Comparative Analysis on Probability Proportional to Size Sampling Scheme in Estimating Population Total of Student Enrolment in Ekiti State University

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ABSTRACT

This study focuses on a comparative analysis of probability proportional to size (PPS) sampling schemes in estimating the population total of student enrollment at Ekiti State University (EKSU), Ado-Ekiti. The study population consists of all ten faculties in EKSU, with data on student enrollment for five academic years (2017–2022) obtained from the Directorate of Academic Planning. Secondary data were utilized, and five faculties were sampled using the recommended sampling techniques for each method. The results revealed that all three methods provided reliable estimates for the total population, but there were notable differences in efficiency. PPS sampling with replacement was found to be relatively simple and robust for ensuring representation from unequal population units. The Horvitz-Thompson method produced unbiased estimates but with higher variance compared to PPS. The Rao-Hartley-Cochran scheme was less efficient, making it less suitable for such analyses.

Keywords: Probability proportional to size, Horvitz-Thompson, Rao-Hartley-Cochran, population total, student enrollment.

INTRODUCTION

Probability proportional to size is an important sampling method in survey research as it addresses the issues and problems associated with traditional probability sampling methods, most especially those which are in the category of equal probability sampling. One drawback of the traditional sampling procedures is that none of these sampling procedures consider the size of the population units, in the process of selecting the units from the population (Ila, Raj & Joshi, 2020). If the size of the population units varies significantly, then it may not be appropriate to select the population units with equal probabilities, as in the population larger units may have some important information and this kind of selection ignores the significance of the larger units. This problem can be solved by assigning different selection probabilities to different units of the population (Pamplona, 2019). Thus, when the size of population units varies considerably and the variance is highly correlated with the size of the unit, then the selection probabilities can be assigned in proportion to the size of the population units. The essence of probability proportional to size is claimed to be superior amidst unbiased sampling procedure mainly due to involvement of auxiliary information. Developments in sampling theory with the introduction of proportional to size, were brought by the emphasis on the need and use of auxiliary information in improving precision of estimates (Homa, Maurya, and Singh, 2013). Sampling scheme is an important aspect most considered in statistics, most especially in survey research, given that it is possible to get a sufficiently good estimate of the parameter of interest at a reasonably low cost (Grafstrom, 2010). Sampling is defined as a procedure to select a sample from individual or from a large group of population for certain kind of research purpose (Shardwaj, 2019). Abdullah, et al (2014) examined the selection of samples in probability proportional to size sampling using cumulative relative frequency method. They used data of a village with 10 holdings applied to probability proportional to size under cumulative relative frequency method, cumulative total method, and Lahiri's Method, result showed that relative frequency to select samples in probability proportional to size takes less time and easy to apply than Cumulative Total Method and Lahiri's Method. Hence, the study recommended engaging the method of selecting samples in probability proportional to size which use relative frequency among others.

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Maleka, Si and Gelman (2017) in their study checked Bayesian inference under cluster sampling with probability proportional to size. They revealed that challenges arise when the sizes of nonsampled cluster are unknown, and that integrated Bayesian approach outperforms classical methods with efficiency gains. They use Stan for computing and apply the proposal to the Fragile Families and Child Wellbeing study as an illustration of complex survey inference in health surveys. esponse probabilities are non-uniform and a sampling fraction can be both negligible and not negligible where both circumstances are more realistic in practice under the reverse framework using simulation and real data sets. Result revealed that consider under less restricted situations where response probabilities are non-uniform and a sampling fraction can be both negligible and not negligible where both circumstances are more realistic in practice under the reverse framework.

METHODS OF ESTIMATION

This study used three different method of estimating total population and variance under probability proportional to size sampling scheme, as identified in the objective of the study which are probability proportional size with replacement, Horvitz-Thompson sampling scheme and Rao-Hartley-Cochran's sampling scheme. The procedures for the three methods are discussed extensively below:

Probability Proportional to Size with Replacement

Let units $U_1, U_2, \dots\dots\dots\dots U_N$ have sizes $X_1, X_2, X_3 \dots\dots\dots\dots X_N$ respectively; where X_i, i=1, 2,…., N is an integer. If X_i's are not integers they should be multipled by appropriate power of 10 to make them integers. If a sample of size n units is to be selected from a population of N units $\{U_i, i = 1,2, \dots, N\}$ by probability proportional to size with replacement (ppswr), we proceed as follows:

Step 1: Form cumulative totals of the sizes; assign ranges to all the population units using the cumulative totals as in Table 1

Table 1: Probability Proportional to Size selection

Units	Sizes	Cumulative Totals	Ranges
U_1	X_1	X_1	$1 - X_1$
U_2	X_2	$X_1 + X_2$	$X_1 + 1 - X_1 + X_2$
U_3	X_3	$X_1 + X_2 + X_3$	$X_1 + X_2 + 1 - X_1 + X_2 + X_3$
:	:	:	:
U_i	X_i	$X_1 + X_2 + \cdots + X_i$	$X_1 + \cdots + X_{i-1} + 1 - X_1 + X_2 + \cdots + X_i$
:	:	:	:
U_N	X_N	$X = \sum_{i=1}^{N} X_i$	$X_1 + X_2 + \cdots + X_{N-1} + 1 - X$

Step 2: Using random table select a number d between 1 and $X = \sum_{i=1}^{N} X_i$ inclusive. If the number d falls in the range of U_2, say, then it is selected in the sample. Another random number is drawn between 1 and X inclusive, and if the number selected falls this time in the range of U_i, unit $U_i$is selected. In other words the unit selected in the sample is the unit in whose range the selected random number falls. The process of drawing a random number is repeated independently (i.e. the selected random number is returned into the pool before the next selection) until n units are selected in the sample. With this selection procedure the n units selected with ppswr, and the probability of selecting the ith unit in the population is $P_i = {X_i}/{X}$, and gives $\sum_{i=1}^{N} P_i = 1$.

Step 3: Estimation of Population Total

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An unbiased estimator of the population total under probability proportional to size with replacement, Y is given by

$$\hat{Y}_{PPS} = \frac{1}{n}\sum_{i=1}^{N} {y_i}/{p_i}$$

Where $p_i = {x_i}/{X}$ is the probability of selecting the i^th unit in the sample; x_i is the measure of size of the i^th sample unit.

Step 4: Estimation of Variance

Since sampling is with replacement the values ${y_i}/{p_i}$, i= 1,2…,n are independent and the covariance between any pair of values $({y_i}/{p_i}, {y_j}/{p_j}; i \neq j)$ is zero.

