# Mathematical Modeling of Corruption Dynamics in Zimbabwe

**Kudzanai Katandika, Martina Mthombeni** *

**National University of Science and Technology, Zimbabwe**

***Corresponding Author**

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## ABSTRACT

Corruption has become one of the persistent worsening problems affecting Zimbabwe. This study presents the formulation of a basic corruption mathematical model and its analysis. We extend the corruption model and analysis revealed that the model is globally and asymptotically stable. The analysis revealed that corruption free equilibrium is locally asymptotically stable if $R_0 < 1$ and the endemic equilibrium is locally stable whenever $R_0 > 1$. To verify the theoretical analysis , numerical simulations were carried out using MATLAB and Python. The numerical simulations reinforced the analytic solutions and we concluded that, the combination of the proposed control strategies which are religious teaching and mass education combined with use of law enforcement successfully control levels of corruption in the country.

**Keywords and Phrases:** Corruption; Basic/Effective reproduction number; differential equations.

## INTRODUCTION

Corruption in Zimbabwe is complex and challenging to define and measure. It is generally understood as the misuse or abuse of entrusted power and public office for private gain(8). Various definitions highlight its nature as a betrayal of trust and its impact on governance and resource allocation(1). Corruption significantly hampers Zimbabwe's economic development and governance. Transparency International and the World Bank have ranked Zimbabwe poorly on corruption indices, with individuals often engaging in corrupt practices such as petty corruption, nepotism, embezzlement, fraud, and extortion(15). This pervasive issue undermines good governance, distorts public policy, and misallocates resources, severely impacting the country's progress. Local authorities struggle with service delivery due to financial constraints and corruption, forcing residents to pay bribes for services and preferential . Despite various anti-corruption initiatives, Zimbabwe continues to struggle with corruption, impacting economic decline and governance(18). Understanding the transmission dynamics and designing appropriate measures for control are crucial for combating this persistent issue(4). The purpose of the study is to evaluate the spread of corruption during Zimbabwe's economic crisis. It aims to develop mathematical models for understanding corruption dynamics, both with and without control measures, and to assess how the economic downturn affects corruption. The study is based on the assumption of a heterogeneous population that can be divided into compartments with equal death rates due to natural causes. This research is significant as it contributes to the existing literature on Zimbabwe, helps policymakers comprehend the dynamics of corruption, and raises awareness about its impact. It focuses on various sectors, including education, public services, healthcare, judiciary, customs, and taxation, providing a mathematical framework for further research on understanding and controlling corruption through applied mathematics.

While extensive theoretical and econometric research, such as the works of Chiweshe (2017) on political corruption and Muzurura (2017) on its economic impacts, has documented the devastating effects of corruption in the country, a critical methodological gap persists: a lack of dynamic, quantitative models to

M Mthombeni understand its transmission dynamics (8). This gap is addressed by the epidemiological modeling approach, which conceptualizes corruption as a contagious disease. Foundational studies, including the deterministic models of (9; 5) , have successfully applied compartmental frameworks (e.g., SIR-type models) to analyze stability, calculate the basic reproduction number (R0), and demonstrate that corruption can be controlled, though not entirely eradicated. Subsequent research has expanded this model to evaluate interventions; for instance, Fantaye and Birhanu (2022) incorporated compartments for jailed and honest individuals, while (6) specifically modeled the impact of religious teachings and mass education in Tanzania.By

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incorporating factors like economic pressure and evaluating the synergistic effect of combined control strategies—mass education and religious teachings this research moves beyond theoretical description to provide a predictive, analytical tool. It thereby offers a quantitative basis for informing the strategic efforts of bodies like the Zimbabwe Anti-Corruption Commission (ZACC) in their fight against corruption.

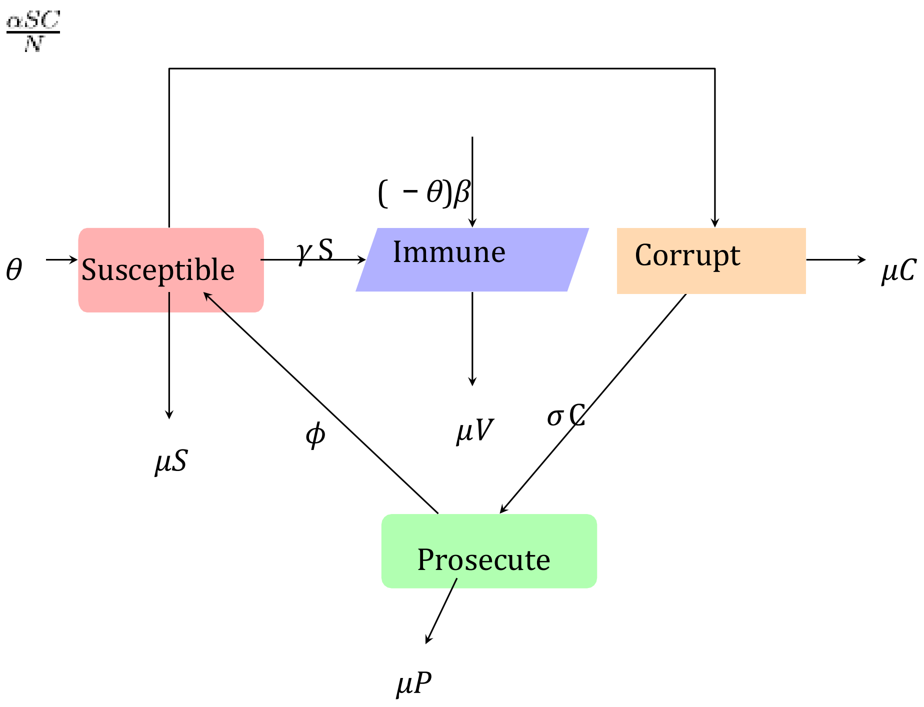
**Mathematical Models formulations**

As we formulate the corruption model, the total population is divided into four categories namely Susceptible (S), immune (V), corrupt (C), prosecuted (P) and Removed (R). Susceptible individuals are those not yet involved in any corrupt activities. Immune class are the individuals exposed to corrupt activities but are not infected. Corrupt individuals are those engaged in corrupt activities and have the ability to infect other individuals. Prosecuted class are those who are serving few sentences in prison after being found guilty of corrupt acts. The dynamics of the basic model can be described , at time t, by the following system of non linear ordinary differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= \theta\beta - \frac{\alpha SC}{N} - (\mu+\gamma)S + \phi P \\
\frac{dV}{dt} &= (1-\theta)\beta + \gamma S - \mu V \\
\frac{dC}{dt} &= \frac{\alpha SC}{N} - (\sigma+\mu)C \\
\frac{dP}{dt} &= \sigma C - (\phi+\mu)P
\end{aligned} \tag{1}$$

