

# Correlation Measure for Neutrosophic Hesitant Fantastic Filter for Supply Chain Management

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## ABSTRACT

In order to address the complexity of decision-making processes that incorporate uncertainty, hesitation, probabilistic aspects, and the Fantastic Filter method, we present the Correlation Measure (CM) for Neutrosophic Hesitant Fantastic Filter (NHFF). By taking into account the Truth Membership Hesitancy Degree (TMHD), Indeterminacy-Membership Hesitancy Degree (IMHD), and Falsity-Membership Hesitancy Degree (FMHD), the suggested measures provide a methodical way to compute the CM between probability neutrosophic hesitant fuzzy set and neutrosophic hesitant fantastic filter. The study also proposed the Weighted Correlation Measure (WCM) approach, which enables varied weighting according to the relative importance of truth, indeterminacy, and falsity degrees, as well as the risk preferences of decision-makers (DMs).

**Keywords:** Probability neutrosophic hesitant set, Neutrosophic hesitant fantastic filter, correlation measure.

## INTRODUCTION

Smarandache presented the conception of Neutrosophic Set (NS) in 2010 [16], an expansion of the concepts of Intuitive Fuzzy Sets (IFs) and Fuzzy Sets (FS). They, have three membership functions Truth(  $T$  ), Indeterminate(  $I$  ) and Falsity (  $F$  ) respectively. These three membership functions compose the Neutrosophic Set(NS)  $\langle T, I, F \rangle$ , where  $T, I$  and  $F \in [0,1]$ . Wang introduced Single-Valued Neutrosophic Sets (SVNs). As an example, the hybrid structures of this theory have been valuable in a variety of domain. Rahul Thakur et al, Correlation Coefficient Measures for Probabilistic Single Valued Neutrosophic Hesitant Fuzzy Sets and Its Application in Supply Chain Management 2025[10]. Font. J. M, Antonio. J. R and Torrense. A introduced the notion is called implicative filter, Basheer ahamaed.M and Ibrahim presented by the Intuitionistic fuzzy implicative filter and zhang developed the Neutrosophic Filter (NF). The concept of NF generalized by a fuzzy filter and Intuitionistic Filters (IF). A. Ibrahim and V. Nirmala proposed Implicative Filters (IF) of Resituated Lattice Wajsberg Algebras in 2018[10].

It has been demonstrated that the conventional correlation analysis, which is based on statistics and probabilities, is inadequate for handling failure data and modeling uncertainty. The challenge to classical statistical theory is the fuzziness-based method for measuring the correlation between two variables. Ning, B., Wei, C., & Wei, G. presented by Some novel correlation coefficients of probabilistic dual hesitant fuzzy sets and their application to multi-attribute decision-making [14] in 2024. Mehmood, A. et al. Entropy and similarity measurements are discussed, along with their limited applicability due to ambiguous soft sets [13] in 2024. Recently Hesami, F. explained in A case study of the electronics sector using a hybrid ANP-TOPSIS approach for strategic supplier selection in reverse logistics with rough uncertainty [9] in the year 2025. Malik, S. C., et al developed in Cosine entropy measure-based weighted correlation coefficient metric for intuitionistic fuzzy sets [12] in 2023. In the same year Bhat, S. A. presented Neutrosophic trapezoidal numbers in an improved AHP group decision-making model [6]. And Banihashemi, S. A. et al proposed Using the fuzzy BWM method to identify and rank the difficulties and barriers associated with green supply chain management in the construction sector [5].

The aim of this article is to propose the notion of Correlation Measure for Neutrosophic Hesitant Fantastic Filter for Supply Chain Management. In Section 2, the basic definitions of Neutrosophic Set (NS), Probabilistic Neutrosophic Hesitant set (PNHs), and fantastic Filter (FF). In Section 3, Neutrosophic Hesitant Fantastic Filter (NHFF) with example. Section 4 Correlation measure for Probabilistic Neutrosophic Hesitant Fantastic Filter (PNHFF) describe in this section. Additionally, section 5 algorithm of correlation measure for PNHFF. finally, section 6 describes the application of supply chain management system.

## Preliminaries

**Definition 1:** Let the universe be  $\mathcal{U}$ . A neutrosophic set  $\mathcal{E}$  in  $\mathcal{U}$  is characterized by a truth ( $T_{\mathcal{E}}$ ), an indeterminacy ( $I_{\mathcal{E}}$ ), and falsity membership function ( $F_{\mathcal{E}}$ ), the functions are real standard or non-standard subset of  $]0, 1^+ [$ . Then the NS  $\mathcal{E}$  can be denoted by,

$$\mathcal{E} = \{ \langle f, T_A(f), I_A(f), F_A(f) \rangle : f \in \mathcal{U}, T_A(f), I_A(f), F_A(f) \in [0, 1] \}$$

Thus, the total sum is unconstrained. Then  $0 \leq \sup T_A(f) + \sup I_A(f) + \sup F_A(f) \leq 3^+$ .

**Definition 2:** Let  $\mathcal{U}$  be a lattice wajsberg algebra. A subset  $\mathcal{P}$  of  $\mathcal{U}$  is called an implicative filter of  $\mathcal{U}$ , if it satisfies the following axioms for all  $x, y \in \mathcal{U}$ ,  
(i)  $1 \in \mathcal{P}$   
(ii)  $x \in \mathcal{P}$  and  $x \rightarrow y \in \mathcal{P}$  for all  $y \in \mathcal{P}$ .

