Weighted Rayleigh Distribution

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ABSTRACT

This paper introduces the Weighted Rayleigh (WR) distribution by inducing inverted weight function into the existing Rayleigh distribution. Statistical and mathematical expressions of its properties such as Survival Function, Hazard Function, Moments, Moment Generating Function, Mean Deviation and Renyi entropy were explicitly derived. The model's parameter was estimated using maximum likelihood method of estimation. Two real life data sets on cancer and waiting time before service were considered to assess the flexibility of the Weighted Rayleigh distribution over existing distributions. The distributions performance were compared using Log-likelihood and Akaike Information Criteria (AIC). The Weighted Rayleigh distribution fits the real life data better than the Rayleigh, Inverse Weibull (IW) and Weighted Inverse Weibull (WIW) distributions.

Keywords: Rayleigh distribution, Generalization, Inversion, Statistical Properties, Weighted distributions, Azzalini.

INTRODUCTION

Weighted probability distribution has been considered in several real life fields. The approach of weighted distributions made an adjustment to the probabilities of the events by introducing weight function into the baseline distribution. It also provides a way of dealing with specifying an appropriate and effective model with data interpretation problems. In environmental science, observations can possibly fall in the non-experimental category Patil (2002). The weighted distribution addresses problems of specifying an appropriate and effective distribution.

Badmus, Bamiduro, and Ogunobi, (2014) modified the method used on weighted weibull model proposed by Azzalini (1985) using the logit of Beta function to have Lehmann Type II weighted weibull model with a view to obtaining a distribution which has an improvement over both weighted weibull and weibull distribution in terms of estimate of their characteristics and their parameter. The weighted weibull distribution is proposed by slightly modifying the method of Azzalini (1985) on weighted distribution with additional shape (β) and scale (λ) parameters. Mahdavi, (2015) also presented a paper, where new classes of weighted distributions was proposed by incorporating exponential distribution in Azzalini's method. Resulting weighted models generated by exponential distribution are: the weighted gamma-exponential model and the weighted generalized exponential-exponential model. The hazard rate function of these distributions has different shapes including increasing, decreasing and unimodal.

This study focuses on proposing a new method of extends an existing Rayleigh distribution, thereby developing

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unifying distribution that are flexible for analysing lifetime data with extraneous variation

The concept of weighted distribution was unified by Rao (1965) after being introduced by Fisher (1934). Let X denote a non negative continuous random variable with its probability density function *f(x)*, then the probability density function of the weight random variable $f_w(x)$ is given by:

$$f_w(x) = \frac{w(x) f(x)}{w_d}$$

where *w(x)* is the weight function and $w_d = \int_0^\infty w(x) f(x) dx$

The CDF and PDF of Rayleigh (RD) distribution with parameter β are defined respectively by:

$$F(x);= e^{-\frac{x^2}{2\beta^2}} \quad x > 0, \beta > 0 \tag{1}$$

$$f(x) = \frac{x}{\beta^2} e^{-\frac{x^2}{2\beta^2}}; \quad x > 0, \beta > 0 \tag{2}$$

THE WEIGHTED RAYLEIGH (WR) DISTRIBUTION

Suppose weight function $w(x) = x^{-1}$ and considering one-parameter Raiyeh distribution as defined in Equation (1) and (2) then the PDF and CDF of Weighted Rayleigh distribution are given by:

$$f_w(x) = \frac{2^{\frac{1}{2}}}{\beta\Gamma\left(\frac{1}{2}\right)} e^{-\frac{x^2}{2\beta^2}} \quad x > 0, \beta > 0 \tag{3}$$

and

$$F_w(x) = \frac{\Gamma\left(\frac{1}{2}, \frac{x^2}{2\beta^2}\right)}{\Gamma\left(\frac{1}{2}\right)} \quad x > 0, \beta > 0 \tag{4}$$

and β is the scale parameter

Derivation of WR Distribution

$$f_w(x) = \frac{w(x) f(x)}{w_d} \tag{5}$$

where *f(x)* is the pdf of Rayleigh distribution as defined in Equation (2) and $w(x) = x^{-1}$

also $w_d = \int_0^\infty w(x) f(x) dx$

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$$w_d = \int_0^\infty \frac{1}{\beta^2} e^{\frac{x^2}{2\beta^2}} dx \qquad (6)$$

Substitute $k = \frac{x^2}{2\beta^2}$ such that $x = 2^{\frac{1}{2}}\beta k^{\frac{1}{2}}$ and $dx = \frac{1}{2} 2^{\frac{1}{2}} \beta k^{\frac{1}{2}-1} dk$

$$w_d = \frac{\Gamma\left(\frac{1}{2}\right)}{2^{\frac{1}{2}}\beta} \qquad (7)$$

Therefore; $f_w(x) = \frac{w(x) f(x)}{w_d}$ (8)

$$f_w(x) = \frac{x^{-1}\beta 2^{\frac{1}{2}} x e^{-\frac{x^2}{2\beta^2}}}{\beta^2 \Gamma\left(\frac{1}{2}\right)} \qquad (9)$$

$$f_w(x) = \frac{2^{\frac{1}{2}}}{\beta\Gamma\left(\frac{1}{2}\right)} e^{-\frac{x^2}{2\beta^2}} \qquad (10)$$

Equation (10) is the pdf of the Weighted Rayleigh distribution.

