Modeling Nigeria Crude Oil Price with Selected Error Distribution

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ABSTRACT

The crude oil sector plays a vital role in Nigeria's economy, accounting for the majority of government revenue and foreign exchange earnings. However, the inherent volatility in crude oil prices, driven by global factors and market shocks, poses significant challenges to economic stability and fiscal planning. This study aims to model Nigeria's crude oil price dynamics by incorporating selected error innovations to enhance the accuracy of volatility modeling and forecasting.

Using a dataset spanning 1960 to 2024, econometric models such as GARCH, EGARCH, and TGARCH were employed to analyze the impact of error innovations on oil price volatility. The research evaluates the performance of these models in capturing stylized facts, such as volatility clustering and asymmetries, while considering different error distributions. Findings highlight the critical role of error innovations in explaining price fluctuations and demonstrate the superior forecasting accuracy of models that incorporate these shocks.

The study provides both theoretical and practical contributions, advancing econometric methodologies for volatility modeling and offering insights for policymakers, investors, and industry stakeholders. Accurate forecasts can aid in mitigating economic risks, improving fiscal policy formulation, and guiding investment decisions in Nigeria's oil sector. Despite limitations related to model assumptions, data quality, and structural changes in the oil market, the research underscores the importance of error innovations in understanding and managing crude oil price volatility.

Keywords: Modelling, Nigeria, Crude Oil Price

INTRODUCTION

Background to the Study

The crude oil sector is a critical component of Nigeria’s economy, accounting for approximately 90% of its foreign exchange earnings and over 70% of government revenue [11]. Nigeria is the largest oil producer in Africa and holds significant reserves, making oil a major factor in its economic development. However, the volatility in crude oil prices has had profound impacts on the nation’s economy, affecting government revenues, exchange rates, inflation, and overall economic stability These price fluctuations are driven by various global factors, including supply and demand dynamics, geopolitical tensions, and technological advancements [13].

Over the past decades, crude oil prices have shown significant volatility, particularly during periods of economic shocks such as the 2008 global financial crisis and the 2020 COVID-19 pandemic [22]. These events have disrupted the global oil supply and demand, causing sharp price movements that have affected the economies of oil-exporting countries like Nigeria. As a result, understanding and modelling the dynamics of crude oil price fluctuations is critical for policymakers, investors, and stakeholders in the Nigerian economy [10].

Statement of the Problem

Nigeria’s heavy dependence on crude oil revenues makes it vulnerable to the global oil market’s volatility. Fluctuations in crude oil prices can lead to severe economic consequences, such as fiscal instability, inflation, and exchange rate volatility [6] and [8] and [4]. Policymakers often face challenges in stabilizing the economy due to the unpredictable nature of oil price movements. Inaccurate forecasts of crude oil prices can result in poor economic planning, budget deficits, and disruptions in government spending.

Despite the significance of error innovations in influencing crude oil price volatility, there is limited research on how these shocks can be incorporated into econometric models for Nigeria's crude oil market. Therefore, this study seeks to fill this gap by developing a model that accounts for selected error innovations, with the goal of enhancing the predictive power of crude oil price forecasts for Nigeria.

Aim and Objectives of the Study

This study aims to model Nigeria's crude oil prices by incorporating selected error innovations to provide a more accurate understanding of volatility patterns and improve forecasting performance. The objectives are to:

i. examines the historical trend of Nigeria crude oil prices; ii. analyze the impact of error innovations on the volatility of Nigeria's crude oil prices; iii. develop econometric model that incorporates selected error innovations to predict

Nigeria's crude oil prices; iv. Evaluate the forecasting performance of the model;

Significance of the Study

This study holds both theoretical and practical significance for the fields of econometrics, energy economics, and policy formulation. From a theoretical standpoint, this research contributes to the existing literature on financial time series modeling by integrating selected error innovations into volatility models. Incorporating these innovations will help explain the non-linear behaviors of crude oil prices more effectively, leading to more accurate forecasts and a better understanding of volatility dynamics (Bollerslev, 1986).

Scope and Limitation of the Study

The scope of this study focuses on modelling Nigeria's crude oil prices using advanced econometric techniques, specifically GARCH, EGARCH, and TGARCH models. The data utilized for this analysis spans from 1960 to 2024, providing a comprehensive historical perspective on crude oil price fluctuations. The dataset is sourced from the World Bank Development Indicators, ensuring reliable and globally recognized data for analysis. The study aims to explore the volatility of crude oil prices, providing insights into market behavior, price dynamics, and forecasting accuracy.

Limitations:

i. **Model Selection:** While GARCH, EGARCH, and TGARCH are robust volatility models, their performance may be limited by certain assumptions about error distributions or volatility persistence. More complex models or machine learning approaches might provide better results, but are outside the scope of this research.

ii. **Data Quality and Availability:** Although the World Bank Development Indicators provide reliable data, the potential for gaps or inaccuracies in historical data could affect model performance and forecast accuracy. Additionally, certain economic shocks or geopolitical events may not be fully reflected in the data.

iii. **Temporal Limitations:** The data from 1960 to 2024 covers a long period, but oil markets and economic conditions have changed dramatically over time. Structural breaks or regime shifts may not be fully accounted for by the models.

iv. **Model Assumptions:** Each of the models used (GARCH, EGARCH, TGARCH) relies on certain assumptions about price volatility and error terms, which may not hold true under all conditions, potentially affecting the validity of the results.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Introduction

The modelling of crude oil prices is a critical area of research, particularly for oil-dependent economies like Nigeria. Accurate forecasting of oil prices is essential for economic planning and stability, as fluctuations can have significant impacts on national revenues and overall economic health [3]. This literature review synthesizes various methodologies and findings from recent studies focused on crude oil price prediction, with an emphasis on the integration of error innovations and advanced time series techniques.

Current Approaches to Crude Oil Price Prediction

Machine Learning Techniques

Recent studies have increasingly turned to machine learning methodologies for crude oil price forecasting. For instance, a study by [17] proposed a multi-step prediction model using Long Short-Term Memory (LSTM) networks optimized by a Salp Swarm Algorithm, demonstrating significant improvements in prediction accuracy. The authors emphasized the necessity of employing advanced machine learning techniques to navigate the complexities of crude oil price movements, which is especially relevant for Nigeria's volatile oil market.

Similarly, [14] compared different machine learning models, specifically AdaBoost-LSTM and AdaBoost-GRU, to enhance forecasting performance. Their findings indicate the potential of ensemble learning techniques in refining prediction results, underscoring the need for innovative methodologies in Nigeria’s oil price modelling.

studies have explored various machine learning approaches for modeling and predicting crude oil prices in Nigeria. Neural network-based models, such as the Neural Network Autoregressive (NNETAR) model and Autoregressive Neural Network, have shown promising results in forecasting oil prices [21]. The Fuzzy Time Series (FTS) model using Chen's algorithm has outperformed traditional statistical methods like ARIMA and machine learning models like Artificial Neural Network and Random Forest in modeling Nigerian Bonny Light crude oil prices [9]. For daily crude oil price modeling during the Russia-Ukraine war, a log-linear regression model demonstrated the best fit among six regression models tested [18]. These studies emphasize the importance of advanced computational techniques in accurately predicting oil prices, which is crucial for economic planning and risk management in oil-dependent economies like Nigeria [15].

