Dynamics of Anisotropic Cosmological Model with Wet Dark Fluid

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ABSTRACT

The work is based on anisotropic Bianchi type-V space-time filled with wet dark fluid in the framework of f(R, T) gravity, the functional form $f(R,T) = R + 2f(T)$ with $f(T) = \gamma T$, where γ is a constant. Exact solutions of the gravitational field equations are solved by taking exponential expansion. Various physical parameters like the Hubble parameter, deceleration parameter, anisotropy parameter, state-finder parameters, etc. are discussed. The results obtained are found to be consistent with the recent observations on the present day universe.

Keywords: Exponential expansion, f(R, T) gravity, deceleration parameter.

INTRODUCTION

The most astonishing discovery from the contemporary cosmological study is that the universe is undergoing expansion, rather than accelerating expansion. The accelerating behaviour is newly observed [1,2,3]. Two types of hypotheses have been proposed to account for the unintended accelerating expansion of the universe. One of them is the main problem of detecting the unknown and toxic nature repulsive force, termed as dark energy, which is the force behind the acceleration of the universe. To better understand its underlying nature, various DE models such as cosmological constant [4], quintessence [5], phantom energy [6], tachyons [7], k-essence [8], etc. models have been proposed by cosmologists from time to time.

The problem of late-time cosmic acceleration has also been turned up with some alternative theories of gravity, well known as modified theories of gravity, which are developed by modifying the geometric part of the Einstein-Hilbert action. Among the various modified theories of gravity, the simplest and the most studied one is the f(R) gravity. The other most interesting and viable alternative to General Relativity is the f(R,T) gravity proposed by Harko et al. [9]. Since then, many cosmologists have studied cosmological dynamics in f(R,T) gravity, as it takes care of the early-time inflation as well as the late-time cosmic acceleration.

The primary objective of this study is to investigate the wet dark fluid (WDF) within the framework of f(R,T) gravity. Introduced by Holman and Naidu [10] as a novel candidate for DE, the WDF is governed by the equation of state

$$p_{WDF} = \alpha(\rho_{WDF} - \rho_*) \tag{1}$$

where p_{WDF} and ρ_{WDF} represent the pressure and energy density of the WDF, respectively, and the associated parameters are strictly positive with the restriction $0 < \alpha < 1$. The energy conservation equation for the WDF is given by

$$\dot{\rho}_{WDF} + 3H(\rho_{WDF} + p_{WDF}) = 0 \tag{2}$$

By using Eq. (1), the conservation Equation (2) can be rewritten as:

$$\rho_{WDF} = \frac{\alpha}{1+\alpha}\rho_* + \frac{c_1}{V^{1+\alpha}} \tag{3}$$

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where C_1 is the constant of integration and V denotes the volume expansion. Inherently, the WDF consists of two distinct components: one acts as a cosmological constant, while the other behaves as a standard fluid (with an equation of state p=ωρ) that can effectively model dark matter. Consequently, the WDF serves to unify these two dark sectors. Several researchers such as Singh and Chaubey [11], Chaubey [12], Adhav et al. [13], Jain et al. [14], Samanta [15], Sarma and Bucha [16] have extensively explored cosmological models incorporating WDF within both General Relativity and various modified theories of gravitation.

In this paper, we develop cosmological models within a Bianchi type-V space time containing wet dark fluid in f(R,T) gravity. The work is structured as follows: Section 2 derives the field equations for the Bianchi type-V metric within this modified gravity framework. In Section 3, we present the exact solutions to these field equations and analyze the physical properties of the resulting models. Finally, our conclusions are summarized in Section 4. Throughout this paper, we adopt a natural system of units where G=c=1 (or 8πG=c=1), with G representing the Newtonian gravitational constant and c denoting the speed of light in a vacuum.

Metric and field equations

The Bianchi type-V space-time is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2\lambda x}B^2(t)dy^2 - e^{-2\lambda x}C^2(t)dz^2 \tag{4}$$

where A, B, C are cosmic scale factors.

