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Transmission Dynamics of Peste des petit ruminant (PPR) in sheep and goats: A Mathematical Modelling Approach

***Bashir S., Muhammad A., and Garba I. D.**

Department of Mathematics, Federal University Dutsinma, Katsina State, Nigeria.

**Corresponding author*

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Abstract: The study is mainly concerned with mathematical modelling of peste des petit ruminant (PPR) disease using deterministic approach. A system of differential equations was formulated. Disease free and epidemic equilibria were calculated and used Jacobian Matrix to carried out stability analysis of the model. We then perform numerical simulations using Euler's method. Sensitivity analysis with basic reproduction number were finally conducted to identify the most important parameters in the model. It was finally recommended that animal suffering from peste des petit ruminant (PPR) diseases should be immediately quarantined so as to reduce the contact rate between the infected and the susceptible and other items that have been in contact with the sick animals must be disinfected with common disinfectants.

Keyword: small ruminants, mathematical model, deterministic approach, sensitivity analysis.

I. Introduction

Livestock are very important for both the subsistence and economic development of the African continent. They provide a flow of essential food products throughout the year. In some countries, like Sudan, they are a major source of government revenue and export earnings. They also sustain the employment and income of millions of people in rural areas. Contribute to energy and manure for crop production and are the only food and cash security available to many Africans.

In many African countries, small ruminants animal (sheep and goats) constitute an important proportion of the meat supply.Peste des Petits Ruminants (PPR) is an infectious disease affecting goats and sheep (Michael, *et al.*, 2017).Researches show that a large number of fatalities may be resulted in a herd due to quick spread of PPR disease(Mbyuzi,*et al.*, 2014).

For identifying strategies to mitigate the spread of PPR disease propagation in ruminant herds, a mathematical model is found very vital for understanding how the disease spreads (Schloeder & Jacobs, 2010). The need for a model to address uncertainty in parameter values arises due to lack of firsthand observations on how PPR spreads through a herd. This model allows us to perform Sensitivity Analysis (SA) on environment and disease parameters for which we do not have empirical data.

A system of differential equations has benn used in modelling problems in the field of electrical circuits, mechanics, vibrations, chemical reactions, kinetic and population growth (Muhammad, Bashir and Mustapha, 2021). Several modelling techniques and methods were used to model infectious diseases. Ordinary Differential Equation (ODE) modeling and stochastic lattice modeling are the two common techniques (Li, *et al.*, 1999). In deterministic (ODE) SI, SIS, or SIR model, the number of individuals in each infection-related class is calculated through a set of ODEs that stand for average transition rates between the classes (Boccara and Cheong, 1992). Magal and Ruan (2014) design traditional SEIR models in which they ignore demographic changes, such as births and natural deaths. The performance of different models can be compared to select the best model that fits a given data set (Chinenye and Bashir, 2021).

In this paper, we describe a PSIR epidemic mathematical model that has compartments; Pre-susceptible, Susceptible, Infectious and Recovered. We prefer this compartmental model over others because it generalise some of the models such as SI SIS and SIR as it takes care of the P class which is left in those models. It does not also make things too complicated as in the models with more and more compartments. For many infections, the new born animals are not born into the susceptible compartment but are immune to the disease for the first four months of life due to protection from maternal antibodies. This new detail can be shown by including a P (pre-susceptible) class, for maternally derived immunity at the beginning of the model.

II. Assumptions

The following assumptions are considered:

1. The population is mixing in a homogeneous manner. That is everyone has equal chances of contacting the disease.
2. Birth and death occur at constant rate.

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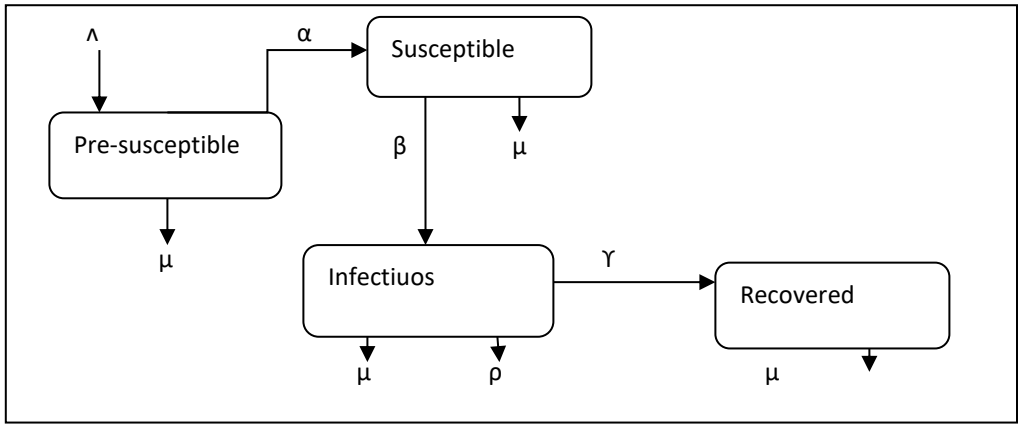
3. That animal in the compartment have equal natural death rate.
4. Recovery occurs at a constant rate;

