Mathematical Modeling and Analysis of Corruption Dynamics in Kenya

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Abstract: This study presents a mathematical model that aims to study corruption in Kenya. The model is validated both epidemiologically and mathematically, with all solutions demonstrating positivity and boundedness within a meaningful set of initial conditions. By investigating unique corruption-free and endemic equilibrium points, as well as computing the basic reproduction number, we assess the system's behavior. Our analysis reveals that a locally asymptotically stable corruption-free equilibrium point is achieved when the reproduction number is below one, while a locally asymptotically stable endemic equilibrium point is attained when the reproduction number exceeds one. Simulation results confirm the agreement with analytical findings. This research enhances our understanding of corruption dynamics and provides valuable insights for designing effective anti-corruption strategies in Kenya.

1. Introduction

Corruption has continued to be a pervasive problem around the globe and more so in Kenya. This vice has seen several studies being conducted with the goal of comprehending it. Globally, the World Bank has established an estimate of the international bribery to exceed 1.5 trillion US dollar. In Kenya, an estimated ksh.608 billion which is an approximate of 7.8% of Kenya's GDP is lost to corruption yearly 30. These figures continue to rise and Kenya continues to suffer great loses where based on audit and prosecution results, it was claimed that an approximate of 2 billion Kenya shillings is lost to corruption every day according to a news outlet published by 24. In Kenya, the efforts to combat corruption have encountered obstacles such as ineffective strategies, opposition from politicians, and an inability to maintain long-term reforms in the public sector. Additionally, there is a lack of awareness regarding effective methods for creating systemic change 22. The World Bank through the Transparency International 12 has averaged the Corruption Index (CI) in Kenya to be 23.97 points as of 1996 up until 2022. Fighting corruption at an individual level is difficult, but it becomes even more challenging when a country's systems force individuals to engage with corrupt practices. This is particularly true when corruption is perceived as a means to access resources and opportunities that are otherwise unavailable due to inequality or social exclusion. According to 29, greed strongly motivates and enables corruption, while need creates a vulnerable population prone to corruption. Understanding these dynamics is crucial for developing effective strategies to address the root causes of corruption and combat it.

A study by Kinyanjui 13, shows the effects of corruption can and have been felt in areas such as the life expectancy, education, and income per capita. Corruption is also seen to undermine the fairness of institutions, distorts policies and priorities leading to a damaged credibility of governments. It's hence clear that corruption is a threat that can become an outbreak if appropriate control measures are not put into place to mitigate it and as Abdulrahman 1 puts it, a population where corruption exists, individuals could be exposed to corruption and later on, may end up being the very corrupt individuals in the society. It's for this reason that 2 developed a new mathematical model and analysed with unvarying recruitment rate and standard incidence for the transmission of corruption dynamics where an analysis was able to show that if a two-fifths of the individuals engaging in corrupt practices are embarrassed due to social media coverage, most of them will become semi-corrupt which can aid in the control of corruption, though, it may take several years before being achieved. In 26 the authors also proposed a model on corruption dynamics where an epidemiological threshold of the vice, corruption, was observed that was in regards to the approximation of the honest population. Another study 27 presented and analysed the non-linear mathematical model and the model was found to be helpful to society in reduction of the burdens of corruption and the study considered job transfers and suspension as punishments for individuals. Another study focused on increasing minimum wage and the amount of tax revenue to monitor corruption 8.

A study by 14 formulated a model that took into account anti-corruption awareness as well as counselling while in jail as the control measures and from it, a person that would lose immunity obtained via counselling in jail did not directly transfer into being corrupt but instead became susceptible due to the human behaviour. In another research, 4 developed a corruption control model which was extended to incorporate optimal control and the findings obtained showed the corruption level in the population can be minimized if the control approaches of corruption through media or punishments are raised and incorporated.

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In another study, a model was used to describe the prevention and disengagement strategies of fighting corruption and it showed that through media and campaigns against corruption as well as the political will of the government to fight this vice, then the persons engaging in corrupt practices would shift to the susceptible compartmental class 21. Another study by 7 also got to formulate a mathematical model for the corruption dynamics by incorporation of the control measures where the intervention/ control strategies were mass education and religious teaching. In another research by 20, another model was formulated and it focused on corruption of morals with an encompassing age-appropriate sexual information and also availability of guidance and counselling as interventions to the vice. In a study by 9, a model on the dynamics of corruption's spread was developed and two additional control strategies—prevention and punishment—were added to the model. The study established the existence of an ideal control strategy, and the results of the numerical simulation demonstrated that the two control measures were the most successful strategy for lowering corruption in the populace. In another study the authors in 6 developed a mathematical model for the dynamics of corruption transmission, including media coverage. The numerical simulation demonstrated that the number of susceptible people rises in the presence of media coverage, while the number of exposed and corrupted people falls. This suggests that while corruption is more quickly eliminated when there is media coverage of it, the number of corrupt people in the population actually rises when there is insufficient media coverage of how corruption is transmitted and controlled. Most of these studies done on this vice have open a gateway to understand corruption and it’s clear that this vice is such a complex and multifaceted one, and a better understanding of the dynamics of corruption is needed in order to design effective policy interventions to reduce its prevalence and impact. Understanding and analyzing the factors that contribute to corruption is crucial, particularly in identifying the variables that can be modified to assess a population and determine the most effective measures to combat corruption.

