Patient Scheduling with Approximate Dynamic Programming for Optimization of Health Care Services

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ABSTRACT

Model that prescribes the optimal appointment date for a patient at the moment this patient makes his request at the outpatient clinic is developed. We categorized patients into two. The first category is concerned with patients with a maximum recommended waiting time. For these types of patient, the sooner these patients are scheduled the better and when the maximum recommended waiting time is exceeded, extra costs are incurred. The other category is characterized by a specific appointment time. The closer the scheduled appointment time is to the specific appointment time, the lower the costs. The objective is to minimize the long-run expected average cost. We modelled the scheduling process as a Markov Decision Process (MDP). we then apply the Bellman Error Minimization (BEM) method as an Approximate Dynamic Programming technique in order to derive an estimate of the optimal value function of our MDP of which the optimal policy (appointment date) can be derived. To determine the set of representative states, which is an element of the BEM method, we use the k-means algorithm. We test several approximation functions and find an approximation function that outperforms all other functions in the scheduling process over four, six, and eight working days. The Approximation Function B gives the near optimal appointment date for patients when appointments are requested. In general, it holds that the higher the arrival rate of patients at the outpatient clinic, the better our BEM method performs. But if the arrival rate reaches a certain value the load of the system becomes that high that it does not matter what policy is applied, since many patients have to be rejected.

Keywords: Markov Decision Process, Approximate Dynamic Programming, Arrival rate, Policy Improvement

INTRODUCTION

Long waiting line is a major challenge in the health care system. Appointment scheduling systems are faced with the challenge of ensuring efficiency of health care services as well as timely access to health care services (Gupta and Denton, 2008). Being able to access services on time plays a major role in realizing good medical outcomes. Difficulty in the accessing health care services e.g. being in the waiting line for a long time can lead to complications medically. For instance, a certain health condition may be in its early stage and could be treated easily if the patient gets access to the necessary health care service on time, failure to get such service on time would lead to the patient having to wait much longer which may result in an aggravation of the patient medical condition. More critically is that situations like this could eventually result to the death of the patient.

Customer's satisfaction is also an important point to note, as no one enjoys having to wait for a long time in order to get a particular service. A better organization of the health care system would therefore yield more customer satisfaction. Optimizing appointment system in the health care facility would reduce waiting lines (direct and indirect) in the system.

In optimizing appointment system, we would relatively help to reduce idleness and overtime with specialist. The effect of overtime for specialist could be more substantial as it could lead to inefficient rendering of services to patient which is so undesirable in a delicate field like medicine. A good appointment system is also a good tool for eliminating tardiness (laziness) amongst specialist.

The overall goal of a well-designed appointment system is to facilitate efficiency and effectiveness in the delivery of health services for all patients in the health care facility. It facilitates smooth work flow, eliminates long waiting lines and allows patient's preference when requested.

The global pandemic has had a lot of impact on the need for a good appointment system. To lower its spread, protocols has been set in place such as social distancing. Social distancing cannot be underemphasized in the health sector. It is the environment that needs the most of its implementation. An appointment system presents a way to ensure this. Appointment systems can be used to ensure a maximum number of patients are in the health facility at the same time.

The need for a less crowded health facility has always been a pressing issue-though ignored. Appointment system plays a vital role in decongesting clinics and hospitals, as they ensure short wait time and an optimized use of health facilities.

Covid19 has affected a lot of things which include the regular operation of many healthcare service providers. Many services have been halted while some have been shifted, leaving patient in wait for a new appointment date (Charlton, 2020; COVIDSurg Collaborative, 2020).

In order to save many lives, an effective and efficient appointment system cannot be overemphasized.

In other to evaluate the optimization in the health facility, researchers make use of various performance measure. Cost-based measures are the performance measure many literatures adopted

Liu et al. (2010) proposed dynamic policies for appointment system considering no-shows and cancellations and discovered that the heuristic policies performed better than other policies. Feldman et al. (2014) advanced Liu et al. (2010) and developed a model that considers that a patient may have preference, thereby choosing one of the days offered to them by the clinic or leaving without an appointment. Feldman et al. (2014) also paid attention to cancellations and no-shows with an objective to maximize the net profit per day.