Its associated cdf is obtained as follows:

$$F_w(x) = \int_0^x f_w(x) dx \qquad (11)$$

$$F_w(x) = \frac{2^{\frac{1}{2}}}{\beta\Gamma\left(\frac{1}{2}\right)} \int_0^x e^{-\frac{x^2}{2\beta^2}} dx \qquad (12)$$

Substitute $k = \frac{x^2}{2\beta^2}$ such that $x = 2^{\frac{1}{2}}\beta k^{\frac{1}{2}}$ and $dx = \frac{1}{2} 2^{\frac{1}{2}} \beta\, k^{\frac{1}{2}-1} dk$

$$F_w(x) = 1 - \frac{1}{\Gamma\left(\frac{1}{2}\right)} \int_{\frac{x^2}{2\beta^2}}^\infty k^{\frac{1}{2}-1} e^{-t} dk \qquad (13)$$

$$F_w(x) = 1 - \frac{\Gamma\left(\frac{1}{2}, \frac{x^2}{2\beta^2}\right)}{\Gamma\left(\frac{1}{2}\right)} \qquad (14)$$

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Proof of validity of WR Distribution

For the PDF to be valid, it suffices that; $\int_{0}^{\infty} f_w(x)\,dx = 1$

$$\frac{2^{\frac{1}{2}}}{\beta\Gamma\left(\frac{1}{2}\right)}\int_{0}^{\infty} e^{-\frac{x^2}{2\beta^2}}dx = 1 \tag{15}$$

Substitute $k = \frac{x^2}{2\beta^2}$ such that $x = 2^{\frac{1}{2}}\ \beta\ k^{\frac{1}{2}}$ and $dx = \frac{1}{2}2^{\frac{1}{2}}\beta k^{\frac{1}{2}-1}dk$

$$\frac{2^{\frac{1}{2}}}{\beta\Gamma\left(\frac{1}{2}\right)}\int_{0}^{\infty} e^{-k}\frac{1}{2}2^{\frac{1}{2}}\beta k^{\frac{1}{2}-1}dk \tag{16}$$

$$\frac{1}{\Gamma\left(\frac{1}{2}\right)}\int_{0}^{\infty} k^{\frac{1}{2}-1}e^{-t}dk \tag{17}$$

$$\frac{1}{\Gamma\left(\frac{1}{2}\right)}\left[\Gamma\left(\frac{1}{2}\right)\right] = 1 \tag{18}$$