II. The Mathematical Model Formulation and Description

The model to be used in this research will be an ***SCRS*** model. In this model, the total population is compartmentalized into three epidemiological classes. These three compartmental classes will be the susceptibles, corrupt, and recovered. At any given time t , the quantities of individuals in these categories are represented as $S(t)$,$C(t)$, and $R(t)$, respectively. The three compartmental classes will be formulated where each of them are described as follows:

Susceptible class involves individuals that have never engaged in corruption but are vulnerable. It also includes individuals that were once corrupt, have served their time in jail and have now recovered but they still are vulnerable to corruption.

Corrupt class involves individuals that are already engaging in corruption and are well able of influencing the susceptibles or individuals in the population into becoming corrupt.

Recovered class involves individuals that were once exposed to corruption or were corrupt that are now reformed in that they neither have an interaction with the corrupt nor can they transmit corruption; though they could become susceptible to corruption within a given stipulated time.

Some of the model assumptions include:

a) Once an individual recovers, they don't necessarily gain immunity but are still prone to being corrupt hence become susceptible after some time.

b) Through interaction, the corrupted may influence the susceptibles into their practices only if there's an interaction.

c) Each and every model parameter is non negative.

d) The individuals in the susceptible compartments are recruited either through birth or immigration.

e) There are no corruption related deaths.

f) The population increases through birth and immigration at the same rate in which it decreases through natural death. Thus, the model assumes that the size of the population remains constant over time.

g) This model assumes that there is no immunity even after recovery from corruption and that individuals who were previously corrupt may not necessary shift to being corrupt instantly, rather they are repeatedly just vulnerable / susceptible.

Thus, based on our model, it can be inferred that an individual who has recovered from corruption does return to the susceptible compartment and does not necessarily gain immunity to corruption. This is because there is no definite way to prove that an individual is immune to corruption, and thus the individual remains in the susceptible compartment. In other words, being corrupted once does not guarantee that an individual will not be corrupted again in the future.

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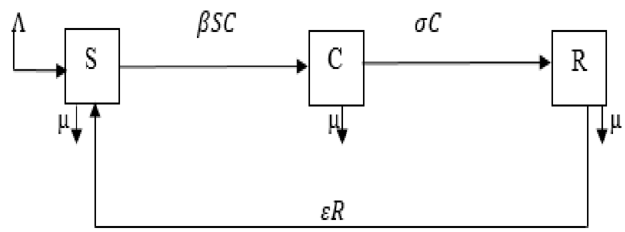

Figure 1: Flow Chart

The above model assumes the following parameters:

The susceptible individuals are recruited at the rate of Λ. These susceptible individuals due to some external factors such as greed and/or later contact with corrupted individuals, end up being corrupt at the rate of β. The corrupt individuals recover from the disease at the rate of σ. The recovered individuals may later loose 'immunity' after some time and return to being susceptible at the rate of ϵ. The natural removal rate of the entire individual compartment is μ.

The equations of the model are:

$$\frac{dS}{dt} = \Lambda + \epsilon R - \beta SC - \mu S$$
$$\frac{dC}{dt} = \beta SC - \sigma C - \mu C$$
$$\frac{dR}{dt} = \sigma C - \mu R - \epsilon R \qquad (1)$$

With initial conditions,

$$S>0, C \geq 0, R \geq 0 \qquad (2)$$

III. Model Analysis

Positivity of Solutions

Here, we look for the positivity of the model solution. Since the state variables change continuously over time, we make the assumption that all the state variables are continuous and hence from the model equations above, we deduce:

$$\frac{dS}{dt} \geq -\beta SC - \mu S$$
$$\frac{dC}{dt} \geq -(\sigma + \mu)C$$
$$\frac{dR}{dt} \geq -(\mu + \epsilon)R \qquad (3)$$

Solving the differential equations at $t \geq 0$, we have:

$$S(t) \geq S(0)e^{(-\mu t - \int \beta C dt)} \geq 0$$
$$C(t) \geq C(0)e^{-(\sigma+\mu)t} \geq 0$$
$$R(t) \geq R(0)e^{-(\mu+\epsilon)t} \geq 0 \qquad (4)$$

Boundedness of Solutions

Here, we add all the model's equations so as to check for the boundedness of the equations

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$$\frac{dS}{dt}+\frac{dC}{dt}+\frac{dR}{dt}= \Lambda + \epsilon R - \beta SC - \mu S + \beta SC - \sigma C - \mu C + \sigma C - \mu R - \epsilon R$$

$$\frac{d}{dt}(S+C+R) = \Lambda - \mu(S+C+R) \quad (5)$$

we recall that the total population at a time t is N(t). Thus,

$$S+C+R=N \quad (6)$$

Hence;

$$\frac{dN}{dt}= \Lambda - \mu N \quad (7)$$

The integrating factor is given by: $I.F = e^{\int \mu t} = e^{\mu t}$

Now, we multiply $e^{\mu t}$ to both sides of (7)