**Definition 3:** Let  $\mathcal{U}$  be a lattice Wajsberg Algebra (WA). A subset  $\mathcal{P}$  of  $\mathcal{U}$  is called an Fantastic filter of  $\mathcal{U}$ , the subsequent axioms are satisfying for any  $a, b, c \in \mathcal{U}$ ,

1.  $1 \in \mathcal{P}$
2.  $c \rightarrow (b \rightarrow a) \in \mathcal{P}$  and  $c \in \mathcal{P}$  imply  $((a \rightarrow b) \rightarrow b) \rightarrow a \in \mathcal{P}$ .

**Definition 4:** Let universe be  $\mathcal{U}$ . A hesitant fuzzy set  $\mathcal{A}$  on  $\mathcal{U}$  is defined in terms of a function  $\mathcal{h}_{\mathcal{A}}(x_i)$  that returns a subset of  $[0, 1]$  is denoted by

$$\mathcal{A} = \{ (x_i, \mathcal{h}_{\mathcal{A}}(x_i)) \mid x_i \in \mathcal{U} \}.$$

Where  $\mathcal{h}_{\mathcal{A}}(x_i) \in [0, 1]$  is a membership degree of all  $x_i \in \mathcal{U}$

**Definition 5:** Let  $\mathcal{U}$  be a lattice Wajsberg Algebra (WA). A subset  $\mathcal{A}$  of  $\mathcal{U}$  is called a hesitant fantastic filter the following condition  $a, b, c \in \mathcal{U}$

1.  $\mathcal{h}_{\mathcal{A}}(1) \geq \mathcal{h}_{\mathcal{A}}(a)$
2.  $\mathcal{h}_{\mathcal{A}}((a \rightarrow b) \rightarrow b) \rightarrow a \geq \min\{\mathcal{h}_{\mathcal{A}}(c \rightarrow (b \rightarrow a)), \mathcal{h}_{\mathcal{A}}(c)\}$

**Definition 6:** Let  $\mathcal{U}$  be a universal set. Then neutrosophic hesitant fuzzy set  $\mathcal{A}$  on  $\mathcal{U}$  is defined by

$$\mathcal{A} = \{ x_i, \bar{t}_{\mathcal{A}}(x), \bar{i}_{\mathcal{A}}(x), \bar{f}_{\mathcal{A}}(x) / x \in \mathcal{U} \},$$

In which  $\bar{t}_{\mathcal{A}}(x), \bar{i}_{\mathcal{A}}(x), \bar{f}_{\mathcal{A}}(x) \in p([0, 1])$ , denoting the possible truth membership hesitant degrees, indeterminacy membership hesitant degrees and falsity membership hesitant degrees of  $x_i \in \mathcal{U}$  to the set  $\mathcal{A}$ , respectively, with the conditions  $0 \leq \delta, \gamma, \beta \leq 1$  and  $0 \leq \delta^+ + \gamma^+ + \beta^+ \leq 3$ , where  $\delta \in \bar{t}_{\mathcal{A}}(x)$ ,  $\gamma \in \bar{i}_{\mathcal{A}}(x_i)$ ,  $\beta \in \bar{f}_{\mathcal{A}}(x)$ ,  $\delta^+ \in \bigcup_{\delta \in \bar{t}_{\mathcal{A}}(x)} \max(\delta)$ ,  $\gamma^+ \in \bigcup_{\delta \in \bar{i}_{\mathcal{A}}(x)} \max(\gamma)$ ,  $\beta^+ \in \bigcup_{\delta \in \bar{f}_{\mathcal{A}}(x)} \max(\beta)$  for  $x \in \mathcal{U}$ .

**Definition 7:** Let  $\mathcal{U}$  be a universal set. Then the Probabilistic neutrosophic hesitant fuzzy set  $\mathcal{A}$  defined as,

$$\mathcal{A} = \{ x, \bar{t}_{\mathcal{A}}(x) | P_I(x), \bar{i}_{\mathcal{A}}(x) | P_{II}(x), \bar{f}_{\mathcal{A}}(x) | P_{III}(x) / x \in \mathcal{U} \}$$

Where  $P_I(x)$ ,  $P_{II}(x)$ ,  $P_{III}(x)$  are the corresponding probabilistic information for the three types of degree and  $0 \leq \alpha, \vartheta, \pi \leq 1$ ;  $0 \leq \alpha^+, \vartheta^+, \pi^+ \leq 1$ ;  $P_I(x), P_{II}(x), P_{III}(x) \in [0,1]$ ;  $\sum_{i=1}^{*\bar{t}_{\mathcal{A}}} P_I(x) = \sum_{i=1}^{*\bar{t}_{\mathcal{A}}} P_{II}(x) = \sum_{i=1}^{*\bar{t}_{\mathcal{A}}} P_{III}(x) = 1$ ;  $\alpha \in \bar{t}_{\mathcal{A}}(x)$ ,  $\vartheta \in \bar{t}_{\mathcal{A}}(x)$ ,  $\pi \in \bar{t}_{\mathcal{A}}(x)$ ;  $\alpha^+ \in \bigcup_{\alpha \in \bar{t}_{\mathcal{A}}(x)} \max(\alpha)$ ,  $\vartheta^+ \in \bigcup_{\vartheta \in \bar{t}_{\mathcal{A}}(x)} \max(\vartheta)$ ,  $\pi^+ \in \bigcup_{\pi \in \bar{t}_{\mathcal{A}}(x)} \max(\pi)$  for  $x \in \mathcal{U}$ . where  $*\bar{t}_{\mathcal{A}}(x)$ ,  $*\bar{t}_{\mathcal{A}}(x)$ ,  $*\bar{t}_{\mathcal{A}}(x)$  are the  $\bar{t}_{\mathcal{A}}(x)|P_I(x)$ ,  $\bar{t}_{\mathcal{A}}(x)|P_{II}(x)$ ,  $\bar{t}_{\mathcal{A}}(x)|P_{III}(x)$  respectively.