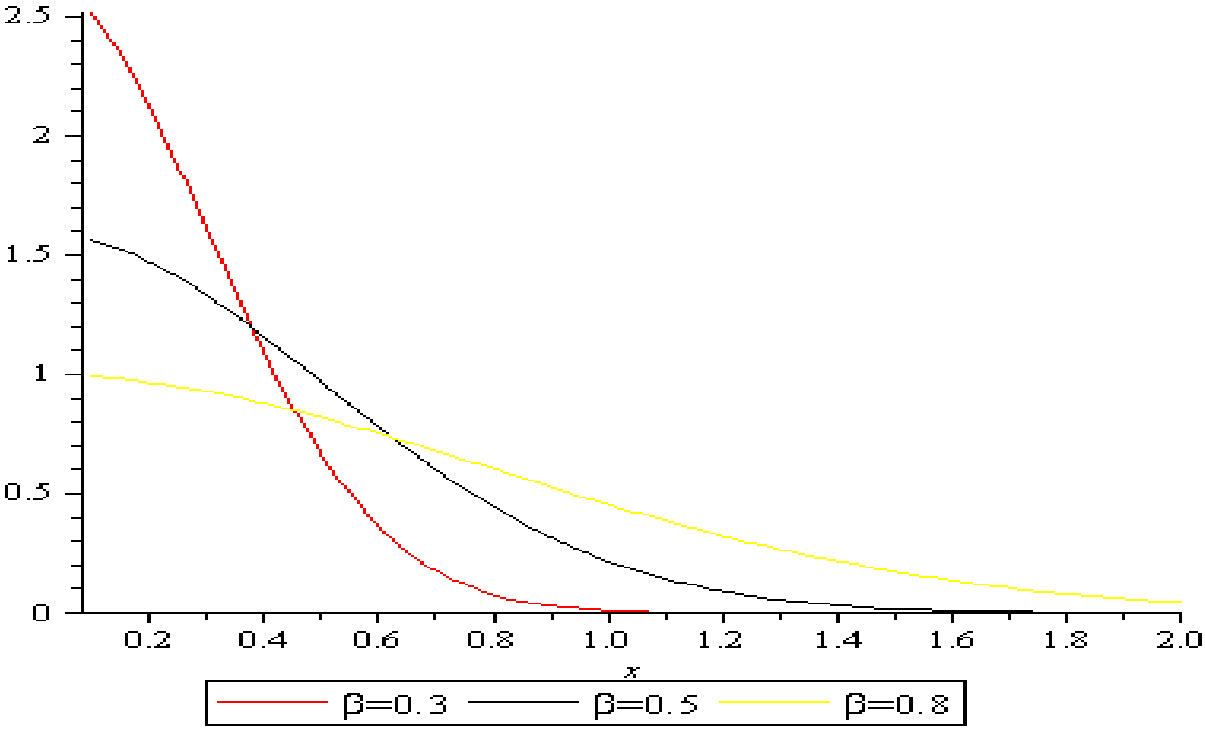

Figure 1: Plot of probability density function of WR distribution

The plot in Figure 1 show that the shape of the WR distribution is unimodal (inverted bathtub) and decreasing shapes depending on the value of the shape parameter.

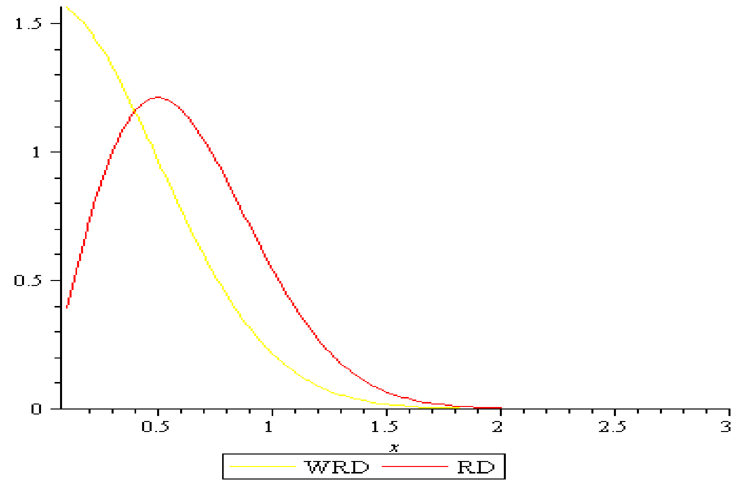

Figure 2: Plot of probability density function of WR and Rayleigh distributions

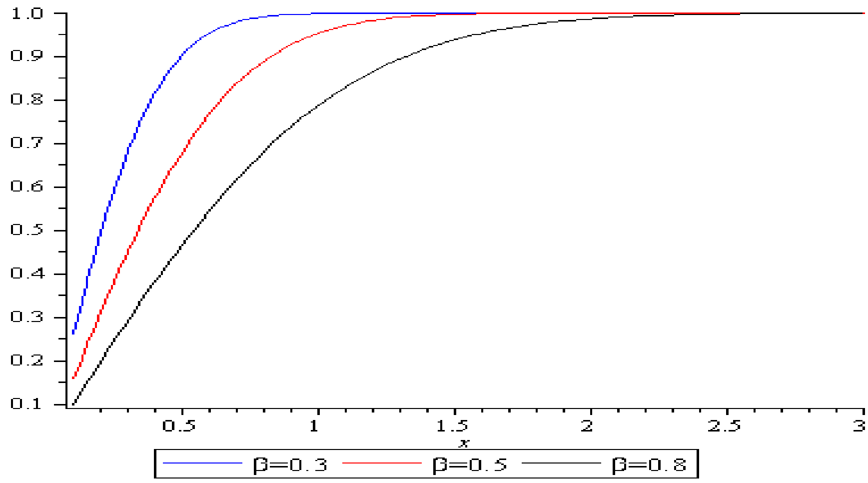

Figure 3: Plot of cumulative density function of WR distribution

Figure 2 shows shape of Weighted Rayleigh and Rayleigh distributions at the same parameter value ($\beta = 0.5$). It revealed that the distribution is unimodal. Figure 3 illustrates cumulative density function shapes of Weighted Rayleigh at different parameter values and further established the validity of the distribution as none of the shapes exceeded one

RELIABILITY ANALYSIS

Survival Function

The Survivor function indicates the probability that the event of interest has not yet occurred by time x; thus, if X denotes time until failure, *S(x)* denotes probability of surviving beyond x.

