

# Finite Element Analysis of Pressure Vessels Subjected to Uniform Internal Pressure Using Ansys Software

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**Abstract:** This paper discusses the stresses developed in the pressure vessels having various thicknesses. Pressure vessels are intended to hold gases or liquids at a pressure substantially different from atmospheric pressure. Equations of static equilibrium along with free body diagrams will be used to determine the normal stress  $\sigma_1$  in the hoop direction and  $\sigma_2$  in the longitudinal direction also the von mises' stresses. Also we will discuss the deformations and displacements to the pressure vessel using commercial finite element software called ANSYS for modelling and analyzing of the vessels. And the main results we found is that there is stress and deformation variation in the vessel according to thickness.

**Key words:** Analysis, pressure vessel, ANSYS, stresses, APDL, cylindrical, spherical, vertical

## I. INTRODUCTION

A pressure vessel is defined as a container with a pressure differential between inside and outside. The inside /internal pressure is usually higher than the outside, except for some isolated situations.

Pressure vessels are compressed gas storage tanks intended to hold gases or liquids at a pressure substantially different from the ambient pressure. The analysis of pressure vessels encompasses many combinations of stresses. The safe design, installation, operation, and maintenance of pressure vessels are in accordance with codes such as American Society of Mechanical Engineers boiler and pressure vessel code. Longitudinal and hoop stresses are produced due to internal pressure and variable bending stress due to bending moment caused by wind loads. Therefore, a designer has to keep all the probable loads and moments on a pressure vessel [1].

Tanks, vessel and pipelines that carry, store or receive fluids are called pressure vessel. A pressure vessel is defined as a container with a pressure differential between inside and outside. The inside pressure is usually higher than the outside. The fluid inside the vessel may undergo a change in state as in the case of steam boiler or may combine with other reagent as in the case of chemical reactor. Pressure vessel often has a combination of high pressure together with high temperature and in some cases flammable fluids or highly radioactive material. Because of such hazards it is imperative that the analysis be such that no leakage can occur. Pressure vessels are

usually spherical or cylindrical with dome end. The cylindrical vessels are generally preferred because of the present simple manufacturing problem and make better use of the available space. Boiler, heat exchanger, chemical reactor and so on, are generally cylindrical [2].

In paper we will focus on cylindrical pressure vessels. And they are of two types namely

Thin-walled cylindrical and thick-walled cylindrical pressure vessels.

### 1.1 Thin Walled Cylindrical Pressure Vessel

If the wall thickness is less than about 7% of the inner diameter, then the cylinder may be treated as a thin one. Thin walled cylinders are used as boiler shells, pressure tanks, pipes and in other low pressure processing equipment. In general, three types of stresses are developed in pressure cylinders viz. circumferential or hoop stress, longitudinal stress in closed end cylinders and radial stresses. These stresses are demonstrated in figures below

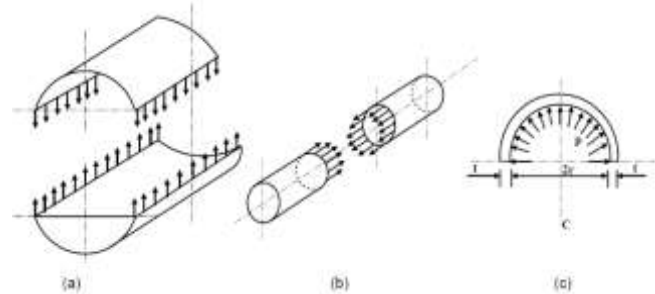


Fig.1.1.1 a) Circumferential stress b) Longitudinal stress and c) Radial

In a thin walled cylinder, the circumferential stresses may be assumed to be constant over the wall thickness and stress in the radial direction may be neglected for the analysis. Considering the equilibrium of a cut out section the circumferential stress  $\sigma_\theta$  and longitudinal stress  $\sigma_z$  can be found.