### Neutrosophic hesitant fantastic (NHF) filter of wasjberg algebra

**Definition 8:** Let  $\mathcal{U}$  be a lattice W-algebra. A Neutrosophic hesitant fuzzy set  $\mathcal{A} = \{a, \bar{t}_{\mathcal{A}}(a), \bar{t}_{\mathcal{A}}(a), \bar{t}_{\mathcal{A}}(a)/a \in \mathcal{U}\}$  of  $\mathcal{U}$  is called an NHFF, if it fulfils the inequalities for all  $a, b, c \in \mathcal{U}$ ,

- $\bar{t}_{\mathcal{A}}(a) \leq \bar{t}_{\mathcal{A}}(1), \bar{t}_{\mathcal{A}}(a) \leq \bar{t}_{\mathcal{A}}(1), \bar{t}_{\mathcal{A}}(a) \geq \bar{t}_{\mathcal{A}}(1)$
- $\bar{t}_{\mathcal{A}}((a \rightarrow b) \rightarrow b) \rightarrow a \geq \min\{\bar{t}_{\mathcal{A}}(c \rightarrow (b \rightarrow a)), \bar{t}_{\mathcal{A}}(c)\}$
- $\bar{t}_{\mathcal{A}}((a \rightarrow b) \rightarrow b) \rightarrow a \geq \min\{\bar{t}_{\mathcal{A}}(c \rightarrow (b \rightarrow a)), \bar{t}_{\mathcal{A}}(c)\}$
- $\bar{t}_{\mathcal{A}}((a \rightarrow b) \rightarrow b) \rightarrow a \leq \max\{\bar{t}_{\mathcal{A}}(c \rightarrow (b \rightarrow a)), \bar{t}_{\mathcal{A}}(c)\}$

**Example 1:** Let  $\mathcal{U} = \{0, a, b, c, d, 1\}$  with binary operation  $\rightarrow$  is a wasjberg algebra.

Let  $\mathcal{A} = \{(0: ([0.40|0.23], [0.08|0.34], [0.24|0.44]), a: ([0.42|0.49], [0.05|0.26], [0.90|0.25]), b: ([0.34|0.09], [0.90|0.61], [0.37|0.30]), c: ([0.24|0.61], [0.24|0.15], [0.27|0.23]), d: ([0.43|0.26], [0.31|0.61], [0.16|0.14])\}$  be a PNHF

Then  $\mathcal{A}$  is neutrosophic hesitant fantastic filter of  $\mathcal{U}$

| $\rightarrow$ | 0 | a | b | c | d | 1 |
|---------------|---|---|---|---|---|---|
| 0             | 1 | 1 | 1 | 1 | 1 | 1 |
| a             | c | 1 | b | c | b | 1 |
| b             | d | a | 1 | b | a | 1 |
| c             | a | a | 1 | 1 | a | 1 |
| d             | b | 1 | 1 | b | 1 | 1 |
| e             | 0 | a | b | c | d | 1 |

Let  $\mathcal{A} = \{a, b, c\}$  is a subset of  $\mathcal{U}$ , Hence  $\mathcal{A} = ([0.42|0.49], [0.05|0.26], [0.90|0.25])$  be a NHFF.

### Correlation measure for probabilistic Neutrosophic hesitant fantastic filter

Let  $G = \{\bar{t}_G(x)|P_{IG}(x), \bar{t}_G(x)|P_{IIG}(x), \bar{f}_G(x)|P_{IIIG}(x)\}$  and  $H = \{\bar{t}_H(x)|P_I(x), \bar{t}_H(x)|P_{IIH}(x), \bar{f}_H(x)|P_{IIIH}(x)\}$  be a two PNHF, and we assign the weights  $w_i > 0$  to each elements of  $\mathcal{U}$  such that  $\sum_{i=1}^n w_i = 1$ , then the correlation measure for neutrosophic hesitant fantastic filter is denoted by

$$\check{\rho}_{wNHFF}(G, H) = \left( \frac{1}{\xi+2} * \check{\rho}_{wthff}(G, H) + \frac{\xi}{\xi+2} * \check{\rho}_{wihff}(G, H) + \frac{1}{\xi+2} * \check{\rho}_{wfhhff}(G, H) \right)$$

Where  $\xi$  is a risk preference coefficient.

$$\check{\rho}_{wthff}(G, H) = \frac{C_{wthff}(G, H)}{\sqrt{C_{wthff}(G, G) * C_{wthff}(H, H)}}, \check{\rho}_{wihff}(G, H) = \frac{C_{wihff}(G, H)}{\sqrt{C_{wihff}(G, G) * C_{wff}(H, H)}} \text{ and}$$

$$\check{\rho}_{wf hff}(G, H) = \frac{C_{wf hff}(G, H)}{\sqrt{C_{wf thff}(G, G) * C_{wf hff}(H, H)}}$$