**The Corruption Basic Model**

**Figure 1:** Corruption Basic Model without control strategies.




**Corruption Extended Model**

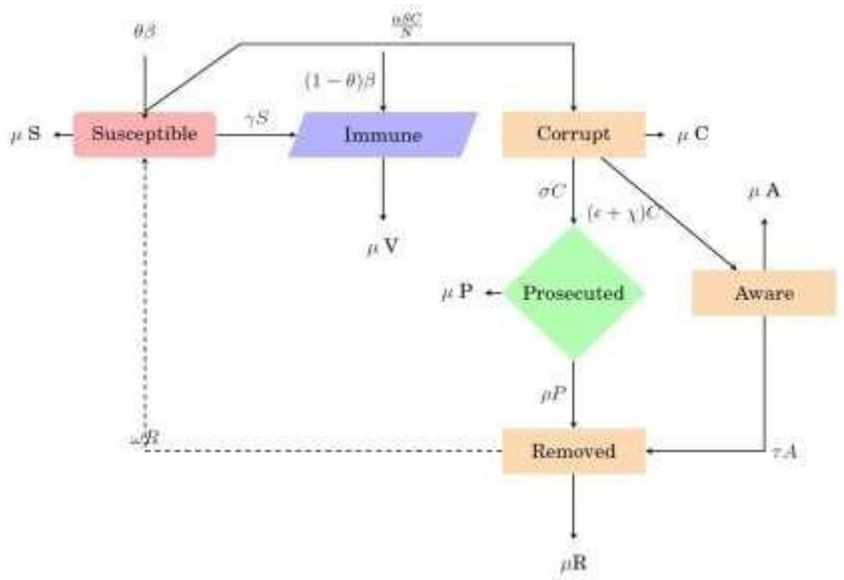
As we formulate the extended corruption model, the total population is divided into six categories namely Susceptible (S), immune (V), corrupt (C), prosecuted (P), aware class(A) and Removed (R). Susceptible individuals are those not yet involved in any corrupt activities. Immune class are the individuals exposed to corrupt activities but are not infected. Corrupt individuals are those engaged in corrupt activities and have the ability to infect other individuals. Prosecuted class are those who are serving few sentences in prison after being found guilty of corrupt acts.The aware class are individuals who now have attended the awareness campaigns and educations and have the knowledge on the dangers of practicing corrupt activities on the country's economy.

The removed class are those individuals who have bailed out of practicing corrupt activities and some of them have been released from prison.

**Table 1:** Description of Model Parameters.

| Parameter | Description |
|---|---|
| θ | Proportion of individuals not recruited immune |
| β | Recruitment Rate |
| α | Rate at susceptible individuals get corruption |
| γ | Rate at which susceptible individuals become immune to corruption |
| ϕ | Rate at which prosecuted are released form prison into the society |
| μ | Natual Death Rate |
| σ | Rate at which corrupt individuals are prosecuted |
| ρ | Rate at which prosecuted individuals are removed from prison |
| ω | Rate at which removed individuals become susceptible |
| τ | Rate at which aware individuals move to the remove class |
| ϵ | Rate of change of corrupt individuals due to mass education |
| χ | Rate at which corrupt individuals change due to religious teachings |

**Figure 2:** Corruption Extended Model




The dynamics of corruption transmission is explained by the following non linear differential equations:

$$
\begin{aligned}
\frac{dS}{dt} &= \theta\beta - \frac{\alpha SC}{N} - (\mu + \gamma)S + \omega R \\
\frac{dV}{dt} &= (1-\theta)\beta + \gamma S - \mu V \\
\frac{dC}{dt} &= \frac{\alpha SC}{N} - (\sigma + \epsilon + \chi + \mu)C \\
\frac{dA}{dt} &= (\epsilon + \chi)C - (\tau + \mu)A \\
\frac{dP}{dt} &= \sigma C - (\rho + \mu)P \\
\frac{dR}{dt} &= \rho P + \tau A - (\mu + \omega)R
\end{aligned} \tag{2}
$$

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**Boundedness of solutions:**

The solutions of the extended model 2 are bounded in Ψ. We prove the boundedness of our extended corruption model for the biologically feasible region.

**Proof.**: Let $N(t) = (S(t) + V(t) + C(t) + A(t) + P(t) + R(t)) \in \mathbb{R}^6_+$ denote the total population. Thus:

$$\frac{dN}{dt} = \beta - \mu N. \tag{3)(4}$$

.

Therefore in conclusion the mathematical model 2 is positive and are bounded in the domain Ψ ,∀t > 0

**Positivity of solutions:**

**Theorem 2.1.** The positive invariant region D = S(0), V (0), C(0), A(0), P(0), R(0) are positive, the solutions of S(t), V (t), C(t), A(t),P(t), R(t) are also non negative ∀t ≥ 0.

**Proof.** : From the system of equations,

$$\begin{aligned}
\frac{dS}{dt} &\geq -\frac{\alpha SC}{N} - (\mu + \gamma)S \\
\frac{dV}{dt} &\geq -\mu V \\
\frac{dC}{dt} &\geq \frac{\alpha SC}{N} - (\sigma + \epsilon + \chi + \mu)C \\
\frac{dA}{dt} &\geq -(\tau + \mu)A \\
\frac{dP}{dt} &\geq -(\rho + \mu)P \\
\frac{dR}{dt} &\geq -(\omega + \mu)R
\end{aligned} \tag{5}$$

Using the method of separation of variables and integrating both sides ,solving the equations and introducing the initial conditions we obtain the following solutions,:

$$\begin{cases}
S(t) \geq S_0 e^{-\left(\frac{\alpha C}{N} + \mu + \gamma\right)t} \\
V(t) \geq V_0 e^{-\mu t} \\
C(t) \geq C_0 e^{-\left(\sigma + \epsilon + \chi + \mu - \frac{\alpha S}{N}\right)t} \\
A(t) \geq A_0 e^{-(\tau + \mu)t} \\
P(t) \geq P_0 e^{-(\rho + \mu)t} \\
R(t) \geq R_0 e^{-(\omega + \mu)t}
\end{cases} \tag{6}$$

Taking the limit a t → ∞, the exponential term decays and the variables S(t), V (t), C(t), A(t), P(t),R(t) become zero. Therefore the variables are non negative for∀t ≥ 0. □

**Corruption Free Equilibrium:**

The corruption free equilibrium is obtained by taking time derivatives of the part of population free from corruption and equating them to zero. Only susceptible and immune class exists in the population which implies that C = 0, P = 0, R = 0, A = 0.