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{n^2}\sum_{i=1}^{N} V({y_i}/{p_i})$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{n}\{E({y_i}/{p_i} - Y)^2\}$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{n}\sum_{i=1}^{N} P_i\,({Y_i}/{P_i} - Y)^2$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{n}\left(\sum_{i=1}^{N} {Y_i}^2/{P_i} - Y^2\right)$$

Its sample estimator

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}({y_i}/{p_i} - \hat{Y}_{pps})^2$$

Horvitz-Thompson Sampling Scheme

Horvitz-Thompson Sampling Scheme is a general theory of sampling scheme without replacement under unequal probability sampling. Under this form of estimator, there are steps to be considered to ensure proper estimation as recommended by these scholars which are as follows:

Step 1: This involves the selection of samples. To achieve this, there is need to first create a table for cumulative total and range of the units (departments) in each of the grouped population (faculties), Afterward, select a sample of size n without replacement, a random start between 1 and k inclusive (k= x\n is the S.I.) is selected using a table of random numbers. If the number selected is r then the units in the sample are those in whose ranges the numbers r, r + k, r + 2k,…,r + (n-1)k fall. The probability of selecting the unit U_i in the sample of size n is

$$Pr\ (U_i) = \pi_i = {X_i}/{k} = nP_i$$

If the unit in the population has its size greater than or equal to k such unit is removed before sampling, and then taken into the sample with probability one. The probability that any pair of units $(U_i\ U_j)$ is together in the sample is

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$\pi_{ij} = {}^{nq_{ij}}/_{X} = {}^{q_{ij}}/_{k} \quad i \neq j$

q_{ij} is the number of the random number between 1 and k inclusive, which will select U_i and $U_j$simultaneously in the sample. Given the r, r + k, r + 2k,…,r + (n-1)k, for the first selected number and second selected number, without replacement, U_i has the range of $U_{imin} \leq r \leq U_{imax}$ and the process of getting q_{ij} generally involves calculating the range of U_j, based on inequality as expressed thus $U_{jmin} \leq r + k \leq U_{jmax}$, where U_{jmin} *and* U_{jmax} are the minimum and maximum limit of the range of cumulative total for U_j. If the minimum values and maximum values of the U_j range falls within the range of U_i, we can use these values and subtract maximum from minimum to get the q_{ij}. But in cases where any of minimum or maximum values or both for U_j does not falls within the range of U_i, the minimum or maximum values or both of the range U_i is used for q_{ij} respectively.

Step 2: Estimation of Population total

An unbiased estimator of the population total for ppswor sampling as given by Horvitz-Thompson is

$$\hat{Y}_{HT} = \sum_{i=1}^{N} {}^{y_i}/_{\pi_i}$$

Step3: Estimation of variance is

$$\hat{V}(\hat{Y}_{HT}) = \sum_{i=1}^{n} \left(\frac{1 - \pi_i}{\pi_i^2} \, y_i^2 \right) + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_{ij}} \, \frac{y_i}{\pi_i} \frac{y_j}{\pi_j}$$

Provided that $\pi_{ij} > 0$

Rao-Hartley-Cochran's Sampling Scheme

Rao-Hartley-Cochran's Sampling Scheme in relation to probability proportional to size is proposed as a unequal probability sampling scheme estimator that entails selection without replacement with special cases. The main processes of this estimator are as follows:

Step 1: divide a population of N units into n groups at random with group g containing $N_g$units (g = 1,2,…,n) such that $N_1 + N_2 + \cdots + N_n = N$.

Step 2: select one unit with ppswor independently from each group. This gives a total of n units selected in the sample ppswor. The probability of selecting U_i in the sample in g^{th} group is

$$p_i^* = \frac{x_i}{X_1} = \frac{x_i / X}{X_g / X} = \frac{p_i}{\sum_{i=1}^{N_g} P_i} = \frac{p_i}{P_g}$$

Where $X_g = \sum_{i=1}^{N_g} X_i$ is the total measure of size in g^{th} group; $P_g$is the sum of the initial probabilities in g^{th} group.

Step 3: Estimation of Population Total

The Rao-Hartley-Cochran estimator of the population total is

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} {}^{y_{gi}}/_{p_i^*} = \sum_{g=1}^{n} P_g \frac{y_{gi}}{p_i} = \sum_{g=1}^{n} \hat{Y}_g$$

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Where y_{gi} is the value of the study variate for i^{th} unit in g^{th} group

Step 4: Estimation of variance

The unbiased estimator of variance of $\hat{Y}_{RHC}$ is

$$\hat{V}(\hat{Y}_{RHC}) = \frac{\sum_{g=1}^{n} N_g^2 - N}{N^2 - \sum_{g=1}^{n} N^2}\left[\sum_{g=1}^{n} \frac{y_{gi}^2}{p_i p_i^*} - \hat{Y}_{RHC}^2\right]$$

$$\hat{V}(\hat{Y}_{RHC}) = \frac{\sum_{g=1}^{n} N_g^2 - N}{N^2 - \sum_{g=1}^{n} N^2}\left[\sum_{g=1}^{n} P_g \frac{y_{gi}^2}{p_I^2} - \hat{Y}_{RHC}^2\right]$$

$$\hat{V}(\hat{Y}_{RHC}) = \frac{\sum_{g=1}^{n} N_g^2 - N}{N^2 - \sum_{g=1}^{n} N^2}\left[\sum_{g=1}^{n} P_g \left(\frac{y_{gi}}{p_i} - \hat{Y}_{RHC}\right)^2\right]$$

Rao-Hartley-Cochran Estimator

Rao et al. (1962) proposed a sampling strategy for use with unequal probability sampling and the estimator of population total. The population units are divided randomly into n groups, where the group sizes are predetermined. Then one unit is selected from each group. Their estimator is

$$y'_{RHC} = \sum_{i=1}^{n} \frac{\pi_i y_{iT}}{p_{iT}} \quad (23)$$

where p_{iT} is the probability of the Tth unit being selected from the ith group. Also $\pi_i = \sum_{i=1}^{n} p_{iT}$ and $\sum_{i=1}^{n} \pi_i = 1$. The Rao-Hartley-Cochran estimator can be used for any sample size. The variance of (23) is

$$Var(y'_{RHC}) = \frac{n\,(\sum_{i=1}^{n} {N_i}^2 - N)}{N(N-1)}\left[\sum_{i=1}^{n} \sum_{T=1}^{N_j} \frac{{Y_{iT}}^2}{P_{iT}} - \frac{Y^2}{n}\right] \quad (24)$$

RESULTS AND DISCUSSION

The population of the study consist all the faculties in Ekiti State University (EKSU) Ado-Ekiti. There are ten faculties in EKSU, which are Sciences, Management Sciences, Social Sciences, Law, Engineering, Agricultural Science, Art, Education, Basic Medical Science, and Medicine. The data on students' enrolment were collected from Directorate of Academic Planning, and data covered the students' enrolment for 5 years spanning from 2017 to 2022 for all the ten faculties in Ekiti State University. Hence, data used for this study is gathered from secondary source. The sample size of this study is five faculties selected from all the faculties in Ekiti State University. The major sampling technique is the technique appropriate and recommended under each of the probability proportional to size sampling scheme, Horvitz-Thompson sampling scheme and Rao-Hartley-Cochran's sampling scheme.