Trung (2015) considered a canonical model of dynamic scheduling without considering patients' preference for a specialist and derived an algorithm which assures efficient computation of the policy. Wang et al. (2020) developed an optimization model to optimize the appointment system while paying attention to potential walk-ins. The study reveals that we cannot consider walk-ins as a reward for no-shows from patients. Diamant et al. (2018) looked into how health care schedule patients for multi stage programs such as elective surgery. It was observed that high rates of no shows has an effect on the system such as treatment delays. The problem was formulated as a Markov decision process and solved using approximate dynamic programming techniques.

Patrick et al. (2008) were interested in a scheduling multi-priority patient dynamically. They formulated the problem as a Markov Decision Process (MDP) and transformed to a linear program in order to solve. However, it wasn't solvable due to a large state space. Approximate Dynamic Programming (ADP) was then used to derive an approximate linear program which was then solvable. Erdelyi and Topaloglu (2010) also made use of Approximate Dynamic Programming to solve an allocation problem.

Appointment process as a Markov Decision Process

We consider two categories of patient. The first category is characterized by waiting time limit. For example, a patient that requests for an appointment and wants it scheduled latest five days ahead. The second category is characterized by appointment request for a specific date e.g. a patient that needs an appointment with the specialist on Saturday. Emergency patient is a type that would fall under the first category while usual patient (that sees the specialist regularly) falls under the second category.

Mathematically, we take patient to be of type (x, y); $x \in \{1, 2\}$, $y \in \{1, \ldots, Y_x\}$; x denote the category while y denote the type under each category.

We define several variable for our MDP model

$\boldsymbol{P_{xy}}$: probability that a patient of type (x, y) will enter the system during a time interval, $P_{xy} > 0$

$\boldsymbol{m_{xy}}$: when x = 1, i.e m_{1y}, this denote the waiting time limit for a patient of type y and when x = 2, i.e m_{2y}, it would denote the specific time for a patient of type y. we express m_{xy}in working days, so $m_{xy} \in \{1,2, \ldots\}$

$\boldsymbol{z_{xy}}$: service time required during scheduling for a patient of type (x, y); z_{xy} is expressed in sections(blocks), e.g. if we have appointment time for a day for two(2) hours(e.g. 10 a.m to 12 a.m); we divide the whole time into sections. For instance, a section can be of 5 minutes; this means the whole appointment time would be divided into 24 sections. A patient that requires 10 minutes would require two (2) sections.

$\boldsymbol{q_{1y}}$: extra cost for not scheduling a patient of type (1, y) within the requested limit. $q_{1y} > 0$.

$\boldsymbol{\alpha_{xy}}$: rejection cost for a patient of type (x, y), $\alpha_{xy} > 0$. The penalty cost of rejecting a patient of type (x, y) reflects when a patient is not scheduled in the n-days appointment system, but instead appointed to N+1 days, which is not on the present schedule. We would make α_{xy} high enough to ensure that the planner would not consider pushing patients to a new appointment schedule for certain reasons such as maybe the patient require a long service time. Patient should only be rejected when there is no opportunity on the present schedule.

Formulation of the components of our MDP

State space

A good appointment system should have a stipulated time horizon it would span for. We define that time horizon as N which is expressed in working days. n = 1 represent the next working day, n = 2 as the next two working days and so on. In our model, we are interested in scheduling patients on days of appointment and not the time in that particular day.

For every day, we have a fixed amount of capacity available denoted as B. B is expressed in the number of available sections. Service time is expressed in this regard.

To absorb arrival into the system, we divide the day into intervals, these intervals allow only one event to occur in them, which is either a patient comes in or not. We determine the number of intervals mathematically.

Since we know that arrival during one day follows a Poisson process with parameter λ. Dividing the day into D interval, that means we say arrival in each of the interval, d occur according to a poisson process with parameter $\frac{\lambda}{D}$