$$0 = \theta\beta - (\mu + \gamma)S$$

(7)

$$0 = (1 - \theta)\beta + \gamma S - \mu V$$

Hence the corruption free equilibrium point is given by ,

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$$E_*=\begin{pmatrix} S_* \\ V_* \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}=\begin{pmatrix} \frac{\theta\beta}{\mu+\gamma} \\ \frac{(1-\theta+\gamma\theta)\beta}{\mu} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

**The Effective Reproduction Number,** $R_e$

In this study, the effective reproduction number represents the anticipated count of new cases of corruption that one corrupt person can cause in a completely susceptible population over the constant period. Following the approach used by (1; 3) , we compute our effective reproduction number using the next generation matrix approach. The reproduction number is given by :

Re = κFV −1

where F is the frequency of new infections, and V is the rate at which corruption is transmitted from the corrupt class to other classes

$$F=\begin{pmatrix} \frac{\alpha SC}{N} \\ 0 \end{pmatrix} V=\begin{pmatrix} -(\sigma+\epsilon+\chi+\mu)C \\ \sigma C-(\rho+\mu)P \end{pmatrix}$$

$$F_i=\begin{pmatrix} \frac{df_1}{dC} & \frac{df_1}{dP} \\ \frac{dg_1}{dC} & \frac{dg_1}{dP} \end{pmatrix} V_i=\begin{pmatrix} \frac{df_2}{dC} & \frac{df_2}{dP} \\ \frac{dg_2}{dC} & \frac{dg_2}{dP} \end{pmatrix}$$

The final F and V matrices are:

$$F=\begin{pmatrix} \frac{\alpha S}{N} & 0 \\ 0 & 0 \end{pmatrix} V=\begin{pmatrix} -(\sigma+\epsilon+\chi+\mu) & 0 \\ \sigma & -(\rho+\mu) \end{pmatrix}$$

The inverse of Matrix V is given by

$$V^{-1}=\frac{1}{(\sigma+\epsilon+\chi+\mu)(\rho+\mu)}\begin{pmatrix} -(\rho+\mu) & 0 \\ -\sigma & -(\sigma+\epsilon+\chi+\mu) \end{pmatrix}$$

$$\begin{bmatrix} -\frac{1}{(\sigma+\epsilon+\chi+\mu)} & 0 \\ -\frac{\sigma}{(\sigma+\epsilon+\chi+\mu)(\rho+\mu)} & -\frac{1}{(\rho+\mu)} \end{bmatrix}$$

Finally, Z = FV $^{-1}$ is given as follows

$$\begin{bmatrix} \frac{\alpha S}{N(\sigma+\epsilon+\chi+\mu)} & 0 \\ 0 & 0 \end{bmatrix}$$

Given the matrix has one nonzero eigenvalue, its spectral radius ( also known as the dominant eigenvalue

) is denoted by κFV $^{-1}$ is

$$R_e=\kappa FV^{-1}=\frac{\alpha S}{N(\sigma+\epsilon+\chi+\mu)}=\frac{\alpha\theta\beta}{N(\sigma+\epsilon+\chi+\mu)}$$

Sensitivity Analysis for effective reproduction Number,$R_e$

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$$= \frac{\theta\beta}{N(\sigma+\epsilon+\chi+\mu)} > 0$$

$$= \frac{\alpha\beta}{N(\sigma+\epsilon+\chi+\mu)} > 0$$

$$= \frac{\alpha\theta}{N(\sigma+\epsilon+\chi+\mu)} > 0$$

$$= -\frac{\alpha\theta\beta}{N(\sigma+\epsilon+\chi+\mu)^2} < 0$$

$$= \frac{\alpha\theta\beta}{N(\sigma+\epsilon+\chi+\mu)^2} < 0$$

$$= \frac{\alpha\theta\beta}{N(\sigma+\epsilon+\chi+\mu)^2} < 0$$

$$= \frac{\alpha\theta\beta}{N(\sigma+\epsilon+\chi+\mu)^2} < 0$$

$$= 0 \qquad (8)$$

$$\frac{dR_e}{d\tau} = 0$$

After computations, the sensitivity of α , θ , β is found positive that is greater than zero. This indicates that a significant impact on the model's outcome can be changed if noticeable changes have been made in the parameter value. In other words, the parameter is influential, changes in it can affect the dynamics of corruption. Also, the sensitivity of σ , ϵ ,χ , μ is negative that is less than zero. This implies an increase in the parameter value reduces the model predicted outcomes. σ , ϵ, μ have an inverse relationship with the dynamics of the model. τ , ϕ, ω have the sensitivity equal to zero, meaning variations in the respective parameter values won't significantly affect the model behavior.

**Local Stability of Corruption Free Equilibrium**

We take into account all model equations and then compute the Jacobian matrix to evaluate the local stability of the equilibrium points.

**Theorem 2.2.** The corruption free equilibrium is locally stable if the eigenvalues of the jacobian matrix are negative and $R_e \leq 1$ if not then it is unstable.

**Proof.**

$$J(E_0) = \begin{pmatrix} -(\frac{\alpha C}{N}+\mu+\gamma) & 0 & -\frac{\alpha S}{N} & 0 & 0 & \omega \\ \gamma & -\mu & 0 & 0 & 0 & 0 \\ \frac{\alpha C}{N} & 0 & -(\sigma+\epsilon+\chi+\mu) & 0 & 0 & 0 \\ 0 & 0 & (\epsilon+\chi) & -(\tau+\mu) & 0 & 0 \\ 0 & 0 & \sigma & 0 & -(\rho+\mu) & 0 \\ 0 & 0 & 0 & \tau & \rho & -(\mu+\omega) \end{pmatrix}$$

We calculate the eigenvalues by applying the linearisation method to the jacobian matrix that is ;

$|J_{E0} - \lambda I| = 0$

We obtain the following eigenvalues;

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$\lambda_1$ $\lambda_2$ $\lambda_3$

$\lambda_4$(9) $\lambda_5$ $\lambda_6$

$$= -(\frac{\alpha C}{N} + \mu + \gamma)$$
$$= -\mu$$
$$= -(\sigma + \epsilon + \chi + \mu)$$
$$= -(\tau + \mu)$$
$$= -(\rho + \mu)$$
$$= -(\omega + \mu)$$

Since the eigenvalues are negative then the model is locally asymptotically stable. This suggests that corruption can be eliminated if the initial number of corrupt people falls within the corruption free equilibrium point's basin of attraction.