The weighted mean of the truth membership hesitant degree (TMHD) of PNHFFs G and H are

$$C_{wthff}(G, H) = \frac{1}{n} \sum_{i=1}^n w_i \left( \sum_{j=1}^m (\bar{t}_G(x_j) | P_{IG}(x_j)) (\bar{t}_H(x_j) | P_{IH}(x_j)) \right)$$

$$C_{wthff}(G, G) = \frac{1}{n} \sum_{i=1}^n w_i \left( \sum_{j=1}^m (\bar{t}_G(x_j) | P_{IG}(x_j)) (\bar{t}_G(x_j) | P_{IG}(x_j)) \right)$$

And

$$C_{wthff}(H, H) = \frac{1}{n} \sum_{i=1}^n w_i \left( \sum_{j=1}^m (\bar{t}_H(x_j) | P_{IH}(x_j)) (\bar{t}_H(x_j) | P_{IH}(x_j)) \right)$$

Similarly, we find the indeterminacy membership hesitant degree (IMHD) and falsity membership hesitant degree (FMHD) of PNHFFs G and H.

### Algorithm of PNHFF

**Step 1:** collect the data and apply the following equation to normalize the decision matrices  $\mathcal{M}_{ij} = \{t_{ij} | P_{Iij}, i_{ij} | P_{IIij}, f_{ij} | P_{IIIij}\}$ . Then the matrices  $\mathcal{M} = [\mathcal{M}_{ij}]_{u \times v}$  is a PNHF's where  $i=1, 2, 3, \dots, u$  and  $j=1, 2, 3, \dots, v$ .

**Step 2:** find the PNHFF based on the PNHF's using by the below matrices  $\mathcal{N}^+ = [\mathcal{D}_j^+]_{1 \times v}$

Let  $\mathcal{N} = \{\mathcal{N}_1, \mathcal{N}_2, \dots, \mathcal{N}_u\}$  are a set of alternatives and  $\mathcal{D} = \{e_1, e_2, \dots, e_v\}$  are a set of alternatives.

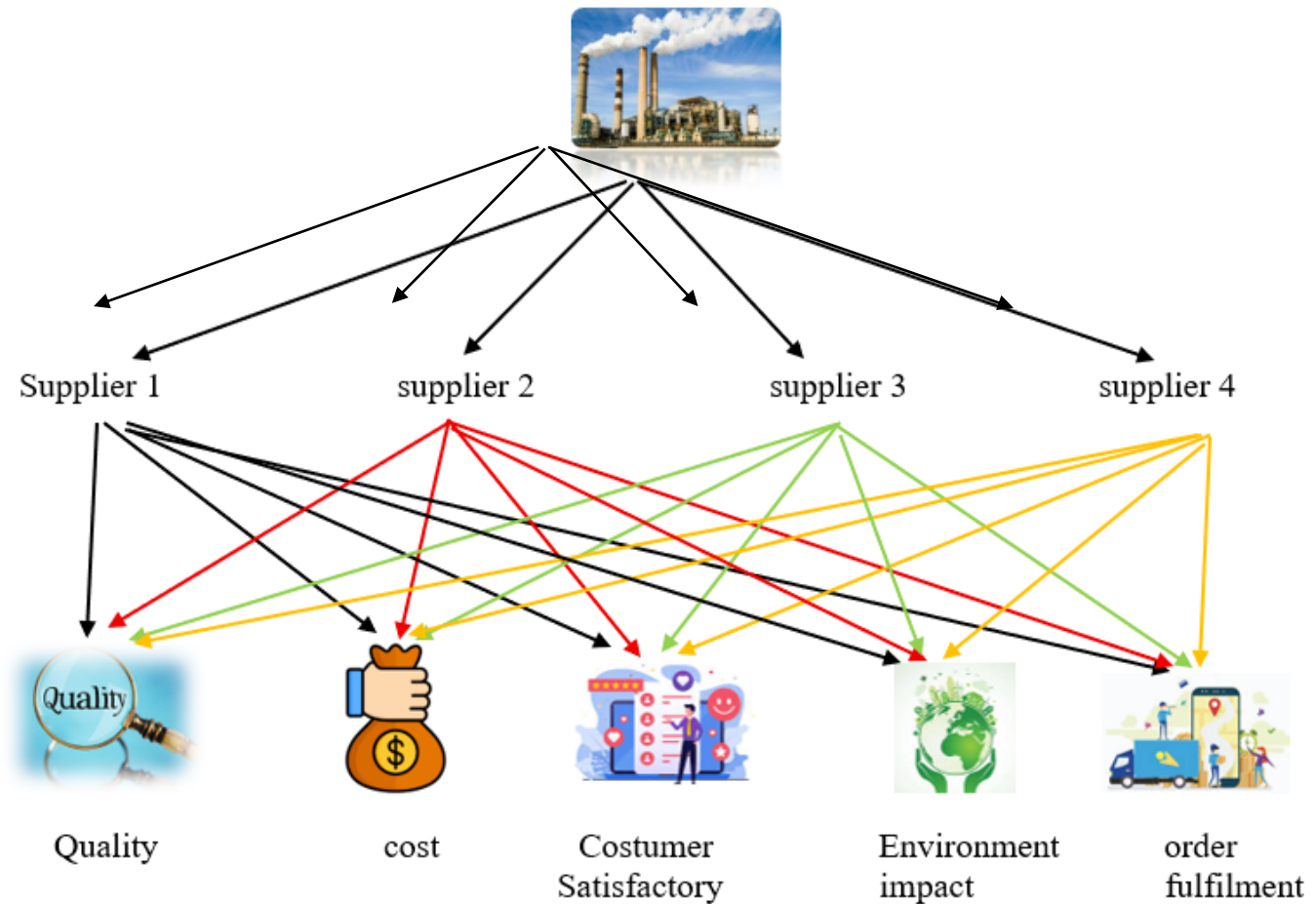
**Step 3:** Find the value of weight  $w_i$  is a weight vector of attributes such that  $w_i \in [0, 1]$  and  $\sum_{i=1}^n w_i = 1$ .