Table 1 Range and Selection of Samples among the Faculties

S/N	Faculty	No of Dept.	Cum Total	Ranges	2017	2018	2019	2021	2022
1	Agricultural science*	6	6	1 – 6	1391	1468	1506	1526	1275
2	Art	7	13	7 – 13	2673	3053	3936	4293	4656
3	Basic Medical Science*	3	16	14 – 16	984	1014	1150	1194	1550

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4	Education*	9	25	17 – 25	3476	3218	3065	3625	3123
5	Engineering	4	29	26 – 29	1622	1903	2341	2469	2507
6	Law	1	30	30	768	817	664	606	592
7	Management Science	3	33	31 – 33	3335	3925	4272	5068	4367
8	Medicine	1	34	34	288	447	1098	262	292
9	Science**	12	46	35 – 46	4884	5591	7725	9205	8053
10	Social Science	6	52	47–52	3710	3894	4629	4484	4375

Note: Faculties were selected based on random sampling with replacement

Source: Author's, (2023)

Estimation of Total Population

An unbiased estimator of the population total under probability proportional to size with replacement, Y is given by

$$\hat{Y}_{PPS} = \frac{1}{n}\sum_{i=1}^{N} {y_i}/{p_i}$$

Table 2: Probabilities and Data Yearly data of students' enrolment for the sampled faculties

Faculties	**Agricultural Science**	**Art**	**Science**	**Education**	**Science**
$\boldsymbol{p_i}$	${}^{6}/_{52}$	${}^{7}/_{52}$	${}^{12}/_{52}$	${}^{9}/_{52}$	${}^{12}/_{52}$
$\boldsymbol{y_{i(2017)}}$	1391	2673	4884	3476	4884
$\boldsymbol{y_{i(2018)}}$	1468	3053	5591	3218	5591
$\boldsymbol{y_{i(2019)}}$	1506	3936	7725	3065	7725
$\boldsymbol{y_{i(2021)}}$	1526	4293	9205	3625	9205
$\boldsymbol{y_{i(2022)}}$	1275	4656	8053	3123	8053

Source: Author's Computation, (2023)

Total Population Estimate for 2017

$$\hat{Y}_{PPS} = \frac{52}{5}\left[\frac{1391}{6} + \frac{2673}{7} + \frac{4884}{12} + \frac{3476}{9} + \frac{4884}{12}\right]$$

$$\hat{Y}_{PPS} = 10.4(23183 + 381.86 + 407 + 386.22 + 407)$$

$$\hat{Y}_{PPS} = 18864.66 \sim 18865\ students$$

Total Population Estimate for 2018

$$\hat{Y}_{PPS} = \frac{52}{5}\left[\frac{1468}{6} + \frac{3053}{7} + \frac{5591}{12} + \frac{3218}{9} + \frac{5591}{12}\right]$$

$$\hat{Y}_{PPS} = 10.4(244.67 + 436.14 + 465.92 + 357.56 + 465.92)$$

$$\hat{Y}_{PPS} = 20490.18 \sim 20480\ students$$

Total Population Estimate for 2019

$$\hat{Y}_{PPS} = \frac{52}{5}\left[\frac{1506}{6} + \frac{3936}{7} + \frac{7725}{12} + \frac{3065}{9} + \frac{7725}{12}\right]$$

$$\hat{Y}_{PPS} = 10.4(251 + 562.29 + 643.75 + 340.56 + 643.75)$$

$$\hat{Y}_{PPS} = 25390.04 \sim 25390\ students$$

Total Population Estimate for 2021

$$\hat{Y}_{PPS} = \frac{52}{5}\left[\frac{1526}{6} + \frac{4293}{7} + \frac{9205}{12} + \frac{3625}{9} + \frac{9205}{12}\right]$$

$$\hat{Y}_{PPS} = 10.4(254.33 + 613.29 + 767.08 + 402.78 + 767.08)$$

$$\hat{Y}_{PPS} = 29167.42 \sim 29167\ students$$

Total Population Estimate for 2022

$$\hat{Y}_{PPS} = \frac{52}{5}\left[\frac{1275}{6} + \frac{4656}{7} + \frac{8053}{12} + \frac{3123}{9} + \frac{8053}{12}\right]$$

$$\hat{Y}_{PPS} = 10.4(212.5 + 665.14 + 671.08 + 347 + 671.08)$$

$$\hat{Y}_{PPS} = 26694.72 \sim 26695\ students$$

Estimation of Variance

$$\hat{V}\left(\hat{Y}_{pps}\right) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left({}^{y_i}/_{p_i} - \hat{Y}_{pps}\right)^2$$

Variance Estimate for 2017

$$\hat{V}\left(\hat{Y}_{pps}\right) = \frac{1}{5(4)}\left[(12055.33)^2 + (19856.56)^2 + (21164)^2 + (200.833)^2 + (21164)^2 - 5(18864.66)^2\right]$$

$$\hat{V}\left(\hat{Y}_{pps}\right) = \frac{1}{20}(1838793528 - 1779376985)$$

$$\hat{V}\left(\hat{Y}_{pps}\right) = 2970827.17$$

Standard Error (SE) is

$$SE\left(\hat{Y}_{pps}\right) = \sqrt{2970827.17}$$

$$SE\left(\hat{Y}_{pps}\right) = 1723.61$$

Variance Estimate for 2018

$$\hat{V}\left(\hat{Y}_{pps}\right) = \frac{1}{5(4)}\left[(12722.67)^2 + (22679.43)^2 + (24227.67)^2 + (18592.89)^2 + (24227.67)^2 - 5(20490.18)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(2195878423 - 2099237382)$$

$$\hat{V}(\hat{Y}_{pps}) = 4832052.04$$

Standard Error (SE) is

$$SE(\hat{Y}_{pps}) = \sqrt{4832052.04}$$

$$SE(\hat{Y}_{pps}) = 2198.19$$

Variance Estimate for 2019

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(13052)^2 + (29238.86)^2 + (33475)^2 + (17708.89)^2 + (33475)^2 - 5(25390.04)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(3580021673 - 3223270656)$$

$$\hat{V}(\hat{Y}_{pps}) = 17837550.85$$

Standard Error (SE) is

$$SE(\hat{Y}_{pps}) = \sqrt{17837550.85}$$

$$SE(\hat{Y}_{pps}) = 4223.45$$

Variance Estimate for 2021

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(13225.33)^2 + (31890.86)^2 + (39888.33)^2 + (20944.44)^2 + (39888.33)^2 - 5(29167.43)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(4812763612 - 4253691947)$$

$$\hat{V}(\hat{Y}_{pps}) = 27953583.24$$

Standard Error (SE) is

$$SE(\hat{Y}_{pps}) = \sqrt{27953583.24}$$

$$SE(\hat{Y}_{pps}) = 5287.11$$

Variance Estimate for 2022

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(11050)^2 + (34587.43)^2 + (34896.33)^2 + (18044.44)^2 + (34896.33)^2 - 5(26694.78)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(4079486445 - 3563040379)$$

$$\hat{V}(\hat{Y}_{pps}) = 25822363.28$$

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Standard Error (SE) is

$$SE(\hat{Y}_{pps}) = \sqrt{25822363.28}$$

$$SE(\hat{Y}_{pps}) = 5081.57$$

Horvitz-Thompson Sampling Scheme

Table 1 is also applicable for the selection of sample units under this scheme. With the intended sample of size 5, k = $\frac{52}{5}$ = 10.4, and using table of random numbers, a number 007 is selected, which is between 1 and 10.4. Based on this random number, the other numbers to be used in the selection are $7 + 10.4 = 17.4;\ 17.4 + 10.4 = 27.8;\ 27.8 + 10.4 = 38.2;\ 38.2 + 10.4 = 48.6.$

The first number, which is random number 007 falls within the range of 7-13 which is for faculty of Art, the second number 17.4 lies in range 17-25 which stand for faculty of education, the third number, 27.8 within 26-29 which is for faculty of engineering, forth number and fifty number fall in the range of 35-46 and 47-52 which are for faculty of sciences and social sciences respectively. Hence, the five elected faculties under Horvitz-Thompson sampling scheme are Art, education, engineering, sciences and social sciences.

