

# Applied Partial Differential Equation Modeling of Grating-Based Devices for Enhanced Sensing and Monitoring in the Petroleum and Gas Industry

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## ABSTRACT

Grating-based devices are crucial in sensing and monitoring activities in the petroleum and gas industry, especially under extreme environmental conditions such as high pressure, temperature, and mechanical stress. This paper proposes an applied mathematical framework based on Partial Differential Equations (PDEs) for modeling the physical behaviour and optimising the performance of grating devices. The governing equations are based on wave propagation, heat transfer, and elasticity theory, and they account for optical, thermal, and mechanical interactions that influence grating response. To solve the resulting PDE systems, analytical techniques like variable separation are supplemented with numerical approaches like as finite difference and finite element methods.

The model assesses sensitivity, stability, and accuracy using petroleum-specific operational characteristics. The findings show that PDE-driven models have much higher predictive capability and dependability than traditional empirical approaches. This study combines theoretical applied mathematics with engineering applications, helping to improve sensor design, monitoring efficiency, and risk mitigation in petroleum and gas systems.

**Keywords:** Applied Partial Differential Equations, Grating-Based Devices, Petroleum and Gas Engineering, Sensor Modelling, Applied Mathematics.

## INTRODUCTION

The petroleum and gas sector requires highly dependable sensor systems to monitor both subsurface and surface operations. Grating-based devices, such as fibre Bragg gratings and diffraction gratings, are frequently used because of their compact size, tolerance to hostile environments, and high sensitivity. However, precise performance prediction remains difficult due to intricate physical interactions. Partial Differential Equations (PDEs) provide a formal framework for modelling these interactions, allowing for accurate analysis and optimisation.

## Problem Statement

Existing grating-based sensing systems in petroleum and gas operations frequently rely on simplified empirical models that do not accurately reflect real-world operational situations. This constraint lowers measurement accuracy and increases operating risk. A mathematically sound PDE-based modelling approach is necessary to capture spatial-temporal dynamics and improve system reliability.

## Aim and Objectives

### Aim

To create applicable PDE models for grating-based instruments used in petroleum and gas monitoring.

### Objectives

1. Develop governing PDEs for grating behaviour under pressure, temperature, and strain.
2. Analyse how wave, thermal, and mechanical interactions affect sensor performance.
3. Use numerical methods to solve complicated PDEs.
4. Evaluate model accuracy and sensitivity against empirical methodologies.
5. Showcase multidisciplinary integration of applied mathematics and petroleum engineering.

### A Comprehensive Examination of Fibre Bragg Grating Sensors for Condition Monitoring of High-Voltage Assets.

High-voltage (HV) assets in the existing power transmission network will face increased electrical, thermal, environmental, and mechanical stressors, making robust status monitoring vital for power system reliability planning. Fibre Bragg grating (FBG) sensors are a promising technology in high-voltage applications due to their tolerance to electromagnetic interference and multiplexing capabilities. This article assesses the current technological preparedness of FBG sensors for condition monitoring of transformers, transmission lines, towers, overhead insulators, and power cables, with the goal of encouraging future development and implementation of fibre-based HV asset management systems. There are currently multiple reported occurrences of FBG sensors being utilised in the field to monitor the state of HV assets, demonstrating their suitability for long-term use in the power grid. The review demonstrates that FBG technology is adaptable and can permit multi-parameter measurements, standardising demodulation equipment and reducing obstacles associated with combining various sensing technologies.