Time Series Analysis

Traditional statistical methods, including ARIMA and GARCH models, remain relevant in the context of crude oil price forecasting. [2] conducted a comparative analysis of these traditional models against Support Vector Machines (SVM), concluding that SVM outperformed conventional methods in forecast accuracy. This suggests that integrating advanced forecasting methods with traditional approaches could yield better results in modelling Nigeria's crude oil prices.

Environmental Factors and Market Dynamics

[8] introduced the Index of Cryptocurrency Environmental Attention (ICEA) and its correlation with Brent crude oil prices, suggesting that environmental concerns could significantly influence commodity prices. This finding opens up new avenues for research, particularly in understanding how environmental factors and public perceptions impact crude oil price volatility and forecasting in Nigeria. The adaptation of methodologies like the Vector Error Correction Model (VECM) could enhance the insights into these dynamics.

Furthermore, [7] examined the lead-lag relationships between crude oil prices and stock returns in oil-producing nations. Their findings suggest that local market dynamics significantly influence crude oil prices, highlighting the necessity of incorporating local economic indicators into forecasting models.

Recent studies have explored various models for predicting crude oil prices in Nigeria. [14] and [13] compared GARCH models with different error innovations, finding the AP-ARCH (1,1) model with skewed Student's t-distribution to be the most effective. [19] evaluated ARIMA models, concluding that ARIMA (2,1,1) performed best based on error variance.

METHODOLOGY

Research Design

The study adopts a quantitative research design, specifically focusing on time series analysis. This approach is suitable for analyzing the historical patterns and predicting future prices of crude oil in Nigeria. The study utilizes monthly secondary data from 1980 to 2024. The primary econometric framework employed in this study is based on Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models and its extensions, with the incorporation of error innovations such as skewness, kurtosis, and fat tails.

Data Collection

Data Source

The data used in this study was sourced from Central Bank of Nigeria (CBN) bulletin, and World Bank Commodity Price Data. The dataset included monthly prices of Nigerian crude oil, covering a period of 1980 to 2024, sufficient to capture significant market dynamics, such as fluctuations due to global crises and local production shifts.

Data Description

The data consists of monthly crude oil prices variables. These prices have huge influence on microeconomic variables such as exchange rates, GDP, Money supply and inflation rates.

Model Specification

The GARCH model is designed to capture volatility clustering in financial time series, where large changes tend to be followed by large changes, and small changes by small changes. The GARCH model assumes that the error variance follows a conditional heteroskedasticity process, which makes it suitable for modelling the volatility of crude oil prices.

A generalized autoregressive conditional heteroskedasticity (GARCH) model is considered if an autoregressive moving average (ARMA) model is designated for the random error variance (Bollerslev, 2021). In this case, the GARCH (p, q) model (where p is the order of the GARCH terms σ^2 and q is the order of the ARCH terms ε^2), that follows the notation, is given by

$$y_t = x'_t\beta + \varepsilon_t \tag{3.1}$$

$$\varepsilon_t \mid \Psi_{t-1} \sim N(0,\sigma^2_t) \tag{3.2}$$

$$\sigma^2_t = w + \alpha_1 \varepsilon^2_{t-1} + \cdots + \alpha_{1j} \varepsilon^2_{t-q} + \beta_1\sigma^2_{t-1} + \cdots + \beta_p\sigma^2_{t-p} \tag{3.3}$$

$$\sigma^2_t = w + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-1} + \sum_{i=1}^{p} \beta_i \sigma^2_{t-i} \tag{3.4}$$

In the context of econometric models, the White test has emerged as the most effective method for assessing heteroskedasticity. When working with time series data, it is necessary to do tests for ARCH and GARCH faults. In lieu of employing GARCH modelling The exponentially weighted moving average (EWMA) is a model belonging to a distinct class of exponential smoothing models. It possesses intriguing characteristics, such as assigning greater significance to more recent observations. However, it is not without its limitations, including the presence of an arbitrary decay factor that introduces subjectivity into the estimation process.

The lag length of order p of a GARCH (p, q) process is deep-rooted in three steps:

i. Estimating the best fit AR(q) model

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \cdots + \alpha_q y_{t-q} + \varepsilon_t \tag{3.5}$$

$$y_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i} + \varepsilon_t \tag{3.6}$$

Compute and plot the autocorrelations of ε^2 by

$$\rho_k = \frac{\sum_{t=i+1}^{T}(\hat{e}^2{}_t - \sigma^2{}_t)(\hat{e}^2{}_{t-1} - \sigma^2{}_{t-1})}{\sum_{t=1}^{T}(\hat{e}^2{}_t - \sigma^2{}_t)^2} \tag{3.7}$$

iii. The asymptotic, for large samples, with standard deviation of $\rho(i)$ is $\frac{1}{\sqrt{T}}$. Individual values that are greater than this denote GARCH errors. Estimating the total number of lags, the Ljung–Box test is used until the value of these are lesser than 10% significant. The Ljung–Box Q-statistic follows X^2 distribution with n degrees of freedom if the squared residuals $\varepsilon^2{}_t$ are uncorrelated. It is recommended to consider up to T/4 values of n. The null hypothesis (H_0) states that there are no ARCH / GARCH errors. Rejecting the null hypothesis (H_0) thus denotes that there exist errors in the conditional variance.

Symmetric Effects of GARCH Models

Considering that Y_t is a stationary time series, if Y_t, $=\sigma_t\,{}_t$, where $\sigma_t \geq 0$, is generated by Yt_{-k}, $k \geq 1$ variable $_t$ represents an independent and randomly distributed variable with a mean of zero and a variance of one, as defined by a volatility model.

The value-added model was defined by ARCH through the temporal relationships while modelling exchange rate return series. It is the first series to account for heteroscedasticity and volatility clustering, and it was first introduced by Engle (1982) as a function of historical squared returns. Here is an example of the ARCH(q) model:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 \tag{3.8}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}(\alpha_i \varepsilon_{t-1}^2) \tag{3.9}$$

Were,

$\omega > 0, \alpha_1 \geq 0$ for $i = 1,2,3,\ldots, q$ and $\sum_{i=1}^{q}(\alpha i) < 1$, Presently, it is acknowledged that the

return process exhibits limited stationarity, as defined by the unconditional variance:

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$$E\left(\varepsilon_{\varepsilon}^{2}\right)=\frac{\omega}{1-\alpha_{1}-\alpha_{2}-\alpha_{3,\ldots,}\alpha_{q}} \tag{3.10}$$

ARCH (1) can be derived from ARCH(q) model, $\sigma^2{}_t$

Then ARCH (1) is showed as, denote as the Conditional Variance of Random Variable, then

ARCH (1) can be denoted by,

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \tag{3.11}$$

The GARCH model can be typically defined as:

GARCH Models

$$\sigma_t^2 = \propto_0 + \sum_{i=1}^{q} \propto_i x_{t-1}^2 \tag{3.12}$$

GARCH (1,1) is given by

$$\sigma_t^2 = \propto_0 + \propto x_{t-1}^2 + \beta x_{t-1}^2 \tag{3.13}$$

Where $\alpha_0 > 0, \alpha \geq 0,\ and\ \beta \geq 0\ for\ \sigma^2{}_t\ to\ be\ positive$

$$\mu_t = \varepsilon_t \varepsilon_t^2 = \varepsilon_t \sqrt{ht} \tag{3.15}$$

Where Y_t denotes the exchange rate returns and $\mu\ as\ mean\ value,\ \mu \geq 0$;

Given that Y_t represents the returns on the exchange rate and μ stands for the mean value, with ≥ 0;

$$\mu_t = \quad \alpha^2{}_t = \quad {}_t\sqrt{ht} \tag{3.15}$$

Where, ${}_t{\sim}N(0,1)$

Conditional variance equation of GARCH (p, q) can be defined as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\left(\alpha_i \varepsilon_{t-1}^2\right) + \sum_{j=1}^{p}(\beta_j \alpha_{t-1}^2) \tag{3.16}$$

The mean value, denoted as $\omega > 0$; $\alpha_i \geq 0\ for\ i = 1,2,3,\ldots, q\ and\ \beta j \geq 0\ for\ j =$

1,2,3,… p

So, $\sigma_t^2 \geq 0$

Condition for the stationary is derived from:

$$\sum_{i=1}^{q}(\alpha_i) + \sum_{j=1}^{p}(\beta j) < 1 \quad (3.17)$$

Combining the ARCH and GARCH terms yields the expected variance from the prior period, with the ARCH term representing the lag of squared residuals. The predicted value is β> α.