Probability of Inclusion

Given the number of unit under each of the selected faculties to be 7, 9, 4, 12, and 6, based on the presentation in table 4.1, as well as k which is 10.4, the probability for each of the faculties selected is given in Table 3 below;

Table 3: Probability of the selected samples among all the faculties

Faculties	Art	Education	Engineering	Sciences	Social Science
$\boldsymbol{\pi_i}$	$^7/_{10.4}$	$^9/_{10.4}$	$^4/_{10.4}$	$^{12}/_{10.4}$	$^6/_{10.4}$

Source: Author's Computation, (2023)

Computation of probability of selecting two faculties ($\boldsymbol{\pi_{ij}}$)

For Faculty of Art and Education (Art=i; Education=j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 7 \le r \le 13;\ F_j: 17 \le r + 10.4 \le 25 \to 6.6 \le r \ \le 14.6$$

Since 6.6 and 14.6 are not within $7 \le r \le 13$

$$q_{ij} = 13 - 7 + 1 = 7$$

$$\boldsymbol{\pi_{ij}} = {}^{7}/_{10.4}$$

For Faculty of Art and Engineering (Art = i; Engineering = j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 7 \le r \le 13;\ F_j: 26 \le r + 20.8 \le 29 \to 5.2 \le r \ \le 8.2$$

Since 5.2 is not within $7 \leq r \leq 13$, while 8.2 is within the range $7 \leq r \leq 13$

$$q_{ij} = 8.2 - 7 + 1 = 2.2$$

$$\boldsymbol{\pi_{ij}} = {}^{2.2}/_{10.4}$$

For Faculty of Art and Science (Art = i; Science = j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 7 \leq r \leq 13;\ F_j: 35 \leq r + 31.2 \leq 46 \rightarrow 3.8 \leq r \ \leq 14.8$$

Since 3.8 and 14.8 are not within $7 \leq r \leq 13$

$$q_{ij} = 13 - 7 + 1 = 7$$

$$\boldsymbol{\pi_{ij}} = {}^{7}/_{10.4}$$

For Faculty of Art and Social Science (Art=i; Social Science=j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 7 \leq r \leq 13;\ F_j: 47 \leq r + 41.6 \leq 52 \rightarrow 5.4 \leq r \ \leq 10.4$$

Since 5.4 is not within the range, $7 \leq r \leq 13$, while 10.4 is within the range $7 \leq r \leq 13$

$$q_{ij} = 10.4 - 7 + 1 = 4.4$$

$$\boldsymbol{\pi_{ij}} = {}^{4.4}/_{10.4}$$

For Faculty of Education and Engineering (Education=i; Engineering=j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 17 \leq r \leq 25;\ F_j: 26 \leq r + 10.4 \leq 29 \rightarrow 15.6 \leq r \ \leq 18.6$$

Since 15.6 is not within the range, $17 \leq r \leq 25$, while 18.6 is within the range $17 \leq r \leq 25$

$q_{ij} = 18.6 - 17 + 1 = 2.6$

$$\boldsymbol{\pi_{ij}} = {}^{2.6}/_{10.4}$$

For Faculty of Education and Science (Education=i; Science=j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 17 \leq r \leq 25;\ F_j: 35 \leq r + 20.8 \leq 46 \rightarrow 14.2 \leq r \leq 25.2$$

Since 14.2 and 25.2 are not within $17 \leq r \leq 25$

$$q_{ij} = 25 - 17 + 1 = 9$$

$$\boldsymbol{\pi_{ij}} = {}^{9}/_{10.4}$$

For Faculty of Education and Social Science (Education=i; Social Science=j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 17 \leq r \leq 25;\ F_j: 47 \leq r + 31.2 \leq 52 \rightarrow 15.8 \leq r \leq 20.6$$

Since 15.8 is not within the range $17 \leq r \leq 25$, while 20.6 is within the range $17 \leq r \leq 25$

$q_{ij} = 20.6 - 17 + 1 = 4.6$

$$\boldsymbol{\pi_{ij}} = {}^{4.6}/_{10.4}$$

For Faculty of Engineering and Science (Engineering=i; Science=j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 26 \leq r \leq 29;\ F_j: 35 \leq r + 10.4 \leq 46 \rightarrow 24.6 \leq r \leq 35.6$$

Since 24.6 and 35.6 are not within the range $26 \leq r \leq 29$

$$q_{ij} = 29 - 26 + 1 = 4$$

$$\boldsymbol{\pi_{ij}} = {}^{4}/_{10.4}$$

For Faculty of Engineering and Social Science (Engineering=i; Social Science=j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

Where

$$F_i: 26 \leq r \leq 29;\ F_j: 47 \leq r + 20.8 \leq 52 \rightarrow 26.2 \leq r \leq 31.2$$

Since 26.2 is within the range $26 \leq r \leq 29$ while 31.2 is not within the range $26 \leq r \leq 29$

$$q_{ij} = 29 - 26.2 + 1 = 3.8$$

$$\boldsymbol{\pi_{ij}} = {}^{3.8}/_{10.4}$$

For Faculty of Science and Social Science (Science=i; Social Science=j)

$$\boldsymbol{\pi_{ij}} = {}^{\boldsymbol{q_{ij}}}/_{k}$$

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Where

$$F_i: 35 \leq r \leq 46;\ F_j: 47 \leq r + 10.4 \leq 52 \rightarrow 36.6 \leq r\ \leq 41.6$$

Since 36.6 and 41.6 are within the range $35 \leq r \leq 46$

$$q_{ij} = 41.6 - 36.6 + 1 = 6$$

$$\boldsymbol{\pi_{ij}} = {}^{6}\!/_{10.4}$$

Estimation of Total Population

An unbiased estimator of the population total for ppswor sampling as given by Horvitz-Thompson is

$$\hat{Y}_{HT} = \sum_{i=1}^{N} {}^{y_i}\!/_{\pi_i}$$

Table 4: Probability of the selected samples among all the faculties with annual enrolment data

Sample faculties		Serial no	$\boldsymbol{\pi_i}$	2017	2018	2019	2021	2022
Art		1	${}^{7}\!/_{10.4}$	2673	3053	3936	4293	4656
Education		2	${}^{9}\!/_{10.4}$	3476	3218	3065	3625	3123
Engineering		3	${}^{4}\!/_{10.4}$	1622	1903	2341	2469	2507
Sciences		4	${}^{12}\!/_{10.4}$	4884	5591	7725	9205	8053
Social Science		5	${}^{6}\!/_{10.4}$	3710	3894	4629	4484	4375

Source: Author's Computation, (2023)

Based on the Horvitz-Thompson sampling scheme criteria, Table 4 reveals the probability of selecting each of the faculties which are sampled, art, education, engineering, science and social sciences based on the number 007 selected from the random. In the Table 4.4 probabilities stood at ${}^{7}\!/_{10.4}$, ${}^{9}\!/_{10.4}$, ${}^{4}\!/_{10.4}$, ${}^{12}\!/_{10.4}$ and ${}^{6}\!/_{10.4}$ for art, education, engineering, sciences and social sciences respectively, in regards to k and the number of units (departments) in each of the faculties.