Consider a section of thin cylinder of radius  $r$ , wall thickness  $t$  and length  $L$  and subjected to an internal pressure  $p$  as shown in **figure-1.1.1 (a)**. Consider now an element of included angle  $d\theta$  at an angle of  $\theta$  from vertical. For equilibrium we may write

$$2 \int_0^{\pi} p r d\theta L \cos \theta - 2 \sigma_z t L$$

This gives  $\sigma_0 = \frac{Pr}{t}$  (1)

Considering a section along the longitudinal axis as shown in figure-1.1.1. (b) we may write

$$p \pi r^2 = \sigma_z \pi (r_o^2 - r_i^2)$$
 (2)

where  $r_i$  and  $r_o$  are internal and external radii of the vessel and since  $r_i \approx r_o = r$  (say)

and  $r_o - r_i = t$  we have  $\sigma_z =$

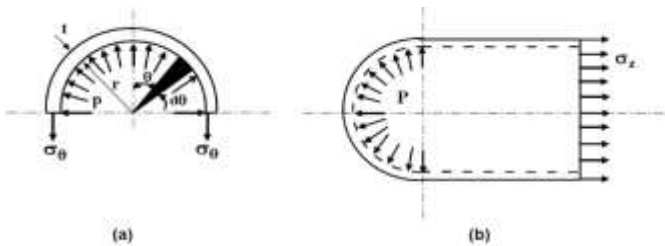


Fig-1.1.2 (a) Circumferential stress in a thin cylinder (b) Longitudinal stress in a thin cylinder [3]

1.2 Thick-Walled Pressure Vessels

A thick walled pressure vessel is generally considered to be one of whose walls are greater than about 1/20 of the radius of the vessel. Remember that a catastrophic failure of the vessel could result in serious injury or death. [4].

For thick cylinders such as guns, pipes to hydraulic presses, high pressure hydraulic pipes the wall thickness is relatively large and the stress variation across the thickness is also significant. In this situation the approach made in the previous section is not suitable. The problem may be solved by considering an axisymmetric about z-axis and solving the differential equations of stress equilibrium in polar co-ordinates [5].

Consider a thick walled cylinder with open ends as shown above. It is loaded by internal pressure  $P_i$  and external pressure  $P_o$  as seen below. It has inner radius  $r_i$  and outer radius  $r_o$ .

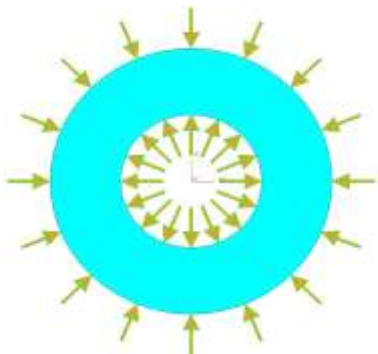


Fig-1.2.1 Thick walled pressure vessel

When calculating the strains and stresses of the pressure vessel we use the formula below

$$\epsilon_r = \frac{u + du - u}{dr} = \frac{du}{dr}$$
 (3)

Giving the stresses as a function of radius in equation 4 and 5

$$\sigma_r = \frac{r_i^2 P_i - r_o^2 P_o}{(r_o^2 - r_i^2)} - \frac{(P_i - P_o) r_i^2 r_o^2}{(r_o^2 - r_i^2) r^2} \quad \sigma_\theta = \frac{r_i^2 P_i - r_o^2 P_o}{(r_o^2 - r_i^2)} + \frac{(P_i - P_o) r_i^2 r_o^2}{(r_o^2 - r_i^2) r^2}$$

(a) Internal Pressure only ( $P_o=0$ ): then we get equation 6 and 7

$$\sigma_\theta = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left[ 1 + \frac{r_o^2}{r^2} \right] : \sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left[ 1 - \frac{r_o^2}{r^2} \right] : \sigma_z = \frac{P_i r_i^2}{r_o^2 - r_i^2}$$

(b) External Pressure only ( $P_i=0$ ): then we get equation 8 and 9

$$\sigma_\theta = \frac{-P_o r_o^2}{r_o^2 - r_i^2} \left[ 1 + \frac{r_i^2}{r^2} \right] : \sigma_r = \frac{-P_o r_o^2}{r_o^2 - r_i^2} \left[ 1 - \frac{r_i^2}{r^2} \right] : \sigma_z = \frac{-P_o r_o^2}{r_o^2 - r_i^2}$$

These are known as simplified Lamé's equations [6].

II. BACKGROUND

Pressure vessels used in industry are leak-tight pressure containers, usually cylindrical or spherical in shape, with different head configurations. They are usually made from carbon or stainless steel and assembled by welding. Early operation of pressure vessels and boilers resulted in numerous explosions, causing loss of life and considerable property damage. Some 80 years ago, the American Society of Mechanical Engineers formed a committee for the purpose of establishing minimum safety rules of construction for boilers. In 1925 the committee issued a set of rules for the design and construction of unfired pressure vessels.

