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$b-I_S-$ Open Sets and Decomposition of Continuity Via Idealization

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ABSTRACT

In this paper, we introduce the notion of $b-I_S-$ open sets and strong $B_{IS}-$sets to obtain decomposition of continuity via idealization. Additionally, we investigate properties of $b-I_S-$ open sets and strong $B_{IS}-$sets

Key words and Phrases: semi -I_S- open sets, pre-I_S- open sets, $\alpha-I_S-$ open sets, $b-I_S-$ open sets and strong $B_{IS}-$sets

AMS Subject Classification: 54A05, 54A10.

INTRODUCTION

Ideal in topological spaces have been considered since 1966 by Kuratowski[9]and Vaidyanathaswamy[16].After several decades,in 1990, Jankovic and Hammlet [7] investigated the topological ideals which is the generalization of general topology.Whereas in 2010,Khan and Noiri [8] introduced and studied the concept of semi local functions.In 2014,Shanthi and Rameshkumar [14] introduced semi -I_S- open sets, pre-I_S- open sets and $\alpha-I_S-$ open sets.In this paper we introduce the notions of $b-I_S-$ open sets and strong $B_{IS}-$sets to obtain decomposition of continuity. Let (X,τ) be a topological space and I is an ideal of subset of X. An ideal I on a topological space $(X,\tau)$is a collection of nonempty subsets of X which satisfies (i) $A\in I$ and $B\subset A$implies $B\in I$ and(ii) $A\in I$ and $B\in I$ implies $A\cup B\in I$.Given a topological space (X,τ) with an ideal I on X and if $\wp(X)$is the set of all subsets of X, a set operator $(.)^*:\wp(X)\rightarrow\wp(X)$,called the local function of A with respect to τ and I , is defined as follows:for $A\subset X, A^*(I,\tau)=\{x\in X/U\cap A\notin I foreveryU\in\tau(x)\}$ where $\tau(x)=\{U\in\tau/x\in U\}$(Kuratowski 1966). A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I,\tau)$, called the $*-$topology, finer than τ is defined by $cl^*(A)=A\cup A^*(I,\tau)$(Vaidyanathaswamy,1945).When there is no chance for confusion ,we will simply write A^* for $A^*(I,\tau)$and τ^* or $\tau^*(I)$for $\tau^*(I,\tau)$.If I is an ideal on X,then $(X,\tau,I)$is called an ideal space. $\beta=\{G-A/G\in\tau,A\in I\}$ is a basis for τ^* (Jankovic and Hamlett,1992). If $A\subset X$, cl(A) and int(A)will respectively denote the closure and the interior of A in (X,τ) and $\operatorname{int}^*(A)$ will denote the interior of A in (X,τ^*).

Definition1.1. Let(X,τ) be a topological space .A subset A of X is said to be semiopen[10] if there exists an open set U in X such that $U\subset A\subset cl(U.)$The complement of a semi open set is said to be semi-closed.The collection of semi open(resp.semiclosed)sets in X is denoted by SO(X)(resp.SC(X)).The semi closure of A in (X,τ)is denoted by the intersection of of all semiclosed sets containing A and is denoted by scl(A).

Definition1. 2. For $A\subset X$, $A_*(I,\tau)=\{x\in X/U\cap A\notin I foreveryU\in SO(X)\}$is called the semi-local function [8] of A with respect to I and τ ,where $SO(X,x)=\{U\in SO(X):x\in U\}$.We simply write A_* instead of $A_*(I,\tau)$ .It is given in [1] that $\tau^{*s}(I)$ is a topology on X ,generated by the sub basis $\{U-E:U\in SO(X) andE\in I\}$or

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equivalently $\tau^{*s}(I)=\{U \subset X : cl^{*s}(X-U)=X-U\}$.The closure operator $cl^{*s}(A)=A \cup A_*$ and $\text{int}^{*s}(A)$ denote the interior of the set A (X,τ^{*s},I).It is known that $\tau \subset \tau^*(I) \subset \tau^{*s}(I)$.

Lemma1.3[8]. Let (X,τ,I) be an ideal topological space and $A,B \subset X$.