$i.e., \ e^{\mu t}\left(\frac{dN}{dt}+ \mu N\right) = (\Lambda)e^{\mu t}$

And later after integration, the solution will be;

$$N e^{\mu t} = \frac{\Lambda}{\mu}(e^{\mu t}) + c_1$$

$$\Rightarrow N = \frac{\Lambda}{\mu} + c_1 e^{\mu t} \quad (8)$$

We then take the limit of (8);

$$i.e., \lim_{t\to\infty}\left(\frac{\Lambda}{\mu} + c_1 e^{\mu t}\right) = \frac{\Lambda}{\mu}$$

∴

$$N = \frac{\Lambda}{\mu} \quad (9)$$

for large t values and we have a bounded system where N can be treated as a constant, $\frac{\Lambda}{\mu}$.

Basic Reproduction Number

By using the next generation matrix method,

$$R_0 = \frac{New}{existing} infections$$

$$R_0 = F V, \quad (10)$$

where $R_0 < 1$, corruption dies out and when $R_0 > 1$, then corruption spreads out.

Taking the equation $\frac{dC}{dt} = \beta SC - \sigma C - \mu C$ from (1);

$$f = \beta SC$$
$$v = (\sigma + \mu) C \quad (11)$$

Now, to find F and V we find the partial derivatives of f and v.

i.e.,

$$F = \frac{\partial f}{\partial C} = \frac{\partial}{\partial C}(\beta SC) = \beta S \quad (12)$$

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$$V = \frac{\partial v}{\partial C} = \frac{\partial}{\partial C}(\sigma + \mu)C = \sigma + \mu$$

Now, we know that at corruption free equilibrium, the whole population N is in the susceptible compartment and hence $= \frac{\Lambda}{\mu}$. This will also imply that $C = R = 0$

$$\therefore R_0 = FV = \frac{\beta S}{(\sigma + \mu)} \quad (13)$$

But at corruption free equilibrium the whole population, N is susceptible hence, $S = \frac{\Lambda}{\mu}$

Thus,

$$R_0 = \frac{\beta \Lambda}{\mu(\sigma + \mu)} \quad (14)$$

Equilibrium Points

At equilibrium, the right hand side of the system (1) is equal to zero, i.e.,

$$\Lambda + \epsilon R - \beta SC - \mu S = 0$$
$$\beta SC - \sigma C - \mu C = 0$$
$$\sigma C - \mu R - \epsilon R = 0 \quad (15)$$

For the **Corruption Free Equilibrium C_0,** corruption is not present. Hence, $C = R = 0$ and $S = S_0$.

$$\Longrightarrow \quad \Lambda + \epsilon(0) - \beta(S_0)(0) - \mu S_0 = 0$$
$$\Longrightarrow \Lambda - \mu S_0 = 0$$
$$\Longrightarrow S_0 = \frac{\Lambda}{\mu} \quad (16)$$

∴ The Corrupt Free Equilibrium is given by;

$$S, C, R = \left(\frac{\Lambda}{\mu}, 0, 0\right) \quad (17)$$

For the **Endemic Equilibrium E_e** we investigate the existence of Endemic Equilibrium E_e points. Here, we will substitute the S, C, R in the system of equations with S_e, C_e, R_e in that order. We hence get the following system of equations:

$$\Lambda + \epsilon R_e - \beta S_e C_e - \mu S_e = 0$$
$$\beta S_e C_e - \sigma C_e - \mu C_e = 0$$
$$\sigma C_e - \mu R_e - \epsilon R_e = 0 \quad (18)$$

We then solve for S_e, C_e and R_e:

Taking the second equation in the above system, (18) we get to solve for S_e which results to:

$$S_e = \frac{\sigma + \mu}{\beta} \quad (19)$$

Now, taking the third equation in (18), and making R_e the subject will result to:

$$R_e = \frac{\sigma C_e}{\mu + \epsilon} \quad (20)$$

Next, we substitute S_e and R_e in the first equation in the above system of equations in this subsection and making C_e the subject of the equation will result to:

$$C_e = \frac{(\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma)}{\beta(\mu + \epsilon)} \quad (21)$$

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We then substitute C_e into (20) and hence the resulting R_e will be:

$$R_e = \frac{\sigma}{\mu + \epsilon}\left[\frac{\Lambda\,\beta - 2\,\mu\,(\,\sigma + \mu + \epsilon) - 2\,\epsilon\,\sigma}{\beta\,(\,\mu + \epsilon)}\right] \tag{22}$$

∴ the resulting Endemic Equilibrium will be given by:

$$E_e\begin{pmatrix} S_e \\ C_e \\ R_e \end{pmatrix} = E_e\begin{pmatrix} \frac{\sigma + \mu}{\beta} \\ \frac{\Lambda\,\beta - 2\,\mu\,(\,\sigma + \mu + \epsilon) - 2\,\epsilon\sigma}{\beta\,(\,\mu + \epsilon)} \\ \frac{\sigma}{\mu + \epsilon}\left[\frac{\Lambda\,\beta - 2\,\mu\,(\,\sigma + \mu + \epsilon) - 2\,\epsilon\sigma}{\beta\,(\,\mu + \epsilon)}\right] \end{pmatrix} \tag{23}$$

Stability Analysis

Here, we linearize the model system of equations as seen in (1):

$$\frac{dS}{dt} = \Lambda + \epsilon R - \beta\,SC - \mu\,S$$

$$\frac{dC}{dt} = \beta\,SC - \sigma\,C - \mu\,C$$

$$\frac{dR}{dt} = \sigma\,C - \mu\,R - \epsilon R$$

We linearize by partially differentiating each equation on the right hand side of system with respect to S, C, R to obtain the Jacobian matrix.