**Global Stability of Corruption Free Equilibrium**

To establish the global stability of the system , we implement the technique used by (7). Rewriting the system of equations as follows;

$$\frac{dX}{dt} = F(X,Z) \qquad (10)$$
$$\frac{dZ}{dt} = G(X,Z), G(X,0) = 0$$

$$\frac{dX}{dt} = F(X,Z)$$
$$\frac{dZ}{dt} = G(X,Z), G(X,0) = 0 \qquad (11)$$

Where X represent the uncorrupted classes, that is S, V . While Z represent the corrupted classes that is C, A, P.

The corruption free equilibrium is now denoted but $E_0 = (X^0,0)$. For the existence of the global stability of the corruption free equilibrium , the conditions a and b are necessary to be satisfied:

1. $\frac{dX}{dt} = F(X,0)$ , $X^0$ is globally asymptotically stable.
2. $G(X,Z) = AZ - \hat{G}(X,Z)$ , $\hat{G}(X,Z) \geq 0$.

$A = D_Z G(X^0,0)$ is a M matrix (the off diagonal entries of matrix A are non negative) and Ω is the region where the model is biological feasible.

**Theorem 2.3.** The corruption free equilibrium is globally asymptotically stable when $R_e < 1$ and the conditions a; b are satisfied

**Proof.** Considering the following;

$$\frac{dX}{dt} = F(X,Z)$$
$$\frac{dZ}{dt} = G(X,Z), G(X,0) = 0$$

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$$F(X,Z)=\begin{pmatrix} \theta\beta-\frac{\alpha SC}{N}-(\mu+\gamma)S+\omega R \\ (1-\theta)\beta+\gamma S-\mu V \\ \rho P+\tau A-(\mu+\omega)R \end{pmatrix}$$

$$G(X,Z)=\begin{pmatrix} \frac{\alpha SC}{N}-(\sigma+\epsilon+\chi+\mu)C \\ (\epsilon+\chi)C-(\tau+\mu)A \\ \sigma C-(\rho+\mu)P \end{pmatrix}$$

At equilibrium point C = A = P = R = 0,therefore;

$$F(X,0)=\begin{pmatrix} \theta\beta-\frac{\alpha SC}{N}-(\mu+\gamma)S+\omega R \\ (1-\theta)\beta+\gamma S-\mu V \\ 0 \end{pmatrix}=\begin{pmatrix} \frac{\theta\beta}{\mu+\gamma} \\ \frac{(1-\theta+\gamma\theta)\beta}{\mu} \\ 0 \end{pmatrix}$$

From the above computations, $X^0=(\frac{\theta\beta}{\mu+\gamma},\frac{(1-\theta+\gamma\theta)\beta}{\mu}$'0). This can be verified from the solutions as t 7−→ ∞ the solutions $S(\infty),V(\infty)\Longrightarrow\frac{\theta\beta}{\mu+\gamma},\frac{(1-\theta+\gamma\theta)\beta}{\mu}$. This implies that $X^0$ is a globally asymptotically point and is globally convergent in Ψ. Therefore condition 1 is satisfied.

For the second condition, we have ;

G(X,Z) = AZ − Gˆ(X,Z),Gˆ(X,Z) ≥ 0

Therefore,

Gˆ(X,Z) = AZ − G(X,Z)

Computing the first partial derivative of G(X,Z), with respect to C,A,P at corruption free equilibrium point result in producing a matrix A. Z is the column vector derived from the system of equations.

$$Z=\begin{pmatrix} C \\ A \\ P \end{pmatrix}$$

$$A=\begin{pmatrix} \frac{\alpha\theta\beta}{N}-(\sigma+\epsilon+\chi+\mu) & 0 & 0 \\ (\epsilon+\chi) & -(\tau+\mu) & 0 \\ \sigma & 0 & -(\mu+\rho) \end{pmatrix}$$

$$G(X,Z)=\begin{pmatrix} \frac{\alpha SC}{N}-(\sigma+\epsilon+\chi+\mu)C \\ (\epsilon+\chi)C-(\tau+\mu)A \\ \sigma C-(\rho+\mu)P \end{pmatrix}$$

from the expression we have,

$$G(X,Z)=\begin{pmatrix} \frac{\alpha\theta\beta}{N}-(\sigma+\epsilon+\chi+\mu) & 0 & 0 \\ (\epsilon+\chi) & -(\tau+\mu) & 0 \\ \sigma & 0 & -(\mu+\rho) \end{pmatrix}\begin{pmatrix} C \\ A \\ P \end{pmatrix}-\begin{pmatrix} \frac{\alpha SC}{N}-(\sigma+\epsilon+\mu)C \\ (\epsilon+\chi)C-(\tau+\mu)A \\ \sigma C-(\rho+\mu)P \end{pmatrix}$$

$$=\begin{pmatrix} \frac{\alpha\beta\theta}{N}-\frac{\alpha SC}{N} \\ 0 \\ 0 \end{pmatrix}$$

$$=\begin{pmatrix} \frac{\alpha C}{N}(\theta\beta-S) \\ 0 \\ 0 \end{pmatrix}\geq 0$$

From Gˆ(X,Z), its clear that $\hat{G}_1(X,Z)\geq 0$ ,$\hat{G}_2(X,Z)=0$ ,which inturn lead to Gˆ(X,Z) ≥ 0.This means that condition 2 is satisfied. We conclude that the corruption free equilibrium is globally asymptotically stable provided that $R_0<1$. □

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**Existence of Corruption Endemic Equilibrium**

The endemic equilibrium point denoted by $E^* = S^* + V^* + C^* + A^* + P^* + R^*$, is a situation in the society where the rate of corruption is stable and odes not change over time. The corruption endemic equilibrium is reached when the system of equations describing corruption model reach a steady state. All sub-populations are positive (9).