Total Population Estimate for 2017

$$\hat{Y}_{HT} = \frac{2673}{{}^{7}\!/_{10.4}} + \frac{3476}{{}^{9}\!/_{10.4}} + \frac{1622}{{}^{4}\!/_{10.4}} + \frac{4884}{{}^{12}\!/_{10.4}} + \frac{3710}{{}^{6}\!/_{10.4}}$$

$$\hat{Y}_{HT} = 3971.31 + 4016.71 + 4217.2 + 4232.8 + 6430.67$$

$$\hat{Y}_{HT} = 22868.69$$

Total Population Estimate for 2018

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$$\hat{Y}_{HT} = \frac{3053}{7/10.4} + \frac{3218}{9/10.4} + \frac{1903}{4/10.4} + \frac{5591}{12/10.4} + \frac{3894}{6/10.4}$$

$$\hat{Y}_{HT} = 4535.89 + 3718.58 + 4947.8 + 4845.53 + 6749.6$$

$$\hat{Y}_{HT} = 24797.4$$

Total Population Estimate for 2019

$$\hat{Y}_{HT} = \frac{3936}{7/10.4} + \frac{3065}{9/10.4} + \frac{2341}{4/10.4} + \frac{7725}{12/10.4} + \frac{4629}{6/10.4}$$

$$\hat{Y}_{HT} = 5847.78 + 3541.78 + 6086.6 + 6695 + 8023.6$$

$$\hat{Y}_{HT} = 30194.76$$

Total Population Estimate for 2021

$$\hat{Y}_{HT} = \frac{4293}{7/10.4} + \frac{3625}{9/10.4} + \frac{2469}{4/10.4} + \frac{9205}{12/10.4} + \frac{4484}{6/10.4}$$

$$\hat{Y}_{HT} = 6378.17 + 4188.89 + 6419.4 + 7977.67 + 7772.27$$

$$\hat{Y}_{HT} = 32736.4$$

Total Population Estimate for 2022

$$\hat{Y}_{HT} = \frac{4656}{7/10.4} + \frac{3123}{9/10.4} + \frac{2507}{4/10.4} + \frac{8053}{12/10.4} + \frac{4375}{6/10.4}$$

$$\hat{Y}_{HT} = 6917.49 + 3608.8 + 6518.2 + 6979.27 + 7583.33$$

$$\hat{Y}_{HT} = 31607.09$$

4.2.2. Estimation of Variance

$$\hat{V}(\hat{Y}_{HT}) = \sum_{i=1}^{n}\left(\frac{1-\pi_i}{\pi_i^2}\, y_i^2\right) + 2\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_{ij}}\,\frac{y_i}{\pi_i}\frac{y_j}{\pi_j}$$

Variance Estimate for 2017

$$\sum_{i=1}^{n}\left(\frac{1-\pi_i}{\pi_i^2}\, y_i^2\right)$$
$$= \frac{1-0.6731}{0.4531}(7144929) + \frac{1-0.8654}{0.7489}(12082576) + \frac{1-0.3846}{0.1479}(2630884) + \frac{1-1.1538}{1.3313}(23853456) + \frac{1-0.5769}{0.3328}(13764100)$$

$$= 5154882.57 + 2171604.66 + 10946896.64 - 2755698.59 + 17498770.16$$

$$= 33016455.44$$

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$$2\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_{ij}}\frac{y_i}{\pi_i}\frac{y_j}{\pi_j} = 2\Big[\frac{0.6731-0.5825}{0.6731}(3971.18)(4016.64) + \frac{0.2115-0.2588}{0.2115}(3971.18)(4217.37) + \frac{0.6731-0.7766}{0.6731}(3971.18)(4232.97) + \frac{0.4231-0.3883}{0.4231}(3971.18)(6430.92) + \frac{0.2500-0.3328}{0.2500}(4016.64)(4217.37) + \frac{0.8654-0.9985}{0.8654}(4016.64)(4232.92) + \frac{0.4423-0.4992}{0.4423}(4016.64)(6430.92) + \frac{0.3846-0.4438}{0.3846}(4217.37)(4232.97) + \frac{0.3654-0.2218}{0.3654}(4217.37)(6430.92) + \frac{0.5769-0.6656}{0.5769}(4232.97)(6430.92)\Big]$$

$$= 2[2146995.27 - 3753438.00 - 2584791.52 + 2100530.05 - 5610414.41 - 2614985.37 - 3323007.65 - 2747889.86 + 10658613.39 - 4185442.47]$$

$$= 2(-991380.61) = -19827661.22$$

$$\hat{V}(\hat{Y}_{HT}) = 33016455.44 - 19827661.22$$

$$\hat{V}(\hat{Y}_{HT}) = 13188794.22$$

Standard error (SE) is

$$\mathrm{SE}(\hat{Y}_{HT}) = \sqrt{13188794.22}$$

$$\mathrm{SE}(\hat{Y}_{HT}) = 3631.64$$

Variance Estimate for 2018

$$\sum_{i=1}^{n}\left(\frac{1-\pi_i}{\pi_i^2}\,y_i^2\right) = \frac{1-0.6731}{0.4531}(9320809) + \frac{1-0.8654}{0.7489}(10355524) + \frac{1-0.3846}{0.1479}(3621409) + \frac{1-1.1538}{1.3313}(31259281) + \frac{1-0.5769}{0.3328}(15163236)$$

$$= 6724724.04 + 1861201.14 + 15487248.77 - 3611265.24 + 19277539.52$$

$$= 39739448.23$$

$$2\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_{ij}}\frac{y_i}{\pi_i}\frac{y_j}{\pi_j} = 2\Big[\frac{0.6731-0.5825}{0.6731}(4535.73)(3718.51) + \frac{0.2115-0.2588}{0.2115}(4535.73)(4947.99) + \frac{0.6731-0.7766}{0.6731}(4535.73)(4845.73) + \frac{0.4231-0.3883}{0.4231}(4535.73)(6749.87) + \frac{0.2500-0.3328}{0.2500}(3718.51)(4947.99) + \frac{0.8654-0.9985}{0.8654}(3718.51)(4845.73) + \frac{0.4423-0.4992}{0.4423}(3718.51)(6749.87) + \frac{0.3846-0.4438}{0.3846}(4947.99)(4845.73) + \frac{0.3654-0.2218}{0.3654}(4947.99)(6749.87) + \frac{0.5769-0.6656}{0.5769}(4845.73)(6749.87)\Big]$$

$$= 2[2270203.32 - 5029721.95 - 3379614.49 + 2518133.91 - 6093798.58 - 2771336.93 - 3228937.86 - 3690629.53 + 13125326.59 - 5028954.42]$$

$$= 2(-11309329.94) = -22618659.88$$

$$\hat{V}(\hat{Y}_{HT}) = 39739448.23 - 22618659.88$$

$$\hat{V}(\hat{Y}_{HT}) = 17120788.35$$

Standard error (SE) is

$\mathrm{SE}(\hat{Y}_{HT}) = \sqrt{17120788.35}$

$\mathrm{SE}(\hat{Y}_{HT})$ =4137.73

Variance Estimate for 2019

$$\sum_{i=1}^{n}\left(\frac{1-\pi_i}{\pi_i^2}\ y_i^2\right)$$
$$= \frac{1-0.6731}{0.4531}(15492096) + \frac{1-0.8654}{0.7489}(9394225) + \frac{1-0.3846}{0.1479}(5480281) + \frac{1-1.1538}{1.3313}(59675625) + \frac{1-0.5769}{0.3328}(21427641)$$