That is:

$$\therefore \quad J = \begin{pmatrix} -\mu - \beta C & -\beta S & \epsilon \\ \beta C & \beta S - (\sigma + \mu) & 0 \\ 0 & \sigma & -(\mu + \epsilon) \end{pmatrix} \tag{24}$$

but at corruption free equilibrium;

$$S = \frac{\Lambda}{\mu}, C = R = 0 \tag{25}$$

Hence, the Jacobian at CFE will be:

$$J_0^{CFE} = \begin{pmatrix} -\mu & -\beta\frac{\Lambda}{\mu} & \epsilon \\ 0 & \beta\frac{\Lambda}{\mu} - (\sigma + \mu) & 0 \\ 0 & \sigma & -(\mu + \epsilon) \end{pmatrix} \tag{26}$$

Lemma 1. *The corruption-free equilibrium is locally asymptotically stable whenever* $R_0 < 1$ *and unstable when* $R_0 > 1$.

Now, to investigate whether the system is locally stable we will use Routh-Hurwitz criteria where if;

i) The trace (τ) of J_0^{CFE} is negative and

ii) The determinant of J_0^{CFE} is positive,

Then the system is stable.

Now, with this criteria of determining stability, we will be finding the

i) **Trace (τ):**

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$\tau\,(J_0^{CFE}) = -\,\mu \,+\, \beta\left(\frac{\Lambda}{\mu}\right) - (\,\sigma \,+\, \mu\,) - (\,\mu \,+\, \epsilon)$

$\tau\,(J_0^{CFE}) \,= -\,\mu \,+\left(\beta\,\frac{\Lambda}{\mu\,(\sigma+\mu\,)} - 1\right)(\,\sigma \,+\, \mu\,) -$
$(\,\mu \,+\, \epsilon)$

$but\; R_0 = \frac{\beta\Lambda}{\mu\,(\sigma+\mu\,)}$

$\therefore, \tau\,(J_0^{CFE}) \,= -\,\mu \,+ (\,R_0 - 1\,)(\,\sigma \,+\, \mu\,) - (\,\mu \,+\, \epsilon)$

$\tau\,(J_0^{CFE}) = -(\,2\,\mu \,+\, \epsilon) + (\,R_0 - 1\,)(\,\sigma \,+\, \mu\,)$

(27)

Thus trace is negative provided $R_0 < 1$.

ii) **Determinant (det) :**

$$\det(J_0^{CFE}) = -\,\mu\left(\beta\frac{\Lambda}{\mu} - (\,\sigma \,+\, \mu\,)\right)(-\,\mu \,-\, \epsilon)$$

$$\det(J_0^{CFE}) = -\,\mu\left(\left(\beta\frac{\Lambda}{\mu\,(-\,\sigma \,-\, \mu\,)} + 1\right)(-\,\sigma \,-\, \mu\,)\right)(-\,\mu \,-\, \epsilon)$$

$$but\; R_0 = \,\beta\frac{\Lambda}{\mu\,(\,\sigma \,+\, \mu\,)},$$

$$\det(J_0^{CFE}) = -\,\mu\,\big((-\,R_0 + 1\,)(-\,\sigma \,-\, \mu\,)\big)(-\,\mu \,-\, \epsilon)$$

$$\det(J_0^{CFE}) = \,\mu\,(\,\mu \,+\, \epsilon)\big((-\,R_0 + 1\,)(-\,\sigma \,-\, \mu\,)\big)$$

$$\det(J_0^{CFE}) = \,\mu\,(\,\mu \,+\, \epsilon)(-)(\,\sigma \,+\, \mu\,)\big((-\,R_0 + 1\,)\big)$$

$$\det(J_0^{CFE}) = -\,\mu\,(\,\mu \,+\, \epsilon)(\,\sigma \,+\, \mu\,)(-\,R_0 + 1\,)$$

$$\therefore det(J_0^{CFE}) = -(\,\mu^2 \,+\, \mu\,\epsilon)(\,\sigma \,+\, \mu\,)(\,1 - \,R_0)$$

(28)

Thus determinant is positive provided $R_0 < 1$.

This therefore means that the stability of the disease-free equilibrium in the model is locally asymptotically stable as both the trace and determinant satisfy the Routh-Hurwitz criterion. The model will have a disease-free equilibrium when $R_0 < 1$.