III. Schematic Diagram



IV. Governing Equations

From the above epidemiological diagram, the transition between compartments can be expressed by the following system differential equations:

$$\left.\begin{aligned}\frac{dP}{dt} &= \wedge - \alpha P - \mu p \\ \frac{dS}{dt} &= \alpha P - \beta SI - \mu S \\ \frac{dI}{dt} &= \beta SI - \mu I - \rho I \\ \frac{dR}{dt} &= \gamma I - \mu R\end{aligned}\right\} 1$$

At steady states. $\frac{dP}{dt} = \frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$

From the above system of equations we get:

$\wedge - \alpha P - \mu p = 0$ ….....2

$\alpha P - \beta SI - \mu S = 0$ ….....3

$\beta SI - \mu I - \rho I = 0$ …….....4

$\gamma I - \mu R = 0$ .....….5

From equation (2): $\wedge - \alpha P - \mu p = 0$

Making P subject of the relation we have;

$P = \frac{\wedge}{(\alpha+\mu)}$…….....… 6

From equation (4): $(\beta S - (\mu + \rho))I = 0$

Either $I = 0$ or $S = \frac{\mu+\rho}{\beta}$ ……...7

From equation (5):

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$R = \frac{\Upsilon I}{\mu}$ ……8

At $I = 0$ then $R = 0$

Substitute for $I = 0$ and $P = \frac{\Lambda}{(\alpha+\mu)}$ in equation (3) and make S subject of the relation to get:

$S = \frac{\alpha\Lambda}{\mu(\alpha+\mu)}$ …… 9

Hence the disease free steady states are as follow

$P_1^* = \frac{\Lambda}{(\alpha+\mu)}$ …… 10

$S_1^* = \frac{\alpha\Lambda}{\mu(\alpha+\mu)}$ ……...11

$I_1^* = 0$ ……....12

$R_1^* = 0$ ……....13

Substituting equation (7) and equation (6) in to equation (3) and make I the subject of the relation to get:

$I = \frac{\alpha\beta\Lambda - \mu(\mu+\rho)(\alpha+\mu)}{\beta(\alpha+\mu)(\mu+\rho)}$ …….....14

Substitute equation (12) in equation (4) and making R subject of the relation we get:

$R = \frac{\gamma(\alpha\beta\Lambda - \mu(\mu+\rho)(\alpha+\mu))}{\beta\mu(\alpha+\mu)(\mu+\rho)}$…….....15

Hence the epidemic steady states are as follows:

$P_2^* = \frac{\Lambda}{(\alpha+\mu)}$ …….....16

$S_2^* = \frac{\alpha\Lambda}{\mu(\alpha+\mu)}$ …….....17

$I_2^* = \frac{\alpha\beta\Lambda - \mu(\mu+\rho)(\alpha+\mu)}{\beta(\alpha+\mu)(\mu+\rho)}$ …….....18

$R_2^* = \frac{\gamma(\alpha\beta\Lambda - \mu(\mu+\rho)(\alpha+\mu))}{\beta\mu(\alpha+\mu)(\mu+\rho)}$ …….....19

4.1 Stability Analysis

Theorem 1:

(a) If $R_e(\lambda) < 0$ for all eigenvalues, then the steady state is stable.

(b) If $R_e(\lambda) > 0$ for any of the eigenvalues, then the steady state is unstable.

(c) If $R_e(\lambda) = 0$ for all eigenvalues, then the steady state is cyclic.