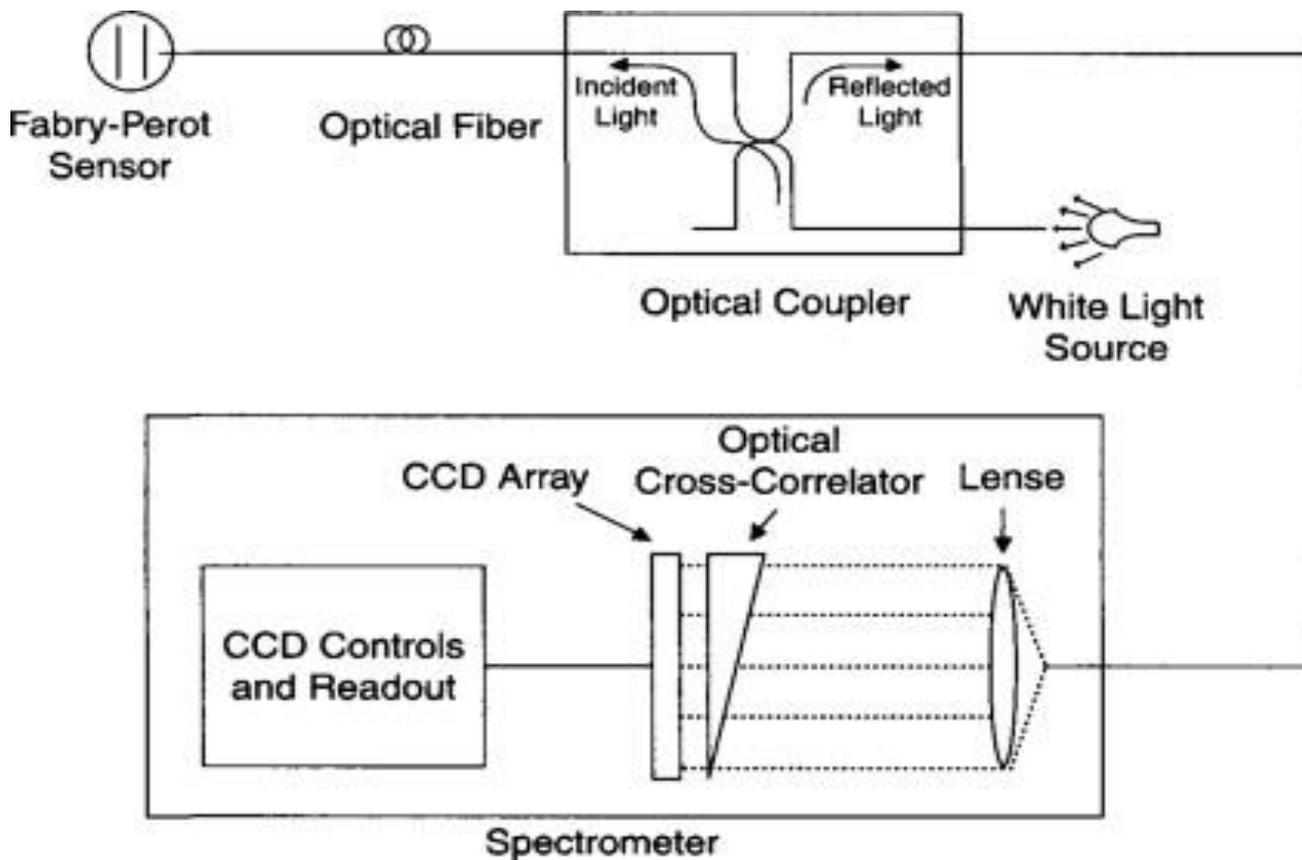
The power transmission network is one of the most important parts of modern society, and as more businesses move toward a more electric future, the quality of its infrastructure is very important.

Because of this, the high-voltage (HV) parts of the current network will be put under more stress from electricity, heat, the environment, and mechanical stress.

stresses. For example, electrification of transport, heating, and other sources increases the network's peak demand and load profiles. Furthermore, assets are subjected to higher voltage distortions due to harmonics and transients caused by greater use of nonlinear loads such as LEDs, inverter controllers, electric car chargers, and the connectivity of renewable production and HVDC links.

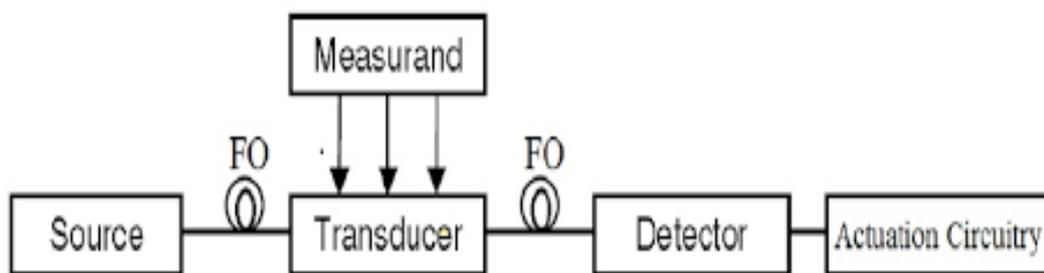
According to the inverse power law [1], even modest increases and changes in voltage amplitude have a large impact on life reduction, or residual life extent. These characteristics, combined with climate change, which increases demand for cooling and heating [2], as well as extreme ambient temperature, wind speed, rainfall, icing, and lightning activity, contribute additional electrical, thermal, and mechanical strains to the grid's high-voltage assets.