Lemma 2. *The Corruption-free equilibrium of the system is globally asymptotically stable when the reproduction number* $R_0 < 1$ *and unstable when* $R_0 > 1$

For this study, we will use the LaSalle's Invariance Principle:

We thus let:

$V = \,½\,C\text{^}2$

$\therefore\; dv/dc = C$ (29)

From the model system of differential equations in (1), we will take the the second equation, which is :

$$\frac{dC}{dt} = \,\beta\,SC - \,\sigma\,C - \,\mu\,C$$

And by chain rule;

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$$\frac{dv}{dt} = \frac{dv}{dc}.\frac{dc}{dt}$$

$$\therefore \frac{dv}{dt} = C.(\beta SC - \sigma C - \mu C)$$

$$\Rightarrow C^2(\beta S - \sigma - \mu) \qquad (30)$$

From the reproduction number, which is given by: $R_0 = \beta \frac{\Lambda}{\mu(\mu+\sigma)}$

This implies that : $\beta \Lambda = \mu R_0(\mu + \sigma)$ or $\beta S = R_0(\mu + \sigma)$

Therefore,

$$\frac{dv}{dt} = C^2(R_0(\mu + \sigma) - \sigma - \mu)$$

$$\frac{dv}{dt} = C^2(\mu + \sigma)(R_0 - 1) \qquad (31)$$

When $R_0 < 1$, $\frac{dv}{dt}$ **is** $-ve$

Hence, in this case, the Corruption Free Equilibrium is globally asymptotically stable if $R_0 < 1$.

Lemma 3. *The endemic equilibrium of the model is locally asymptotically stable whenever* $R_0 > 1$.

We investigate the system's endemic equilibrium stability using the Routh-Hurwitz criterion where if;

i) The trace (τ) of J_{EE}^* is negative and

ii) The determinant of J_{EE}^* is positive,

then the Endemic Equilibrium (EE) is stable.

The Jacobian matrix at J_{EE}^* is given by:

$$J_{EE}^* = \begin{pmatrix} -\mu - \beta C_e & -\beta S_e & \epsilon \\ \beta C_e & \beta S_e - (\sigma + \mu) & 0 \\ 0 & \sigma & -(\mu + \epsilon) \end{pmatrix}$$

$$where\ S_e = \frac{\sigma + \mu}{\beta}, C_e = \frac{\Lambda \beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma}{\beta(\mu + \epsilon)} \qquad (32)$$

Now, with this criterion of determining stability, we will be finding the

i) Trace (τ) of J_{EE}^* :

$$\tau = -\mu - \beta\frac{\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma}{\beta(\mu + \epsilon)} + \beta\frac{\sigma + \mu}{\beta} - (\sigma + \mu) - (\mu + \epsilon)$$

$$\tau = -(2\mu + \epsilon) - \frac{\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma}{(\mu + \epsilon)}$$

$$\tau = -(2\mu + \epsilon) - \frac{\Lambda\beta - 2(\epsilon + \mu)(\mu + \sigma)}{(\mu + \epsilon)}$$

$$\tau = -(2\mu + \epsilon) - \left\{\frac{\Lambda\beta}{(\mu + \epsilon)} - 2(\mu + \sigma)\right\}$$

$$\tau = -(2\mu + \epsilon) - \left\{\left(\frac{\Lambda\beta}{2\mu(\mu+\sigma)(\mu+\epsilon)} - \frac{1}{\mu}\right)2\mu(\mu + \sigma)\right\}$$

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$$but,$$

$$R_0 = B\frac{\Lambda}{\mu(\mu + \sigma)}$$

$$\therefore \tau = -(2\mu + \epsilon) - \mu\left(\frac{R_0}{(\mu + \epsilon)} - \frac{1}{\mu}\right)(\mu + \sigma)\}$$

(33)

and provided $R_0 > 1$, then the **trace is negative**.

ii) **Determinant (det)** :

$$\det(J_{EE}^*) = (-\mu - \beta C_e)(\beta S_e - (\mu + \sigma))(-(\mu + \epsilon)) + \beta S_e(\beta C_e)(-(\mu + \epsilon)) + \epsilon(\sigma\beta C_e)$$

(34)

Substituting

$S_e = \frac{\sigma + \mu}{\beta}, C_e = \frac{\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma}{\beta(\mu + \epsilon)}$,

$$\det(J_{EE}^*) = \left(-\mu - \frac{\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma}{(\mu + \epsilon)}\right)\left(\beta\frac{\sigma + \mu}{\beta} - (\mu + \sigma)\right)(-(\mu + \epsilon)) + (-1)(\sigma + \mu)(\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma) + \epsilon\left(\sigma\frac{\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma}{(\mu + \epsilon)}\right)$$

$$\det(J_{EE}^*) = 0 + (-1)(\sigma + \mu)(\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma) + \frac{\epsilon\sigma}{(\mu + \epsilon)}(\Lambda\beta - 2\mu(\sigma + \mu + \epsilon) - 2\epsilon\sigma)$$

$$\det(J_{EE}^*) = -(\sigma + \mu)(\Lambda\beta - 2(\epsilon + \mu)(\mu + \sigma)) + \frac{\epsilon\sigma}{(\mu + \epsilon)}(\Lambda\beta - 2(\epsilon + \mu)(\mu + \sigma))$$

$$And\ we\ know,$$

$$R_0 = B\frac{\Lambda}{\mu(\mu + \sigma)},$$

$$\Longrightarrow \det(J_{EE}^*) = -(\sigma + \mu)(R_0\mu(\mu + \sigma) - 2(\epsilon + \mu)(\mu + \sigma)) + \frac{\epsilon\sigma}{(\mu + \epsilon)}(R_0\mu(\mu + \sigma) - 2(\epsilon + \mu)(\mu + \sigma)) \quad (35)$$