Then for the semi-local function the following properties hold:

(i) If $A \subset B$,then $A_* \subset B_*$.

(ii) If $U \in \tau$,then $U \cap A_* \subset (U \cap A)_*$.

Definition1.4.

A subset A of a topological space X is said to be

(i) $\alpha -$open [12] if $A \subset \text{int}(cl(\text{int}(A)))$.

(ii) pre-open [11] if $A \subset \text{int}(cl(A))$.

(iii) semi-open [10] if $A \subset cl(\text{int}(A))$.

(iv) t-set [13] if int(A)=int(cl(A)).

(v) b-open set [3] if $A \subset \text{int}(cl(A)) \cup cl(\text{int}(A))$.

(vi) strong B-set [4] if $A=U \cap V$,where U is open, V is t-set and int(cl(A)) =cl(int(A)).

Definition1.5.

A subset A of an ideal topological space (X,τ,I) is said to be

(i) $\alpha - I -$ open [6] if $A \subset \text{int}(cl^*(\text{int}(A)))$.

(ii) $\text{pre} - I -$open [5] if $A \subset \text{int}(cl^*(A))$.

(iii) semi $- I -$open [6] if $A \subset cl^*(\text{int}(A))$.

(iv) $b - I -$ open [2] if $A \subset \text{int}(cl^*(A)) \cup cl^*(\text{int}(A))$

(v) t-I-set [6]if $\text{int}(cl^*(A))=\text{int}(A)$.

(vi) $B_I -$set [6] if $A=U \cap V$, $U \in \tau$ and V is a t-I-set.

(vii) Strong $B_I -$set [2] if $A=U \cap V$, $U \in \tau$ and V is a t-I-set and $\text{int}(cl^*(V))=cl^*(\text{int}(V))$.

Definition1.6.

A subset A of an ideal space (X,τ,I) is said to be

(i) $\alpha - I_S -$ open [14] if $A \subset \text{int}(cl^{*s}(\text{int}(A)))$.

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(ii) pre $-I_S-$open [14] if $A\subset \text{int}(cl^{*s}(A))$.

(iii) semi $-I_S-$open [14] if $A\subset cl^{*s}(\text{int}(A))$.

(iv) $t-I_S-$set [14] $\text{int}(cl^{*s}(A))=\text{int}(A)$.

(v) $B_{IS}-$set [14] if $A=U\cap V$,where $U\in\tau$ and V is an $t-I_S-$set.

The family of all $\alpha-I_S-$ open (resp. Semi $-I_S-$open, Pre$-I_S-$open) sets an ideal topological space $(X,\tau,I)$is denoted by $\alpha ISO(X)(resp.SISO(X), PISO(X))$.

Lemma1.8[15**].**Let (X,τ,I) be an ideal topological space and $A\subset X$.

If U is open in (X,τ,I),then $U\cap cl^{*s}(A)\subset cl^{*s}(U\cap A)$.

2.$b-I_S-$open set

Definition2.1A subset A of an ideal space (X,τ,I) is said to be a $b-I_S-$open set

if $A\subset \text{int}(cl^{*s}(A))\cup cl^{*s}(\text{int}(A))$.

Proposition 2.1Le A be a $b-I_S-$open set such that $\text{int}(A)=\phi$,then A is pre$-I_S-$open.

Proof :Let A be a $b-I_S-$open set. Then we have $A\subset \text{int}(cl^{*s}(A))\cup cl^{*s}(\text{int}(A))$.

If $\text{int}(A)=\phi$,then $cl^{*s}(\text{int}(A))=\phi$ .Therefore, $A\subset \text{int}(cl^{*s}(A))\cup cl^{*s}(\text{int}(A))$becomes

$A\subset \text{int}(cl^{*s}(A))$.

Proposition 2.2 For a sub set of an ideal space (X,τ,I) the following hold.

(i)Every open set is $b-I_S-$open.

(ii)Every semi$-I_S-$open set is $b-I_S-$open.

(iii)Every pre$-I_S-$open set is $b-I_S-$open.