We can now define the Generalised ARCH model as:

$$\sigma_t^2 = E\left(x_t^2 \middle| x_{t-1}^2, x_{t-2}^2, x_{t-3}^2, \dots,\right)$$

(3.18)

$$\sigma_t^2 = d_0 + \sum_{j=1}^{\infty} d_j x_{t-j}^2$$

where ,constant $d_j \geq 0$

The GARCH (1, 1) model is derived from the GARCH (p, q) model, where terms (1, 1) represent the first order autoregressive GARCH term and the first order moving ARCH term, respectively. Next, the model is defined in the following manner:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \varepsilon_{t-1}^2 \quad (3.19$$

Where,

$\omega > 0, \alpha_1 > 0, \beta_1 \geq 0$ *and* $\alpha_1 + \beta_1 < 1$

$E(\ _{t-1}|\Omega) = 0$

If σ_t^2 *denotes as* h_t, then

Mean equation,

$$Y_t = \mu + \varepsilon_t, \quad \varepsilon_t\,|\mathrm{I}_{t-1} \sim N(0, h_t) \quad (3.20)$$

Conditional variance equation

$$h_t = \omega + + \beta h_{t-1} \quad (3.21)$$

The locations where the terms are defined are forecast variance, N-conditional normal density with mean zero (0) variance; Y_t, ε_t – residual term; ht, denoted by ω–mean, conditional variances expressed as ht-1, and previous period's news, $^2{}_{t-1}$The α represents the volatility observed in the previous period, while the α and β signifies the variance of the previous lag/period forecast.

Asymmetric effects of GARCH Models

There were some problems with the ARCH and GARCH models that Nelson fixed in 1991 with the EGARCH model. These problems were non-negativity limits and leverage effects.

The leverage effect is an example of an uneven volatility trait [12].

Conditional Variance are being expressed as,

$$\log\left(\alpha_t^2\right) = \omega + \sum_{i=1}^{q}(\alpha_i)\frac{|\varepsilon_{t-1}|}{|\alpha_{t-1}|} + \sum_{k=1}^{r}(\gamma k)\frac{\varepsilon_{t-k}}{\alpha_{t-k}} + \sum_{j=1}^{p}\beta_j \log\left(\alpha_{t-j}^2\right) \quad (3.22)$$

Where, y_k denotes an asymmetry parameter. Hence, an asymmetry effect is clearly visible when yk ≠ 0, and an increase in volatility is shown by yk < 0, εt-1 < 0, and the conditional variance is denoted by εt-1 > 0. The non-negativity criteria is guaranteed when the logarithm of the conditional variance is taken into account, specifically when the leverage effect is exponential in the EGARCH model and Y < 0. With regard to the symmetric effect Y≠ 0. The model facilitates the automatic asymmetry of the lagged error. Consequently, the regression residuals that represent the asymmetric response do not exhibit equal negative residuals. To accept the hypothesis, it is necessary to assign a value of zero to the condition that there is no substantial difference in the positive or negative impacts. So, the conditional variance is not affected by any asymmetry. Black did more digging into this later on (1976).

The TGARCH model can be expressed as:

The Threshold GARCH (TGARCH) model, proposed by [20] and [16] is a variant of the GJR GARCH model that focuses on the conditional standard deviation rather than the conditional variance.

$$\alpha_t^2 = \omega + \sum_{i=1}^{q}(\alpha_i)\varepsilon_{t-j}^2 + \sum_{i=1}^{q}\left(\gamma id\left(\varepsilon_{t-1} < 0\right)\varepsilon_{t-1}^2\right) + \sum_{j=1}^{p}\beta_j\left(\alpha_{t-j}^2\right) \quad (3.23)$$

Were,

$_{t-1} < 0$ denotes the good news, the total effects are $(\alpha_i + \gamma_i)\,^2{}_{t-i}$ *and* $_{t-1} >$

0 *denotes the bad news, then total effects are* $\alpha_i{}^2{}_{t-i}$.

Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH)

An alternative version of the GARCH model is the EGARCH model, which was proposed by Nelson and Cao (1991). In formal terms, the Exponential GARCH (EGARCH) model is known as an EGARCH (p,q): Asymmetric GARCH model. The linear GARCH model's non-negativity restrictions are limiting, according to an argument between Nelson and Cao from 1992. There are no parameters limitations in the EGARCH model. The EGARCH model posits that the conditional variance is asymmetric with respect to latent disturbances. This is due to the fact that volatility tends to increase more significantly following a negative return as opposed to a positive return. The leverage effect is modelled using this model.

EGARCH is stated by:

$$\text{In}\left(\alpha_t^2\right) = \propto_0 + \sum_{i=1}^{q} \propto_i g\left(Z_{t-1}\right) + \sum_{j=1}^{q}\beta j\, \text{In}\left(\sigma^2{}_{t-j}\right), \quad (3.24)$$

$g(Z_t)$ *value depends on some several components.*

Hence,

$$g(Z_t) = \theta_1(Z_t) + \theta_2[|Z_t| - E|Z_t|], \quad (3.25)$$

Where, $\theta_1(Z_t)$ *is the sign effect while* $\theta_2[|Z_t| - E|Z_t|]$*is the magnitude effect.* An

additional advantage of this statement is that there is no need for any stationary constraint.

EGARCH and the Changes in the Distributions

Diverse distributions have a specific impact on EGARCH model. For the normal (Gaussian) density function expectation in the $g(Zt)$ function is

$$E(|Z_t|) = \sqrt{\frac{2}{\pi}} \quad (3.26)$$

For the student's t-density expectation in the (Z_t)*function is*

$$E(|Z_t|) = \frac{\Gamma(\frac{v+1}{2}2\sqrt{v-2})}{\sqrt{\pi(v-2)\Gamma(\frac{v}{2})}}, \quad (3.27)$$