Survival (or reliability) function is derived from:

$$S(x) = 1 - F(x)$$

$$S(x) = \left[\frac{\Gamma\left(\frac{1}{2}, \frac{x^2}{2\beta^2}\right)}{\Gamma\left(\frac{1}{2}\right)} \right] \quad (19)$$

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Hazard function

Hazard function is a conditional probability that the event of interest (device failure) will occur during the time t and dt under the condition that the device is safe until time t. hazard function is defined as the ratio of probability density function *f(x)* to the survival function *S(x)* given by:

$$h(x)=\frac{2^{\frac{1}{2}}e^{-\frac{x^2}{2\beta^2}}}{\beta\Gamma\left(\frac{1}{2},\frac{x^2}{2\beta^2}\right)} \quad (20)$$

Reversed Hazard Function of WR Distribution

The Reverse Hazard function can be expressed as the ratio of the pdf *f(x)* to the cdf *F(x)*

Therefore, the RH function for the WR is given by:

$$r(x)=\frac{2^{\frac{1}{2}}e^{-\frac{x^2}{2\beta^2}}}{\beta\left(\Gamma\left(\frac{1}{2}\right)-\Gamma\left(\frac{1}{2},\frac{x^2}{2\beta^2}\right)\right)} \quad (21)$$

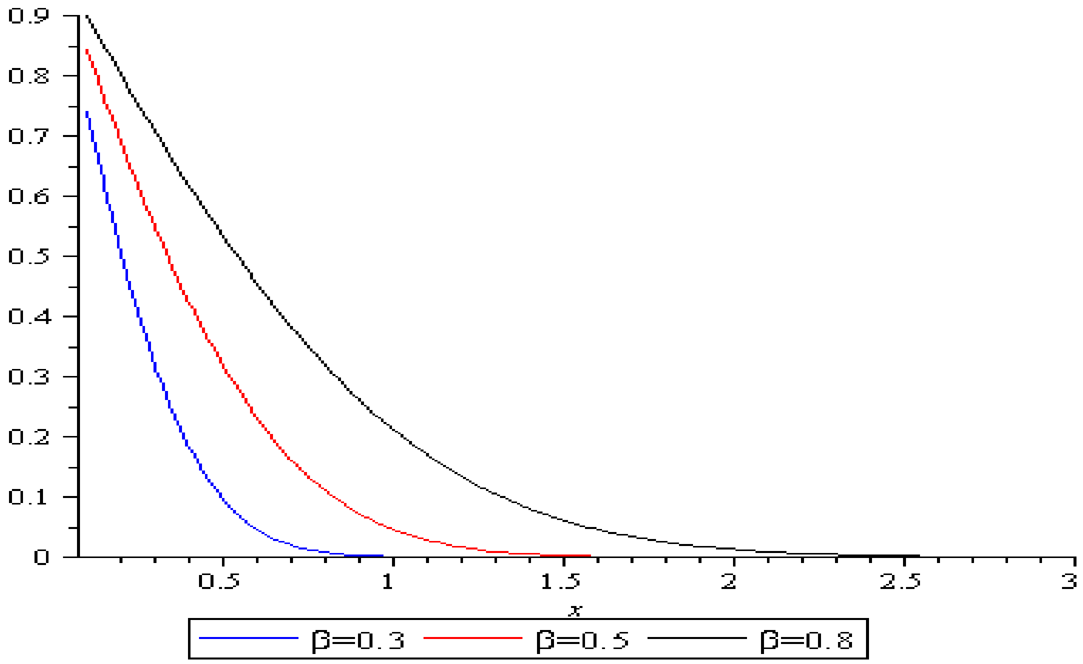

Figure 4: Survival plot of the Weighted Rayleigh Distribution

Figure 4 shows survival function of the Weighted Rayleigh distribution at different parameter values. It further revealed that parameter value increases alongside with survival time.

