

Application of Soft Set Theory in Choice Selection and Purchase

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ABSTRACT

We study the fundamentals of soft set theory which includes soft set operations, soft set relations and functions with applications in purchase and decision making. Selection from different varieties of phones is considered in this paper.

Keywords: Application; Choice; Fuzzy set; Phones; Purchase; Soft set

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INTRODUCTION

The notion of theory of soft set as a means of handling uncertainties was introduced by Molodtsov [6]. The majority of real-life activities are associated with imprecise data, and their solutions are based on mathematical principles that deals with uncertainty. Some mathematical approaches such as fuzzy set, vague set, rough set, interval mathematics theory and probability theory are used in handling uncertainty before the introduction of theory of soft set. The limitation of fuzzy set was that it could not handle uncertainty problems properly because of its difficulties in setting membership function in the set, although it was found to be appropriate to some extent. Considering the inefficiency of the existing problems of uncertainty, soft set theory has emerged as an area of research to proffer solutions to uncertainty problems. Soft set theory, unlike traditional mathematics, demands an approximate description of an item as its starting point, rather than a perfect solution of a mathematical model. Furthermore, soft set theory is particularly efficient and straightforward to implement in reality due to the use of appropriate parameterization tools such as words, real numbers, and functions.

In recent times, studies on fundamentals and applications of soft set theory have been conducted. A systematic and critical study of the fundamentals of soft set theory which include operations on soft sets and their properties was studied by Onyeozili and Gwary [7]. In 2020, Borzooei *et al.* [2], introduced the concept of soft set theory to hoop and investigated some of its properties. However, some fundamental properties related to soft set theory and its application in purchase has not been studied. Recently, various other applications of soft set theory especially in decision making by some researchers like Maji *et al.* [5], Cagman *et al.* [3], Sut [8], Babitha and Sunil [1] have been developed. Thus, the study of soft set theory and its applications become imperative.

In this paper, phone is considered because it unites the world as a whole and solving a problem concerning it, means solving a world problem, and it is a convenient and common gadget in continuous demand.

MAIN RESULTS

Firstly, we shall give the definition of fuzzy set, soft set and its application in decision making especially in phone purchase for a particular family.

Definition 2.1: [4] Let Y be a subset of X , the indicator function μ_Y is defined as

$$\mu_Y = \begin{cases} 1, & \text{if } x \in Y \\ 0, & \text{if } x \notin Y \end{cases}$$

Let U be a universal set, a fuzzy set X over U is a mapping $\mu_X: U \rightarrow [0,1]$ defined by

$$X = \{(\mu_X(u)/u) : u \in U, \mu_X(u) \in [0,1]\}.$$

Denote μ_X as the membership function of X , and call the value $\mu_X(u)$ the grade of membership of $u \in U$ which represents the degree of u belonging to the fuzzy set X .

Example 2.1: Let $F(\alpha) = \{x \in U | \mu_Y(x) \geq \alpha\}$, $\alpha \in [0,1]$ be a family of α –level sets for function μ_Y . If the family F is known, $\mu_Y(x)$ is to be determined by

$$\mu_Y(x) = \text{Sup}_{\alpha \in [0,1]} \alpha \text{ where } x \in F(\alpha).$$

Thus, every fuzzy set Y may be considered as the soft set $(F, [0,1])$.

Soft set theory is a generalization of fuzzy set theory (sets whose elements have degrees of membership), that was proposed by Molodtsov [6] to deal with uncertainty in a parametric manner.

A soft set is a parameterised family of sets - intuitively, this is "soft" because the boundary of the set depends on the parameters. The main advantage of soft set theory is that, it is very easy to apply and it provides the optimum solution to results. To apply the soft set theory method, it is necessary to state all the constraints in the universe (U) and all the set of parameters (S), it is necessary to know one's choice (a constraint) which seems to be the most suitable for a particular Mr. 'K' family. Then, the soft set theory is said to be applicable.

Definition 2.2. [6] Let $p(U)$ denotes the power set of a universal set U . A pair (f, S) is called a soft set over U , if f is a mapping of a set of parameters S , into the power set of U . That is, $f: S \rightarrow p(U)$. In other words, a soft set over U is a parameterized family of subsets of the universal set U . Also, for any $a \in S$, $f(a)$ is considered as the set of a –approximates element of the soft set (f, S) .