Structures such as pipes or bottles capable of holding internal pressure have been very important in the history of science and technology. Although the ancient Romans had developed municipal engineering to a high order in many ways, the very need for their impressive system of large aqueducts for carrying water was due to their not yet having pipes that could maintain internal pressure. Water can flow uphill when driven by the hydraulic pressure of the reservoir at a higher elevation, but without a pressure-containing pipe an aqueduct must be constructed so the water can run downhill all the way from the reservoir to the destination. Airplane cabins are another familiar example of pressure-containing structures. They illustrate very dramatically the importance of proper analysis, since the atmosphere in the cabin has enough energy associated with its relative pressurization compared to the thin air outside that catastrophic crack growth is a real possibility.

A number of fatal commercial tragedies have resulted from this, particularly famous ones being the Comet aircraft that disintegrated in flight in the 1950's and the loss of a 5-meter section of the roof in the first-class section of an Aloha Airlines B737 in April 1988. [ 7]

2.1 Materials

The materials to be used in pressure vessels must be selected from Code-approved material specifications. This requirement is normally not a problem since a large catalogue of tables listing acceptable materials is available. Factors that need to be considered in picking a suitable table are:

- Cost
- Fabricability
- Service condition (wear, corrosion, operating temperature)
- Availability and Strength requirements. [8]

III. ANSYS CASE STUDY: AXISYMMETRIC ANALYSIS OF A VERTICAL PRESSURE VESSEL

The pressure vessel shown below is made of stainless steel ( $E = 2 \times 10^5$  Pa,  $\nu = 0.3$ ) and contains an internal pressure of  $p = 4000$  pa. The cylindrical vessels have inner diameters of 180 mm and 160 mm and thicknesses of 10mm and 20 mm for thin and thick cylinders respectively.

3.1 Formulation using ANSYS

Start > Programs > ANSYS Mechanical APDL Product Launcher

Utility Menu >File >Change Job name...> Enter 'Pressure Vessel' > OK

Utility Menu > File >Change Title... > Enter 'Stress Analysis of an Axisymmetric Pressure Vessel' > OK

ANSYS Main Menu > Preferences > Preferences for GUI Filtering >Select 'Structural' and 'h-method' > OK

3.2 Enter the Preprocessor to define the model geometry and do up to post processor

3.2.1 Thin-walled pressure vessel

Define Element Type (Axisymmetric Option) and Material Properties.

ANSYS Main Menu > Preprocessor > Element Type > Add/Edit/Delete > Add... > Structural Solid 8 node 183 (PLANE183) (define 'Element type reference number' as 1) > OK > Click Options... > Select 'Axisymmetric' for K3 (Element behavior) > OK > Close

ANSYS Main Menu > Preprocessor > Material Props > Material Models > Double Click Structural > Linear > Elastic > Isotropic > Enter  $2e5$  for EX and 0.3 for PRXY > Click OK > Click Exit (under 'Material') and Begin creating the geometry by defining the keypoints

ANSYS Main Menu Preprocessor > Modeling > Create>keypoints>in active coordinate system.

Table 1: shows the key points of thin pressure vessel

K1(0,0) apply	K2 (100,0) apply
K3(100,300) apply	k4(20, 330) apply
k5(20, 350) apply	k6(10,350) apply
k7(10,290) apply	k8(90,290) apply
k9(90,10) apply	k10(0,10) ok

After creating Keypoints then create the area

ANSYS Main Menu Preprocessor > Modeling > Create>areas>rectangles>through keypoints then connect the keypoints by area>ok

Then we mesh the finer meshing

ANSYS Main Menu Preprocessor >Meshing>mesh tool>tick smart size>reduce to fine mesh of 3>mesh>pick all.

Now applying The loading boundary conditions

ANSYS Main Menu processor(solution)>Analysis type>tick new.

ANSYS Main Menu processor(solution)>Define load>apply>structural>displacement > on lines >select UY to support vertical reaction,

ANSYS Main Menu processor(solution)>Define load>apply>structural>pressure>on lines>enter 4000pa>ok.