MOMENTS AND ASSOCIATED MEASURES OF WR DISTRIBUTION

The moment of distribution is very important, it is used to describe the characteristics of a distribution such as measurement of central location, dispersion, asymmetry and peakedness. The moments helps to determine the

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mean, dispersion, coefficient of skewness and kurtosis of the Weighted Rayleigh distribution. The jth moments of a non-negative random variable X is defined as

$$E\left(X^{j}\right)=\int_{0}^{\infty} x^{j} f_{w}\left(x\right) dx \tag{22}$$

$$E\left(X^{j}\right)=\int_{0}^{\infty} x^{j} \frac{2^{\left(\frac{1}{2}\right)}}{\Gamma\left(\frac{1}{2}\right)} e^{-\frac{x^{2}}{2\beta^{2}}} dx \tag{23}$$

Substitute $k = \frac{x^{2}}{2\beta^{2}}$ such that $x = 2^{\frac{1}{2}} \beta\, k^{\frac{1}{2}}$ and $dx = \frac{1}{2} 2^{\frac{1}{2}} \beta k^{\frac{1}{2}-1} dk$

$$E\left(X^{j}\right)=\frac{2^{\left(\frac{1}{2}\right)}}{\beta\Gamma\left(\frac{1}{2}\right)} \int_{0}^{\infty}\left(2^{\frac{1}{2}} \beta k^{\frac{1}{2}}\right)^{j} e^{-k}\, 2^{\frac{1}{2}} \beta\; k^{\frac{1}{2}-1} dk \tag{24}$$

$$E\left(X^{j}\right)= \frac{2^{\frac{j}{2}}}{\Gamma\left(\frac{1}{2}\right)} \int_{0}^{\infty} k^{\left(\frac{j-1}{2}\right)} e^{-k} dk \tag{25}$$

$$E\left(X^{j}\right)=\frac{2^{\frac{j}{2}} \beta^{j} \Gamma\left(\frac{1+j}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} \tag{26}$$

Equations (27), (30), (32), (34) and (36) give the Mean, Standard Deviation (SD), Coefficient of Variation (CV), Coefficient Skewness (CS) and Coefficient Kurtosis (CV) of WR respectively

$$\mu=\frac{2^{\frac{1}{2}} \beta}{\Gamma\left(\frac{1}{2}\right)} \tag{27}$$

$$\sigma^{2}=E\left(X^{2}\right)-\left(\mu\right)^{2} \tag{28}$$

$$\sigma^{2}=\frac{2\beta^{2}\left[\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right)-1\right]}{\Gamma^{2}\left(\frac{1}{2}\right)} \tag{29}$$

$$\sigma=\frac{2^{\frac{1}{2}} \beta\left[\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right)-1\right]^{\frac{1}{2}}}{\Gamma\left(\frac{1}{2}\right)} \tag{30}$$

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume IX Issue VIII August 2024

$$CV = \frac{\sigma}{\mu} \tag{31}$$

$$CV = \left[\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right) - 1 \right]^{1/2} \tag{32}$$

$$CS = \frac{E\left(X^3\right) - 3\mu\sigma^2 - \mu^3}{\sigma^3} \tag{33}$$

$$CS = \frac{\Gamma^2\left(\frac{1}{2}\right) - 3\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right) + 2}{\left[\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right) - 1 \right]^{3/2}} \tag{34}$$

$$CK = \frac{E\left(X^4\right) - 4\mu E\left(X^3\right) + 6\mu^2\sigma^2 + 3\mu^4}{\sigma^4} \tag{35}$$

$$CK = \frac{\Gamma^3\left(\frac{1}{2}\right) - 4\Gamma^2\left(\frac{1}{2}\right) + 6\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right) - 3}{\left[\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right) - 1 \right]^{2}} \tag{36}$$

MOMENT GENERATING FUNCTION OF WR DISTRIBUTION

Following (Cordeiro, 2011) the expression for moment generating function for x having a weighted Rayleigh distribution is obtained as

$$M_x(t) = \sum_{j=0}^{n} \left[\frac{t^j}{j!} \frac{2^{\frac{j}{2}} \beta^j \Gamma\left(\frac{1+j}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} \right] \tag{37}$$

The moment generating function is the expected value of exponential function of tX, i.e, the moment generating function of random variable X is given as:

$$M_x(t) = E\left(e^{tX}\right) \tag{38}$$

and $$E\left(e^{tX}\right) = \int_0^\infty e^{tX} f_w(x)\, dx \tag{39}$$

substitude Equation (39) into (38), we have :

$$M_x(t) = \int_0^\infty e^{tX} f_w(x)\, dx \tag{40}$$

with the use of Taylor’s series, Equation (40) becomes

$$M_x(t)=\int_0^\infty\left(1+\frac{tx}{1!}+\frac{t^2x^2}{2!}+...+\frac{t^jx^j}{j!}...\right)f_w(x)\,dx \quad (41)$$

$$M_x(t)=\sum_{j=0}^{\infty}\frac{t^j}{j!}E\left(X^j\right) \quad (42)$$

and $E(X^j)$ is defined in Equation (37) above, then

$$M_x(t)=\sum_{j=0}^{n}\left[\frac{t^j}{\text{j!}}\frac{2^{\frac{j}{2}}\beta^j\Gamma\left(\frac{1+j}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}\right] \quad (43)$$