Proof:

To investigate the stability of the steady states, we compute the Jacobian matrix of the system of equations(1) and get:

$$J = \begin{bmatrix} -(\alpha+\mu) & 0 & 0 & 0 \\ \alpha & -(\beta I+\mu) & -\beta S & 0 \\ 0 & \beta I & \beta S-(\gamma+\mu+\rho) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

The characteristic equation of the Jacobian matrix will be:

$$(J-\lambda I) = \begin{bmatrix} -(\alpha+\mu) & 0 & 0 & 0 \\ \alpha & -(\beta I+\mu) & -\beta S & 0 \\ 0 & \beta I & \beta S-(\gamma+\mu+\rho) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix} - \lambda \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}$$

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$$\begin{bmatrix} -(\alpha+\mu+\lambda) & 0 & 0 & 0 \\ \alpha & -(\beta I+\mu+\lambda) & -\beta S & 0 \\ 0 & \beta I & \beta S-(\gamma+\mu+\rho+\lambda) & 0 \\ 0 & 0 & \gamma & -(\mu+\lambda) \end{bmatrix} = 0$$

$$(\alpha+\mu+\lambda)\{-(\beta I+\mu+\lambda)[-(\mu+\lambda)(\beta S-(\gamma+\mu+\rho+\lambda))]\}=0$$

At the disease free equilibrium, we have

$$\lambda_1 = -\alpha-\mu$$
$$\lambda_2 = -\mu$$
$$\lambda_3 = -\mu$$
$$\lambda_4 = \beta\left(\frac{\alpha\Lambda}{\mu(\alpha+\mu)}\right)-(\gamma+\mu+\rho)$$

Hence the values for the eigenvalues of the characteristics polynomials, λ_1, λ_2 and λ_3 are lessthan zero and λ_4 will also be negative if

$$\beta\left(\frac{\alpha\Lambda}{\mu(\alpha+\mu)}\right) < (\gamma+\mu+\rho)$$

Thus, for if $\frac{\beta\alpha\Lambda}{\mu(\alpha+\mu)} < (\gamma+\mu+\rho)$, then the zero equilibrium point is conditionally stable

But if $\frac{\beta\alpha\Lambda}{\mu(\alpha+\mu)} > (\gamma+\mu+\rho)$ then, we say that the zero equilibrium point is generally unstable. This satisfy theorem 1(b) above.

For non-zero equilibrium state: $\lambda_1 = -\alpha-\mu$

$$\lambda_2 = -\frac{\alpha\beta\Lambda}{(\alpha+\mu)(\mu+\rho)}$$
$$\lambda_3 = -\mu$$
$$\lambda_4 = -\gamma$$

These values are all lessthan zero. That is all eigenvalues of the characteristics polynomials have negative real part, thus implying asymptotic stability as applied in (Bashir, Shehu, and Chinenye, 2021). The results satisfy theorem 1(a)above and it shows that the non-zero equilibrium state is stable.

V. Numerical Simulations

According to Muhammad, *et al.*, (2022) analytical and Numerical methods are the two common methods used to solve Ordinary differential equations. The solutions generated by the analytical method are generally exact values, whereas those with the numerical method of an approximation is given as a solution approaching the real value. A PSIR model was formulated and analyzed. Numerical simulations of the model are carried out using set of parameter values as given in table 1.1below. The Euler method has been used in Microsoft excel to perform numerical simulations.

Graphical representation showing the variation in reproduction numbers with respect to contact rate between susceptible and contact rate as provided in figures 1.1, 1.2 and 1.3. Since most of the parameter values were not readily available; we estimatedly choose some values.

Table 1.1 show the set of parameter values which were used in the simulation.

parameters	Values	Source
Λ	0.00100	estimated
$\boldsymbol{\alpha}$	0.02000	estimated
$\boldsymbol{\beta}$	2.74725	Estimated
$\boldsymbol{\Upsilon}$	0.50000	Estimated
$\boldsymbol{\mu}$	0.00039	Estimated
ρ	0.10000	Estimated

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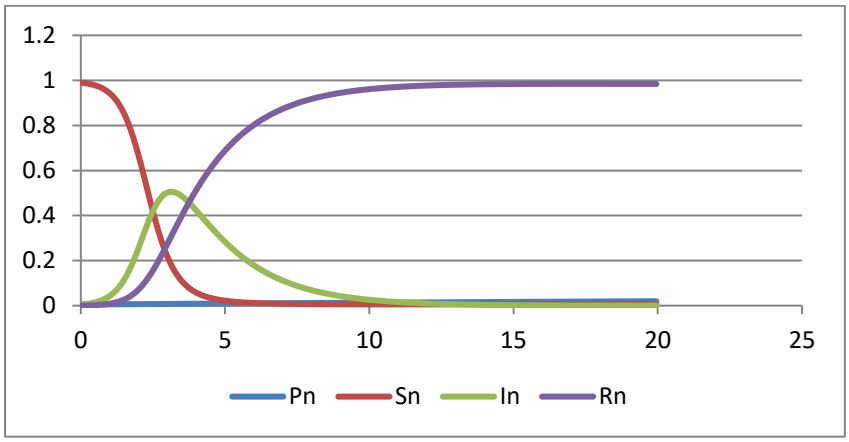

Figure 1.1 : At $\boldsymbol{\beta = 2.74725}$, almost half of the population became infected.