For the skew Student's t-density expectation in the $g(Zt)$ function is

$$E(|Z_t|) = \frac{2\varepsilon^2\Gamma(\frac{1+v}{2})2\sqrt{v-2})}{\varepsilon\Gamma\left(\frac{1}{\varepsilon}\right)\sqrt{\pi(v-1)\Gamma\left(\frac{v}{2}\right)}}, \quad (3.28)$$

student'st − distribution

Parameter Estimation

Parameter estimation in the GARCH model is often done using Maximum Likelihood

Estimation (MLE). To forecast a time series' volatility, this is the initial step. The popular Maximum Likelihood Estimation (MLE) method is investigated in this study. In most cases, when approximating parameters using Maximum Likelihood Estimation (MLE), a likelihood function is being established. The likelihood function in most cases is crucially a joint probability density function, but can be rather thought of as a function of the data ($x_1, x_2, ..., x_n$) given the set of parameters, f ($\propto_0, \propto_1, ..., \propto_q, \beta_1, ..., \beta_p|\theta$), the likelihood function as a set of function of the parameters given the data is given as L f (θ |$\propto_0, \propto_1, ..., \propto_q, \beta_1, ..., \beta_p$), maximizing the likelihood function with respect to the parameters. GARCH model returns, are not independent, however, the joint probability density function can be written as a product of the conditional density. The model for financial returns $X_t = \sigma_t Z_t$ with $Z_t \leftarrow N(0, 1)$ which is the assumption of being independent and identically distributed (i.i.d.) and normal shock (Z_t) shows that the density of time t observations is:

$$P_t = \frac{1}{\sqrt{2\pi\alpha_t^2}} e^{\frac{-(Z_t-\mu)^2}{2\alpha_t^2}} \quad (3.29)$$

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Recall the model:

$$\begin{cases} X_t = \sigma_t Z_t \\ \sigma_t^2 = \propto_0 + \sum_{i=1}^{q} \propto_1 X_{t-i}^2 + \beta_j \sigma_{t-j}^2 \end{cases}$$

Where $\propto_0, \ldots, \propto_q, \beta_1,, \beta_p$ *are unknon parameters. the likelihood function is*

$L(\propto_0,\ldots, \propto_q, \beta_1,\ldots, \beta_p)p(X_1,\ldots, X_n|\propto_0,\ldots, \propto_q, \beta_1,\ldots, \beta_p)$

$p(X_n|X_1,\ldots, X_{n-1},|\propto_0,\ldots, \propto_q, \beta_1, \ldots, \beta_p)p(X_1,\ldots, X_{n-1}|\propto_0,\ldots, \propto_q, \beta_1, \ldots, \beta_p$

$=\ldots=\prod^n_{t=2} p(X_t|X_1,\ldots, X_{t-1},\propto_0,\ldots, \propto_q, \beta_1,\ldots, \beta_p)p(X_1|\propto_0,\ldots, \propto_q, \beta_1, \ldots, \beta_p)$

$$= \prod_{t=2}^{n} p(X_t|\sigma_t\, p(X_1) \tag{3.30}$$

It is known that each shock is independent of the others.

we can now write the total probability of the entire sample is the product of T such densities,

$-(Z_t-\mu)^2$

$\overline{2\sigma 2t}$

$2\pi\sigma$ $\prod_{t=2}^{n} \frac{e_{\,1}}{\sqrt{\quad {}^2{}_t}}$

while this is called the likelihood function. Nevertheless, it is much simpler to work with sums than a product. Therefore, we consider log-likelihood function,

$$l(\theta) = \sum_{t=z}^{n} \left(In\frac{1}{\sqrt{2\pi\sigma^2{}_t}} - \frac{1}{2}\frac{(Z_t-\mu)^2}{\sigma^2{}_t} \right) \tag{3.31}$$

$$= \sum_{t=2}^{n} - \frac{In(2\pi)}{2} - \frac{1}{2}\text{In}\sigma^2{}_t - \frac{1}{2}\frac{(Z_t-\mu)^2}{\sigma^2{}_t} \tag{3.32}$$

$$= -\frac{TIn(2\pi)}{2} - \frac{T}{2}In\sigma_t^2 - \frac{1}{2}\sum_{t=2}^{n}\left(\frac{(Z_t-\mu)^2}{\sigma^2{}_t}\right) \tag{3.33}$$

Where for GARCH (1,1) we can substitute

$$\sigma_t^2 = \propto_0 + \propto X_{t-1}^2 + +\beta\sigma_{t-1}^2 \tag{3.34}$$

MLE-based estimates are defined as:

$$\Theta = \arg\max_{\Theta} L(\Theta) \quad (3.35)$$

LOG-LIKELIHOOD Function for the students' t distribution

$$l_n = \sum_{t=1}^{n}\left\{\log r\left(\frac{v+1}{2}\right) - \log r\left(\frac{v}{2}\right) - \frac{1}{2}\log\left(\pi(v-2)\right) - \frac{1}{2}log\sigma_t^2 - \left(\frac{v+1}{2}log\left(1+\frac{(r_i-\mu)^2}{\sigma_t^2(v-2)}\right)\right)\right\}$$

(3.36)

Log-likelihood for skew student's distribution

$$l_n = log\left[r\left(\frac{v+1}{2}\right)\right] - log\left[r\left(\frac{v}{2}\right)\right] - \frac{1}{2}log[\pi(v-2)] + log\left(\frac{2}{\xi+\frac{1}{\xi}}\right) + \log(\mathcal{S})$$

$$-\frac{1}{2}\sum_{t=1}^{T}\left[log\sigma_t^2 + (1+v)log\left(\frac{1+sX_t+m}{v-2}\xi^{-I_t}\right),\right] \quad (3.37)$$

where ξ is the asymmetry parameter, v represents the degrees of freedom of the distribution and the gamma function $\Gamma(\cdot)$,

$$I_t = \begin{cases} 1 & if\ Z_t \geq -\frac{m}{s} \\ -1 & if\ Z_t < -\frac{m}{s} \end{cases} \quad (3.38)$$

Where,

$$m = \frac{r\left(\frac{v+1}{2}\sqrt{v-2}\right)}{\sqrt{\pi r\left(\frac{v}{2}\right)}}\left(\xi - \frac{1}{\xi}\right) \quad (3.39)$$

And

$$\mathcal{S} = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}. \quad (3.40)$$

Unit Root Tests

Prior to modeling the stock prices, we determine the order of integration of the variables. Unit root test of the stock returns is essential because any meaningful econometrics time series modeling requires stationarity of the series. If the series are not stationary, the important test statistics used in the evaluation of the econometric results become unreliable. We employ the following unit root tests to examine the order of integration of the six banks equity return series.

i. Augmented Dickey-Fuller (ADF) Test: The ADF unit root test is applied to determine if the daily stock index returns y_t is stationary based on the following regression:

$$\Delta y_t = \varphi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \mu_t \qquad (3.41)$$

Where $\mu_t$is a white noise error term and Δy_{t-1}=$y_{t-1} - y_{t-2}$, Δy_{t-2}= $y_{t-2} - y_{t-3}$ etc. equation 1 test the hypothesis of a unit root against a trend stationary alternative.

ii. The Philips-Perron (PP) Test: It uses model similar to the Dickey-Fuller test but with Newey-West non-parametric correction for possible autocorrelation rather than the lagged variable method employed in the ADF test. The Philips-Perron equation modifies the Dickey-fuller test (Philips and Perron, 1988). The PhilipsPerron test is computed from the equation below:

$$y_t = \delta_t + \gamma y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_p \Delta y_{t-p} + \mu_t \qquad (3.42)$$

Where δ_t may be 0, φ or $\varphi + \beta_t$

Having determined the asymptotic distributions of the test statistic, the null hypothesis is rejected if the estimated value of the ADF and the PP test statistic are less than the critical values.

Error Innovation

The probability distribution of stock returns often exhibits fatter tails than the standard normal distribution. The existence of heavy-tailedness is probably due to a volatility clustering in stock markets. In addition, another source for heavy-tailedness seems to be the sudden changes in stock returns. An excess kurtosis also might be originated from fat-tailedness. Mostly, in practice, the returns are typically negatively skewed [1] and [5]. In order to capture this phenomenon, the normal distribution, student-t, and skewed student-t distributions are also considered in this analysis.