Definition 2.3. The class of all value sets of a soft set (f, S) is called valued-class of the soft set and is denoted by $Cl_{(f,S)}$.

Let $Cl_{(f,S)} = \{v_1, v_2, \dots, v_n\}$. Clearly $Cl_{(f,S)} \subseteq p(U)$.

Definition 2.4. Let (f, A) and (g, B) be soft sets over a common universal set U , (f, A) is a soft subset of (g, B) if

- (i) $(A \subset B)$, and
- (ii) for all $\varepsilon \in A$, $f(\varepsilon)$ and $g(\varepsilon)$ are identical approximations.

We write $(f, A) \supseteq (g, B)$.

(f, A) is said to be a soft super set of (g, B) , if (g, B) is a soft subset of (f, A) . We denote it by $(f, A) \supseteq (g, B)$.

We state without proof some of the basic properties of soft set operations by Onyeozili and Gwary [7] as follows:

1. Idempotent properties

- (i) $(f, A) \sim \mathcal{U}(f, A) = (f, A) \cup_R (f, A)$

$$(ii) (f, A) \tilde{\cap} (f, A) = (f, A) \cap_s (f, A)$$

2. Domination properties

$$(i) (f, A) \tilde{\cup} U = \tilde{U} = (f, A) \cup_R \tilde{U}$$

$$(ii) (f, A) \tilde{\cap} \tilde{\Phi} = \tilde{\Phi} = (f, A) \cap_s \tilde{\Phi}$$

3. Complementation properties

$$(i) \tilde{\Phi} = \tilde{U} \tilde{\Phi}$$

$$(ii) \tilde{\theta} = \tilde{\Phi} \tilde{U}$$

4. Involution property

$$((f, A)^c)^c = (f, A) = ((f, A)^r)^r.$$

5. Exclusion property

$$(i) (f, A) \tilde{\cup} (f, A)^r = \tilde{U} = (f, A) \cup_R (f, A)^r$$

6. Contradiction property

$$(ii) (f, A) \tilde{\cap} (f, A)^r = \tilde{\Phi} = (f, A) \cap_s (f, A)^r.$$

For the purpose of this research work, data was collected through structured phone purchase survey questionnaires. The main advantage of questionnaire is that they are affordable unlike telephone surveys and it has standard answers for easy data compilation. Questionnaire is more applicable because it gives room for speedy result, anonymity and it is appropriate for a large audience. The data used in this research was collected from a particular phone company where the ages of respondents are known.

The Table below gives the age range of the respondents.

Table 1: Age of Respondents.

Age	Frequency	Percentage
18-22	11	24.4
23-27	21	46.7
28-32	6	13.3
33-37	4	8.9
38-42	1	2.2
Above 42	2	4.4
Total	45	100

The larger part of the respondents is between the ages of 23-27.

Table 2: Features determining purchase

S/N	Popular Phone Brands	Reliability	Battery Life	Best added Technology	Best features	Cheapest	Score rating	Ranking
1	Nokia	1	1	1	2	0	5	11

2	Samsung	10	5	13	15	2	45	4
3	iPhone	16	3	17	14	4	54	3
4	Huawei	2	3	4	3	3	15	6
5	Oppo	4	6	6	7	2	25	5
6	Infinix	12	18	8	14	4	56	2
7	Itel	1	2	1	2	7	13	7
8	Tecno	15	21	9	9	26	80	1
9	Vivo	1	3	3	3	0	10	8
10	Xiaomi	2	1	1	3	0	7	9
11	Redmi	1	2	1	2	0	6	10
	Total	65	65	64	74	48		

The phone companies sell eleven varieties of phones. Five phones will be considered under the universal set U and S to be set of parameters which describes the features of the phones under consideration.

The topmost brands of mobile phones sold by the companies based on the above factors of reliability, battery life, best added technology, and affordability are as follows (according to ranking): Tecno, Infinix, iPhone, Samsung and Oppo. The mobile phones with the lowest patronage were (according to ranking) Nokia, Redmi, Xiaomi, Vivo, Itel and Huawei.