The probability that more than one arrival would happen in an interval is as small as less than $0.05 \equiv 5\%$. mathematically,

$$P(arrival > 1) < 0.05$$

$$1 - (P(arrival = 0) + P(arrival = 1)) < 0.05$$

$$1 - \frac{\left(\frac{\lambda}{D}\right)^0 e^{-\frac{\lambda}{D}}}{0!} + \frac{\left(\frac{\lambda}{D}\right)^1 e^{-\frac{\lambda}{D}}}{1!} < 0.05$$

$$1 - \frac{1. e^{-\frac{\lambda}{D}}}{1} + \left(\frac{\lambda}{D}\right)^1 e^{-\frac{\lambda}{D}} < 0.05$$

$$1 - [1 + \frac{\lambda}{D}]\, e^{-\frac{\lambda}{D}} < 0.05$$

$$1 - 0.05 < [1 + \frac{\lambda}{D}] e^{-\frac{\lambda}{D}}$$

$$0.95 < [1 + \frac{\lambda}{D}] e^{-\frac{\lambda}{D}}$$

$$[1 + \frac{\lambda}{D}] e^{-\frac{\lambda}{D}} \geq 0.95 \quad (1)$$

Therefore to determine D, we solve equation (1) using an arrival rate, λ

Action space

The scheduler accepts or rejects a patient. By rejecting a patient, he/she can assign the patient to a date outside the N-day schedule. i.e N + 1

$$A_{\vec{\iota},d} = (\vec{a}) = (a_1, a_2, \dots, a_N;\ a_{N+1})$$

a_n is the action of rejecting or accepting the patient n working days ahead; $a_n \in (0,1)$, $n \in N$

a_{N+1} is the action of accepting the patient in a new plan or totally reject the patient from the system $a_{N+1} \in (0,1)$

For this model, to ensure we do not go above the available capacity on a particular day

$$i_n + z_{xy} a_n \leq B, n \in N$$

And also $\sum_{n=1}^{N+1} a_n = 1$(patient can only be scheduled on at most one day)

Cost function

The cost function would be formulated based on the two category of patient. For the first category, we defined already m_{1y} as the waiting time limit for a patient of type y, q_{1y} as the penalty cost for not scheduling such patient within the requested limit and α_{1y} as the rejection cost. Cost function is written as

$$C_{1y}(\vec{a}) = \left[\sum_{n \in N} (n - 1 + q_{1y}(n - m_{1y})^{+}) a_n\right] + \alpha_{1y} a_{N+1}$$

For the second category, we define m_{2y} as the specific time request by the patient of type y and α_{2y} as the rejection cost

$$C_{2y}(\vec{a}) = \left[\sum_{n \in N} (m_{2y} - n)^2 a_n\right] + \alpha_{2y} a_{N+1}$$

Transition probabilities

Transition occurs in the system in time. The system shifts from one state to another in time depending on d (time interval). After an event occur(which could either be an arrival or no arrival), d moves to the next time interval. If $d < D$, it moves to the next, however if $d = D$, this means it is the end of the day. A new day comes in and the previous day disappears from the plan. This continues till the end of N-time horizon planned for the appointment system. The planner then makes a new N-day plan and starts the same process

When $d < D$ and we have an arrival, the transition is from

$$(i_1, i_2, \dots, i_N;\ d) \rightarrow (i_1 + z_{xy} a_1, \dots, i_N + z_{xy} a_N; d + 1)$$

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If no arrival occurs

$$(i_1, i_2, \dots, i_N;\ d) \ \rightarrow\ (i_1, i_2, \dots, i_N; d+1)$$

whereas if $d = D$ which signals the end of the day, If there is an arrival, it can only be put in the next day

$$(i_1, i_2, \dots, i_N;\ d) \ \rightarrow\ (i_2 +\ z_{xy}a_2, \dots, i_N +\ z_{xy}a_N, 0; 1)$$

For no arrival

$$(i_1, i_2, \dots, i_N;\ d) \ \rightarrow\ (i_2, i_3, \dots, i_N, 0; 1)$$

Having stated the basics of our model, we formulate our optimality equation

$$\mathrm{V}(\vec{\imath}, d) + \mathrm{g} =$$

$$\mathbb{I}_{\{d<D\}}\left[\tau \sum_{x,y} P_{xy} \min_{\vec{a}\,\in\, A_{\vec{\imath},d}} \left\{C_{xy}(\vec{a}) + V\left(i_0 +\ z_{xy}a_0, \dots, i_N +\ z_{xy}a_N; d+1\right)\right\} +\ \tau\left(1 - \sum_{x,y} P_{xy}\right) V(\vec{\imath}, d+1) \right.$$
$$\left. + (1-\tau)V(\vec{\imath}, d)\right]$$

