$$

Therefore the Lorentz force experienced is given by

$$\vec{F} = -U\sigma\mu_e^2 H_y^2 \quad (16)$$

Substituting equation (16) into equation (9) we get

$$v_\circ\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{U\sigma\mu_e^2 H_y^2}{\rho} \quad (17)$$

c) Energy equation

The tensor form of this equation is written as

$$\rho\left(\frac{\partial r}{\partial t}\right) + \frac{\partial}{\partial x_j}(\rho u_j r) = \frac{\partial P}{\partial t} + \frac{\partial}{\partial x_j}(u_i P) - \frac{\partial q_j}{\partial x_j} + \varphi \quad (18)$$

Substituting heat equation and considering that the plates are in x- and y- directions, equation (18) becomes

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_\circ\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (19)$$

Since the flow is steady, equation (19) results to

$$\rho C_p v_\circ \frac{\partial T}{\partial y} = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (20)$$

Effecting electrical resistance of the fluid due to Ohmic heating which is $\frac{J^2}{\sigma}$ equation (20) gives

$$\rho C_p v_\circ \frac{\partial T}{\partial y} = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{J^2}{\sigma} \quad (21)$$

From equation (13)

$$J = \sigma U B_y \quad (22)$$

So

$$\frac{J^2}{\sigma} = \frac{\sigma^2 U^2 {B_y}^2}{\sigma} = \sigma U^2 {B_y}^2 \quad (23)$$

However ${B_y}^2 = {\mu_e}^2 {H_y}^2$, therefore, Ohmic heating is given by

$$\frac{J^2}{\sigma} = \sigma {\mu_e}^2 {H_y}^2 U^2 \quad (24)$$

Substituting equation (24) to equation (21) results to

$$\rho C_p v_\circ \frac{\partial T}{\partial y} = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma {\mu_e}^2 {H_y}^2 U^2 \quad (25)$$

The initial boundary conditions used in this study are

$$\left.\begin{array}{ll} \mathrm{t}=0, \mathrm{u}=0 & \mathrm{T}=0 \text{ at} -\mathrm{a} \le \mathrm{y} \le \mathrm{a} \\ \mathrm{t}>0, u=0 & T=T_w \text{ when } \mathrm{y}=-\mathrm{a} \\ \mathrm{t}>0, u=U & T=T_\infty \text{ when } \mathrm{y}=\mathrm{a} \end{array}\right\} \quad (26)$$

Where ρ is the density of the fluid, u, v, w is the velocity components in x, y, z axis, x, y, z are the Cartesian coordinates, P is the pressure, U is the characteristic velocity, σ is the electrical conductivity, μ_e is the magnetic permeability, $\vec{H}$ is he magnetic field strength, C_P is the specific heat capacity, T is the temperature, K is the thermal conductivity, μ is coefficient of viscosity, t is time, T_∞ is the characteristic free stream temperature, T_W is the characteristic temperature on the plate, $v_\circ$ is the suction velocity of upper plate and a is the distance between plates.

To non- dimensionalize the equation (17), equation (25) and also the initial boundary conditions in equation (26), we use transformations below where the values with asterisks represented the dimensionless variables,

$$X^* = \frac{X}{L}, \quad Y^* = \frac{Y}{L}, \quad P^* = \frac{P}{\rho u^2}, \quad u^* = \frac{u}{U}, \quad T^* = \frac{T - T_\infty}{T_W - T_\infty},$$
$$t^* = \frac{tU}{L}, E_C = \frac{U^2}{C_p \Delta T}, \quad P_r = \frac{C_p \mu}{K}, H_a = L\mu_e H\sqrt{\frac{\sigma}{\mu}}, \quad R_e = \frac{\rho u L}{\mu},$$
$$S_\circ = \frac{v_\circ}{u}, \; M = \frac{\sigma \mu_e^2 H_y^2 L}{\rho U} = \frac{H_a^2}{R_e} \quad (27)$$

Where L is characteristic length, E_C is Eckert parameter, H_a is the Hartmann number, R_e is the Reynolds number, $S_\circ$ is the Suction parameter, M is the magnetic numer.