After applying the loads and boundary conditions we solve the element

ANSYS Main Menu processor(solution)>solve>current LS>when the solution is done>close the dialog boxes. After completing the processing part, we may start the post processor

ANSYS Main Menu General post processor>Plot results>deformed shapes>tick def +undef>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Nodal sol>DOF>X component>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Nodal sol>DOF>Y component>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Nodal sol>Displacement vector> ok

ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>X stress component>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>Y stress component>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>Z stress component>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>XY stress component>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>von misses stress >ok

*The axisymmetric of the cylinder*

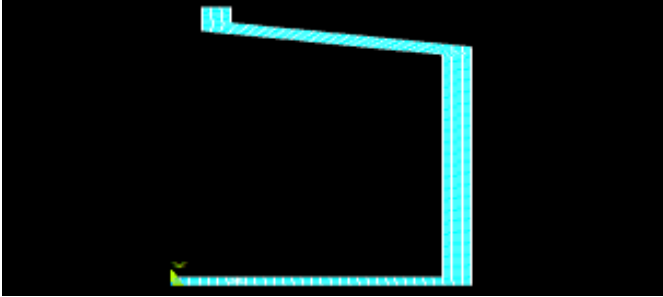


Figure 3.2.1.1 axisymmetric shape of the cylinder

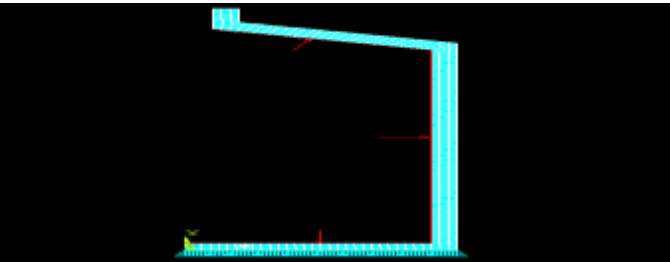


Figure 3.2.1.2 The loading and boundary conditions of the cylinder

*The following are the results obtained in the analysis (Nodal Solution)*