For the GARCH models characterized by GED, student-t, and skewed student-t distributions which help to capture additional skewness and kurtosis in the returns, which are not adequately captured by Normal error distribution. The GED is estimated by maximizing the likelihood function below:

$$L(\theta_t) = -\frac{1}{2}\sum_{t-1}^{T}\left(ln2\pi + ln\sigma_t^2 + \frac{a_t^2}{\sigma_t^2}\right) \qquad (3.43)$$

In the case of t-innovation, the volatility models considered are estimated to maximize the likelihood function of a Student's t distribution as follow:

$$L_{(std)}(\theta_t) = -\frac{1}{2}\ln\left(\frac{\pi(d)\Gamma\left({d}/{2}^2\right)}{\Gamma\left[\left({d+1}/{2}\right)\right]^2}\right) - \frac{1}{2}\ln S_t^2 - \frac{d+1}{2}\left(1 + \frac{(r_t - X_{t,\theta})^2}{S_t^2\,(d-2)}\right) \qquad (3.44)$$

In the case of the skewed student's t-innovation, the volatility models is to maximize the likelihood function of the skewed student's t distribution. The probability density function of the skewed student's t distribution is

$$f(x) = \begin{cases} \frac{bc}{w}\left(1 + \frac{1}{v}\left(\frac{b(x-\varepsilon)}{w}\right)^2\right)^{-\frac{v+1}{2}}, if\ x \geq \varepsilon \\ \\ \frac{bc}{w}\left(1 + \frac{1}{v}\left(\frac{b(x-\varepsilon)}{w}\right)^2\right)^{-\frac{v+1}{2}}, if\ x < \varepsilon \end{cases} \qquad (3.45)$$

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The log of the likelihood function is given as follow: $x \geq 0$:

$$logL(x;\ v,\lambda) = log\left(\frac{2}{v+1}t\left(\frac{x}{v}\right)T\left(\lambda.\frac{x}{v+1},v+1\right)\right)$$

$$= log\left(\frac{2}{v+1}\right) + log\left(t\left(\frac{x}{v}\right)\right) + log\left(T\left(\lambda.\frac{x}{v+1},v+1\right)\right) \quad (3.46)$$

Log-likelihood for $x < 0$:

$$logL(x;\ v,\lambda) = log\left(\frac{2}{v+1}.t\left(\frac{-x}{v}\right)[T-1\left(\lambda.\frac{x}{v+1},v+1\right)]\right)$$

$$= log\left(\frac{2}{v+1}\right) + \log\left(t\left(\frac{-x}{v}\right)\right) + log[T-1\left(\lambda.\frac{x}{v+1},v+1\right)]\Big) \quad (3.47)$$

The Daily Stock Returns

The returns were computed using the equation below.

$$y_t = logk_t - logk_{t-1} \quad (3.48)$$

Where y_t=denotes the continuously compounded return at time t, k_t denotes the asset price at time t,$_{t-1}$= previous asset price, and ln denotes the natural logarithm.

The existence of volatility clustering in the daily stock index returns y, is established by plotting the residual of the equation:

$$y_t = k + \xi_t \quad (3.49)$$

Equation (3.48) tends to shows that prolong period of low volatility are followed by prolong period of high volatility. k is a constant, ξ_t is the residual series and y_t is return series.

The Lagrange Multiplier (LM) Test

The Lagrange Multiplier (LM) test for ARCH in the residuals ξ is used to assess the null hypothesis that there is no ARCH effect (H_0: $\pi_l = 0$) up to order q at a 5% significance level.

This is tested using the following equation:

$$\xi_0^2 = \psi_0 + \sum_{l=1}^{q} \pi_l \xi_{t-l}^2 + \mu_t \quad (3.50)$$

where ψ_0 is a constant and μ_t represents the error term. For the GARCH model to be applicable, the null hypothesis should not be accepted, indicating the presence of an ARCH effect. Since the existence of ARCH effects is a prerequisite for GARCH modeling, GARCH models effectively capture and account for this volatility structure in financial time series.

The mean equation of a stationary return series exhibiting an ARCH effect is expressed in a univariate form as:

$$y_t = \rho + \varpi y_{t-1} + \varepsilon_t \qquad (3.51)$$

Where y_t represents the daily returns, ρ is a constant, ϖ is the estimated autoregressive coefficient, , y_{t-1} is the one-period lag of returns, and $_t$ denotes the standardized residuals of returns at time t.

Model Selection Criteria

Model selection is done using information criteria, and the model with the least information criteria value across the error distributions is adjudged the best fitted. If the number of parameters in the model is denoted as p, then the AIC is defined by:

$$AIC(p) = -2In(Ml) + 2p \qquad (3.52)$$

Where:

Ml is the maximum likelihood estimate.

The Bayesian information criteria (BIC)given by

$$BIC(p) = -2In(Ml) + pIn(N) \qquad (3.53)$$

Where:

N is the number of observations.

For a large data set, the BIC has a heavier penalty for the number of parameters in the model, therefore it will select a more parsimonious.

RESULTS AND DISCUSSION

Introduction

This chapter presents the empirical analysis and results of modeling Nigeria's crude oil prices using selected error innovations. It starts with a descriptive statistical analysis, followed by stationarity testing, model selection, and estimation. We then evaluate the chosen models using various diagnostics and examine the forecast performance. The chapter concludes with an interpretation of the findings and their implications.

Descriptive Statistics of Nigeria Crude Oil Prices

To understand the data's basic characteristics, we begin with a descriptive analysis of Nigeria's crude oil prices and the stock returns over the study period. Table 4.1 shows the descriptive statistics of Nigeria’s crude oil price series and the stock returns from 1980 to 2024. The descriptive statistics provide insights into the data's distribution and potential volatility

Table 4.1: Descriptive Statistics of Nigeria’s Crude Oil Price Series and the Stock Returns

	Crude Oil Price_	**Rcrude**
Mean	45.38831	0.001384
Median	32.88338	0.007932
Maximum	132.8252	0.430394

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Minimum	9.616667	-0.504742
Std. Dev.	29.94400	0.089440
Skewness	0.862134	-0.665680
Kurtosis	2.574864	8.534661
Jarque-Bera	70.04158	718.3106
Probability	0.000000	0.000000
Sum	24191.97	0.736366
Sum Sq. Dev.	477014.1	4.247714
Observations	533	532

The descriptive statistics in table 4.1 for Nigeria's crude oil prices and stock returns (RCrude) reveal substantial volatility and deviations from normality. The average crude oil price is 45.39, with wide-ranging values from 9.62 to 132.83, and a high standard deviation of 29.94, indicating significant fluctuations. The price series is positively skewed and slightly platykurtic, while stock returns show a minimal average of 0.001384, marked negative skewness, and high kurtosis (8.53), signaling frequent extreme returns. Both series fail the Jarque-Bera normality test, suggesting non-normal distribution. These features underscore the need for specialized volatility models to capture and forecast their complex dynamics. Figure 4.1 and 4.2 show the time plot and the return series plot respectively of the crude oil price.