(iv)Every $b-I_S-$open set is b-open.

Proof: (i),(ii),(iii) Obvious.

(iv)Let A be $b-I_S-$open. Then we have

$$A\subset \text{int}(cl^{*s}(A))\cup cl^{*}(\text{int}(A))\subset \text{int}(A^{*s}\cup A)\cup(\text{int}(A))^{*s}\cup \text{int}(A)\subset \text{int}(scl(A)\cup A)\cup scl(\text{int}(A))\cup \text{int}(A)$$
$$\subset \text{int}(cl(A))\cup cl(\text{int}(A))\cup \text{int}(A)\subset \text{int}(cl(A))\cup cl(\text{int}(A)).$$

This shows that A is b-open.

Remark2.1. Converse of the Proposition 2.2 need not be true as seen from the following examples.

Example 2.1Let $X=\{a,b,c,d\}, \tau=\{\phi,\{a\},\{b\},\{a,b\},\{a,d\},\{a,b,d\},X\}$ and $I=\{\phi,\{b\},\{c\},\{b,c\}\}$.Then

(i) $A=\{a,b,c\}$is $b-I_S$ –open but it is not semi$-I_S$ –open.

(ii) $A=\{a,b,c\}$is $b-I_S$ –open but it is not open.

Example2.2. Let $X=\{a,b,c\}, \tau=\{\phi,\{a\},\{a,c\},X\}$and $I=\{\phi,\{c\}\}$.

(i) $A=\{b,c\}$is $b-I_S$ –open but it is not pre$-I_S$ –open.

Example 2.3. Let $X=\{a,b,c,d\}, \tau=\{\phi,\{b\},\{a,d\},\{a,b,d\},X\}$and $I=\{\phi,\{b\}\}$.

Then $A=\{a,c,d\}$is not b open but it is $b-I_S$ –open.

3.Strong B_{IS} –set

Definition3.1. A subset A of an ideal space (X,τ,I) is called strong $B_{IS}-$ set if $A=U\cap V$, where $U\in\tau$ and V is a $t-I_S$ –set and $\text{int}(cl^{*s}(V))=cl^{*s}(\text{int}(V))$.

Proposition 3.1 Let (X,τ,I) be an ideal space and $A\subset X$. If A is a strong $B_{IS}-$ set, then A is a $B_{IS}-$ set.

Proof: Obvious

Remark 3.1. Converse of the Proposition 3.1 need not be true as seen from the following example.

Example 3.1Let $X=\{a,b,c\}, \tau=\{\phi,\{a\},\{a,c\},X\}$and $I=\{\phi\}$.

If $A=\{b\}$,then $\text{int}(cl^{*s}(A))=\{\phi\}$ and $\text{int}(A)=\{\phi\}$ .Hence A is a $t-I_S$ –set.Clearly A is a $B_{IS}-$ set. But $\text{int}(cl^{*s}(A))=\{\phi\}$and $cl^{*s}(\text{int}(A))=\{b\}$.Hence $\text{int}(cl^{*s}(V))\neq cl^{*s}(\text{int}(V))$. So A is not a strong $B_{IS}-$ set.

Theorem3.1 Let (X,τ,I)be an ideal space and $A\subset X$ .Then the following conditions are equivalent:

(i) A is open;

(ii) A is $b-I_S$ –open and a strong $B_{IS}-$ set.

Proof: $(i)\Rightarrow(ii)$ By Proposition 2.2,every open set is $b-I_S$ –open.On the other hand every open set is strong $B_{IS}-$ set, because X is $t-I_S$ –set and $\text{int}(cl^{*s}(X))=cl^{*s}(\text{int}(X))$.