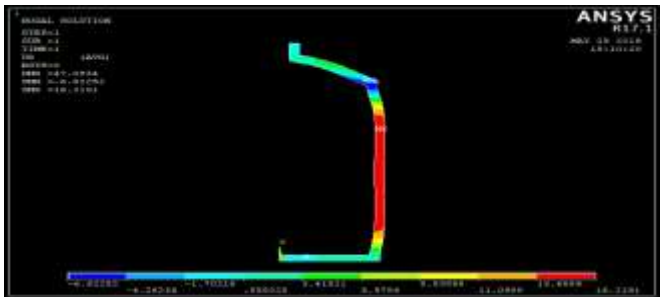


Figure 3.2.1.3 Nodal solution in x direction



Figure 3.2.1.4 Nodal solution in y direction

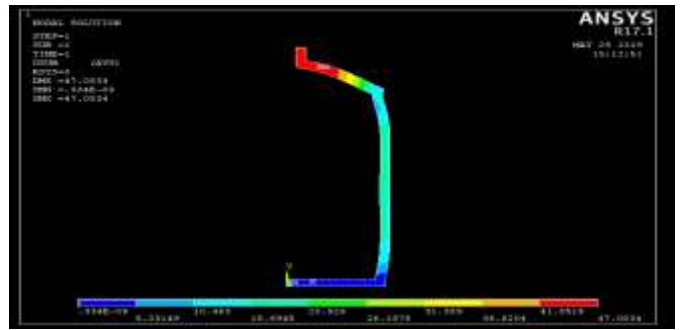


Figure 3.2.1.5 Displacement vector

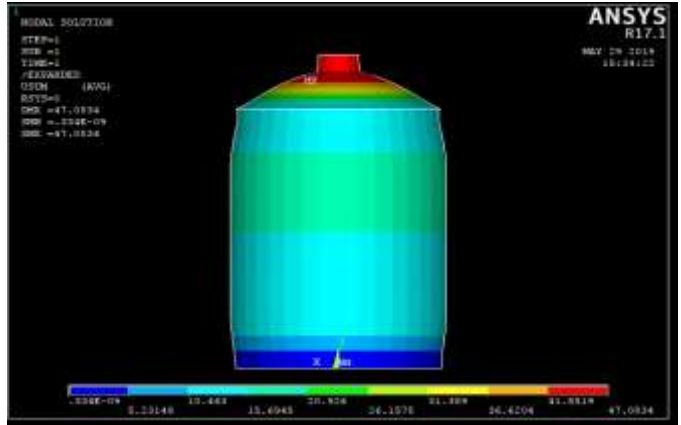


Figure 3.2.1.6 Full symmetry expansion

*Analysis of stresses of the cylinder*



Figure 3.2.1.7 stress in X direction

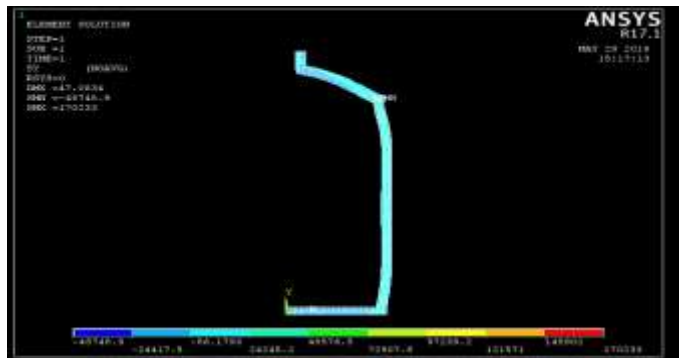


Figure 3.2.1.8 stress in Y direction



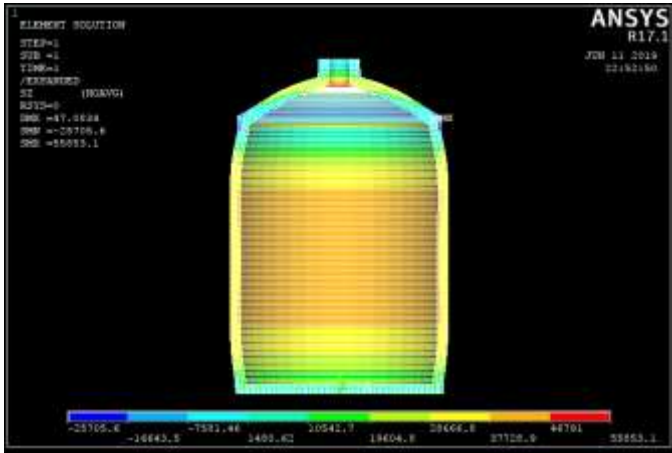


Figure: 3.2.1.9 stress in Z direction

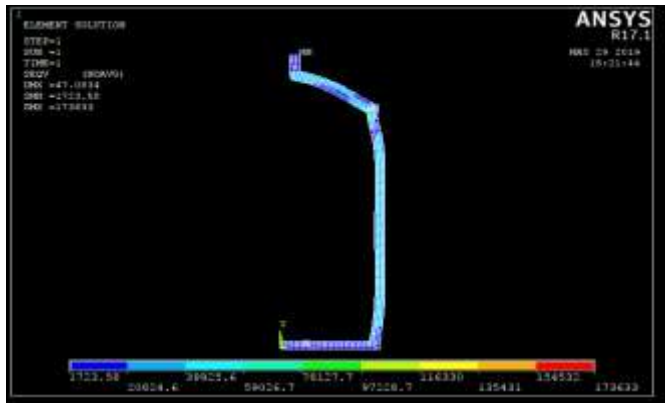


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Table 2: Shows keypoints of thick pressure vessels

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ANSYS Main Menu General post processor>Plot results>contour plots>Nodal sol>Displacement vector> ok

ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>X stress component>ok

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ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>Z stress component>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>XY stress component>ok

ANSYS Main Menu General post processor>Plot results>contour plots>Element sol>stress>von misses stress >ok