From figure 1.1, we see that the light blue curve indicates the proportion of sheeps and goats at birth who enjoy a mertarnally drived immunity for just few months of life and moved to susceptible compartment due to expiry of this immunity. Almost half of the susceptible population (of sheeps and goats) were infected with peste des petit ruminant disease at the contact rate, β , and start decreasing due to treatment at the rate, γ. This increasses the number of recovered animals. The process continuous to the time when all infected animals get recovered and the disease dies out.

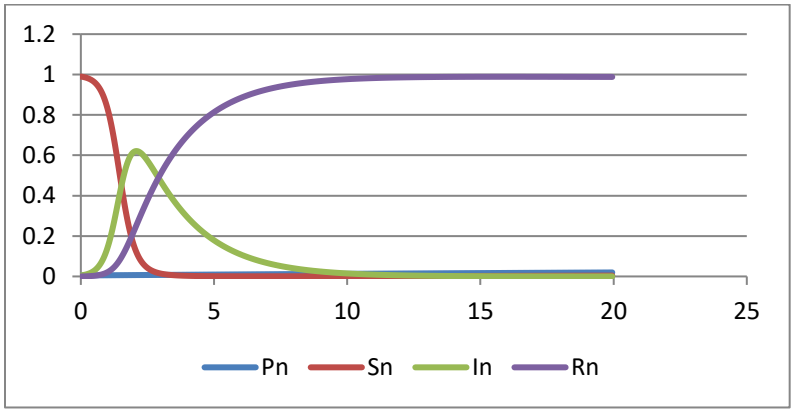

Figure 1.2 : At $\boldsymbol{\beta = 4.12088}$, more than half of the population became infected.

From Figure 1.2, when the value of β increases from $\beta = 2.74725$ to $\beta = 4.12088$ itindicates that the green curve shoots up which shows a considerable increase in the number of infected animals to more than half of the entire population.

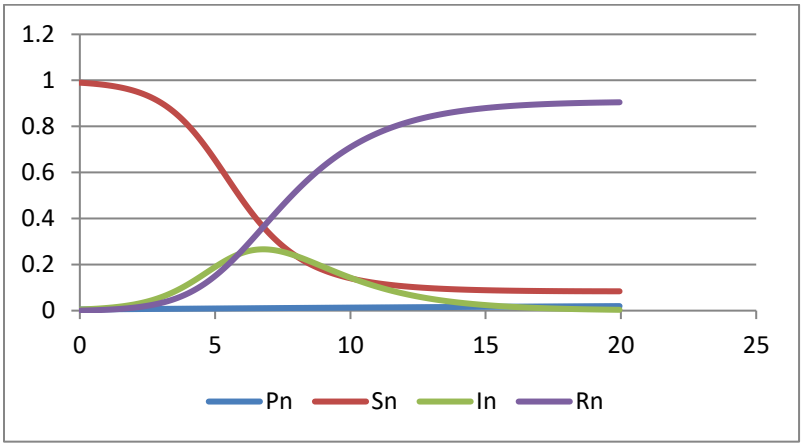

Figure 1.2: At $\boldsymbol{\beta = 1.37363}$, less than half of the population became infected.

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From figure1.3, when the value of β decreases from $\beta = 2.74725$ to $\beta = 1.37363$ itindicates that the green curve relax and fall down which shows a considerable decay in number of infected animals to less than half of the population. That is to say, there is a drastic reduction in disease as compared to both Figure 1 and Figure 2,

Basic Reproduction Number, R_o

The basic reproduction number is an important non-dimensional quantity in epidemiology as it sets the threshold in the study of a disease both for predicting its outbreak and for evaluating its control strategies. Thus, whether a disease becomes persistent or dies out in an area depends on the value of the reproduction number, R_0. Furthermore, stability of equilibrium can be analyzed using R_0. If < 1 it means that every infectious individual will cause less than one secondary infection and hence the disease will die out and when > 1, every infectious individual will cause more than one secondary infection and hence the disease will invade the population. A large number of R_0 may indicate the possibility of a major epidemic.

For the case of a model with a single infected class, R_0 is simply the product of the infection rate and the mean duration of the infection. From this model, the parameter γ refers to the removal rate and μ is the Natural death rates while ρ is the death rate as a result of the disease. Then, the average infections period will be given by: $\frac{1}{\gamma+\mu+\rho}$

Since the rate of transmission per infective is β then the basic reproduction number, R_o will be

$$R_o = \frac{1}{\gamma+\mu+\rho} \times \beta R_o = \frac{\beta}{\gamma+\mu+\rho}$$

Generally the higher the value of R_o, the harder it is to control the epidemic. When$R_o < 1$, the infection will die out in the long run. But when$R_o > 1$, the infection will invade.