$(ii)\Rightarrow(i)$ Let A be $b-I_S-$ open and a strong $B_{IS}-$ set. Then $A\subset\text{int}(cl^{*s}(A))\cup cl^{*s}(\text{int}(A))$ $=\text{int}(cl^{*s}(U\cap V))\cup cl^{*s}(\text{int}(U\cap V))$ where U is open and V is a $t-I_S$ –set and $\text{int}(cl^{*s}(V))=cl^{*s}(\text{int}(V))$.
Hence $A\subset(\text{int}(cl^{*s}(U))\cap\text{int}(cl^{*s}(V)))\cup(cl^{*s}(\text{int}(U))\cap cl^{*s}(\text{int}(V)))$

$\Rightarrow A\subset U\cap(\text{int}(cl^{*s}(V))\cup cl^{*s}(\text{int}(V)))$

$\Rightarrow A\subset U\bigcap\text{int}(cl^{*s}(V))$

$\Rightarrow A\subset U\cap\text{int}(V)=\text{int}(U)\cap\text{int}(V)=\text{int}(U\cap V)=\text{int}(A)$.

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So A is open.

Remark3.2. The notion of A is $b-I_S-$openness is different from that of strong $B_{IS}-$ sets.

(i) In Example2.1 $A=\{b,c\}$is not $b-I_S-$open .But $\mathrm{int}\left(cl^{*s}(A)\right)=cl^{*s}\left(\mathrm{int}(A)\right)=\mathrm{int}(A)=\{b\}$ .So A is a strong $B_{IS}-$ set.

(ii) In Example2.1 $A=\{a,b,c\}$ is $b-I_S-$open .But $\mathrm{int}\left(cl^{*s}(A)\right)=X, cl^{*s}\left(\mathrm{int}(A)\right)=\{a,b,d\}, \mathrm{int}(A)=\{a,b\}$. So A is not a strong $B_{IS}-$ set.

Decomposition of continuity

Definition 4.1.A function $f:(X,\tau)\to((Y,\sigma)$ is said to be b continuous[3] if for every $V\in\sigma, f^{-1}(V)$is b open set of (X,τ).

Definition 4.2. A function $f:(X,\tau,I)\to((Y,\sigma)$ is said to be $B_{IS}-$ continuous[14](resp. semi $-I-$continuous [6], pre$-I-$ continuous[5])if for every $V\in\sigma, f^{-1}(V)$is a $B_{IS}-$ set(resp. semi $-I-$open set, pre$-I-$open set)of (X,τ,I).

Definition 4.3. A function $f:(X,\tau,I)\to((Y,\sigma)$ is said to be $\alpha-I_S-$ continuous[14](resp. semi $-I_S-$ continuous [14], pre$-I_S-$ continuous[14])if for every $V\in\sigma, f^{-1}(V)$is an $\alpha-I_S-$ open set(resp. semi $-I_S-$ open set, pre$-I_S-$open set)of (X,τ,I).

Definition 4.4. A function $f:(X,\tau,I)\to((Y,\sigma)$ is said to be $b-I_S-$ continuous,(resp. strong $B_{IS}-$ continuous) if for every $V\in\sigma, f^{-1}(V)$is a $b-I_S-$ set(resp. a strong $B_{IS}-$ set)of (X,τ,I).

Proposition 4.1 Let (X,τ,I) be an ideal space. If a function $f:(X,\tau,I)\to(Y,\sigma)$ is

semi $-I_S-$continuous(res. pre$-I_S-$ continuous),then f is $b-I_S-$ continuous.

Proof: This is an immediate consequence of Proposition 2.2 (ii) and (iii).

Proposition 4.2 Let (X,τ,I) be an ideal space. If a function $f:(X,\tau,I)\to((Y,\sigma)$

$b-I_S-$ continuous ,then f is b continuous.

Proof: This is an immediate consequence of Proposition 2.2 (iv).

Proposition 4.3 Let (X,τ,I) be an ideal space. If a function $f:(X,\tau,I)\to((Y,\sigma)$

strong $B_{IS}-$continuous, then f is $B_{IS}-$continuous.

Proof: This is an immediate consequence of Proposition 3.1 (i).

Theorem 4.1. Let (X,τ,I) be an ideal space. For a function $f:(X,\tau,I)\to((Y,\sigma)$the following conditions are equivalent:

(i)f is continuous;

(ii) f is $b - I_S -$ continuous and strong B_{IS} –continuous.

Proof: This is an immediate consequence of Theorem3.1.

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