The endemic equilibrium points are as follows:

$$S^* = \frac{(\sigma + \epsilon + \chi + \mu)N}{\alpha}$$

$$V^* = \frac{\gamma S^* + (1-\theta)\beta}{\mu}$$

$$C^* = \frac{\Lambda N + \alpha(\beta\theta + \omega R^*)}{\alpha\Lambda}$$

$$A^* = \frac{(\epsilon + \chi)H}{\alpha\Lambda(\tau + \mu)} \quad (12)$$

$$P^* = \frac{\alpha H}{\alpha\Lambda(\rho + \mu)}$$

$$R^* = \frac{\rho P^* + \tau A^*}{(\omega + \mu)}$$

where

$H = \Lambda N + \alpha(\beta\theta + \omega R^*)$

$\Lambda = \sigma + \epsilon + \chi + \mu$

**Local Stability of Corruption Endemic Equilibrium**

The Jacobian stability method will be applied to establish the local stability of the corruption endemic equilibrium. The model equations' linearization at the equilibrium point yields the Jacobian matrix.

$$J(E^*{}_0) = \begin{pmatrix} -(\frac{\alpha C^*}{N} + \mu + \gamma) & 0 & -\frac{\alpha S^*}{N} & 0 & 0 & \omega \\ \gamma & -\mu & 0 & 0 & 0 & 0 \\ \frac{\alpha C^*}{N} & 0 & -(\sigma + \epsilon + \chi + \mu) & 0 & 0 & 0 \\ 0 & 0 & \epsilon & -(\tau + \mu) & 0 & 0 \\ 0 & 0 & \sigma & 0 & -(\rho + \mu) & 0 \\ 0 & 0 & 0 & \tau & \rho & -(\mu + \omega) \end{pmatrix}$$

We calculate the eigenvalues by applying the linearisation method to the jacobian matrix i.e,

$|J_{E0} - \lambda I| = 0$

We obtain the following eigenvalues ,

$$= -(\frac{\alpha C^*}{N} + \mu + \gamma)$$
$$= -\mu$$
$$= -(\sigma + \epsilon + \chi + \mu)$$
$$= -(\tau + \mu)$$
$$= -(\rho + \mu)$$
$$= -(\omega + \mu)$$

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$\lambda_1$ $\lambda_2$ $\lambda_3$

$\lambda_4$(13) $\lambda_5$ $\lambda_6$

The characteristic polynomial of the above jacobian matrix is :

λ6 + a1λ5 + a2λ4 + a3λ3 + a4λ2 + a5λ + a6

The eigenvalues of the system are seen to have negative real parts, this implies that the system is locally asymptotically stable.

When $R_e < 1$ , the corruption free equilibrium point is locally asymptotically stable otherwise , it it unstable.

**Global Stability of Corruption Endemic Equilibrium**

**Theorem 2.4** (The Lyapunov Stability Theorem:)**.** Let $(x^*,y^*) = (0,0)$ be the equilibrium point of $x' = f(x,y)$ and L(x,y) be a continuously differentiable positive definite function in the neighborhood of the origin(J.Y.T Mugisha,2008),(**?** ).

1. If L (x,y) ≤ 0,∀x,y ∈ U, then the point is stable.
2. If L (x,y) < 0,∀x,y ∈ U, then the origin is uniformly asymptotically stable.
3. If L > 0,∀x,y ∈ U, then the origin is unstable.

The endemic equilibrium is globally stable when the Lyapunov function conditions hold.

**Proof.** Consider the lyapunov function;

$L(S,V,C,A,P,R) = (S - S^*In(S)) + (V - V^*In(V)) + (C - C^*In(C))+$

(14)

$+(A - A^*In(A)) + (P - P^*In(P)) + (R - R^*In(R))$

Taking the derivatives along the solutions of the system of equations:

$$T' = (1 - \frac{S^*}{S})S' + (1 - \frac{V^*}{V})V' + (1 - \frac{C^*}{C})C' + (1 - \frac{A^*}{A})A' + (1 - \frac{P^*}{P})P' + (1 - \frac{R^*}{R})R' \quad (15)$$

′Global stability for corruption endemic equilibrium hold when L ≤ 0, therefore,

$$\begin{aligned} U' = \frac{T'}{T} &= (\frac{\theta\beta}{S} - \frac{\alpha C}{N} - (\mu+\gamma) + \frac{\omega R}{S}) + (\frac{(1-\theta)\beta}{V} + \frac{\gamma S}{V} - \mu) + (\frac{\alpha S}{N} - (\sigma+\epsilon+\chi+\mu)) + \\ &(\frac{(\epsilon+\chi)C}{A} - (\tau+\mu)) + (\frac{\sigma C}{P} - (\rho+\mu)) + (\frac{\rho P}{R} + \frac{\tau A}{R} - (\mu+\omega)) \\ &= -(-\frac{\theta\beta}{S} + \frac{\alpha C}{N} + (\mu+\gamma) - \frac{\omega R}{S}) + -(-\frac{(1-\theta)\beta}{V} - \frac{\gamma S}{V} + \mu) - (-\frac{\alpha S}{N} + (\sigma+\epsilon+\chi+\mu)) - \\ &(-\frac{(\epsilon+\chi)C}{A} + (\tau+\mu)) - (-\frac{\sigma C}{P} + (\rho+\mu)) - (-\frac{\rho P}{R} - \frac{\tau A}{R} + (\mu+\omega)) \le 0 \end{aligned} \quad (16)$$

Since U≤ 0 holds, therefore we conclude that the endemic equilibrium is globally asymptotically stable. □

**Effect of Intervention Strategies**

In this section, we analyze the effectiveness of intervention strategies by investigating $R_e$.

From the basic model ,we concluded that;

$$R_0 = \frac{\alpha\theta\beta}{N(\sigma + \mu)}$$

From the extended model, we comprehended that;

$$R_e = \frac{\alpha\theta\beta}{N(\sigma + \epsilon + \chi + \mu)}$$

Given that $\epsilon = \chi = 0$ , then ;

$$R_e = \frac{\alpha\theta\beta}{N(\sigma + \mu)} = R_0$$

This means that use of law enforcement is also effective in reducing the spread the spread of corruption.

If we let $\chi = 0$ such that the use of mass education is the only intervention strategy present , then our new reproduction number, $R_m$, becomes;