**Step 4:** Calculate the CM between  $\mathcal{F}_i$  and  $\mathcal{F}^+$  for every  $i=1, 2, \dots, u$ .

**Step 5:** Determined the final ranking of alternatives from highest to lowest

### Correlation measure for supply chain management

A manufacturing company's overall operational success, cost, product quality, and Customer satisfaction can all be greatly impacted by the supplier it chooses. When selecting a supplier, a number of aspects that impact the company's success both now and, in the future, must be considered. Selecting the top supplier from a pool of possible applicants is the aim. A number of important characteristics are included in the selection criteria, which influence the company's choice. There is intrinsic uncertainty in each attribute, which is represented by probabilistic neutrosophic hesitant sets. In light of these uncertainties, the corporation seeks to identify the supplier that best suits its requirements. Let's look at four possible suppliers ( $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3, \mathcal{N}_4$ ) and their five characteristics ( $e_1, e_2, e_3, e_4, e_5$ ): quality, cost, Customer satisfaction, Order fulfilment and environmental impact is given in figure 1. The decision matrix in table1 is based on qualities and suppliers.



**Figure 1: supplier and attributes**

**Step 1: Collect the all data converted PNHFs**

|                       | Supplier 1  | Supplier 2  |
|-----------------------|---|---|
| Quality               | $\left( \begin{array}{l} \{0.49 0.42,0.45 0.42,0.42 0.16\} \\ \{0.83 0.41,0.81 0.48,0.7 0.11\} \\ \{0.25 0.46,0.21 0.22,0.18 0.32\} \end{array} \right)$  | $\left( \begin{array}{l} \{0.35 0.54,0.28 0.15,0.25 0.31\} \\ \{0.92 0.47,0.91 0.43,0.87 0.1\} \\ \{0.61 0.3,0.59 0.14,0.5 0.56\} \end{array} \right)$      |
| cost                  | $\left( \begin{array}{l} \{0.20 0.34,0.18 0.13,0.17 0.53\} \\ \{0.60 0.48,0.51 0.49,0.48 0.03\} \\ \{0.78 0.46,0.71 0.42,0.67 0.12\} \end{array} \right)$ | $\left( \begin{array}{l} \{0.83 0.55,0.76 0.16,0.66 0.29\} \\ \{0.46 0.34,0.37 0.48,0.33 0.18\} \\ \{0.87 0.04,0.83 0.65,0.82 0.31\} \end{array} \right)$   |
| Customer satisfaction | $\left( \begin{array}{l} \{0.21 0.23,0.16 0.40,0.08 0.37\} \\ \{0.34 0.75,0.30 0.25\} \\ \{0.14 0.56,0.11 0.26,0.08 0.18\} \end{array} \right)$           | $\left( \begin{array}{l} \{0.58 0.56,0.46 0.17,0.41 0.27\} \\ \{0.94 0.28,0.92 0.54,0.84 0.18\} \\ \{0.63 0.62,0.61 0.24,0.52 0.14\} \end{array} \right)$   |
| Environmental impact  | $\left( \begin{array}{l} \{0.39 0.04,0.34 0.43,0.31 0.53\} \\ \{0.78 0.35,0.66 0.32,0.62 0.33\} \\ \{0.23 0.06,0.13 0.61,0.08 0.33\} \end{array} \right)$ | $\left( \begin{array}{l} \{0.17 0.16,0.15 0.37,0.04 0.47\} \\ \{0.33 0.19,0.28 0.23,0.22 0.58\} \\ \{0.92 0.43,0.87 0.28,0.79 0.29, \} \end{array} \right)$ |