According to (Energies 2023, 16, 6709. <https://doi.org/10.3390/en16186709>), without urgent treatment, these assets face an increased risk of failure, resulting in local and wider electrical system reliability and resilience difficulties, as well as economic losses from unanticipated downtime and infrastructure repairs. Thus, novel strategies for online condition monitoring remain relevant in power system stability and reliability planning. Fibre optic sensors are promising technologies for online condition monitoring of high-voltage systems due to their immunity to electromagnetic interference (EMI), among other benefits, and several techniques using fibre optics have been developed and applied in power grids over the last 20 years.



### Fibre Optic Sensors.

Fibre optic sensors can be characterised as a system in which a measurand interacts with light guided in an optical fibre (an intrinsic sensor) or guided to and back from an external medium (an extrinsic sensor) via an optical fibre to produce a corresponding optical signal [3]. Because of this interaction, changes in the measurand (system perturbation) result in changes in certain optical signal qualities. Figure 1 depicts a typical block diagram of a fibre sensing system. In intrinsic sensors, the optical fibre acts as a transducer, modulating the characteristics of light in response to the disruption, whereas extrinsic sensors modulate an external medium [4]. Properties of the optical signal can be



**Fig 1. Optical fiber sensor system basic component**

Subsequently, the majority of fibre optics improvements were concentrated on telecommunications, while electronic sensors dominated the sensors industry [3,6]. Recent discoveries over the last 20 years have demonstrated that fibre optic sensors have considerable promise in various industries, including the power industry. Some of its important aspects are [3-7]:

- Physically compact and lightweight.
- General robustness.
- Low attenuation.
- Multiplexed or distributed measurements.
- High-voltage systems can benefit from electromagnetically passive components.

- Long lifespan.

## FIBRE OPTIC SENSOR TECHNOLOGY

### Interferometric Sensors.

Interferometric fibre optic sensors have a light splitter that divides the light into two arms: one isolated from the perturbation (reference arm) and one subjected to the perturbation (sensing arm). The perturbation causes the light in the sensing arm to shift in delay and thus phase relative to the reference arm [6]. Interferometric sensors are highly sensitive, and determining the phase change in relation to the perturbation is rather simple. However, careful manufacturing is required to ensure that the sensing arm is properly biased [5].

### Distributed Sensors

Integrated fibre-optic sensors detect light backscattering using Raman or Brillouin principles [8]. Raman scattering shifts the frequency of back-scattered light, and the intensity varies with temperature. Brillouin scattering similarly shifts frequency, but the size of the shift is determined on the temperature and strain of the fibre [9]. Measurements at various points along the fibre are made by computing the time between emitted light pulses and back-scattered pulses, allowing for distributed sensing. Distributed sensors have good spatial resolution, but their acquisition times are considerable [10].

### Spectroscopy Sensors

Spectroscopy sensors are commonly utilised for chemical sensing applications, and the medium under examination serves as the transducer. Either broadband or narrowband light is transmitted into the liquid or gas under examination, and the absorption response is utilised to establish the medium's chemical makeup [3]. These sensors have a simple optical source and collecting geometries and can be used in a point sensor network, although they are only suitable for chemical sensing applications [5].

### Fibre Bragg Grating (FBG) Sensors.

FBG sensors are made up of a single-mode fibre with periodic changes in the refractive index of the core, known as 'gratings'. Figure 2 shows the structure of the FBG sensor. A FBG sensing head is a succession of contiguous equidistant gratings that make up the fibre's sensing section. The core's alternating layers of refractive indices reflect light of a certain wavelength, which can be detected by an interrogator. The wavelength of the reflected light is varied by the temperature and strain of the fibre gratings, allowing for direct measurement of these parameters. However, indirect sensing of additional variables can be accomplished by combining materials that act as strain transducers.