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$$\Rightarrow \det(J_{EE}^{*}) = \left\{ \left(R_0\mu(\mu + \sigma) - 2(\epsilon + \mu) \right\} \left\{ \left(\mu + \sigma \right) \frac{\epsilon\sigma}{(\mu + \epsilon)} - (\sigma + \mu) \right) \right\}$$

Hence, when $R_0 > 1$ each component in the curly braces will be a negative and thus the whole solution will translate to a positive solution.

Thus, provided$R_0 > 1$, the determinant is positive.

∴, the endemic equilibrium is stable.

We can therefore make a conclusion that the model has an asymptotically stable endemic equilibrium as both the trace and the determinant of the Endemic Equilibrium satisfy the Routh-Hurwitz criterion.

3.1 Sensitivity Analysis

Sensitivity analysis is a technique that is used to examine how the changes of certain independent variables, or model parameters, impact the dependent variable. In our current study, the objective is to explore whether manipulating a specific parameter leads to an increase or decrease in the reproduction number.

This approach aids in the selection of corruption control tactics by giving significant consideration to the most sensitive parameters. In the current study, the normalized forward sensitivity index technique is employed to assess the sensitivity of the model parameter included in the basic reproduction number. By determining the impact of changes in the parameters on the reproduction number, the study aims to identify the most effective strategies for controlling corruption.

Sensitivity index on R_0

This is given by:

$$\alpha_{p_0^R} = \frac{\partial R_0}{\partial P} \cdot \frac{P}{R_0} \qquad (36)$$

where:

R_0 = Basic Reproduction Number

P = Parameter of interest

Now, we will assign values to the parameters and use these values to find the sensitivity index of different parameters on R_0 .

Table 1: Description of Values to corruption model’s parameters

Parameter	Value assigned	Description	source
Λ	50	Recruitment rate due to birth or immigration	Danford (2020)
β	0.001	The rate of transmission of corruption to the susceptible individuals	assumed
σ	0.03	Rate at which the corrupt persons shift to the recovered compartment	assumed
μ	0.11	Natural removal rate by death	assumed
ϵ	0.06	The rate the recovered persons shift to the susceptible compartment	Binuyo (2019)

The basic reproduction number is:

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$$R_0 = \beta \frac{\Lambda}{\mu\,\sigma + \mu^2}$$

We thus find the sensitivity index of β , Λ , μ and σ .

$$\alpha_{\beta}^{R_0} = \frac{\partial R_0}{\partial \beta} . \frac{\beta}{R_0}$$

$$\frac{\partial R_0}{\partial \beta} = \frac{\Lambda}{\mu\,\sigma + \mu^2},$$

$$thus:$$

$$\alpha_{\beta}^{R_0} = \frac{\Lambda}{\mu\,\sigma + \mu^2} . \frac{\beta}{\beta \frac{\Lambda}{\mu\,\sigma + \mu^2}} = 1 \qquad (37)$$

This therefore means that there is a direct relationship between β and R_0 and a unit increase in β will result in an increase in R_0 .

$$\alpha_{\Lambda}^{R_0} = \frac{\partial R_0}{\partial \Lambda} . \frac{\Lambda}{R_0}$$

$$\frac{\partial R_0}{\partial \Lambda} = \frac{\beta}{\mu\,\sigma + \mu^2},$$

$$thus:$$

$$\alpha_{\Lambda}^{R_0} = \frac{\beta}{\mu\,\sigma + \mu^2} . \frac{\Lambda}{\beta \frac{\Lambda}{\mu\,\sigma + \mu\,2}} = 1 \qquad (38)$$

This therefore means that there is a direct relationship between Λ and R_0 and a unit increase in Λ will result in an increase in R_0 .

$$\alpha_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} . \frac{\mu}{R_0}$$

$$\frac{\partial R_0}{\partial \mu} = -\beta\,\Lambda \frac{\sigma + 2\,\mu}{(\mu\,\sigma + \mu^2)^2},$$

$$thus:$$

$$\alpha_{\mu}^{R_0} = -\beta\,\Lambda \frac{\sigma + 2\,\mu}{(\mu\,\sigma + \mu^2)^2} . \frac{\mu}{\beta \frac{\Lambda}{\mu\,\sigma + \mu\,2}}$$

$$= -\frac{\sigma + 2}{\mu\,\sigma + \mu}$$

$$\therefore\ -\frac{0.03 + 2(\,0.11\,)}{0.03 + 0.11} = -\frac{0.25}{0.14} = -1.7857 \qquad (39)$$