Substituting equation (27) into equations (17), (25) and (26) and simplifying, we obtain

$$S_\circ \frac{\partial u^*}{\partial y^*} = -\frac{\partial P^*}{\partial X^*} + \frac{1}{R_e}\frac{\partial^2 u^*}{\partial y^{*2}} - MU^* \quad (28)$$

$$S_\circ \frac{\partial T^*}{\partial y^*} = \frac{1}{R_e P_r}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{E_C}{R_e}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{E_C {H_a}^2}{R_e} U^{*2} \quad (29)$$

$$\left.\begin{array}{l} t^* = 0, u^* = 0, \; T^* = 0 \text{ , at} -1 \le y^* \le 1 \\ t^* > 0, u^* = 0, T^* = 1 \; at \; y^* = -1 \\ t^* > 0, u^* = 1, T^* = 0 \; at y^* = 1 \end{array}\right\} \quad (30)$$

III. METHODOLOGY

The governing partial difference equations are presented in their finite difference approximations and solved using finite difference method. For simplicity we consider one dimension domain (space). The finite difference expressions for U and T in equations are

$$\frac{\partial u^*}{\partial y^*} = \frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta y} \quad (31)$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^k - 2U_j^k + U_{j-1}^k}{2(\Delta y)^2} \quad (32)$$

$$\frac{\partial T^*}{\partial y^*} = \frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^k - T_{j-1}^k}{2\Delta y} \quad (33)$$

$$\frac{\partial^2 T^*}{\partial y^{*2}} = \frac{T_{j+1}^{k+1} - 2T_j^{k+1} + T_{j-1}^{k+1} + T_{j+1}^k - 2T_j^k + T_{j-1}^k}{2(\Delta y)^2} \quad (34)$$

Equations (31) and (32) are substituted to equation (28) to get

$$S_\circ\left(\frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta y}\right) =$$
$$\frac{\partial P^*}{\partial X^*} + \frac{1}{R_e}\left(\frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^k - 2U_j^k + U_{j-1}^k}{2(\Delta y)^2}\right) - MU_j^k \quad (35)$$

Making U_j^{k+1} the subject of the formula yields

$$U_j^{k+1} =$$
$$\left\{-\frac{\partial P^*}{\partial X^*} - S_\circ\left(\frac{U_j^k - U_{j-1}^{k+1} - U_{j-1}^k}{2\Delta y}\right) + \right.$$

1ReUj+1k+1+Uj−1k+1+Uj+1k−2Ujk+Uj−1k2Δy2−M Ujk÷S°2Δy+1ReΔy2

(36)

Similarly, equations (33) and (34) are substituted to equation (29)

$$S_\circ\left(\frac{T_j^{k+1} - T_{j-1}^{k+1} + T_j^k - T_{j-1}^k}{2\Delta y}\right) =$$
$$\frac{1}{R_e P_r}\left(\frac{T_{j+1}^{k+1} - 2T_j^{k+1} + T_{j-1}^{k+1} + T_{j+1}^k - 2T_j^k + T_{j-1}^k}{2(\Delta y)^2}\right) +$$
$$\frac{E_C}{R_e}\left(\frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^k - 2U_j^k + U_{j-1}^k}{2(\Delta y)^2}\right) + \frac{E_C {H_a}^2}{R_e}\left(U_j^k\right)^2 \quad (37)$$

Making T_j^{k+1} the subject of the formula

$$T_j^{k+1} =$$
$$\left\{-S_\circ\left(\frac{T_j^k - T_{j-1}^{k+1} - T_{j-1}^k}{2\Delta y}\right) + \frac{1}{R_e P_r}\left(\frac{T_{j-1}^{k+1} + T_{j+1}^{k+1} + T_{j-1}^k - 2T_j^k + T_{j+1}^k}{2(\Delta y)^2}\right) + \right.$$

$$\frac{E_C}{R_e}\left(\frac{U_{j+1}^{k+1}-2U_j^{k+1}+U_{j-1}^{k+1}+U_{j+1}^{k}-2U_j^{k}+U_{j-1}^{k}}{2(\Delta y)^2}\right)+\frac{E_C H_a{}^2}{R_e}\left(U_j^k\right)^2\Big\}\div \left(\frac{S_\circ}{2\Delta y}+\frac{1}{R_e P_r(\Delta y)^2}\right) \quad (38)$$

Equations (36) and (38) are the final set of equations solved simultaneously using a computer code in MATLAB (R2018b) computer software.