The energy momentum tensor (T_{lm}) of the matter source is

$$T_{lm} = (p_{WDF} + \rho_{WDF})u_l u_m - p_{WDF}g_{lm} \tag{5}$$

and the flow vector(u_l) satisfies

$$g_{lm}u^l u^m = 1 \tag{6}$$

From Eq. (5), we obtain

$$T_{00} = \rho_{WDF}; T_{11} = T_{22} = T_{33} = -p_{WDF} \tag{7}$$

The field equations for $f(R,T)$ gravity model are given as

$$f_R(R,T)R_{lm} - \frac{1}{2}f(R,T)g_{lm} + (g_{lm}\Box - \nabla_l\nabla_m)f_R(R,T) = 8\pi T_{lm} - f_T(R,T)T_{lm} - f_T(R,T)\theta_{lm} \tag{8}$$

where ∇_i is the co-variant derivative , $\Box = \nabla^l\nabla_l$,

$$\theta_{lm} = -2T_{lm} - p_W g_{lm} \text{ and } T_{lm} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{lm}} \tag{9}$$

Here $f(R,T)$ is an arbitrary function of the Ricci scalar R and T, trace of the energy momentum tensor T_{lm} and L_m is the matter Lagrangian density.

In the present study, we take the functional form of $f(R,T)$ as

$$f(R,T) = R + 2f(T) \tag{10}$$

Now, the field Eq. (8) takes the form

$$R_{lm} - \frac{1}{2}Rg_{lm} = 8\pi T_{lm} + 2f'(T)T_{lm} + \big(2p_{WDF}f'(T) + f(T)\big)g_{lm} \tag{11}$$

where the overhead prime denotes differentiation with respect to the argument.

We chose $f(T) = \gamma T$ (12)

With the help of Eqs.(5), (6), (7) , (12), the field equation (Eq. (11)) can be written as

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{3\lambda^2}{A^2} = -(8\pi + 3\gamma)\rho_{WDF} + \gamma P_{WDF} \quad (13)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\lambda^2}{A^2} = (8\pi + 3\gamma)p_{WDF} - \gamma\rho_{WDF} \quad (14)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{\lambda^2}{A^2} = (8\pi + 3\gamma)p_{WDF} - \gamma\rho_{WDF} \quad (15)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\lambda^2}{A^2} = (8\pi + 3\gamma)p_{WDF} - \gamma\rho_{WDF} \quad (16)$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (17)$$

Solutions and physical properties

From Eqs. (14) to (16), we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \quad (18)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \quad (19)$$

$$\frac{\ddot{C}}{C} - \frac{\ddot{A}}{A} + \frac{\dot{B}}{B}\left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right) = 0 \quad (20)$$

The spatial volume (V) for the metric (4) is defined by

$$V = ABC \quad (21)$$

and V is a function of cosmic time t .

Using Eq. (21), the Eqs. (18) to (20) can be written as

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0 \quad (22)$$

$$\frac{d}{dt}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\frac{\dot{V}}{V} = 0 \quad (23)$$

$$\frac{d}{dt}\left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right) + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right)\frac{\dot{V}}{V} = 0 \quad (24)$$

On integration equations (22), (23), (24), we get

$$\frac{A}{B} = x_1\, exp\left(d_1 \int \frac{dt}{V}\right) \tag{25}$$

$$\frac{B}{C} = x_2\, exp\left(d_2 \int \frac{dt}{V}\right) \tag{26}$$

$$\frac{C}{A} = x_3\, exp\left(d_3 \int \frac{dt}{V}\right) \tag{27}$$

where x_1 , d_1 , x_2, d_2, x_3 , d_3 are integration constants.

From Eq. (17), we get

$$A^2 = BC \tag{28}$$

In the view of $V = ABC$, we can write A, B, C in the form as

$$A = V^{\frac{1}{3}} \tag{29}$$

$$B = EV^{\frac{1}{3}}\, exp\left(X \int \frac{dt}{V}\right) \tag{30}$$

$$C = E^{-1}V^{\frac{1}{3}}\, exp\left(-X \int \frac{dt}{V}\right) \tag{31}$$

where X and E are integration constants .