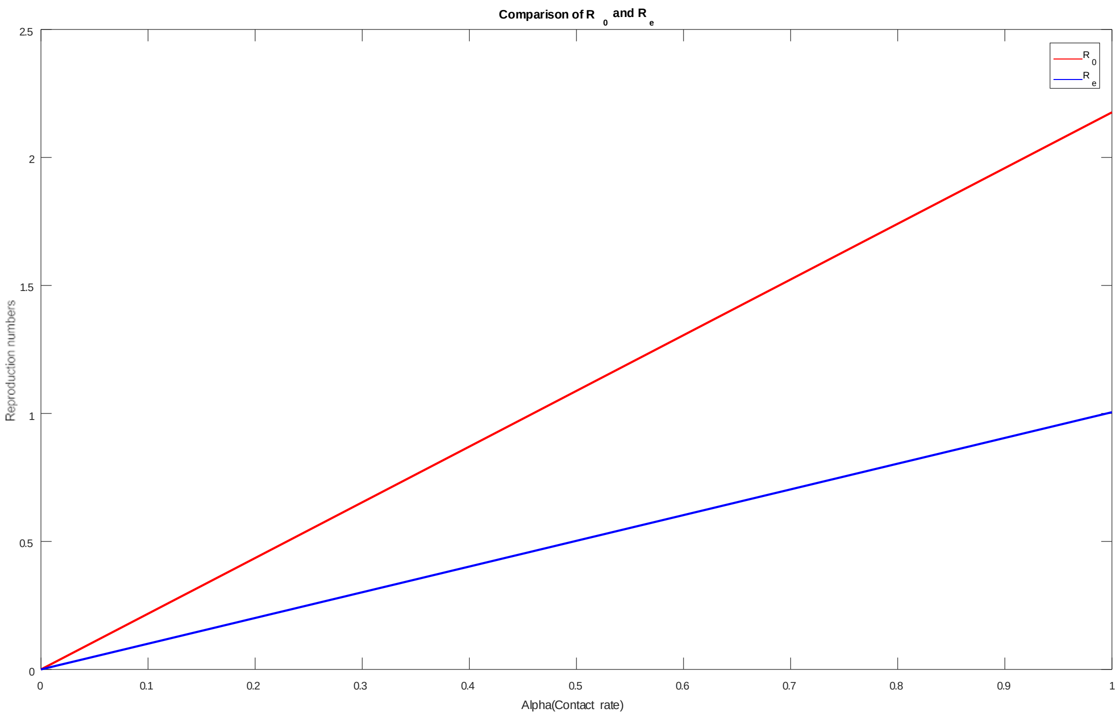
$$R_m = \frac{\alpha\theta\beta}{N(\sigma + \epsilon + \mu)}$$

Letting $\epsilon = 0$ such that use of religious education is the only intervention strategy present , then our new reproduction number, $R_g$, becomes;

$$R_g = \frac{\alpha\theta\beta}{N(\sigma + \chi + \mu)}$$

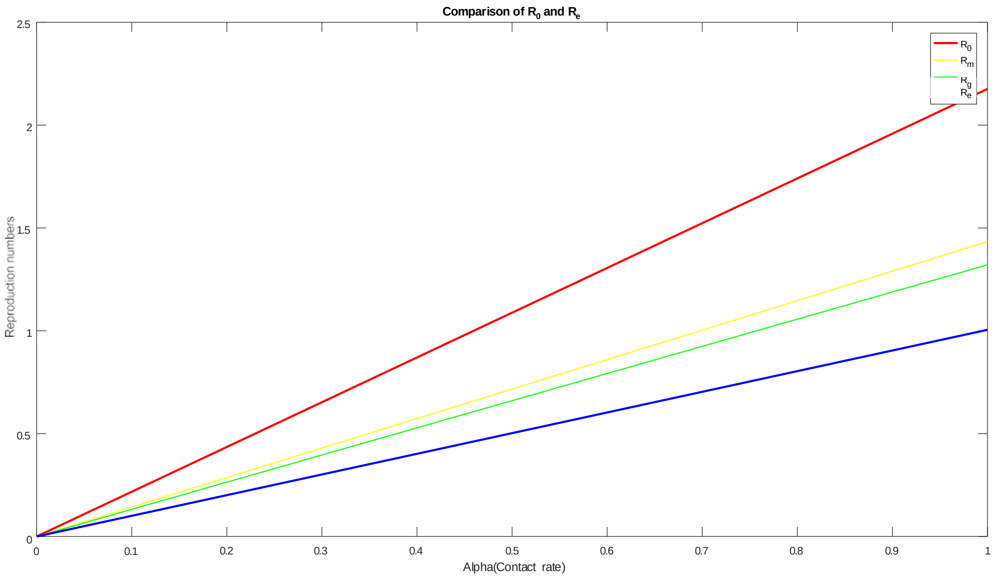
To further explain the computed results above, a graph was fitted comparing $R_0$ and $R_e$. To show the effectiveness of intervention strategies $R_e < R_0$ is plotted against the contact rate , α.Figure 3 shows that $R_e < R_0$.

**Figure 3:** Comparison of $R_0$ and $R_e$.




We also take into consideration the reproduction numbers, $R_m$ and $R_g$ when comparing the reproduction numbers. Figure 4 illustrates that $R_e < R_g < R_m < R_0$ , implying the combination of all intervention strategies have a effect as the combination of strategies yield a result less than $R_0$ . Also, the reproduction number increases as the contact rate,α increases.

**Figure 4:** Comparison of Reproduction Numbers.




## Numerical Simulations And Results

In this section, we present the results of our numerical simulations which build upon the analytical findings established earlier. Using Matlab ODE45 solver software, we solve the equations and investigate the behavior of our systems under different conditions.

**Table 2:** Description of Model Variables.

| Variable | Description | Value | Source |
|---|---|---|---|
| S | Number of Susceptible individuals at time(t) | 900 | Estimated |
| V | Number of Immune individuals at time(t) | 100 | Estimated |
| C | Corrupt individuals at time (t) | 100 | Estimated |
| P | Number of Prosecuted humans at any time(t) | 50 | Estimated |
| A | Number of Aware individuals at time(t) | 100 | Estimated |
| R | Number of Removed individuals at time(t) | 600 | Estimated |

**Table 3:** Parameter Value Description.

| Parameter | Description | Value | Source |
|---|---|---|---|
| $\theta$ | Proportion of individuals not recruited immune | 0.00403 | (9) |
| $\beta$ | Recruitment Rate | 0.042 | (9) |
| $\alpha$ | Rate at susceptible individuals get corruption | 0.5 | (9) |
| $\gamma$ | Rate at which susceptible individuals become immune to corruption | 0.004 | (9) |
| $\phi$ | Rate at which prosecuted are released form prison into the society | 0.143 | Estimated |
| $\mu$ | Natural Death Rate | 0.0189 | (9) |
| $\sigma$ | Rate at which corrupt individuals are prosecuted | 0.0001 | (9) |