$$+\mathbb{I}_{\{d=D\}}\left[\tau \sum_{x,y} P_{xy} \min_{\vec{a}\,\in\, A_{\vec{\imath},d}} \left\{C_{xy}(\vec{a}) + V\left(i_1 +\ z_{xy}a_1, \dots, i_N +\ z_{xy}a_N; 0; 1\right)\right\} \right.$$
$$\left. +\ \tau\left(1 - \sum_{x,y} P_{xy}\right) V(i_1, i_2, i_3, \dots, i_N; 0; 1) + (1-\tau)V(\vec{\imath}, d)\right] \quad (2)$$

3 Applying BEM to the appointment process

From equation (2), we have our Bellman error given as

$$\mathrm{D}\left(\vec{\imath}, d, \vec{t}\right) = -\mathrm{g} - \mho\left(\vec{\imath}, d, \vec{t}\right)$$

$$+\mathbb{I}_{\{d<D\}}\left[\tau \sum_{x,y} P_{xy} \min_{\vec{a}\,\in\, A_{\vec{\imath},d}} \{C_{xy}(\vec{a}) + \mho(i_0 +\ z_{xy}a_0, \dots, i_N +\ z_{xy}a_N; d+1; \vec{t})\} \right.$$
$$\left. +\ \tau\left(1 - \sum_{x,y} P_{xy}\right) \mho\left(\vec{\imath}, d+1, \vec{t}\right) + (1-\tau)\mho\left(\vec{\imath}, d, \vec{t}\right)\right]$$

$$+\mathbb{I}_{\{d=D\}}\left[\tau \sum_{x,y} P_{xy} \min_{\vec{a}\,\in\, A_{\vec{\imath},d}} \{C_{xy}(\vec{a}) + \mho(i_1 +\ z_{xy}a_1, \dots, i_N +\ z_{xy}a_N; 0; 1; \vec{t})\} \right.$$
$$\left. +\ \tau\left(1 - \sum_{x,y} P_{xy}\right) \mho\left(i_1, i_2, i_3, \dots, i_N; 0; 1; \vec{t}\right) + (1-\tau)\mho\left(\vec{\imath}, d, \vec{t}\right)\right] \quad (3)$$

In order to apply this method, we need to determine all the components needed to for its application, which are

1. An initial policy;
2. The long-run expected average cost for the initial policy, g;
3. The set of representative states, Ì $\subset I$
4. The weights, $w(i), i \in$ Ì

Initial policy for the BEM method

We choose a greedy policy which allows patients of type $x = 1$ to be scheduled as soon as possible while patients of type $x = 2$ would be given appointments as closely as possible to their requested date. Rejection under this policy would only happen when the capacity is exhausted for any particular day.

Long run expected average cost

We use simulation to determine the long-run expected average cost, g, belonging to a certain policy. The simulation is performed as follows: starting with the first day, this day is cut into D intervals. In each interval $\mathrm{d} \in \{1, \dots, \mathrm{D}\}$ there is one patient arrival of type (x, y) with probability p_{xy} or no patient arrival with probability $1 - \sum_{x,y} P_{xy}$ If $\mathrm{d} \neq \mathrm{D}$ and there is a patient arrival, this patient is scheduled according to $\min_{\vec{a} \in A_{\vec{\imath},d}} \{C_{xy}(\vec{a}) + \mho(i_1 + z_{xy}a_1, \dots, i_N + z_{xy}a_N; d + 1; \vec{t})\}$ and the corresponding costs, $C_{xy}(\vec{a})$, are incurred. After the patient is scheduled we move to the next interval. If no arrival occurs, we move directly to the next interval. If $\mathrm{d} = \mathrm{D}$ and there is a patient arrival, this patient is scheduled according to $\min_{\vec{a} \in A_{\vec{\imath},d}} \{C_{xy}(\vec{a}) + \mho(i_2 + z_{xy}a_2, \dots, i_N + z_{xy}a_N; 0; 1; \vec{t})\}$ and the corresponding costs, $C_{xy}(\vec{a})$, are incurred. After the patient is scheduled we move to the next day and shift the schedule. The first day disappears from the schedule, the second day becomes the first day, the third day becomes the second day and so on, and finally a new empty day enters the schedule and we start with $\mathrm{d} = 1$. If no arrival occurs, we move directly to the new day and shift the schedule. We let the simulation run over n days. At the end of the simulation we can obtain g by dividing the total costs incurred by the length of the simulation, $n * \mathrm{D}$. Note that the initial policy can be achieved by setting the parameter vector $\vec{t}$ to zero. In this case $\mho(\vec{\imath}, d, \vec{0})$ equals zero, $\forall\, \vec{\imath}, d$ and hence, the actions to choose only depends on the cost function. To make sure we only reject patients if there is no sufficient capacity available in any day, the rejection costs must be chosen higher than the highest costs that can be obtained when patients are scheduled.