|                       |  |  |
|-----------------------|--|--|
| Order fulfilment      | $\left( \begin{array}{c} \{0.39 0.32,0.38 0.08,0.31 0.60\} \\ \{0.72 0.64,0.68 0.36\} \\ \{0.15 0.53,0.04 0.18,0.03 0.29\} \end{array} \right)$            | $\left( \begin{array}{c} \{0.37 0.52,0.21 0.48\} \\ \{0.76 0.82,0.74 0.18\} \\ \{0.84 0.8,0.74 0.19,0.72 0.01\} \end{array} \right)$                     |
|                       | Supplier 1   | Supplier 2   |
| Quality               | $\left( \begin{array}{c} \{0.58 0.55,0.57 0.19,0.48 0.26\} \\ \{0.77 0.09,0.73 0.62, 0.05 0.29\} \\ \{0.41 0.27,0.29 0.34,0.26 0.38\} \end{array} \right)$ | $\left( \begin{array}{c} \{0.64 0.17,0.59 0.87\} \\ \{0.72 0.39,0.68 0.26,0.58 0.35\} \\ \{0.35 0.23,0.27 0.57,0.2 0.2\} \end{array} \right)$            |
| cost                  | $\left( \begin{array}{c} \{0.52 0.71,0.5 0.27,0.44 0.02\} \\ \{0.71 0.01,0.7 . 28,0.55 0.71\} \\ \{0.94 0.45,0.83 0.25,0.25 0.30\} \end{array} \right)$    | $\left( \begin{array}{c} \{0.83 0.43,0.78 0.42,0.7 0.15\} \\ \{0.47 0.96,0.43 0.04\} \\ \{0.57 0.1,0.53 0.86,0.44 0.4\} \end{array} \right)$             |
| Customer satisfaction | $\left( \begin{array}{c} \{0.79 0.26,0.74 0.43,0.71 0.32\} \\ \{0.28 0.41,0.2 0.24,0.11 0.35\} \\ \{0.7 0.33,0.63 0.33,0.62 0.35\} \end{array} \right)$    | $\left( \begin{array}{c} \{0.66 0.39,0.58 0.28,0.55 0.33\} \\ \{0.94 0.19,0.93 0.28,0.83 0.53\} \\ \{0.15 0.71,0.03 0.29\} \end{array} \right)$          |
| Environmental impact  | $\left( \begin{array}{c} \{0.89 0.37,0.83 0.63\} \\ \{0.79 0.03,0.78 0.54,0.73 0.43\} \\ \{0.55 0.67,0.49 0.03,0.44 0.3\} \end{array} \right)$             | $\left( \begin{array}{c} \{0.79 0.33,0.71 0.28,0.68 0.39\} \\ \{0.79 0.19,0.7 0.37,0.68 0.44\} \\ \{0.48 0.37,0.45 0.42,0.37 0.21\} \end{array} \right)$ |
| Order fulfilment      | $\left( \begin{array}{c} \{0.26 0.11,0.21 0.82,0.14 0.07\} \\ \{0.57 0.18,0.55 0.28,0.53 0.54\} \\ \{0.72 0.53,0.69 0.01,0.65 0.46\} \end{array} \right)$  | $\left( \begin{array}{c} \{0.78 0.16,0.7 0.44,0.62 0.40\} \\ \{0.49 0.43,0.33 0.43,0.31 0.14\} \\ \{0.59 0.19,0.55 0.38,0.52 0.43\} \end{array} \right)$ |

**Table 1: Decision matrices of PNHF**

Step 2: Find the PNHFF based on PNHF using the five attributes

$$\mathcal{N}^+ = \left( \begin{array}{cc} \left( \begin{array}{c} \{0.49|0.42,0.45|0.42,0.42|0.16\} \\ \{0.77|0.09,0.73|0.62, 0.05|0.29\} \\ \{0.41|0.27,0.29|0.34,0.26|0.38\} \end{array} \right) & \left( \begin{array}{c} \{0.20|0.34,0.18|0.13,0.17|0.53\} \\ \{0.71|0.01,0.7|. 28,0.55|0.71\} \\ \{0.94|0.45,0.83|0.25,0.25|0.30\} \end{array} \right) \\ \left( \begin{array}{c} \{0.21|0.23,0.16|0.40,0.08|0.37\} \\ \{0.28|0.41,0.2|0.24,0.11|0.35\} \\ \{0.7|0.33,0.63|0.33,0.62|0.35\} \end{array} \right) & \left( \begin{array}{c} \{0.39|0.04,0.34|0.43,0.31|0.53\} \\ \{0.78|0.35,0.66|0.32,0.62|0.33\} \\ \{0.55|0.67,0.49|0.03,0.44|0.3\} \end{array} \right) \\ \left( \begin{array}{c} \{0.26|0.11,0.21|0.82,0.14|0.07\} \\ \{0.57|0.18,0.55|0.28,0.53|0.54\} \\ \{0.72|0.53,0.69|0.01,0.65|0.46\} \end{array} \right) & \end{array} \right)$$

Step 3: Let the weight of attributes  $w = (0.36,0.24,0.19,0.14,0.7)$

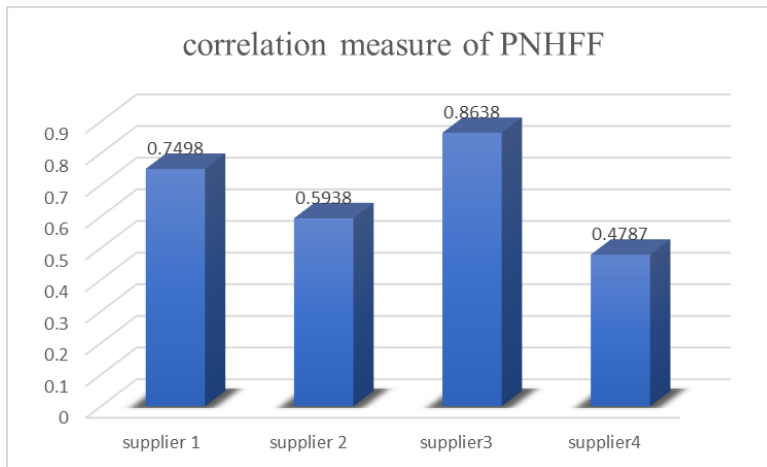
Step 4: Calculate the CM between  $\mathcal{F}_i$  and  $\mathcal{F}^+$  for every  $i=1, 2, 3, \dots, u$  where  $\xi = 1$ .