$$= 11177148.931 + 1688426.61 + 23436865.37 - 6894896.84 + 27241691.43$$

$$= 56650035.5$$

$$2\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_{ij}}\ \frac{y_i}{\pi_i}\frac{y_j}{\pi_j} = 2\Big[\frac{0.6731-0.5825}{0.6731}(5841.57)(3541.71) + \frac{0.2115-0.2588}{0.2115}(5841.57)(6086.84) + \frac{0.6731-0.7766}{0.6731}(5841.57)(6695.27) + \frac{0.4231-0.3883}{0.4231}(5841.57)(8023.92) + \frac{0.2500-0.3328}{0.2500}(3541.71)(6086.84) + \frac{0.8654-0.9985}{0.8654}(3541.71)(6695.27) + \frac{0.4423-0.4992}{0.4423}(3541.71)(8023.92) + \frac{0.3846-0.4438}{0.3846}(6086.84)(6695.27) + \frac{0.3654-0.2218}{0.3654}(6086.84)(8023.92) + \frac{0.5769-0.6656}{0.5769}(6695.27)(8023.92)\Big]$$

$$= 2[2787642.22 - 7976920.89 - 2060107.99 + 3859208.45 - 7139950.68 - 3647054.53 - 3655905.09 - 6272958.41 + 19193950.6 - 8259959.58]$$

$$= 2(-17132052.9) = -34264105.8$$

$$\hat{V}(\hat{Y}_{HT}) = 56650035.5 - 34264105.8$$

$$\hat{V}(\hat{Y}_{HT}) = 22385929.7$$

Standard error (SE) is

$\mathrm{SE}(\hat{Y}_{HT}) = \sqrt{22385929.7}$

$\mathrm{SE}(\hat{Y}_{HT}) = 4731.38$

Variance Estimate for 2021

$$\sum_{i=1}^{n}\left(\frac{1-\pi_i}{\pi_i^2}\ y_i^2\right)$$
$$= \frac{1-0.6731}{0.4531}(18429849) + \frac{1-0.8654}{0.7489}(13140625) + \frac{1-0.3846}{0.1479}(6095961) + \frac{1-1.1538}{1.3313}(84732025) + \frac{1-0.5769}{0.3328}(20106256)$$

$$= 13296662.19 + 2361768.09 + 26069870.74 - 9788766.95 + 25561769.57$$

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$$= 57501303.64$$

$$2\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_{ij}}\frac{y_i}{\pi_i}\frac{y_j}{\pi_j} = 2\Big[\frac{0.6731-0.5825}{0.6731}(6377.95)(4188.81) + \frac{0.2115-0.2588}{0.2115}(6377.95)(6419.66) + \frac{0.6731-0.7766}{0.6731}(6377.95)(7977.99) + \frac{0.4231-0.3883}{0.4231}(6377.95)(7772.58) + \frac{0.2500-0.3328}{0.2500}(4188.81)(6419.66) + \frac{0.8654-0.9985}{0.8654}(4188.81)(7977.99) + \frac{0.4423-0.4992}{0.4423}(4188.81)(7772.58) + \frac{0.3846-0.4438}{0.3846}(6419.66)(7977.99) + \frac{0.3654-0.2218}{0.3654}(6419.66)(7772.58) + \frac{0.5769-0.6656}{0.5769}(7977.99)(7772.58)\Big]$$

$$= 2[3596005.75 - 9176162.74 - 7824117.35 + 4077392.59 - 8906211.77 - 5139789.27 - 4188429.29 - 7883479.48 + 19609346.69 - 9534145.32]$$

$$= 2(-25369590.19) = -50739180.38$$

$$\hat{V}(\hat{Y}_{HT}) = 57501303.64 \ - 50739180.38$$

$$\hat{V}(\hat{Y}_{HT}) = 6762123.26$$

Standard error (SE) is

$$\text{SE}(\hat{Y}_{HT}) = \sqrt{6762123.26}$$

$$\text{SE}(\hat{Y}_{HT}) = 2600.41$$

Variance Estimate for 2022

$$\sum_{i=1}^{n}\left(\frac{1-\pi_i}{\pi_i^2}\ y_i^2\right) = \frac{1-0.6731}{0.4531}(21678336) + \frac{1-0.8654}{0.7489}(9753129) + \frac{1-0.3846}{0.1479}(6285049) + \frac{1-1.1538}{1.3313}(64850809) + \frac{1-0.5769}{0.3328}(19140625)$$

$$= 15640362.04 + 1752932.52 + 26878520.88 - 7492966.07 + 24334129.92$$

$$= 61113979.29$$

$$2\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_{ij}}\frac{y_i}{\pi_i}\frac{y_j}{\pi_j} = 2\Big[\frac{0.6731-0.5825}{0.6731}(6917.25)(3608.74) + \frac{0.2115-0.2588}{0.2115}(6917.25)(6518.46) + \frac{0.6731-0.7766}{0.6731}(6917.25)(6979.55) + \frac{0.4231-0.3883}{0.4231}(6917.25)(7583.64) + \frac{0.2500-0.3328}{0.2500}(3608.74)(6518.46) + \frac{0.8654-0.9985}{0.8654}(3608.74)(6979.55) + \frac{0.4423-0.4992}{0.4423}(3608.74)(7583.64) + \frac{0.3846-0.4438}{0.3846}(6518.46)(6979.55) + \frac{0.3654-0.2218}{0.3654}(6518.46)(7583.64) + \frac{0.5769-0.6656}{0.5769}(6979.55)(7583.64)\Big]$$

$$= 2[3359987.57 - 10105235.67 - 7423721.18 + 4314668.15 - 7790959.14 - 3873862.31 - 3520696.81 - 7003011.74 + 19427256.47 - 8138250.87]$$

$$= 2(-41507651.06) = -19606328.23$$

$$\hat{V}(\hat{Y}_{HT}) = 61113979.29 \ - 19606328.23$$

$$\hat{V}(\hat{Y}_{HT}) = 4427.90$$

Standard error (SE) is

$\mathrm{SE}(\hat{Y}_{HT}) = \sqrt{236680462.4}$

$\mathrm{SE}(\hat{Y}_{HT})$ =15384.42

Rao-Hartley-Cochran's Sampling Scheme

Faculties was selected under Rao-Hartley-Cochran's sampling scheme

Table 5: Categories of faculties under Rao-Hartley-Cochran's sampling scheme

First Random Group				**Second Random Group**				**Third Random Group**			
Faculties	X_i	**Cum** X_i	**Range**	**Faculties**	X_i	**Cum** X_i	**Range**	**Faculties**	X_i	**Cum** X_i	**Range**
Agricultural science	6	6	1-6	Education	9	9	1-9	Medicine	1	1	1
Law*	1	7	7	Engineering	4	13	10-13	Management science*	3	4	2-4
Social Science	6	13	8-13	Art*	7	20	14-20	Basic Medical Science	3	7	5-7
								Science	12	11	8-11

Note: * indicates selected faculties based on random sampling

Source: Author's Computation (2023)

Result presented in Table 5 reveals that there are three random groups for the faculties covered in the study. For the first random group, there are faculties of agricultural science, law and social sciences, in the second random group we have faculties of education, engineering and art, while the third random group consist faculties of medicine, management science, basic medical science and science. In line with selection of one faculty from each of the groups, based on random sampling, the faculties selected were law, art and management science for first, second and third group respectively.