Set of representative states and the corresponding weight vector

The set Ì should contain the most important states in the state space, while w should represent the importance of the states in Ì. The choice of the set of representative states could be to include only the states that have a high probability of being visited.

Step 1 we get a list of sampled state S.

Step 2 K-means clustering From this list, we want to cluster these states into K clusters. As clustering technique, we use Elkan algorithm. The algorithm returns the clusters and the cluster centre.

Step 3 Determine the set of representative states and the corresponding weight vector For each cluster centre we want to find the state with the shortest Euclidean distance as the most representative state of this cluster. Hence, after this step, we have a set of K representative states, which will be our Ì and the number of states in each cluster will determine our weight vector w.

Besides the patient-specific parameters, our model has a few other parameters that need to be determined. These parameters are: N, the number of working days which covers the scheduling process; B, the fixed amount of capacity available on any day; λ, the rate for patient arrivals and from which parameter D can be determined,

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and τ needed for the data transformation to overcome the problem of aperiodicity. We set B to 20 blocks and τ to 0.9.

Table 1: Values for different parameters

Parameters	**Values**
D	{ 31, 33, 36, 38, 40}
S	{1000000, 2000000, 3000000}
K	{50, 100, 150}

Table 2: Different types of basis functions

Number	Basic function
0	$d + \sum_{p=1}^{3}\sum_{n=1}^{N} i_n^p$
1	$\sum_{n=1}^{N-1} i_n * i_{n+1}$
2	$\sum_{n=1}^{N-2} i_n * i_{n+1} * i_{n+2}$
3	d^2
4	$\sum_{n=1}^{N} i_n * d$
5	$\sum_{n=1}^{N} i_n^2 * d$
6	$\sum_{n=1}^{N-1} i_n * i_{n+1} * d$
7	$\sum_{n=1}^{N-2} i_n * i_{n+1} * i_{n+2} * d$

Each approximation function we use contains basis function 0(Roubos, 2010). The other basis functions contain several cross terms between different parts of the state space. If we refer to approximation function 046, then this approximation function consist of the basis functions: 0, 4 and 6. for example If $\mathrm{N} = 3$ and we refer to approximation function 046, then $\mho(\vec{\imath}, d, \vec{t}) = dt_1 + i_1 t_2 + i_2 t_3 + i_3 t_4 + {i_1}^2 t_5 + {i_2}^2 t_6 + {i_3}^2 t_7 + {i_1}^3 t_8 + {i_2}^3 t_9 + {i_3}^3 t_{10} + i_1 d t_{11} + i_2 d t_{12} + i_3 d t_{13} + i_1 i_2 d t_{14} + i_2 i_3 d t_{15}$

Bottom-up Approach

We start with the following approximation functions: {0, 01, 02, 03, 04, 05, 06, 07}. From here we use a so-called bottom up approach. We take the functions that show the best improvements overall and then add the other remaining functions one at a time. For instance, if function 05 performs best, then we make the following new combinations: {051, 052, 053, 054, 056, 057}. This is repeated until no further improvement occurs. At the end we have a set of approximation functions which shows in general the best improvements for a scheduling process over four working days. To test if these set of approximation functions also perform well for a larger scheduling process we expand our model to $N = 6$ and 8. The parameters S and K are set to the value that in general performs best in the model with $N = 4$.

Data Analysis on Four, Six and Eight working days

Four working days

For each combination of the parameters in **Table 3** we apply the BEM method. We can make $5*3*3 = 45$ combinations, for each combination we apply the BEM method with 8 different approximation functions. Each time we apply the BEM method, we compare g obtained from our initial policy with g obtained after the one-step policy improvement and compute the improvement that is made. We refer to this as the improvement of the BEM method.