MEAN DEVIATION OF WR DISTRIBUTION

The deviation from the mean measures spread and deviation of values from the mean. Let x be a random variable from WR distribution with mean $\mu = E(x)$. The expression for mean deviation is defined as

$$MD(\mu)=E\ \left\{\left|x-\mu\right|\right\} \quad (44)$$

$$MD(\mu)=\int_0^\infty\left|x-\mu\right| f_w(x)\,dx \quad (45)$$

$$MD(\mu)=2\mu F_w(\mu)-2\int_0^\mu x f_w(x)\,dx \quad (46)$$

where $F_w(\mu)$ in Equation (46) is obtained as

$$F_w(\mu)=\frac{\Gamma\left(\frac{1}{2},\frac{\mu^2}{2\beta^2}\right)}{\Gamma\left(\frac{1}{2}\right)} \quad (47)$$

also $\int_0^\mu x f_w(x)\,dx$ in Equation (46) is obtained as

$$\int_0^\mu x f_w(x)\,dx=\frac{2^{\frac{1}{2}}\beta\Upsilon(1,\mu)}{\Gamma\left(\frac{1}{2}\right)} \quad (48)$$

where $\Upsilon(1,\mu)$ is expression for lower incomplete gamma function.

Substitute Equation (47) and (48) into (46), then the Mean Deviation (MD) is obtained as;

$$MD(\mu)=2\frac{2^{\frac{1}{2}}\beta}{\Gamma\left(\frac{1}{2}\right)}\frac{\Gamma\left(\frac{1}{2},\frac{\mu^2}{2\beta^2}\right)}{\Gamma\left(\frac{1}{2}\right)}-2\frac{2^{\frac{1}{2}}\beta\Upsilon(1,\mu)}{\Gamma\left(\frac{1}{2}\right)} \quad (49)$$

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume IX Issue VIII August 2024

RENYI ENTROPY OF WR DISTRIBUTION

The entropy of X is a randomness measure of the system, see Renyi (1961).

The expression for the entropy is defined as

$$I_{R(\rho)} = \frac{1}{1-\rho}\log\left(\int_0^\infty f_w(x)^\rho\, dx\right) \tag{50}$$

where $\rho > 0 \quad and \quad \rho \neq 1$

$$I_{R(\rho)} = \frac{1}{1-\rho}\log\left(\int_0^\infty \left(\frac{2^{\left(\frac{1}{2}\right)}}{\beta\Gamma\left(\frac{1}{2}\right)} e^{-\frac{x^2}{2\beta^2}}\right)^\rho dx\right) \tag{51}$$

$$I_{R(\rho)} = \frac{1}{1-\rho}\log\left(\frac{2^{\left(\frac{\rho}{2}\right)}}{\beta^\rho\Gamma^\rho\left(\frac{1}{2}\right)}\int_0^\infty e^{-\frac{x^2}{2\beta^2}} dx\right) \tag{52}$$

$$let \quad t = \frac{x^2\rho}{2\beta^2} \quad then \quad x = 2^{\frac{1}{2}}\beta\, t^{\frac{1}{2}}\rho^{-\frac{1}{2}} \quad and \quad dx = 2^{-\frac{1}{2}}\beta\,\rho^{-\frac{1}{2}}t^{\frac{1}{2}-1}$$

$$I_{R(\rho)} = \frac{1}{1-\rho}\log\left(\frac{2^{\frac{\rho-1}{2}}\beta^{1-\rho}}{\Gamma^\rho\left(\frac{1}{2}\right)\rho^{\frac{1}{2}}}\int_0^\infty e^{-t}t^{\frac{1}{2}-1}dt\right) \tag{53}$$

$$I_{R(\rho)} = \frac{1}{1-\rho}\log\left(\frac{2^{\frac{\rho-1}{2}}\beta^{1-\rho}\Gamma\left(\frac{1}{2}\right)}{\Gamma^\rho\left(\frac{1}{2}\right)\rho^{\frac{1}{2}}}\right) \tag{54}$$

PARAMETER ESTIMATION OF WR DISTRIBUTION

The Estimation of Weighted Rayleigh is obtained using the MLE as follows:

Let $x_1, x_2 \ldots\ldots x_n$ be a random sample of size "n" from Weighted Rayleigh defined in Equation (3) and (4), then the likelihood function $Lw(\beta)$ is expressed by

$$L_w(\beta) = \prod_{i=1}^{n} f_w(\mathrm{x}_i/\beta) \tag{55}$$

Then the likelihood function of Weighted Rayleigh is expressed as:

$$L_w(\beta)=\prod_{i=1}^{n}\left(\frac{2^{\frac{1}{2}}}{\beta\Gamma\left(\frac{1}{2}\right)}e^{-\frac{x_i^2}{2\beta^2}}\right) \quad (56)$$

and therefore, the log of the likelihood function is given as:

$$\log L_w(\beta)=\frac{n}{2}\log(2)-\text{nlog}(\beta)-\frac{1}{2\beta^2}\sum_{i=1}^{n}x^2-n\log\left(\Gamma\left(\frac{1}{2}\right)\right) \quad (57)$$