Probability of Selection under Rao-Hartley-Cochran's sampling scheme

Table 6: Sampled Faculties and Selection Probabilities

Sample Faculty	p_i*	p_i	p_g	**2017**	**2018**	**2019**	**2021**	**2022**
Law	$^1/_{13}$	$^1/_{52}$	$^{13}/_{52}$	768	817	664	606	592
Art	$^7/_{20}$	$^7/_{52}$	$^{20}/_{52}$	2673	3053	3936	4293	4656
Management Science	$^3/_{19}$	$^3/_{52}$	$^{19}/_{52}$	3335	3925	4212	5068	4367

Source: Author's Computation (2023)

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Estimation of Total Population

The Rao-Hartley-Cochran's estimator of the population total is

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} {y_{gi}}/{p_i^*}$$

Total Population Estimate for 2017

$$\hat{Y}_{RHC} = \frac{768}{\frac{1}{13}} + \frac{2673}{\frac{7}{20}} + \frac{3335}{\frac{3}{19}}$$

$$\hat{Y}_{RHC} = 9984 + 7637.14 + 21181.67$$

$$\hat{Y}_{RHC} = 38742.81 \sim 38743\ students$$

Total Population Estimate for 2018

$$\hat{Y}_{RHC} = \frac{817}{\frac{1}{13}} + \frac{3053}{\frac{7}{20}} + \frac{3925}{\frac{3}{19}}$$

$$\hat{Y}_{RHC} = 10621 + 8722.86 + 24858.33$$

$$\hat{Y}_{RHC} = 44202.19 \sim 44202\ students$$

Total Population Estimate for 2019

$$\hat{Y}_{RHC} = \frac{664}{\frac{1}{13}} + \frac{3936}{\frac{7}{20}} + \frac{4272}{\frac{3}{19}}$$

$$\hat{Y}_{RHC} = 8632 + 11245.71 + 27056$$

$$\hat{Y}_{RHC} = 46933.71 \sim 46934\ students$$

Total Population Estimate for 2021

$$\hat{Y}_{RHC} = \frac{606}{\frac{1}{13}} + \frac{4293}{\frac{7}{20}} + \frac{5068}{\frac{3}{19}}$$

$$\hat{Y}_{RHC} = 7878 + 12265.71 + 32097.33$$

$$\hat{Y}_{RHC} = 52241.04 \sim 52241\ students$$

Total Population Estimate for 2022

$$\hat{Y}_{RHC} = \frac{592}{\frac{1}{13}} + \frac{4656}{\frac{7}{20}} + \frac{4367}{\frac{3}{19}}$$

$$\hat{Y}_{RHC} = 7696 + 13302.86 + 27657.67$$

$$\hat{Y}_{RHC} = 48656.53 \sim 48657\ students$$

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Estimation of Variance

The unbiased estimator of variance of $\hat{Y}_{RHC}$ is

$$V\widehat{\left(\hat{Y}_{RHC}\right)}\left[\sum_{g=1}^{n} P_g\left(\frac{y_{gi}}{p_i}-\hat{Y}_{RHC}\right)^2\right]$$

Variance Estimate for 2017

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{10-3}{10(2)}\left[(39936)^2\frac{13}{52}+(19856.57)^2\frac{20}{52}+(57806.67)^2\frac{19}{52}-(38742.81)^2\right]$$

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{7}{20}\left[398721024+151647450.8+1220973285-1501005327\right]$$

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{7}{20}\,(270336432.8)$$

$$\hat{V}\left(\hat{Y}_{RHC}\right)=94617751.48$$

Standard error (SE) is

$\text{SE}\left(\hat{Y}_{RHC}\right)=\sqrt{94617751.48}$

$\text{SE}\left(\hat{Y}_{RHC}\right)=9727.17$

Variance Estimate for 2018

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{10-3}{10(2)}\left[42484^2\frac{13}{52}+(22679.43)^2\frac{20}{52}+(68033.33)^2\frac{19}{52}-(44202.19)^2\right]$$

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{7}{20}\left[451222564+197829440.4+1691195112-1953833601\right]$$

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{7}{20}\,(386413515.4)$$

$$\hat{V}\left(\hat{Y}_{RHC}\right)=135244730.4$$

Standard error (SE) is

$\text{SE}\left(\hat{Y}_{RHC}\right)=\sqrt{135244730.4}$

$\text{SE}\left(\hat{Y}_{RHC}\right)=11629.48$

Variance Estimate for 2019

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{10-3}{10(2)}\left[(34528)^2\frac{13}{52}+(29238.86)^2\frac{20}{52}+(74048)^2\frac{19}{52}-(46933.71)^2\right]$$

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{7}{20}\left[298045696+328811897.7+2003442688-2202773134\right]$$

$$\hat{V}\left(\hat{Y}_{RHC}\right)=\frac{7}{20}\,(427527147.7)$$

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$$\hat{V}(\hat{Y}_{RHC}) = 149634501.7$$

Standard error (SE) is

$\text{SE}(\hat{Y}_{RHC}) = \sqrt{149634501.7}$

$\text{SE}(\hat{Y}_{RHC}) = 12232.52$

Variance Estimate for 2021

$$\hat{V}(\hat{Y}_{RHC}) = \frac{10-3}{10(2)}\left[(31512)^2\frac{13}{52} + (31890.86)^2\frac{20}{52} + (87845.33)^2\frac{19}{52} - (52241.04)^2\right]$$

$$\hat{V}(\hat{Y}_{RHC}) = \frac{7}{20}\left[248251536 + 391164212.1 + 2819600732 - 2729126260\right]$$

$$\hat{V}(\hat{Y}_{RHC}) = \frac{7}{20}(729890220.1)$$

$$\hat{V}(\hat{Y}_{RHC}) = 255461577$$

Standard error (SE) is

$\text{SE}(\hat{Y}_{RHC}) = \sqrt{255461577}$

$\text{SE}(\hat{Y}_{RHC}) = 15983.17$

Variance Estimate for 2022

$$\hat{V}(\hat{Y}_{RHC}) = \frac{10-3}{10(2)}\left[(30784)^2\frac{13}{52} + (34587.43)^2\frac{20}{52} + (75694.67)^2\frac{19}{52} - (48656.53)^2\right]$$

$$\hat{V}(\hat{Y}_{RHC}) = \frac{7}{20}\left[236913664 + 460111659 + 2093538043 - 2367457912\right]$$

$$\hat{V}(\hat{Y}_{RHC}) = \frac{7}{20}(423105454.2)$$

$$\hat{V}(\hat{Y}_{RHC}) = 148086909$$

Standard error (SE) is

$\text{SE}(\hat{Y}_{RHC}) = \sqrt{148086909}$

$\text{SE}(\hat{Y}_{RHC}) = 12169.09$

Comparison of Estimated Population and Variance for the three PPS estimators

Table 7: Comparison of Estimated Population for PPS, HT and RHC

	Estimated Population		
YEAR	**PPS**	**HT**	**RHC**
2017	18864.66	22868.69	38743.00