*The loading and boundary conditions of the cylinder*

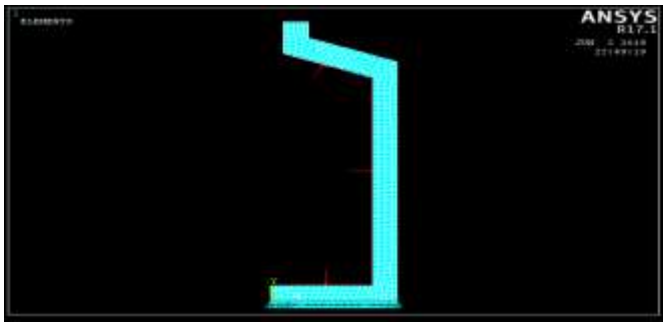


Figure: 3.2.2.1 Loading and the boundary condition

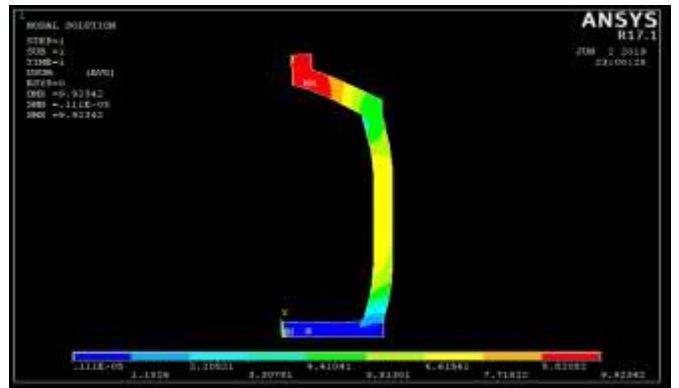


Figure: 3.2.2.5 Nodal solution in displacement vector form

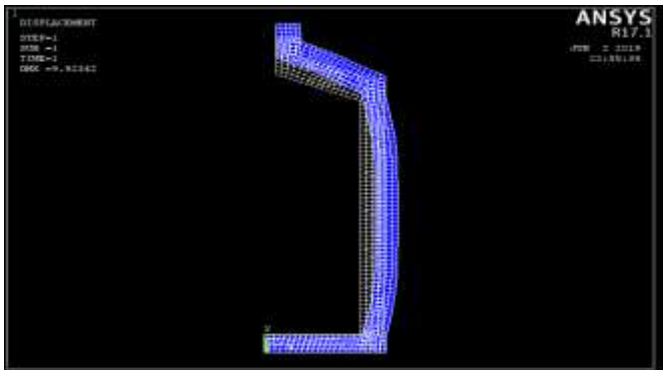


Figure: 3.2.2.2 The deformed shape of the vessel

*Analysis of stresses in the cylinder*

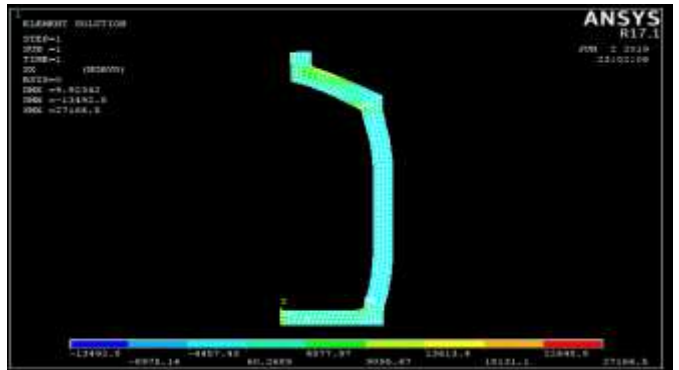


Figure: 3.2.2.6 Stress component in X-direction

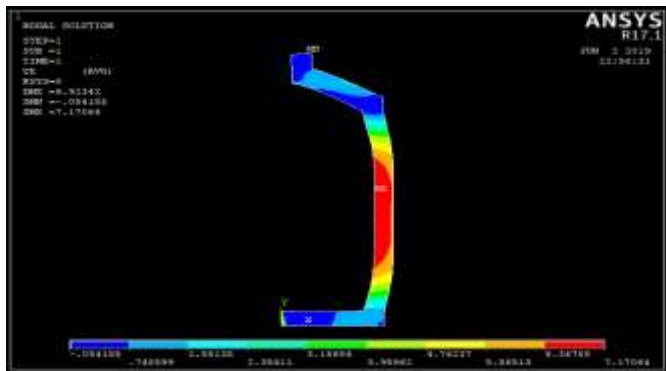


Figure: 3.2.2.3 The nodal solution in the X-direction

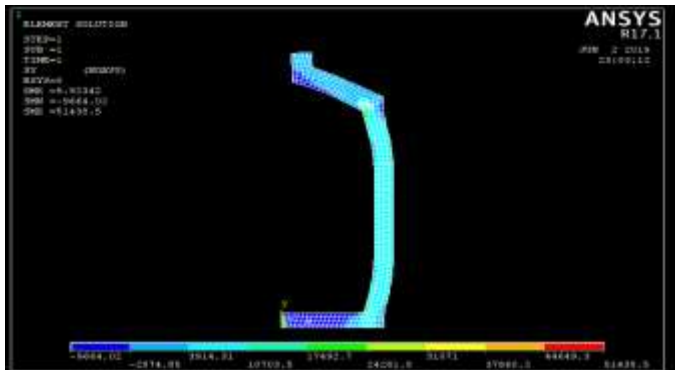


Figure: 3.2.2.7 Stress component in Y-direction

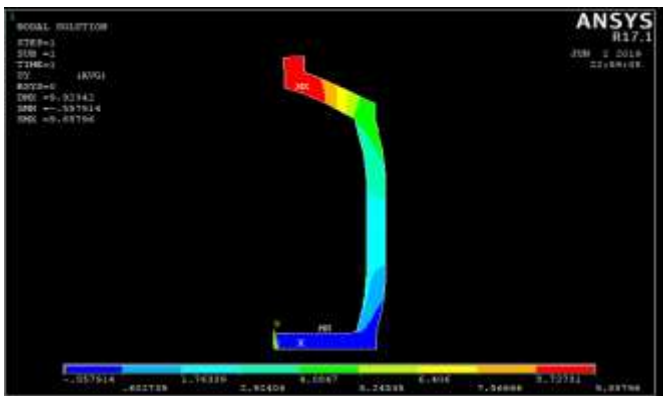


Figure: 3.2.2.4 The nodal solution in the Y-direction

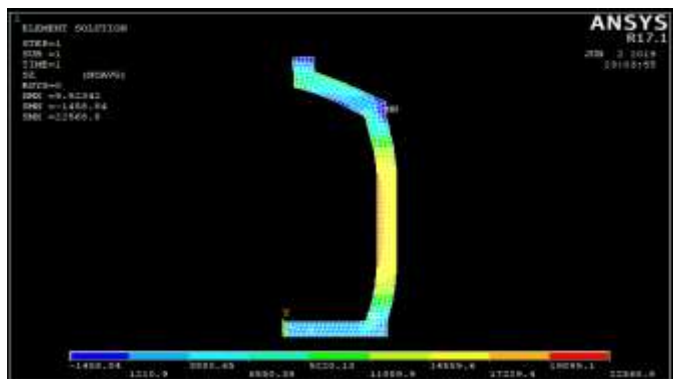


Figure: 3.2.2.8 Stress component in Z-direction

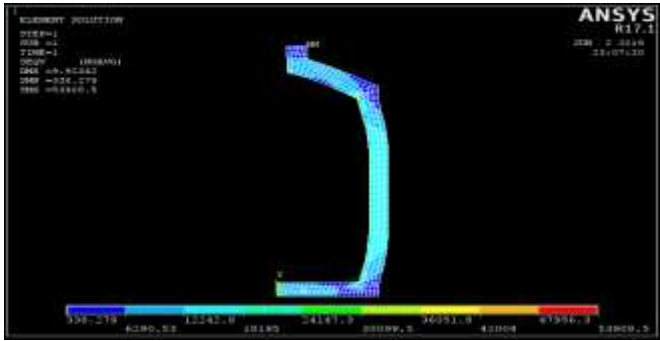


Figure: 3.2.2.9 Von misses stress

Table 3: Summary of results by considering maximum value

Thin walled pressure vessel		Thick walled pressure vessel	
Nodal solution(mm)	Stresses(pa)	Nodal solution (mm)	Stresses (pa)
16.2 in x direction	95204.3 in x direction	7.2 in x direction	27166.5 in x direction
46.99 in y direction	170233 in y direction	9.9 in y direction	51438.5 in y direction
47.1 displacement vector	55853.1 in Z direction	9.923 displacement vector	22568.8 in Z direction
	173633 von misses		53908.5 von misses

IV. CONCLUSION

1. Various parameters of Pressure vessels are analyzed using ANSYS APDL.
2. The difference between the thin and thick pressure vessels were classified.
3. Analysis of cylindrical pressure vessel were performed and maximum stresses found by ANSYS are presented in the postprocessor. It is concluded that there are differences in resisting stresses in the thin and the thick pressure vessel, we get the less deformation in the thick one where as the large deformation in the thin one.

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