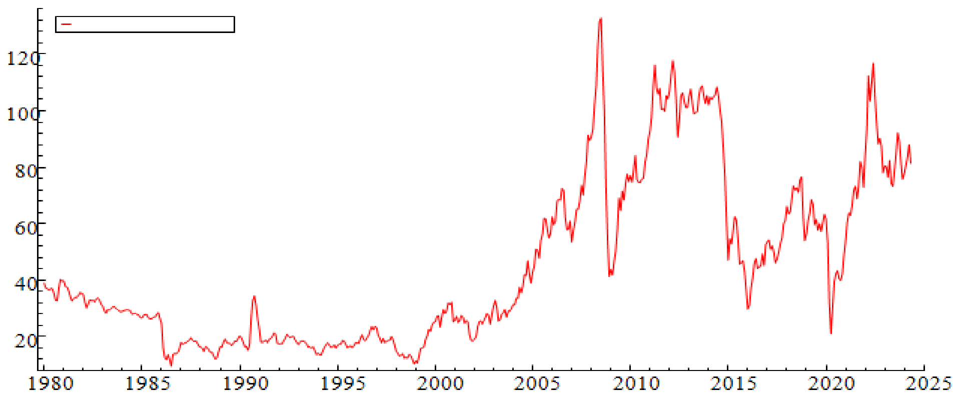

Figure 4.1: Time Plot of Nigeria Crude Oil Price from 1980 to 2024.

Figure 4.1 is a time plot of monthly crude oil prices in Nigeria from 1980 to 2024. The plot shows a decline in the price of crude from 1980 to about 1987 and then the price begins to rise with some slight fluctuation from 1988 to 2024

Stationarity Tests

Before fitting any models, we test the stationarity of the crude oil price series using the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests.

Augmented Dickey-Fuller (ADF) Test

The ADF test assesses whether the series has a unit root. The null hypothesis is that the series is non-stationary. Table 4.2 presents the results of the ADF test.

Table 4.2: Augmented Dickey-Fuller (ADF) Test

Null Hypothesis: CRUDE OIL PRICE has a unit root			
		I(0)	**I(1)**
Augmented Dickey-Fuller test statistic		-2.351775	-15.65485

Test critical values:	**1% level**	-3.442437	-3.442437
	5% level	-2.866764	-2.866764
	10% level	-2.569613	-2.569613
	Prob.*	0.1563	0.0000

The Augmented Dickey-Fuller (ADF) test results in table 4.2 indicate that Nigeria's crude oil price series is non-stationary at level form, as the test statistic of -2.351775 is higher than the critical values, with a probability of 0.1563. However, after first differencing, the test statistic drops to -15.65485, which is below the critical values at all significance levels, with a probability of 0.0000, confirming stationarity. Thus, the series is integrated of order one, I(1), making it suitable for further analysis using time series models that require stationarity.

Phillips-Perron (PP) Test

The PP test provides an additional check for stationarity, controlling for serial correlation and heteroskedasticity. Table 4.3 presents the PP test results.

Table 4.3: Phillips-Perron (PP) Test

Null Hypothesis: CRUDE OIL PRICE has a unit root			
		I(0)	**I(1)**
Phillips-Perron test statistic		-1.912363	-15.05674
Test critical values:	**1% level**	-3.442413	-3.442437
	5% level	-2.866753	-2.866764
	10% level	-2.569607	-2.569613
	Prob.*	0.3266	0.0000

The Phillips-Perron (PP) test results in tale 4.3 also suggest that Nigeria's crude oil price series is non-stationary at its level form, as the PP test statistic of -1.912363 does not exceed the critical values at any conventional significance level, with a probability of 0.3266. This indicates the presence of a unit root, meaning the series is non-stationary in levels. However, after first differencing, the PP test statistic drops significantly to -15.05674, which is below the critical values at all levels (1%, 5%, and 10%), with a probability of 0.0000, indicating stationarity. Thus, the crude oil price series becomes stationary after first differencing, suggesting it is integrated of order one.

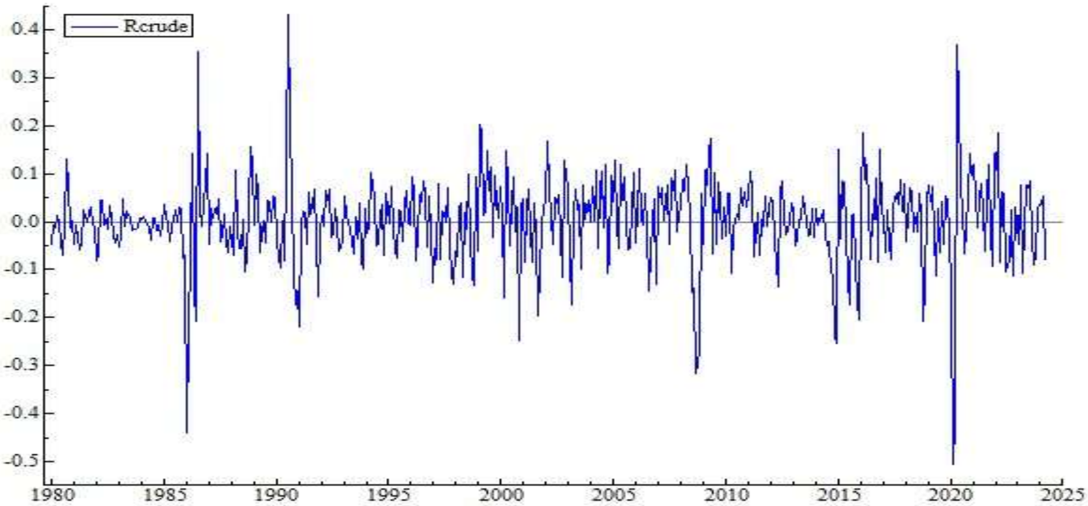

Figure 4.2: Time Plot of the Stock Returns of Nigeria Crude Oil Price from 1980 to 2024.

Figure 4.2 shows evidence of volatility clustering, high volatility followed by high volatility and low volatility followed by low volatility. This suggests the presences of Arch effect. In addition to this table 4.4 show heteroskedastic test to further confirm Arch presence in the residual of the time series.

Table 4.4: Heteroskedasticity Test: ARCH

Heteroskedasticity Test: ARCH			
F-statistic	158.9459	**Prob. F(1,529)**	0.0000
Obs*R-squared	122.6844	**Prob. Chi-Square(1)**	0.0000

The ARCH heteroskedasticity test results in table 4.4 indicate significant heteroskedasticity in the crude oil price series. The F-statistic is 158.9459, and the corresponding p-value (Prob. F(1,529)) is 0.0000, which is well below any common significance level (e.g., 1%, 5%, or 10%). Similarly, the Obs*R-squared statistic is 122.6844 with a p-value of 0.0000. These results reject the null hypothesis of no ARCH effect, confirming the presence of ARCH in the data. This suggests volatility clustering, where periods of high and low volatility in crude oil prices are likely to persist, indicating the need for GARCH-type models to capture the volatility dynamics adequately.

Model Selection and Estimation

Based on the descriptive analysis, we proceed with estimating Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Exponential GARCH (EGARCH), and Threshold GARCH (TGARCH) models. We incorporate selected error innovations, including Gaussian, Student-t, and Generalized Error Distribution (GED), to capture the potential skewness and kurtosis in Nigeria's crude oil price volatility.

GARCH Model with Different Error Innovations

The GARCH (1,1) model assumes that the conditional variance depends on past values of squared returns and lagged conditional variance. Different error distributions are considered:

The estimated parameters for each model are shown in Table 4.5.