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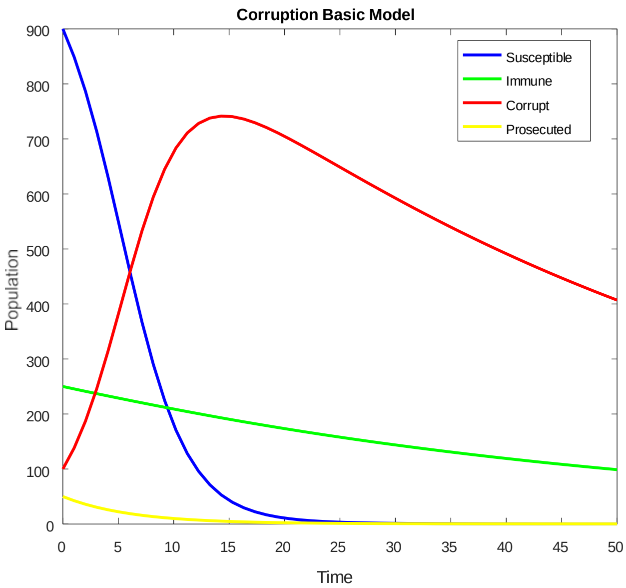
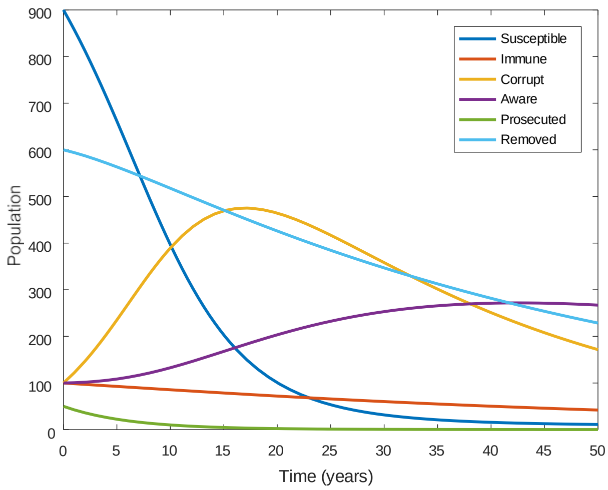
| | | | |
|---|---|---|---|
| $\rho$ | Rate at which prosecuted individuals are removed from prison | 0.143 | (9) |
| $\omega$ | Rate at which removed individuals become susceptible | 0.0021 | (9) |
| $\tau$ | Rate at which aware individuals move to the remove class | 0.000001 | (9) |
| $\epsilon$ | Rate of change of corrupt individuals due to mass education | 0.1 | Estimated |
| $\chi$ | Rate at which corrupt individuals change due to religious teachings | 0.0125 | Estimated |

We demonstrate the dynamics of corruption in the absence of control strategies within the basic corruption model. The numerical simulations were carried out using the parameter values listed in Table 3.

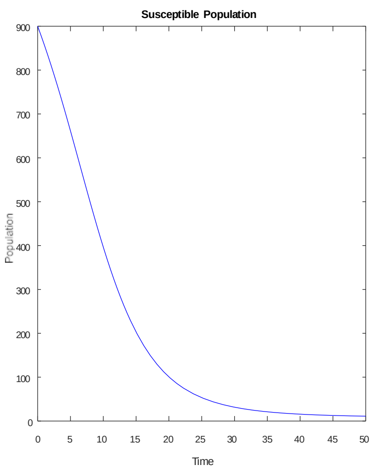
**Figure 5:** Behavior of the Basic model solutions and Trend of the extended model with intervention strategies.



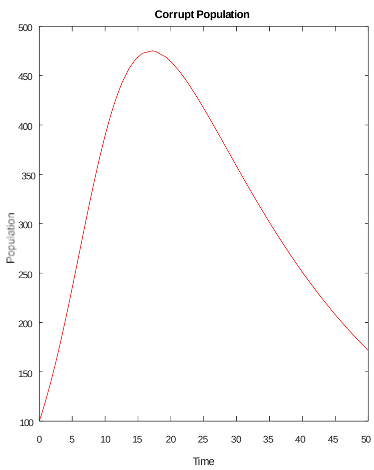




## Simulation of Extended Corruption Model

From Figure above, the blue line represents the susceptible population who are vulnerable to becoming corrupt. The sharp decrease of the line indicates that over a certain period of time , the continuous interaction between the susceptible and corrupt groups reduce the susceptible population to the corrupt group. The green line represents the immune population . The graph remains constant and this means that the number of immune population does not change significantly over time. The increase in the Corrupt class is due to the susceptible population being influenced by the corrupt class to engage in corrupt activities. However, the gradual decrease implies that some corrupt individuals are experiencing natural death $\mu$, and some are being prosecuted. The yellow line indicates individuals who are arrested for breaking the law and are held accountable for practicing corrupt activities.




(a)




(b)

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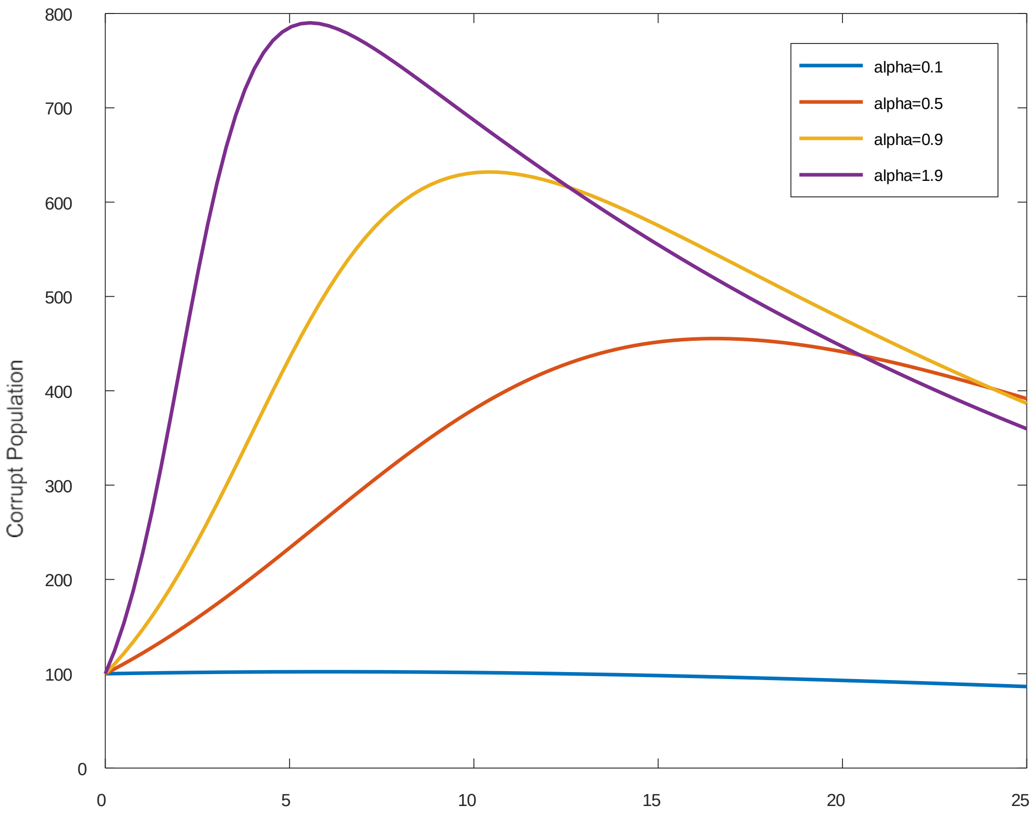
**Contact Rates**