The field Eqs. (13) to (17) are highly non-linear. So, we require one more volumetric expansion relation to solve the equations completely. For this purpose, we take exponential expansion law

$$V = a_1 e^{3nt} \tag{32}$$

Now, from Eqs. (29), (30), (31),(32) we obtain the scale factors as

$$A = a_1{}^{\frac{1}{3}} e^{nt} \tag{33}$$

$$B = E a_1{}^{\frac{1}{3}} e^{nt}\, exp\left(-\frac{X}{3na_1} e^{-3nt}\right) \tag{34}$$

$$C = E^{-1} a_1{}^{\frac{1}{3}} e^{nt}\, exp\left(\frac{X}{3na_1} e^{-3nt}\right) \tag{35}$$

We get the energy density and pressure of the WDF for this model as

$$\rho_{WDF} = \frac{\alpha}{1+\alpha}\rho_* + \frac{c_1}{(a_1 e^{3nt})^{1+\alpha}} \tag{36}$$

$$p_{WDF} = \frac{\alpha c_1}{(a_1 e^{3nt})^{1+\alpha}} - \frac{\alpha}{1+\alpha}\rho_* \tag{37}$$

The geometry of the universe takes the form as

$$ds^2 = dt^2 - \left[a_1{}^{\frac{1}{3}} e^{nt}\right]^2 dx^2 - e^{-2\lambda x}\left[E a_1{}^{\frac{1}{3}} e^{nt}\, exp\left(-\frac{X}{3na_1} e^{-3nt}\right)\right]^2 dy^2 - e^{-2\lambda x}\left[E^{-1} a_1{}^{\frac{1}{3}} e^{nt}\, exp\left(\frac{X}{3na_1} e^{-3nt}\right)\right]^2 dz^2 \tag{38}$$

The directional Hubble parameters and mean Hubble parameter are obtained as

$$H_1 = \frac{\dot{A}}{A} = n \tag{39}$$

$$H_2 = \frac{\dot{B}}{B} = n + \frac{X}{a_1} e^{-3nt} \tag{40}$$

$$H_3 = \frac{\dot{C}}{C} = n - \frac{X}{a_1} e^{-3nt} \tag{41}$$

$$H = \frac{H_1+H_2+H_3}{3} = n \tag{42}$$

The anisotropy parameter(Δ), expansion scalar(θ), shear scalar(σ) and deceleration parameter(q) are obtained for this model as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2}{3}\frac{X^2}{{a_1}^2 n^2} e^{-6nt} \tag{43}$$

$$\theta = 3H = 3n \tag{44}$$

$$\sigma^2 = \frac{1}{3}\sum_{i=1}^{3} {H_i}^2 - \frac{\theta^2}{3} = \frac{2}{3}\frac{X^2}{{a_1}^2} e^{-6nt} - 2n^2 \tag{45}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 \tag{46}$$

The state-finder parameters [17] for this model are found as

$$r = \frac{\ddot{a}}{aH^3} = 1, s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = 0 \tag{47}$$

CONCLUSIONS

In this paper, we investigated anisotropic Bianchi type-V space time filled with a WDF within the framework of the recently established f(R, T) theory of gravity. By assuming an exponential expansion, we successfully derived exact solutions to the corresponding field equations.

Our analysis reveals distinct late-time behaviours depending on the expansion model. In the exponential expansion scenario, the directional Hubble parameters asymptotically decrease to a constant value, m, as cosmic time approaches infinity. Furthermore, the expansion anisotropy is shown to monotonically decay to zero when n>0. We also evaluated key geometrical parameters to test the viability of our models. The deceleration parameter (q) was found to be -1, which strongly aligns with recent Type-Ia Supernovae observations supporting an accelerating universe. Additionally, an analysis of the state finder diagnostic yielded a parameter pair of $r = 1$ and $s = 0$, successfully reproducing the ΛCDM model. Ultimately, the theoretical solutions derived in this study demonstrate excellent consistency with contemporary cosmological data.

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