IV.RESULTS AND DISCUSSION

Effects of variable pressure gradient on Magnetohydrodynamic flow between parallel plates considering variable transverse magnetic fields are analyzed and presented graphically for different parameters. Default values are chosen and used to observe the effect on varying various parameter values.

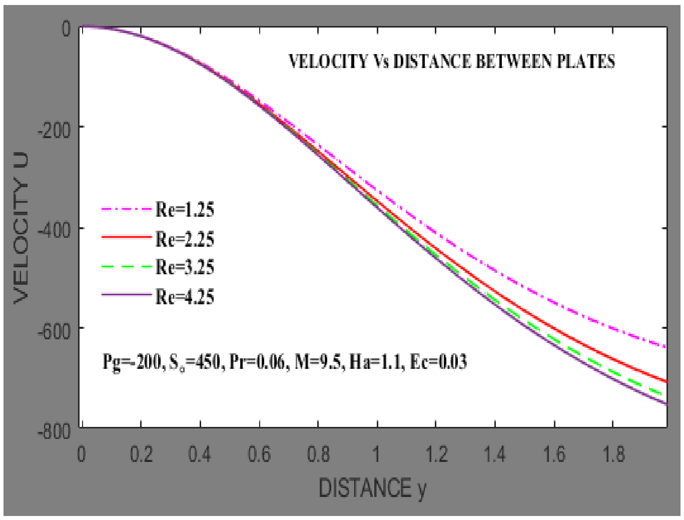

Figure 2a: Effect of Reynolds number on the fluid velocity profile

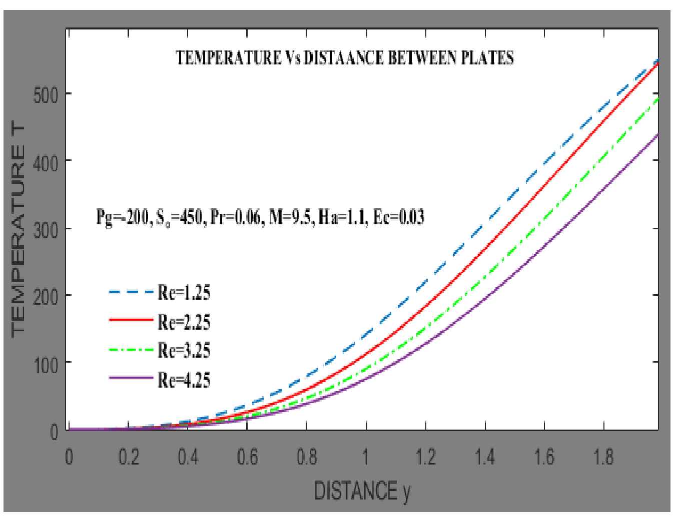

Figure 2b: Effects of Reynolds number on the fluid temperature profile

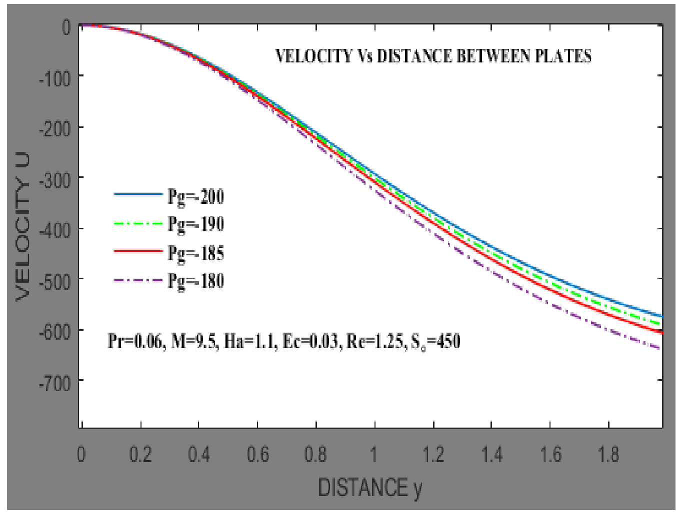

Figure 3a: Effects of pressure gradient on the fluid velocity profile

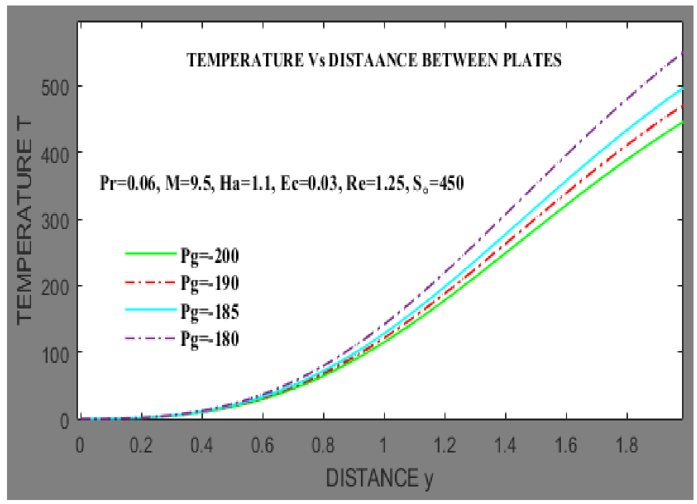

Figure 3b: Effects of pressure gradient on the fluid temperature profile

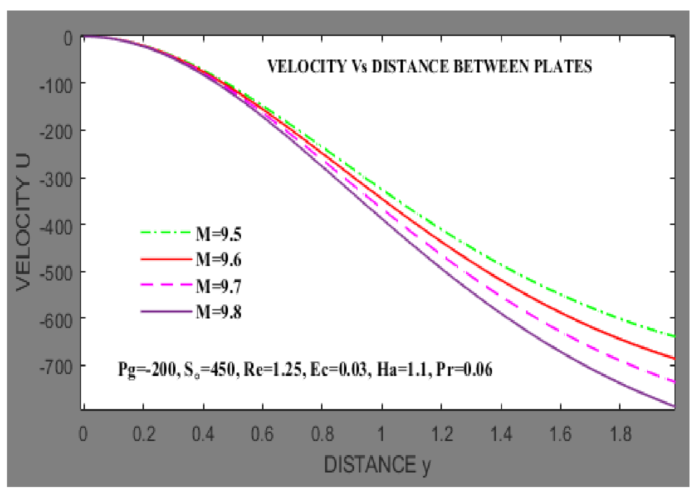

Figure 4a: Effects of magnetic number on velocity profile

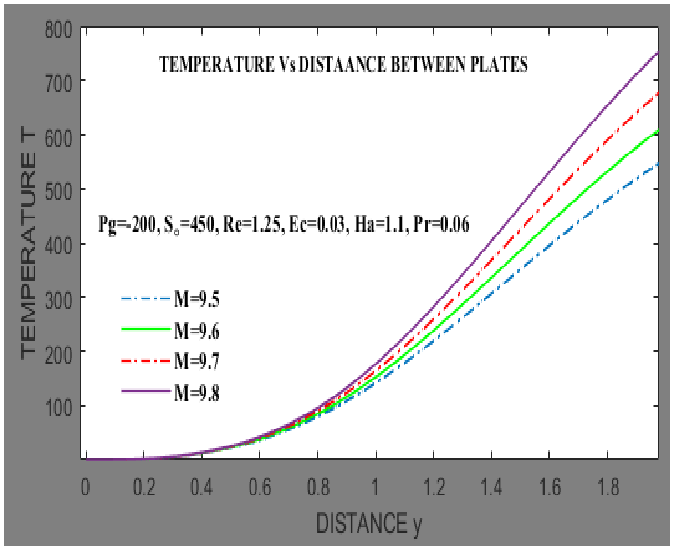

Figure 4b: Effects of magnetic number on the fluid temperature profile

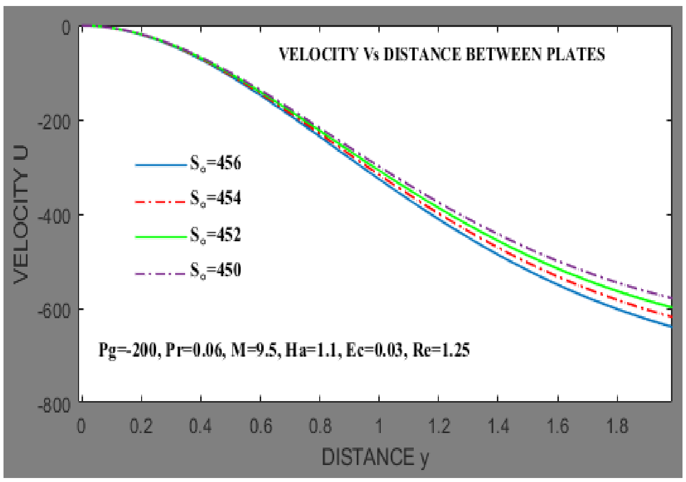