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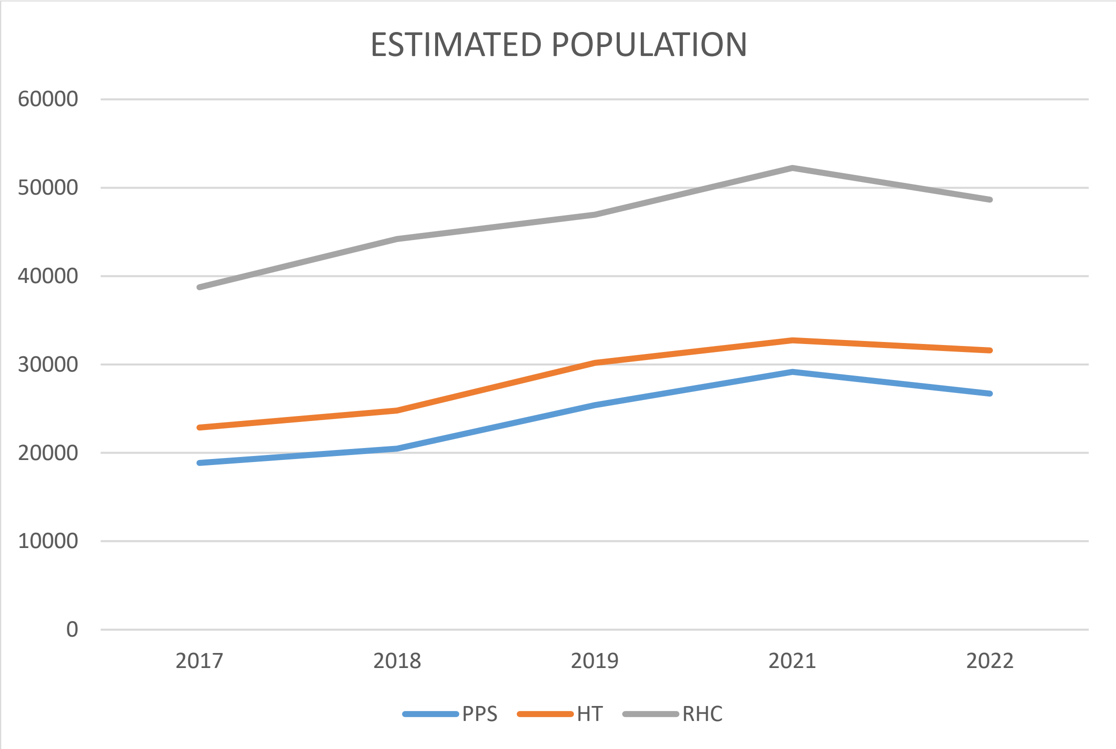
2018	20490.18	24797.40	44202.19
2019	25390.04	30194.76	46933.71
2021	29167.42	32736.40	52241.04
2022	26694.72	31607.09	48656.53

Note: PPS= Probability proportional to size; HT= Horvitz-Thompson; RHC= Rao-Hartley-Cochran's

Figure 1: Estimated Population Total for PPS, HT and RHC



Result in Table 7 and Figure 1 reveals the estimated population total of student's enrolment in Ekiti State University under probability proportional to size sampling scheme, Horvitz-Thompson sampling scheme and Rao-Hartley-Cochran's sampling scheme for the year 2017, 2018, 2019, 2021 and 2022. It is revealed that population proportional to size sampling scheme with replacement has the lowest estimation population while Rao-Hartley-Cochran's sampling scheme without replacement and sample selection has the highest estimated population total of students' enrolment in Ekiti State University over the period covered.

Table 8: Comparison of Estimated Variance for PPS, HT and RHC

	Estimated Variance		
YEAR	**PPS**	**HT**	**RHC**
2017	2970827.17	13188794.22	94617751.48
2018	4832052.04	17120788.35	135244730.4
2019	17837550.85	22385929.7	149634501.7
2021	27953583.24	6762123.26	255461577.0
2022	25822303.28	19606328.23	148086909.0

Figure 2: Estimated Variance for PPS, HT and RHC

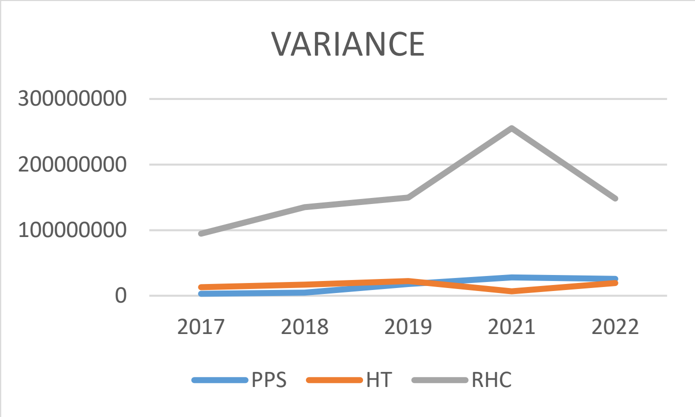


Table 8 and Figure 2 shows estimated variance of the student's enrolment in Ekiti State University under probability proportional to size sampling scheme, Horvitz-Thompson sampling scheme without replacement and Rao-Hartley-Cochran's sampling scheme without replacement and special selection for the year 2017, 2018, 2019, 2021 and 2022. It is revealed that population proportional to size sampling scheme with replacement has the lowest variance in 2017 to 2019, but Hovitz-Thompson has the lowest variance in 2021 and 2022, while Rao-Hartley Cochran's sampling scheme has the highest variance of students' enrolment in Ekiti State University in all the period covered.

DISCUSSION OF FINDING

The study revealed that the probability proportional to size (PPS) sampling scheme is more efficient than the Rao-Hartley-Cochran sampling scheme for estimating the total student enrolment at Ekiti State University. However, PPS showed inconsistencies in efficiency when compared to the Horvitz-Thompson sampling scheme. The results indicated that PPS produced the lowest estimated population total among the three estimators. Regarding variance, PPS had the minimum variance between 2017 and 2019, but the Horvitz-Thompson scheme outperformed it in 2021 and 2022.

A key finding is the difficulty in determining the more efficient estimator between PPS without replacement and Horvitz-Thompson when analyzing continuous (time series) data instead of discrete (point-in-time) data. Nonetheless, PPS remains more efficient than the Rao-Hartley-Cochran sampling scheme.

CONCLUSION

This study concludes that there is a close competition between the probability proportional to size without replacement (PPSWOR) estimator and the Horvitz-Thompson estimator. Both estimators produce population estimates within the total student enrolment, though the PPSWOR estimator generates a lower estimate than the Horvitz-Thompson, but both outperform the Rao-Hartley-Cochran estimator, which overestimates the population. In terms of efficiency, the study finds no clear distinction between PPSWOR and Horvitz-Thompson due to the irregular student enrolment flow in 2020. However, PPSWOR is deemed more efficient than the Rao-Hartley-Cochran estimator.

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