This therefore means that there is a inverse relationship between μ and R_0. The value 1.7857 means that a unit change in μ will result to a decrease in R_0 by 1.7857 .

$$\alpha_{\sigma}^{R_0} = \frac{\partial R_0}{\partial \sigma} . \frac{\sigma}{R_0}$$

$$\frac{\partial R_0}{\partial \sigma} = -\beta\,\Lambda \frac{\mu}{(\mu\,\sigma + \mu^2)^2}$$

$$thus:$$

$$\alpha_{\sigma}^{R_0} = -\beta\,\Lambda \frac{\mu}{(\mu\,\sigma + \mu^2)^2} . \frac{\sigma}{\frac{\beta\,\Lambda}{\mu\,\sigma + \mu^2}} = -\frac{\mu\,\sigma}{\mu\,\sigma + \mu^2} \qquad (40)$$

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$$\therefore\ -\frac{0.11(\,0.03\,)}{0.11(\,0.03\,)+(0.11\,)^2}=-\frac{0.0033}{0.0154}=-0.2143$$

This therefore means that there is a inverse relationship between σ and R_0 . The value 0.2143 means that a unit change in σ will result to a decrease in R_0 by 0.2143 .

Parameter	Sensitivity Index
β	1
Λ	1
μ	-1.7857
σ	-0.2143

Table 4: Sensitivity indices of parameters

Analytical interpretation:

It can be inferred that a decrease in the reproduction number is a key factor in controlling corruption. Parameters that influence this number are crucial in designing corruption control strategies. Now, the parameters that are capable of causing a decrease in R_0 are both μ and σ . In this context, we will thus take the parameter σ into account as the sensible factor for corruption control. This therefore means that, which represents the rate at which corrupt individuals recover, is identified and hence an increase in σ can be an effective control measure.

Certain control strategies updating anti-corruption policies, providing adequate compensation to employees, instituting the death penalty, establishing a truly independent anti-corruption commission, instituting mandatory anti-corruption training at the grassroots level, removing all benefits and opportunities from corrupt individuals, religious education, and reinforcing top-down punishment among many others can be implemented to increase $\boldsymbol{\sigma}$. It is important to note that finding the optimal measure for corruption control is the ultimate goal in the long run

IV. Numerical Simulations

4.1 Numerical simulation of the basic epidemiological model

We will use the assigned values to the parameters from table 1 and use the values to simulate the model that is proposed from figure 1.

Now, using the values assigned to the model parameters, the results of the figure obtained are as shown below.

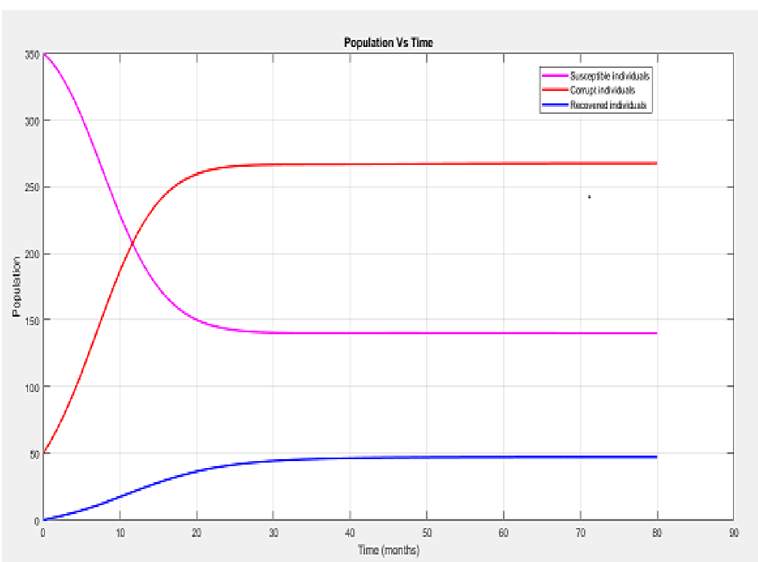

Figure 2: Corruption dynamics of the SCRS model

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From the above figure above, we could see that the number of recovered humans increases from the time stamp 0 to 42 due to an increase in the number of infected humans who naturally recover. This natural recovery, also known as self-change, is caused by a change in the infected individuals themselves and hence leading to an increase in the number of recovered humans. After the time stamp 42, the number of recovered individuals remains constant. It is important to note that the increase in recovered humans also leads to an increase in the number of susceptible humans, as individuals in the recovery compartment only temporarily stay there before either becoming susceptible again at the rate of ϵor naturally being removed at the rate of μR

4.2 Other simulation results

Here, we analyse the behaviour of corruption based off of the sensitivity analysis done previously. We will increase the parameter σ and simulate with the other parameters constant. The parameter is manipulated by a 200% increase and then we could double efforts aimed at increasing the rate of recovery from corrupt deeds to 400% for the purposes of seeing the progression of prevalence of corruption in the population.