Table 3: Median and average of the improvement for each approximation function.

Function	Mean (%)	Median (%)
0	7.26	6.37
01	-3.51	-2.60
02	-3.80	-2.69
03	6.70	5.74
04	7.00	5.70
05	6.41	5.26
06	6.57	5.30
07	7.16	5.75

we remove the value of $\lambda = 11, 12$. It seems plausible logically that a low λ would mean a low load on the system, thereby making the initial policy a good policy. We use the BEM method for a reduced number of D. we have $3*3*3 = 27$ combinations remaining. Table 4 shows for the remaining approximation functions the average and median of the improvement of the BEM method over 27 combinations. As can be seen functions 07 give the best results with an average improvement of 7.16%. we start with our bottom up approach with all of the functions over the 27 combinations.

Table 4: Median and average of the improvement for each approximation function for a reduced number of D.

Function	Mean (%)	Median (%)
0	6.06	6.25
01	6.17	4.62

02	6.28	5.85
03	6.45	6.01
04	5.25	4.95
05	5.94	4.67
06	5.47	3.91
07	7.16	5.36

Table 5 shows the results from the first step of the bottom up approach. For each approximation function the average and median of the improvement of the BEM method are given. For function 04 it holds that the mean increases slightly from 5.25% to 6.20% when function 1 is added. Adding one of the other functions does not improve the average or median. For function 05, an improvement is made when function 3 is added. We also have an improvement made with function 06 when function 1 or 3 is added. Therefore, in our second step of the bottom up approach we start with 041, 053, 061 and 063.

Table 5: Median and average of the improvement for each approximation function after one step.

Function	Mean (%)	Median (%)
021	4.85	4.04
023	5.12	3.68
024	5.08	3.95
025	4.85	4.02
026	4.78	3.88
027	5.44	4.07

(a) Function 01 (b) Function 02

Function	Mean (%)	Median (%)
012	4.66	3.56
013	4.99	3.26
014	4.24	3.24
015	5.22	3.07
016	4.94	3.20
017	4.30	2.95

Function	Mean (%)	Median (%)
041	6.20	6.11

042	4.89	3.75
043	2.37	1.55
045	4.59	3.75
046	5.14	4.16
047	4.72	3.84

(c) Function 03 (d) Function 04

Function	Mean (%)	Median (%)
031	5.06	4.12
032	5.17	4.00
034	4.66	3.71
035	5.53	3.76
036	5.19	4.12
037	5.19	4.24

Function	Mean (%)	Median (%)
061	5.67	4.04
062	5.32	4.22
063	9.21	5.74
064	5.21	4.16
065	5.24	3.58
067	4.86	3.91

(e) Function 05 (f) Function 06

Function	Mean (%)	Median (%)
051	4.95	4.12
052	5.32	4.13
053	8.29	6.48
054	5.49	3.77
056	-4.89	-3.79

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057	4.66	3.68

(g) Function 07

Function	Mean (%)	Median (%)
071	5.00	3.87
072	5.04	3.67
073	1.03	0.95
074	5.09	3.86
075	5.10	4.04
076	5.02	3.54

Table 6 shows the results of the second step of the bottom up approach. As can be seen, no further improvement is obtained.

Table 6: Median and average of the improvement for the different functions after two steps.

Function	Mean (%)	Median (%)
0531	7.80	5.79
0532	4.63	3.42
0534	1.45	1.43
0536	0.78	1.00
0537	5.09	3.77

(a) Function 041 (b) Function 053

Function	Mean (%)	Median (%)
0412	5.37	3.84
0413	5.16	3.73
0415	5.77	4.03
0416	5.34	3.87
0417	5.05	3.76

(c) Function 061 (d) Function 063

Function	Mean (%)	Median (%)
0612	1.00	1.00

0613	-4.04	-2.13
0614	1.00	1.00
0615	1.00	1.00
0617	1.00	1.00

Function	Mean (%)	Median (%)
0631	5.09	3.95
0632	4.77	3.79
0634	5.34	3.26
0635	5.05	3.78
0637	5.55	4.18

Function 041, 053, 061 and 063 are the function that gives the best improvements during the one-step policy improvement, based on the median and average for the scheduling process over four working days. Therefore, we apply these functions to the scheduling process over six, and eight working days. Since function 07 also performs very good for the scheduling process over four working days, we also apply these functions to the scheduling process over six, and eight working days. To simplify the figures in the following sections, we create a translation table, see **Table 7**. From here, if we write about function A, we actually mean function 07.