The Threshold, $\frac{1}{R_o} = \frac{\gamma+\mu+\rho}{\beta}$

This implies, $R_o = \frac{\beta}{\gamma+\mu+\rho}$

Problem 1:

Take parameter values in the model system as ϒ = 0.213, β = 0.235, μ = 0.0034, ρ = 0.142.

For these parameter values, the basic reproduction number for the Disease-Free Equilibrium (DFE) is$R_o = 0.6557 < 1$. This shows that the infection is temporal and the disease dies out in time. If we keep the value of μ unchanged and let ϒ= 0.0213, β = 0.0651, ρ= 0.0142, then the basic reproduction number is calculated as$R_o = 1.6735 > 1$, it indicates that the disease becomes endemic. In this situation, an average infectious individual is able to replace itself and the number of infected rises and an epidemic reveals.

VI. Sensitivity Analysis

In this section, we perform sensitivity analysis to determine the contribution of each parameter to the basic reproduction number. Sensitivity Analysis (SA) is a common scientific modeling technique used to understand uncertainty in parameter estimation (Arendt *et al.*, 2012).

That is to say, sensitivity of a specific parameter identifies how a change in numeric value causes a corresponding impact on the model output.

This analysis determines the level of contribution of each parameter value to the reproduction number (Shaibu, Oluwole, and David 2018).

Since the basic reproductive number is: $R_o = \frac{\beta}{\gamma+\mu+\rho}$

Sensitivity index of the model parameter is given by the relation;

$$S_X^{R_0} = \frac{\partial R_0}{\partial X} \times \frac{X}{R_0}$$

where X represents any parameter in the model.

For β: $S_\beta^{R_0} = \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0}$

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$$= \frac{1}{\gamma+\mu+\rho} \times \frac{\beta(\gamma+\mu+\rho)}{\beta} = 1$$

For γ: $S_{\gamma}^{R_0} = \frac{\partial R_0}{\partial \gamma} \times \frac{\gamma}{R_0}$

$$= -\beta(\gamma+\mu+\rho)^{-2} \times \frac{\gamma(\gamma+\mu+\rho)}{\beta}$$

$$-\frac{\gamma}{(\gamma+\mu+\rho)}$$

For μ : $S_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} \times \frac{\mu}{R_0}$

$$= -\beta(\gamma+\mu+\rho)^{-2} \times \frac{\mu(\gamma+\mu+\rho)}{\beta}$$

$$= -\frac{\mu}{(\gamma+\mu+\rho)}$$

For ρ : $S_{\rho}^{R_0} = \frac{\partial R_0}{\partial \rho} \times \frac{\rho}{R_0}$

$$= -\beta(\gamma+\mu+\rho)^{-2} \times \frac{\rho(\gamma+\mu+\rho)}{\beta}$$

$$= -\frac{\rho}{(\gamma+\mu+\rho)}$$

Table 1.2: Sensitivity analysis of the parameter values

Parameter	Parameter values	Sensitivity Index
β	2.74725	1.00000
γ	0.50000	- 0.83279
μ	0.00039	- 0.00065
ρ	0.10000	- 0.16656

Table 1.2 shows the contribution of each parameter to the basic reproduction number. Given that the reproduction number is greater than unity, a decrease in the contact rate and animal recruitment rates by 50% cause a reasonable decrease in the basic reproduction number by almost 50%. Moreover, increasing the recovery rate by 20% would reduce the basic reproduction number by 14.3%. However, decreasing the recovery rate by 20% would increase the reproduction number by 19.9%.

VII. Conclusion

In this project, we have formulated a deterministic mathematical model for transmission dynamics of peste des petit ruminant disease (PPR).We have derived both the Disease Free Equilibrium (DFE) and the Endemic Equilibrium (EE) points. It was proved that the Disease Free Equilibrium (DFE) is conditionally stable while the Endemic Equilibrium (EE) is stable. The study has found that the disease not only causes animal death but also leads to reduction in milk yield and weight loss as well as meat scarcity, which reflect a huge liability on them and their farm economy.

VIII. Recommendations

It is recommended that animal suffering from peste des petit ruminant (PPR) diseases should be immediately quarantined so as to reduce the contact rate between the infected and the susceptible compartments. Tools and other items that have been in contact with the sick animals must be disinfected with common disinfectants such as phenol, sodium hydroxide 2%, virkon as well as alcohol, ether and detergents.

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