Figure 5a: Effects of suction number on the fluid velocity profile

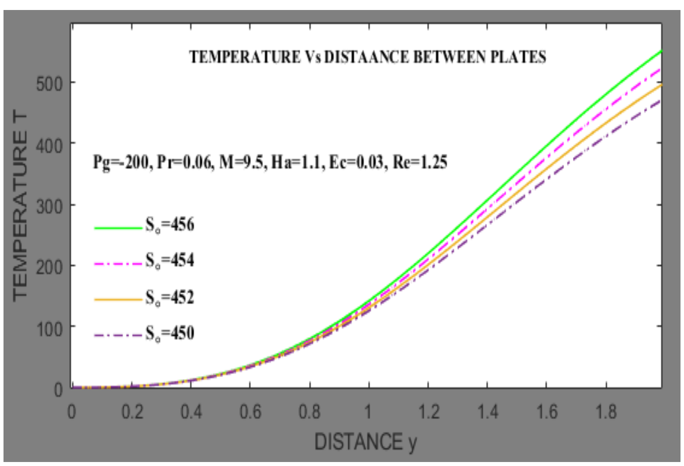

Figure 5b: Effects of suction number on the fluid temperature profile

Figures 2a, 3a, 4a and 5a show that increase in suction number, magnetic number, pressure gradient and Reynold number decreases velocity profile. Increase in Reynold number implies that viscous forces decrease as inertial forces increase hence reduced velocity. It is observed that, at the bottom stationary plate the flow assumes the velocity of the plate. Gradually, the velocity rise to maximum as it reaches the center of the plates and then decrease as it approaches the top plate. Also Lorentz force acting normally to the fluid due to magnetic fields causes resistance to the fluid flow consequently slowing the fluid motion hence reducing the velocity of the flow. Pressure of the fluid reduces due to the convection of the fluid particles caused by suction on the surface of the plates. Reduced pressure results to decreased velocity.

In figures 3b, 4b and 5b, increase in magnetic number, pressure gradient and suction number leads to increase temperature. From figure 2b, it can be observed that increase in Reynold number decreases temperature profile. Decreased viscous forces imply that friction force reduces and hence temperature of the fluid decreases. Also, increase in electromagnetic forces results to increase in joule dissipation leading to increase in fluid temperature. Increase in suction number decreases boundary layer region hence increasing temperature gradient of the fluid at the surface.

V. CONCLUSIONS

Effects of variable pressure gradient on Magnetohydrodynamic flow between two parallel plates considering variable transverse magnetic fields were studied. The results obtained in this research provide useful information to different fields especially in designing and modeling of systems in dyeing industries, cooling of automobile moving parts, purifying crude oil, sprays as well as in extraction of metal industries.

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