Differentiating Equation (57) with respect to β

$$\frac{\log L_w(\beta)}{d\beta}=\frac{n}{\beta}+\frac{1}{\beta^3}\sum_{i=1}^{n}x^2 \quad (58)$$

The MLE of parameter β is obtained by setting Equation (58) to zero and solve.

APPLICATION TO DATA SETS

Two real life data sets were analyzed with Weighted Rayleigh (WRD), Rayleigh (RD), Weighted Inverse Weibull (WIW) and Inverse Weibull (IW) distributions. The performance of the WRD distribution was compared with that of existing distributions using log-likelihood and Akaike Information Criterion as selection criteria.

Data Set I

The first data set has been previously used by Alqallaf *et al,* (2015).

The data set represents waiting time (in minutes) before service of 100 bank customers

Table 1: Waiting time of 100 bank Customers

0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1, 38.5

Table 2: Summary of waiting time (Minutes) before service of bank Customers

N	Mean	Med.	Var.	Skewness	Kurtosis
100	9.877	8.100	52.3741	1.4727	5.5403

Table 3: Analysis of the performance of the competing distributions

Models	Estimates	LL	AIC
WRD	$\hat{\alpha}=12.2231(0.8562)$	-322.9123	647.8247

RD	$\hat{\alpha} = 8.6431(0.4373)$	-329.24	660.4801
WIWD	$\hat{\alpha} = 8.92778\ (0.80856)$ $\hat{\beta} = 0.76940(0.07370)$	-327.8677	659.7354
IWD	$\hat{\alpha} = 6.53228(0.87686)$ $\hat{\beta} = 1.16291(0.0799)$	-334.3810	672.7620

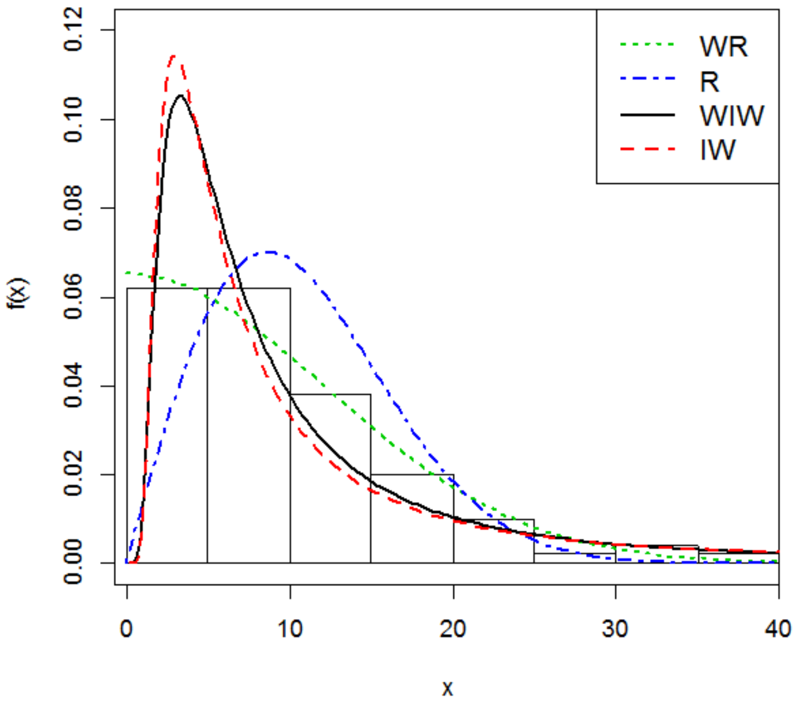

Figure 6: Histogram with competing distributions on waiting time data set

Table 3 shows the estimated parameters values (standard errors), log-likelihood (LL) and Akaike Information Criterion (AIC) values for the Weighted Inverse Weibull, Weighted Rayleigh and other existing distributions. The WR has the highest log-likelihood value of -322.9123 and lowest AIC value of 647.8247, followed by WIW distribution with log-likelihood value of -327.8677 and low AIC value of 659.7354, therefore, WR distribution fit the waiting time before service of bank customers data set better than WIW, IW and R distributions. Figure 6 further revealed that the Weighted Rayleigh distribution has a better curve to the data set than other distributions.