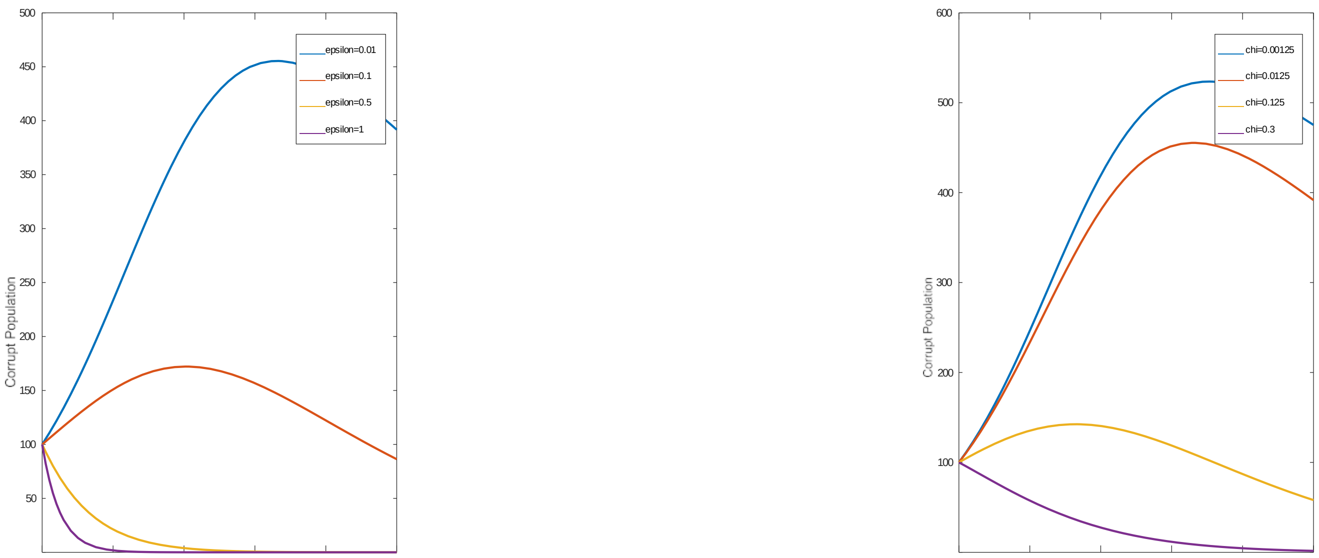



Time (years) (c)

**Figure 6:** From Figure above, its been observed that the corrupt compartment increases, whilst at the same instance the susceptible class decrease due to the interaction and influence of corrupt class and natural death in the compartment. The above figure illustrates that the population susceptible to corruption gradually diminishes approaching a corruption free equilibrium point. This demonstrates a concurrence between the numerical simulations and the local stability analysis lemecha2018modelling.Time series plots varying interaction rates.

## Effects of control strategies against corruption

**Figure 7:** Effect of control stratgies against corruption




(a) (b)

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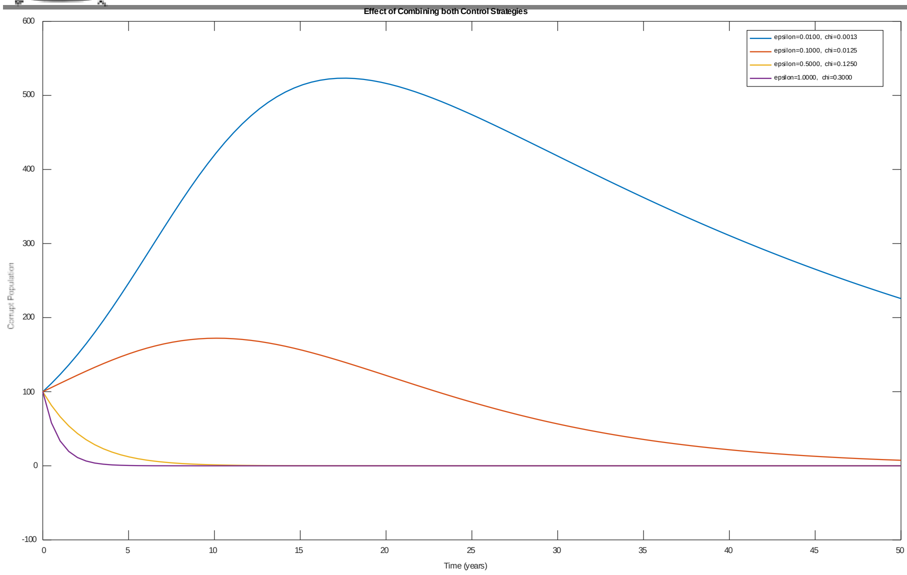


**(c)**

Figure (a) , shows that the more frequently mass education is provided to the communities on corruption and its consequences on economy development, the fewer corrupt people there will be present in the community. Figure (b), illustrates different rates in providing knowledge on corruption in religions. From the graph, the higher the frequency in providing knowledge to the congregation about corruption and its consequences if you caught on practicing reduces the levels of corrupt individuals rapidly. Effective control of corruption levels in Zimbabwe may be obtained from the implementation of intervention strategies such as mass education and religious teachings. From the Figure, it is clearly visible that mass education and religious teachings have great impact when their rates are increased. Due to instability of the country's economy, the control measures are unclear if they can be implemented and attained in Zimbabwe. Basically, the combined implementation of the control strategies even if they are implemented with low effective level can result in reducing corruption in the society and communities.Figure (c) shows the combination of all the control strategies to combat corruption. The implementation of all the control strategies that is mass education and religious teachings as control measures have a significant impact when their rates are increased, corruption levels decrease rapidly. When the control strategies $\epsilon$ , $\chi$ ,are combined and implemented at full potential, corrupt population levels decrease approaching zero, hence corruption will be eradicated.

## CONCLUSIONS

This study has not fully explored all aspects of corruption. Additional research is necessary to delve deeper into understanding corruption dynamics in Zimbabwe. Specifically, Further research is warranted in the following research areas, Social factors impacting reporting behavior of individuals. Studies are needed to explore the factors discouraging and hindering citizens from reporting corruption. Cost effective analysis and Optimal Control. This study will seek to assess the optimal control measures and evaluate the cost effectiveness of anti corruption measures.

## ACKNOWLEDGEMENTS

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