| CM   | $\mathcal{N}_1$ | $\mathcal{N}_2$ | $\mathcal{N}_3$ | $\mathcal{N}_4$ |
|--|-----------------|-----------------|-----------------|-----------------|
| $\check{\rho}_{wTHFF}(\mathcal{F}_i, \mathcal{F}^+)$ | 0.8458          | 0.5467          | 0.6764          | 0.7620          |
| $\check{\rho}_{wIHFF}(\mathcal{F}_i, \mathcal{F}^+)$ | 0.6697          | 0.5182          | 0.9240          | 0.5104          |



|  |        |        |        |        |
|--|--------|--------|--------|--------|
| $\check{\rho}_{wFHFF}(\mathcal{F}_i, \mathcal{F}^+)$ | 0.7342 | 0.7166 | 0.9909 | 0.6137 |
| $\check{\rho}_{wNHFF}(\mathcal{F}_i, \mathcal{F}^+)$ | 0.7498 | 0.5938 | 0.8638 | 0.4787 |

**Table 2: Correlation measure of PNHFF**



**Figure 2: CM for PNHFF**

**Step 5:** The ranking of four suppliers is  $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_2 > \mathcal{N}_4$ . Weight of quality is 36%, the cost 32%, Customer satisfaction 19%, Environmental impact 14% and Order fulfilment 7%, then the supplier 3( $\mathcal{N}_3$ ) is the best choice.

### Evaluation of the Impact of the Parameters on Ranking

#### Evaluation of weights Impact of the Ranking

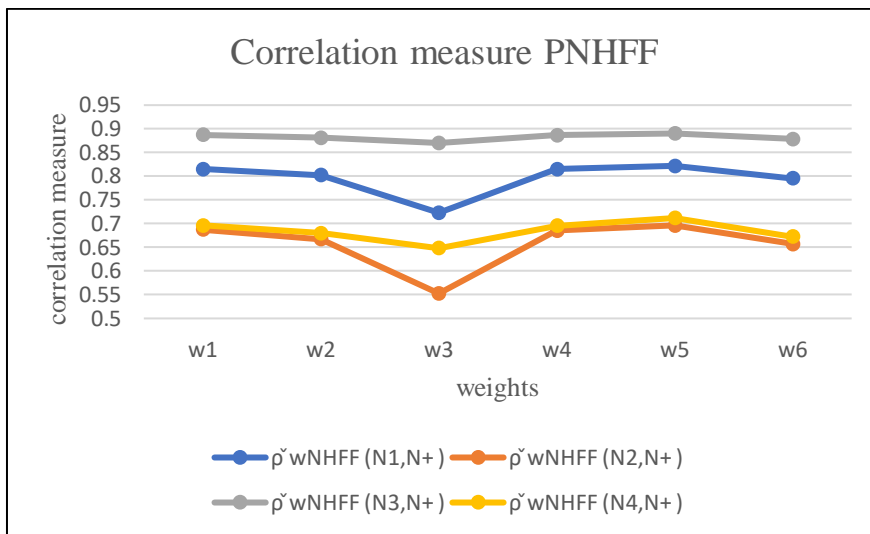
Analyse how changes in attribute weights affect the suppliers' WCCs and ranks in order to determine how weights affect the supplier ranking. At  $\xi = 1$ , we will examine several weight possibilities. The weights are below

|       |   |                                 |
|-------|---|---------------------------------|
| $w_1$ | : | (0.460,0.103,0.029,0.033,0.375) |
| $w_2$ | : | (0.206,0.320,0.055,0.225,0.194) |
| $w_3$ | : | (0.335,0.246,0.025,0.320,0.074) |
| $w_4$ | : | (0.365,0.095,0.011,0.52,0.009)  |
| $w_5$ | : | (0.049,0.316,0.082,0.210,0.343) |
| $w_6$ | : | (0.240,0.360,0.010,0.049,0.341) |
| $w_7$ | : | (0.100,0.210,0.341,0.015,0.334) |

| weights | $\check{\rho}_{wNHFF}(\mathcal{N}_1, \mathcal{N}^+)$ | $\check{\rho}_{wNHFF}(\mathcal{N}_2, \mathcal{N}^+)$ | $\check{\rho}_{wNHFF}(\mathcal{N}_3, \mathcal{N}^+)$ | $\check{\rho}_{wNHFF}(\mathcal{N}_4, \mathcal{N}^+)$ | Ranking   |
|---------|--|--|--|--|---|
| $w_1$   | 0.7743   | 0.6272   | 0.8706   | 0.6495   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| $w_2$   | 0.8153   | 0.6873   | 0.8870   | 0.6961   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_2 > \mathcal{N}_4$ |
| $w_3$   | 0.8023   | 0.6671   | 0.8812   | 0.6800   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| $w_4$   | 0.7229   | 0.5526   | 0.8701   | 0.6480   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| $w_5$   | 0.8148   | 0.6853   | 0.8867   | 0.6953   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |

|       |        |        |        |        |   |
|-------|--------|--------|--------|--------|---|
| $w_6$ | 0.8219 | 0.6963 | 0.8902 | 0.7118 | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| $w_7$ | 0.7957 | 0.6572 | 0.8784 | 0.6725 | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |

**Table 3: correlation measure for different weight impact of the ranking**



**Figure 3: different weights with constant risk preference**

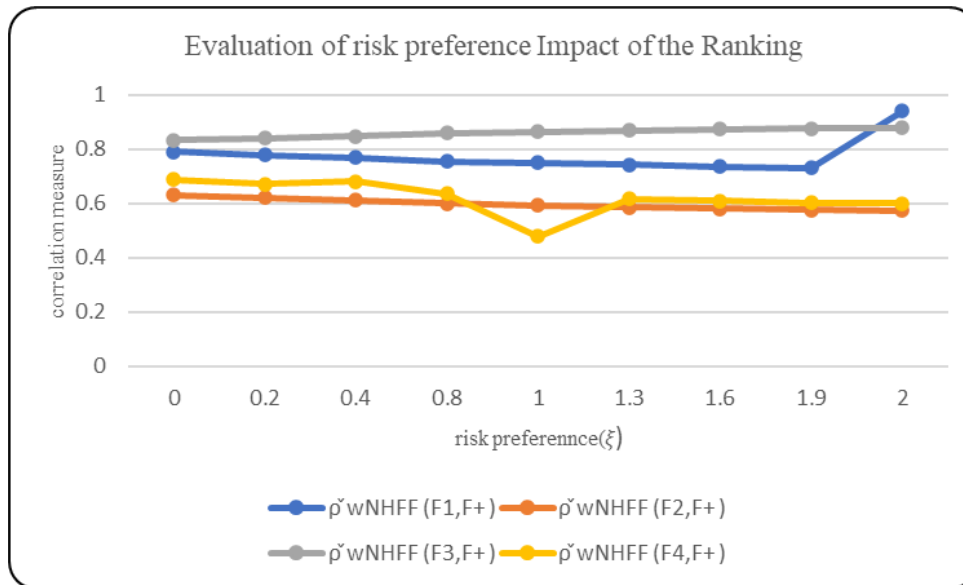
It is evident that there are substantial differences in the rankings of the four options shown in Table 3 and Fig. 3. This variation emphasizes how different ranking outcomes might arise from assigning alternative attribute weights. Thus, it is clear how weights affect the ranking process, emphasizing how crucial it is to properly examine weight allocation. DMs can alter the weights to match with their decision-making scenarios' individual objectives and requirements. By doing so, companies may ensure that the ranking outcomes best reflect their strategic goals and preferences, ultimately leading to more informed and effective decision making.

### Evaluation of risk preference ( $\xi$ ) Impact of the Ranking

The impact of various risk preference coefficients on the rankings is then thoroughly examined. Assuming  $w = (0.36, 0.32, 0.19, 0.14, 0.07)$ , Table 4 presents the outcomes of the ranking with various risk function values.

| Risk preference | $\check{\rho}_{WNHFF}(\mathcal{N}_1, \mathcal{N}^+)$ | $\check{\rho}_{WNHFF}(\mathcal{N}_2, \mathcal{N}^+)$ | $\check{\rho}_{WNHFF}(\mathcal{N}_3, \mathcal{N}^+)$ | $\check{\rho}_{WNHFF}(\mathcal{N}_4, \mathcal{N}^+)$ | Ranking   |
|-----------------|--|--|--|--|---|
| 0               | 0.7900   | 0.6317   | 0.8337   | 0.6879   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| 0.2             | 0.7791   | 0.6213   | 0.8419   | 0.6710   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| 0.4             | 0.7699   | 0.6128   | 0.8487   | 0.6816   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| 0.8             | 0.7556   | 0.5993   | 0.8595   | 0.6371   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| 1               | 0.7498   | 0.5938   | 0.8638   | 0.4787   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_2 > \mathcal{N}_4$ |
| 1.3             | 0.7426   | 0.5870   | 0.8693   | 0.6180   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| 1.6             | 0.7364   | 0.5813   | 0.8739   | 0.6090   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| 1.9             | 0.7315   | 0.5764   | 0.8777   | 0.6015   | $\mathcal{N}_3 > \mathcal{N}_1 > \mathcal{N}_4 > \mathcal{N}_2$ |
| 2               | 0.9414   | 0.5750   | 0.8788   | 0.5991   | $\mathcal{N}_1 > \mathcal{N}_3 > \mathcal{N}_4 > \mathcal{N}_2$ |



**Table 4: correlation measure for different risk presences impact on ranking**

**Fig 4: line chart of different risk preference**

Changes in the risk preference coefficient significantly affect the rankings of the four options, underscoring the importance of risk attitudes in assessment. As  $\xi$  increases, a measure of increased risk aversion, option  $\mathcal{N}_3$  gains popularity while  $\mathcal{N}_2$  and  $\mathcal{N}_4$  lose it. A lower  $\xi$ , on the other hand, indicates a risk-taking attitude and maintains  $\mathcal{N}_1$ 's relative stability in preference. Decision outcomes are more dependable and pertinent when decision-makers are able to match their choices with their risk tolerance thanks to this flexibility.

## CONCLUSION

In this study, we introduce the inclusion relationship of the new Neutrosophic hesitant fantastic filter (NHFF). Since the current correlation measure is mainly designed for the original inclusion relationship, then propose the correlation and probability of neutrosophic hesitant set using by the NHFF. The practical application of the proposed measure is shown in the field of supply chain management.

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