Below is the result obtained when the parameter is manipulated:

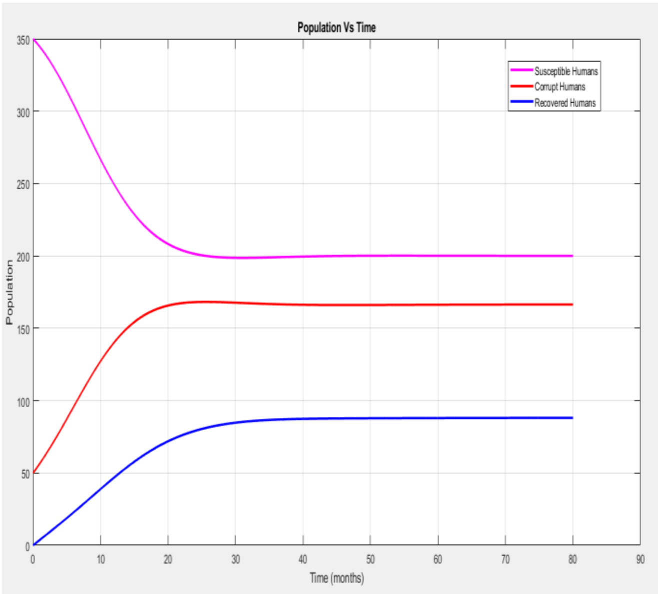

Figure 3: Corruption dynamics of the SCRS model with a 200% of σ increase

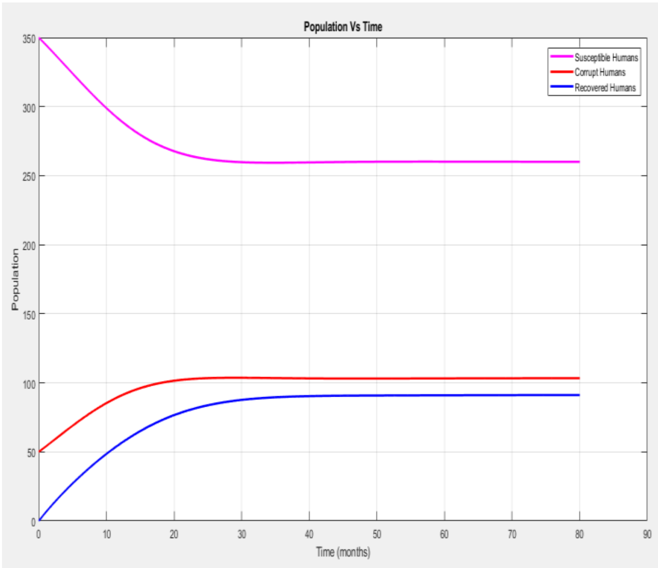

Figure 4: Corruption dynamics of the SCRS model with a 400% of σ increase

From the 2 figures above it is clear that the analytical interpretation of our findings in our sensitivity analysis done, allign with the numerical simulation. Since both the analytical and numerical simulation agree, we could then conclude that indeed measures taken to increase the rate of recovery will tune our model in the right direction.

V. Conclusion and Recommendation

5.1 Conclusion

In conclusion, the mathematical modeling of corruption dynamics using the epidemiological approach has provided insights into the factors that influence the spread and persistence of corruption in a society. The epidemiological approach has shown that the spread and persistence of corruption in a society can be influenced by factors such as the rate of transmission of corrupt practices and the rate of recovery from corruption.

Also, from approaching the problem of corruption from this angles, we could clearly conclude that there are a number of strengths and limitations associated with the epidemiological approach to modeling the dynamics of corruption. The epidemiological approach has the strength of taking into account the complex network of social ties that can facilitate or inhibit the spread of corrupt behaviours. This approach has its roots in the field of network science, and uses the compartmental modeling to understand the spread of contagions through networks. It has been used to study the spread of corruption in political networks, and has provided insights into the role of social ties in facilitating or inhibiting the spread of corrupt behaviours.

However, the epidemiological approach also has some limitations. One limitation is that it can be more difficult to implement, as it requires data on the structure of social networks and the prevalence of corruption within them. Additionally, the model may be more sensitive to changes in parameters and assumptions, which can make it more difficult to interpret the results in some cases.

5.2 Recommendations

We would recommend one potential way of combining the strengths of the epidemiological approach and another approach by potentially using this found approach as a baseline model, and to incorporate additional features from the epidemiological approach to account for the role of social networks. This could provide a more comprehensive understanding of corruption, and may be particularly useful for policy analysis. However, it is important to carefully consider the assumptions and limitations of each approach when combining them, as this may affect the accuracy and precision of the resulting model. Overall, the intention will be that both approaches offer useful insights into the dynamics of corruption, and a combination of these approaches may be the most effective way to study this complex and multifaceted problem.

Also, based on these findings, there are several recommendations that can be made for addressing the issue of corruption. First, policies that aim to reduce the benefits of engaging in corruption, such as stronger penalties for corrupt individuals and organizations, can be effective in reducing the prevalence of corruption. Additionally, policies that aim to increase the costs of engaging in corruption, such as stronger checks and balances, can also help to deter corrupt behaviour.

Data Availability

The data supporting this model are from published articles and are cited at relevant places.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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