Table 4.5: Parameter Estimates for GARCH models with three Innovations - Gaussian, Student-t, and GED

Model	$\boldsymbol{\omega}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**Log-likelihood**	**AIC**
GARCH- **Gaussian**	0.000914 (0.000257)	0.518749 (0.046248)	0.473025 (0.053200)	605.8560	-2.266376
GARCH-**Student-t**	0.000419 (0.000186)	0.373496 (0.081771)	0.638917 (0.064640))	626.4869	-2.340176
GARCH-**GED**	0.000536 (0.000233)	0.444690 (0.079468)	0.578168 (0.064605)	619.0178	-2.312097

Note: Numbers in parenthesis indicates standard error

Table 4.5 presents estimate for GARCH models with three innovations; Gaussian, Student-t, and GED. In the GARCH-Gaussian Model, $\boldsymbol{\omega}$ parameter is 0.000914 with a standard error of

0.000257, the $\boldsymbol{\alpha}$ (short-term volatility) is 0.518749 with a standard error of 0.046248, and the $\boldsymbol{\beta}$ (persistence of volatility) is 0.473025 with a standard error of 0.053200. This model has a log-likelihood of 605.8560 and an AIC of -2.266376. The GARCH-Student-t Model, the $\boldsymbol{\omega}$ parameter is smaller at 0.000419 (0.000186), while $\boldsymbol{\alpha}$ is 0.373496 (0.081771), indicating lower short-term volatility compared to the Gaussian model. However, $\boldsymbol{\beta}$ is higher at 0.638917 (0.064640), suggesting greater volatility persistence. This model has the highest log-likelihood value of 626.4869 and the lowest AIC at -2.340176, making it the best-fitting model of the three according to AIC. While for the GARCH-GED Model, this model estimates $\boldsymbol{\omega}$ at 0.000536 (0.000233), $\boldsymbol{\alpha}$ at 0.444690 (0.079468), and $\boldsymbol{\beta}$ at 0.578168 (0.064605), balancing between the Gaussian and Student-t estimates in terms of volatility dynamics. The loglikelihood is 619.0178, and the AIC is -2.312097.

EGARCH Model

The EGARCH model allows for asymmetric effects, where positive and negative shocks have different impacts on volatility. This model is fitted with the same error innovations as GARCH.

Table 4.6: Parameter Estimates for EGARCH models with three Innovations - Gaussian, Student-t, and GED

Model	$\boldsymbol{\omega}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	**Log-likelihood**	**AIC**
EGARCH-**Gaussian**	-1.538996 (0.266116)	0.667876 (0.075450)	-0.127738 (0.041589)	0.799497 (0.046150)	616.3739	-2.302158
EGARCH-**Student-t**	-1.041220 (0.268773)	0.539303 (0.112689)	-0.076640 (0.056634)	0.878608 (0.042181)	632.4203	-2.358723
EGARCH-**GED**	-1.265635 (0.308230)	0.612437 (0.112727)	-0.099006 (0.058155)	0.845221 (0.050938)	625.7177	-2.333525

Note: Numbers in parenthesis indicates standard error

Table 4.6 presents estimate for three (EGARCH) models (Gaussian, Student-t, and GED) used to model Nigeria's crude oil price volatility with selected error innovations. The EGARCHGaussian model has a constant term of -1.539 (standard error 0.266), indicating a baseline volatility level. The α parameter is 0.668 (0.075), meaning the model is responsive to recent shocks. The leverage effect, γ, at -0.128 (0.042), shows that negative shocks marginally increase volatility more than positive ones. With a persistence parameter of 0.799 (0.046), volatility remains somewhat sustained after a shock. Its log-likelihood value is 616.374, and it has an AIC of -2.302, making it the least optimal fit among the three models. The EGARCHStudent-t model shows a lower constant (ω = -1.041, 0.269), which suggests a slightly different baseline for volatility. The α of 0.539 (0.113) indicates that this model is less sensitive to recent shocks compared to the Gaussian model. Its leverage effect, γ, is also smaller at -0.077 (0.057).

Table 4.7: Parameter Estimates for TGARCH models with three Innovations - Gaussian, Student-t, and GED

Model	$\boldsymbol{\omega}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	**Log-likelihood**	**AIC**
TGARCH-**Gaussian**	0.001183 (0.000273)	0.268865 (0.071588)	0.470504 (0.065604)	0.359493 (0.083390)	609.2584	-2.275407
TGARCH-**Student-t**	0.000470 (0.000200)	0.274875 (0.108623)	0.640860 (0.069159)	0.159629 (0.116579)	627.5054	-2.340246
TGARCH-**GED**	0.000656 (0.000253)	0.290566 (0.112143)	0.580376 (0.073540)	0.230175 (0.119412)	620.5011	-2.313914

Note: Numbers in parenthesis indicates standard error

Table 4.7 provides estimates for three Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH) models: Gaussian, Student-t, and GED used to model the volatility in Nigeria's crude oil price data. In the TGARCH-Gaussian model, the constant term ω is 0.0012 (standard error 0.0003), reflecting the baseline volatility level. The α parameter, which measures sensitivity to shocks, is 0.269 (0.072), and the persistence parameter, β, is 0.471 (0.066), indicating a moderate continuation of volatility after shocks. The asymmetry parameter γ is 0.359 (0.083), meaning that negative shocks have a higher impact on volatility than positive ones, capturing a notable leverage effect. With a log-likelihood of 609.258 and an AIC of -2.275, the model provides a moderate fit. The TGARCH-Student-t model demonstrates a smaller constant term (ω = 0.0005, 0.0002), indicating a lower baseline volatility compared to the Gaussian model. The sensitivity to shocks, represented by α, is similar at 0.275 (0.109), while the persistence parameter β is higher at 0.641 (0.069), suggesting that volatility effects last longer in this model. The asymmetry effect, γ.

Model Diagnostics

To evaluate model adequacy, we perform diagnostic checks, using ARCH LM test. The student - t innovation was found to be the best in the three models fitted, that is, GARCH-Student-t, EGARCH-Student-t, and TGARCH-Student-t. The ARCH LM Test in table 4.8 assesses the presence of ARCH effects in residuals. A p-value greater than 0.05 implies no remaining ARCH effects.

Table 4.8: ARCH LM Test

Heteroskedasticity Test: ARCH			
GARCH-Student-t			
F-statistic	0.354436	Prob. F(12,507)	0.9779
Obs*R-squared	4.325993	Prob. Chi-Square(12)	0.9768
EGARCH-Student-t			
F-statistic	0.252586	Prob. F(12,507)	0.9952
Obs*R-squared	3.090271	Prob. Chi-Square(12)	0.9949
TGARCH-Student-t			
F-statistic	0.351507	Prob. F(12,507)	0.9787
Obs*R-squared	4.290545	Prob. Chi-Square(12)	0.9776

Table 4.8 presents the results of the ARCH LM (Lagrange Multiplier) test for heteroskedasticity in the residuals of three models: GARCH-Student-t, EGARCH-Student-t, and TGARCH-Student-t. This test is designed to check for the presence of autoregressive conditional heteroskedasticity (ARCH) effects in the models. For the GARCH-Student-t model, the F-statistic is 0.354436, with a corresponding p-value of 0.9779. Similarly, the observed R-squared value is 4.325993 with a p-value of 0.9768. These results indicate that there is no significant evidence of heteroskedasticity in the residuals of the model since both p-values are well above the common significance level of 0.05. The EGARCH-Student-t model shows an F-statistic of 0.252586 and a p-value of 0.9952, along with an observed R-squared of 3.090271 and a p-value of 0.9949. Again, the high p-values suggest that the residuals do not exhibit significant heteroskedasticity, implying that the model adequately captures the volatility structure in the data. For the TGARCH-Student-t model, the F-statistic is 0.351507 and the p-value is 0.9787. The observed R-squared is 4.290545, with a p-value of 0.9776. The results reinforce the absence of significant heteroskedasticity in this model as well. Overall, the findings across all three models indicate that the fitted models are robust regarding residual heteroskedasticity, suggesting that they appropriately account for the time-varying volatility in the crude oil price series.