Let the topmost brands of mobile phones sold based on the following factors: reliability, battery life, best added technology, and affordability be represented in the table as follows for soft set application.

Let $U = \{M_1, M_2, M_3, M_4, M_5\}$ be the set of Phones under consideration, S be a set of parameters such that

$$S = \{a_1 = \text{Reliability}, a_2 = \text{Battery life}, a_3 = \text{Best Added technology}, a_4 = \text{Best Features}, a_5 = \text{Affordability}\}.$$

Then the soft set (F, S) describes the affordability of the phones.

Table 3: Tabular representation of a soft set.

U	PHONES	Reliability	Battery life	Best Added technology	Best Features	Affordability
M_1	Tecno	15	21	9	9	26
M_2	Infinix	12	18	8	14	4
M_3	IPhone	16	3	17	14	4
M_4	Samsung	10	5	13	15	2
M_5	Oppo	4	6	6	7	2

Here is an application that makes the theory of soft set very convenient and easy in solving real life problem of decision making. We will be considering a particular family of Mr. K in this research. The family of Mr. K wants to make a phone purchase, and his wife, son and daughter has their own opinion on the parameter Mr. K should consider while making his choice.

Soft set theory is applied here to enable Mr. K makes the right choice which must also cover the opinion of his family members.

Consider the five mobile phones in the universal set U given by

$$U = \{M_1, M_2, M_3, M_4, M_5, \} \quad \text{and a set of parameters}$$

$$S = \{a_1, a_2, a_3, a_4, a_5\}.$$

Parameters

a_1 represents 'Reliability'

a_2 represents 'Battery life'

a_3 represents 'Best Added technology'

a_4 represents 'Best Features'

a_5 represents 'Affordability'

Result analysis

$$f(a_1) = \{ M_1, M_2, M_3, M_4 \}$$

$$f(a_2) = \{M_1, M_2\}$$

$$f(a_3) = \{M_1, M_3, M_4\}$$

$$f(a_4) = \{M_1, M_2, M_3, M_4\}$$

$$f(a_5) = \{M_1\}$$

Sets under consideration

M_1 – Techo

M_2 – Infinix

M_3 – iPhone

M_4 – Samsung

M_5 – Oppo.

The soft set (f, S) is a parameterized family $\{f(a_i), i = 1, 2, 3, 4, 5\}$ of subsets of the set U and gives us a collection of approximate descriptions of an object. Consider the mapping f which is "phones (.)" where dot (.) is to be filled up by a parameter $a \in S$. Therefore, $F(a_1)$ means "phones (Reliable)" whose functional-value is the set $\{M_4, M_5\}$.

Thus, we can view the soft set (f, S) as a collection of approximations as below:

$(f, S) = \{ \text{Reliable mobile phones} = \{M_4, M_5\}, \text{battery life} =, \{M_3, M_5\}, \text{best added technology} = \{M_1, M_4, M_5\}, \text{best features} = \{M_1, M_3, M_5\}, \text{most affordable} = \{M_5\} \}$, where each approximation has two parts:

i) a predicate P ; and

ii) an approximate value-set v .

For example, for the approximation "reliable mobile phones = $\{M_4, M_5\}$ ", we have the following:

i) the predicate name is reliable phones; and

ii) the approximate value set or value set is $\{M_4, M_5\}$.

Thus, a soft set (f, S) can be viewed as a collection of approximations below:

$$(f, S) = \{M_1 = v_1, M_2 = v_2, \dots, M_n = v_n\}.$$

Now, we present the family interest below:

MOTHER (affordable, reliable)

$$f_A = \{(a_1\{M_1, M_2, M_3, M_4\}), (a_5\{M_1\})\}$$

SON (Best added technology, affordable, strong battery life)

$$f_B\{(a_2\{M_1, M_2\}), (a_5\{M_1\}), (a_3\{M_1, M_3, M_4\})\}$$

DAUGHTER (affordability, best features, best technology)

$$f_C = \{(a_5\{M_1\}), (a_4\{M_1, M_2, M_3, M_4\}), (a_3\{M_1, M_3, M_4\})\}.$$

Let us consider the common interest of the family.