Table 7: Translation table for the different functions.

New Function Name	Old Function Name
A	07
B	041
C	053
D	061
E	063

Figure 1 shows for each λ the average improvement of the BEM method by each of the approximation function. By improvement, we refer to the improvement made when we compare g obtained from our initial policy with g obtained after the one-step policy improvement. We see that if $\lambda \leq 12$, the average improvement for each function increases as λ increases. When $\lambda > 12$ the average improvement for each function seem to decrease as λ decreases. The lower λ, the lower the load of the system which infer the better our initial policy performs and hence, less improvement is possible. Whereas on the other hand, the higher λ, the higher the load of the system which infer the worse our initial policy performs and hence, the more important our one-step policy improvement. But if λ reaches a certain value, the load of the system becomes that high that it does not matter what policy is applied, since it will be imperative to reject many patients.

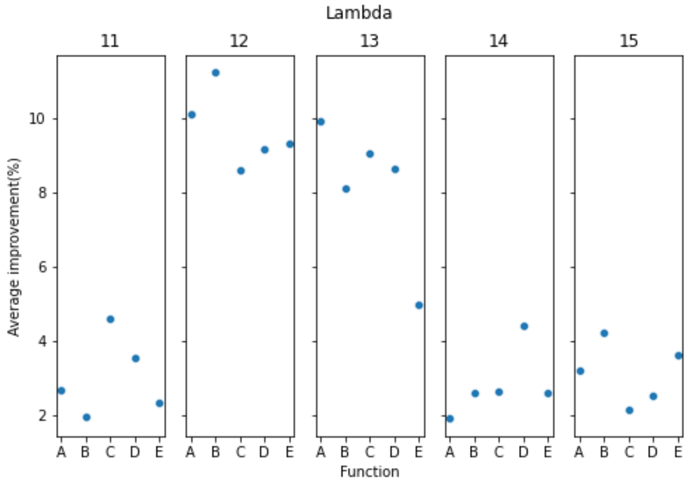

Figure 1: Average improvement by λ for different functions for the scheduling process over four working days

Six, Eight working days

We apply the BEM method over six and eight working days. The results of the approximation functions {A, B, C, D, E} of the scheduling process over six, eight days are given. The parameters S and K needed for the BEM method are fixed to 1000000 and 50 respectively. Figure 2 shows for each λ the average improvement of the BEM method by the different functions for the scheduling process over six working days. It shows more or less the same pattern as the results of the scheduling process over four working days. We see that if $\lambda \leq 12$, the average improvement for each function increases as λ increases. When $\lambda > 12$ the average improvement for each function seem to decrease as λ decreases.

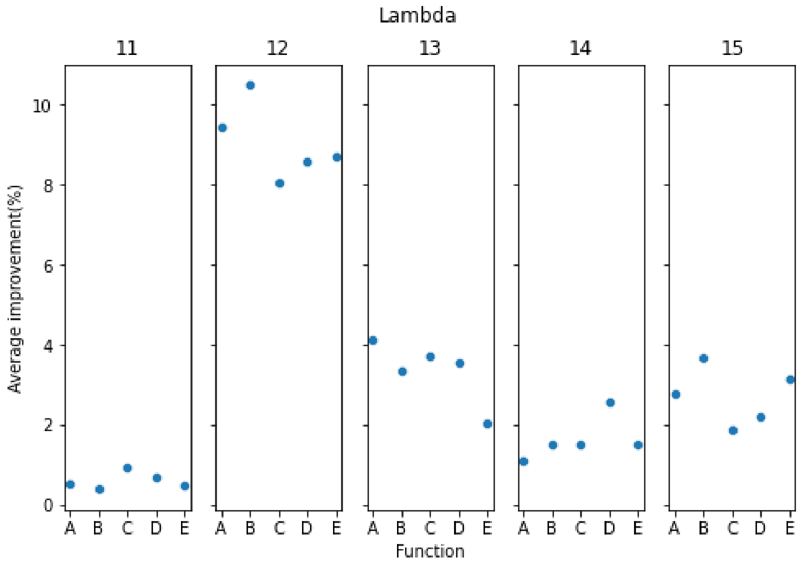

Figure 2: Average improvement by λ for different functions for the scheduling process over six working days