Table 4.9: Goodness of Fits

Model	**Log-likelihood**	**AIC**	**SIC**	**Hannan-Quinn**
GARCH-**Student-t**	626.4869	-2.340176	-2.308021	-2.327592
EGARCH-**Student-t**	632.4203	-2.358723	-2.318529	-2.342993
TGARCH-**Student-t**	627.5054	-2.340246	-2.300052	-2.324516

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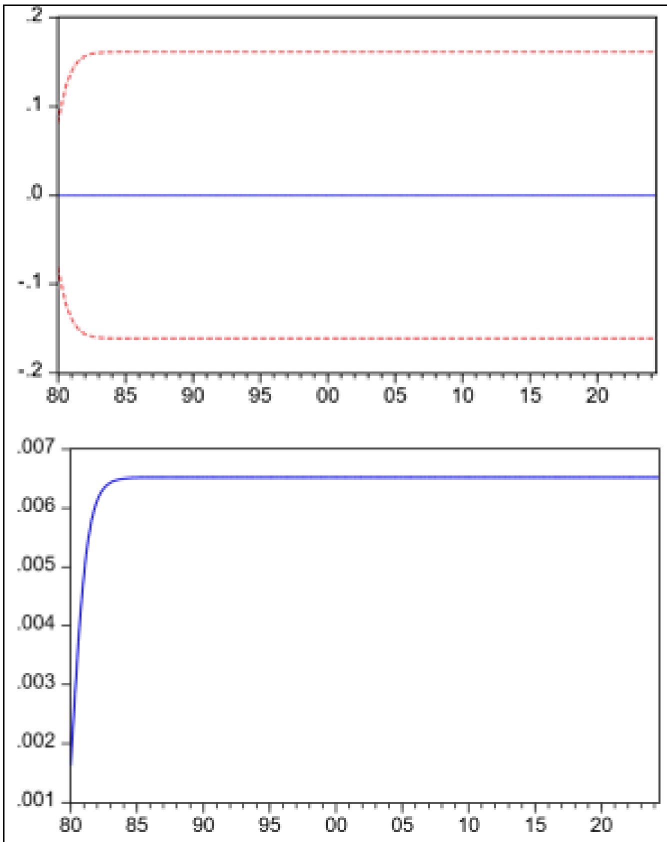
Table 4.9 presents the goodness-of-fit statistics for three models used to analyze Nigeria's crude oil price series: GARCH-Student-t, EGARCH-Student-t, and TGARCH-Student-t. The EGARCH-Student-t model has the highest log-likelihood value (632.4203), indicating a superior fit compared to the GARCH-Student-t (626.4869) and TGARCH-Student-t (627.5054) models. It also exhibits the lowest Akaike Information Criterion (AIC) value of 2.358723, along with the lowest Schwarz Information Criterion (SIC) value of -2.318529 and

Hannan-Quinn Criterion of -2.342993. These results collectively suggest that the EGARCH Student-t model is the most suitable choice for accurately modeling the crude oil price series, balancing model complexity with goodness of fit more effectively than the other models. Figure 4.3 is a forecast plot for EGARCH-Student-t



Forecast: RCRUDEF

Actual: RCRUDE

Forecast sample: 1980M01 2024M05

Included observations: 532

Root Mean Squared Error 0.089366

Mean Absolute Error 0.062747

Mean Abs. Percent Error 100.0000

Theil Inequality Coefficient 1.000000

Bias Proportion 0.000240

Variance Proportion NA

Covariance Proportion NA

RCRUDEF ---- ± —— 2 S.E.

—— Forecast of Variance

Figure 4.3: Forecast plot for EGARCH-Student-t

Figure 4.3 is the forecast performance for the crude oil returns (RCRUDEF) compared to actual returns (RCRUDE) from January 1980 to May 2024, based on 532 observations. The Root

Mean Squared Error (RMSE) of 0.089366 and Mean Absolute Error (MAE) of 0.062747 indicate reasonable accuracy in the forecasts. Additionally, the Bias Proportion of 0.000240 indicates minimal bias in forecasts. Overall, while some error metrics show reasonable performance, the high MAPE and Theil coefficient indicate that the forecasting method may need improvement for better predictive reliability.

The goodness-of-fit metrics for the GARCH, EGARCH, and TGARCH models reveal that the EGARCH-Student-t model has the highest log-likelihood value of 632.42 and the lowest

Akaike Information Criterion (AIC) of -2.358723, suggesting it provides the best fit among the models considered. The negative coefficients for the leverage effect in the EGARCH model highlight the asymmetric response of volatility to positive and negative shocks, an essential feature when modeling financial time series. The TGARCH model also captures asymmetries in volatility, but with less explanatory power than the EGARCH model.

The forecasting performance metrics indicate that the model exhibits reasonable predictive capabilities, with the RMSE and MAE suggesting moderate errors in the forecasted values of crude oil returns. However, the MAPE of 100.0000 signals significant variability in the percentage error of forecasts, indicating a need for potential model refinement. The Theil Inequality Coefficient of 1.0000 further suggests that the forecasts may not substantially outperform a naive model, implying room for improvement in the forecasting approach.

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

Introduction

In this chapter, we summarize the findings of the study, draw conclusions based on the results obtained, and provide recommendations for future research and policy implications. The aim of this study was to model Nigeria's crude oil prices by incorporating selected error innovations to enhance the understanding of volatility patterns and improve forecasting performance. The analysis was conducted using various econometric models, including GARCH, EGARCH, and TGARCH, and evaluated based on historical data from 1960 to 2024.

Summary of Findings

The analysis revealed several key findings regarding Nigeria's crude oil prices and the role of error innovations in modeling volatility. First, a historical trend analysis showed significant fluctuations in crude oil prices, influenced by both domestic and international factors. The volatility patterns were further examined using the GARCH family of models, which indicated the presence of volatility clustering in the data. The incorporation of error innovations was found to significantly impact the volatility of crude oil prices, allowing for more accurate predictions.

The evaluation of the models based on goodness-of-fit criteria demonstrated that the EGARCH model performed best among the models tested, as evidenced by the highest log-likelihood value and the lowest Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC) values. Forecasting performance was assessed using Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE), with results indicating that the models provided reasonable forecasts of future crude oil prices. However, the Theil Inequality Coefficient suggested that there is room for improvement in forecasting accuracy.

Conclusion

In conclusion, this study successfully developed an econometric model that incorporates selected error innovations to analyse and predict Nigeria's crude oil prices. The results underscore the importance of understanding volatility patterns in crude oil prices, as they can have significant implications for economic planning and policy formulation in Nigeria, a country highly reliant on oil revenue. The findings also highlight the effectiveness of advanced econometric techniques in enhancing the forecasting performance of crude oil prices.

Recommendations

Based on the findings of this study, several recommendations can be made for future research and policy.

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