Data Set II

The data consist of death times (in weeks) of patients with cancer of tongue with aneuploid DNA profile. The data set has been previously used by Oguntunde and Adejumo, (2015)

Table 4: Death times (in weeks) of patients with cancer of tongue with aneuploid DNA profile

1, 3, 3, 4, 10, 13, 13, 16, 16, 24, 26, 27, 28, 30, 30, 32, 41, 51, 61, 65, 67, 70, 72, 73, 74, 77, 79, 80, 81, 87, 87, 88, 89, 91, 93, 93, 96, 97, 100, 101, 104, 104, 108, 109, 120, 131, 150, 157, 167, 231, 240, 400

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Table 5: Summary of death times (in weeks) of patients with cancer of tongue

N	**Mean**	**Med.**	**Var.**	**skewness**	**Kurtosis**
52	81.76	78.00	4774.898	2.17917	10.2266

Table 6: Analysis of the performance of the competing distributions

Models	**Estimates**	**LL**	**AIC**
WRD	$\hat{\alpha} = 106.305(2.966)$	-280.3724	562.7449
RD	$\hat{\alpha} = 75.146(4.194)$	-296.3231	594.6462
WIWD	$\hat{\alpha} = 14.16931(1.69216)$ $\hat{\beta} = 0.36911(0.05762)$	-293.6063	591.2125
IWD	$\hat{\alpha} = 9.18719(1.70227)$ $\hat{\beta} = 0.67451\ (0.05861)$	-300.2503	604.5006

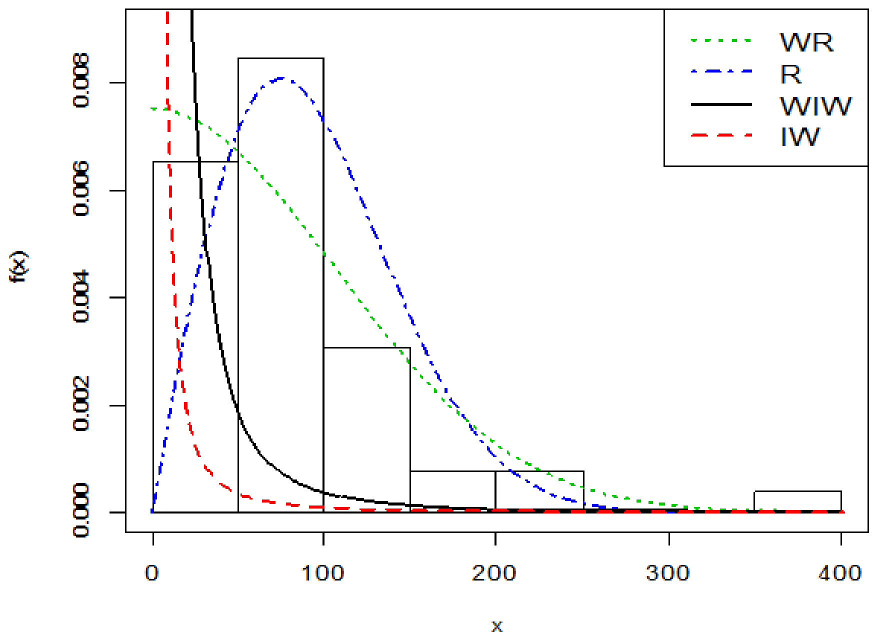

Figure 7: Histogram with competing distributions on death times of patients with cancer of the tongue data set

Table 6 shows the estimated parameters values (standard errors), log-likelihood (LL) and Akaike Information Criterion (AIC) values for the Weighted Inverse Weibull, Weighted Rayleigh and other existing distributions. The WR has the highest log-likelihood value of -280.3724 and lowest AIC value of 562.7449, followed by WIW distribution with log-likelihood value of -293.6063 and low AIC value of 591.2125, therefore, the WR distribution fit the death times of patients with cancer of the tongue data set better than the WIW, IW and R distributions. Figure 7 further revealed that the Weighted Rayleigh distribution has a better curve to the data set than other distributions.

CONCLUSION

The Weighted Rayleigh distribution as derived, has unimodal (inverted bathtub) and decreasing shapes (depending on the value of the parameters). Explicit expressions for its basic statistical properties such as reliability analysis, Moment and Moment Generating Function have been successfully derived. The unimodal and decreasing failure rates properties, imply that the distribution will be suitable to describe and model real life phenomena with unimodal or decreasing failure rates. In the real life application considered, the proposed Weighted Rayleigh distribution performs better than the existing Rayleigh, Weighted Inverse Weibull and Inverse Weibull distributions; hence, it is a good and competitive distribution.

Conflict of Interest

The authors declare that there is no conflict of interest

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