FATHER D = $\{a_1, a_2, a_4, a_5\}$

$$f_D = \{(a_1\{M_1, M_2, M_3, M_4\}), (a_2\{M_1, M_2\}), (a_3\{M_1, M_3, M_4\}), (a_5\{M_1\})\}$$

MOTHER A = $\{a_1, a_2, a_4, a_5\}$

$$g_A = \{(a_1\{M_1, M_2, M_3, M_4\}), (a_2\{M_1, M_2\}), (a_4\{M_1, M_2, M_3, M_4\}), (a_5\{M_1\})\}$$

SON B = $\{a_2, a_3, a_5\}$

$$h_B = \{(a_2\{M_1, M_2\}), (a_3\{M_1, M_3, M_4\}), (a_5\{M_1\})\}$$

DAUGHTER C = $\{a_3, a_4, a_5\}$

$$i_C = \{(a_3\{M_1, M_3, M_4\}), (a_4\{M_1, M_2, M_3, M_4\}), (a_5\{M_1\})\}$$

Now we find the intersection of the family soft set, which is their common interest.

That is, $f_D \cap g_A \cap h_B \cap i_C = j_i$

Where $I = D \cap A \cap B \cap C$.

$$I = \{(a_1\{M_1, M_2, M_3, M_4\}), (a_2\{M_1, M_2\}), (a_3\{M_1, M_3, M_4\})\}$$

$$\cap \{(a_1\{M_1, M_2, M_3, M_4\}), (a_5\{M_1\})\}$$

$$\cap \{(a_2\{M_1, M_2\}), (a_3\{M_1, M_3, M_4\})\} \cap \{(a_3\{M_1, M_3, M_4\}), (a_4\{M_1, M_2, M_3, M_4\})\}$$

$$I = \{M_1\} \Rightarrow a_5.$$

Find $j(a)$ for $a \in I$,

$$j(a) = f(a_5) \cap g(a_5) \cap h(a_5) \cap i(a_5) = \{\{M_1\} \cap \{M_1\} \cap \{M_1\} \cap \{M_1\}\} = \{M_1\}.$$

Since the set $\{M_1\}$ which is the soft set intersection satisfies the family's choice, hence $\{M_1\}$ -Techno phone is the best choice for the family of Mr. K.

CONCLUSIONS

In this paper, we have been able to minimize the cost of purchase by choosing most affordable mobile phone M_1 for Mr. K, and also put his family members' interest into consideration, whereby making decision of maximum satisfaction for him and the member of his family.

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Conflict of interest

The authors declare that they have no competing interests among them during the time of writing this paper.

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REFERENCES

1. Babitha, K. V. and Sunil, J. J. (2010). Soft sets relations and functions, *Comp. Math. Appl.* 60, 1840-1849.
2. Borzooei, R. A., Babaei, E., Jun Y. B., Aaly Kologani M., and Mohseni Takallo M. (2020). Soft set theory applied to hoops, *Sciendo*, Vol. 28 (1), 61-79.
3. Cagman, N., Citak, F. and Enginoglu, S. (2010). Fuzzy parameterized fuzzy soft set theory and its applications, an official journal of Turkish fuzzy systems association. Vol.1, no. 1, pp 21-35.
4. Ibrahim, A. M. and Yusuf, A. O. (2012). Development of Soft Set Theory, *American International Journal of Contemporary Research*, Vol. 2 No. 9; pp. 205-210.
5. Maji, P. K., Roy A. R. and Biswas, R. (2002). An application of soft sets in decision making problem, *Computers and Mathematics with Applications*, 44(8/9), 1077-1083.
6. Molodtsov, D. A. (1999). Soft set theory - First results, *Computers Math. Appl.* 37 (4/5), 19-31.
7. Onyeozili, I. A. and Gwary, T. M. (2014). A study of the fundamentals of soft set theory, *Int. Journal of Scientific and Tech. Research*. Vol. 3. Issue 4, 132-143.
8. Sut, D. K. (2012). An Application of Fuzzy soft Relation in Decision Making Problems, *Int. Journal of Mathematics Trends and Technology*, Vol. 3, Issue 2.