Figure 3 shows for each λ the average improvement of the BEM method by the different functions for the scheduling process over 8 working days. Function {A, B, E} shows the same pattern as the results for four and six working days. For function {A, B, E}, the threshold is at $\lambda = 12$. Function C shows a different pattern than have been seen before. It has its highest result at $\lambda = 11$ and continues with a decline thereafter. At $\lambda = 11$ the average improvement gives 4.19% which apparently shows the importance of the one-step policy with this load of the system. Function D also shows a similar pattern to Function C. It has its highest result at $\lambda = 11$. The average improvement then decreases to 2.76% After which the average improvement is increased at $\lambda = 14$

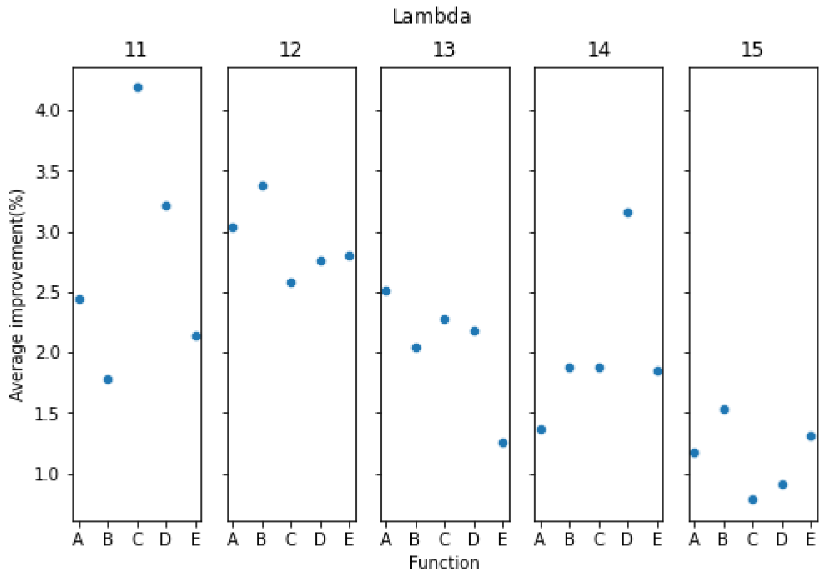

Figure 3: Average improvement by λ for different functions for the scheduling process over eight working days

Table 8: Average of the improvement by functions for four, six and eight working days.

Function	$N = 4$	$N = 6$	$N = 8$
	Avg (%)	Avg (%)	Avg (%)
A	5.571572	3.601719	2.105545
B	5.628243	3.889145	2.123410
C	5.408628	3.225292	2.344520
D	5.660669	3.529022	2.446660
E	4.576536	3.183158	1.874466

Table 8 shows for the scheduling process over four, six and eight working days for each function the average of the improvement of the BEM method relative to the initial policy. Overall, function B give the overall best improvement over four, six and eight working.

CONCLUSION

With the goal to develop a model that prescribes the optimal appointment date for a patient at the moment this patient makes his request. From all combinations of the set of basis functions, the following combination outperforms all other combinations:

$$d + \sum_{p=1}^{3} \sum_{n=1}^{N} i_n^p + \sum_{n=1}^{N} i_n * d + \sum_{n=1}^{N-1} i_n * i_{n+1}$$

The overall average improvement of the Approximation Function B compared to the initial policy over four, six and eight working days is 11.640798%. In general, it holds that the lower λ, the lower the low load of the system, the better our initial policy performs and hence, less improvement is obtained. The higher the load of the system, the worse our initial policy performs and the more important is our one-step policy improvement. But if λ reaches a certain value the load of the system becomes that high that it does not matter what policy is applied, since many patients have to be rejected. we have been able to make use of Approximate Dynamic Programming to solve problems that arises in appointment system in the health care facility by developing a model that prescribes the optimal appointment date for a patient at the moment this patient makes his request.

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