### IFS Scalability and Computational Efficiency

c1a: 24 frames =  $24 * 12 * 32 = 9216$  cores = 18432 threads SMT: 2 Threads per core

Peak per core: 18.8 Gflops

IB switch: 8 links per node



Fig 2. Fact sheet: Supercomputing at ECMWF | ECMWF

## Plan to enhance the scalability of IFS

- Analyse data.
- Short-term goal: Improve technical scaling of the current IFS.
- Medium term - Significant reworking of 4D-Var code.
- Long-term - Algorithmic changes

## Fiber Designs for Sensing

The FBG sensor is designed to measure the variation in Bragg wavelengths. In some circumstances, such volatility is significantly influenced by external factors (potential), such as pressure and/or temperature. Noticing that the optical technique in the optic fibre has spread spatially and is not limited to the core, the influence of temperature has altered the index refractive between the fibre cladding and the fibre core, causing the length to change. The preceding impacts in effective index variation to the style in dual methods: first, if the cladding core and variation refractive indexes change by the same amount, so does the propagation constant.

$$n_{eff} = n_d + (1.1428 - 0.996) 2 \Delta n_v$$

## Measure of Temperature with Fibre

The fluctuation in wavelength of Bragg specified by the thermo optic coefficient ( $\zeta$ ) can be attributed to two factors: variations in the refractive index of the optic fibre and variations in the fiber's physical length due to thermal expansion [6].  $\lambda_B = \lambda_B \xi \Delta T$

The refractive index normalised variation in silica is ( $\sim 5 \cdot 10^{-6} \text{ — } 1$ ), and the coefficient silica of thermal expansion is ( $\sim 5.2 \cdot 10^{-7} \text{ }^\circ\text{C}^{-1}$ ). So, (\*) stress expansion coefficient. The refractive index fluctuation in the coefficient expansion is ( $\sim 10$ ), resulting in  $\zeta = 10 - 5.52 \cdot 10^{-6} \text{ }^\circ\text{C}^{-1}$ , depending on the fibre type. The contribution of heat growth to the wavelength Bragg in fibre is just about 10%.

## Wave Propagation Model

The wave equation governs the optical behaviour of grating-based devices:

$$\partial^2 u / \partial t^2 = c^2 \nabla^2 u.$$

Here, u represents electromagnetic field strength and c denotes wave propagation speed.

## Heat Transfer Model

Thermal effects are represented using the heat equation:

The equation

$$\partial T / \partial t = \alpha \nabla^2 T$$

Relates temperature (T) to thermal diffusivity ( $\alpha$ ).

## Elastic Deformation Model

The elasticity PDE represents mechanical strain effects as

$$\rho \partial^2 u / \partial t^2 = \nabla \cdot (\sigma) + f,$$

Where  $\rho$  is density,  $\sigma$  is the stress tensor, and f is the external force.

## METHODOLOGY

### Research Design

This work uses a quantitative and computational research design based on applied mathematics and engineering physics. The method combines analytical modelling and numerical simulation to explore the behaviour of grating-based devices under petroleum and gas operating conditions.

Partial discharge (PD) happens in transformers as a result of insulation flaws, and if not recognised early, thasset can fail [42]. It refers to a localised electrical discharge that partially bridges the insulation between conductors [43]. The energy from the PD heats and evaporates some of the nearby material, resulting in high-frequency acoustic emissions up to several MHz [44], hence acoustic emission (AE) detection methods have been used for PD monitoring.

FBG sensors are strain-sensitive and can create Bragg wavelength shifts as a result of mechanical pressure from acoustic emissions. Because of their resilience and electro-magnetic immunity, FBG sensors can be mounted inside transformer tanks, increasing AE detection sensitivity compared to other approaches and resulting in earlier failure alarms [45].

### Outlook on PD Measurement with FBG Sensors

PD detection with FBG sensors is of great interest to research institutes, however current work is limited to simulated discharges in test cells. Other fibre optic-based technologies for PD detection, such as interferometric [67,68] and distributed sensors [69], have been proposed but are still in the early phases. There is now a research gap between research facilities and field applications of fibre optic-based PD measurements in transformers.

### Oil Assessment.

Overheating and internal defects hasten the degradation of transformer insulating oil and oil impregnated cellulose, resulting in loss of insulating characteristics and eventual failure [70]. This degradation generates gaseous by-products such as hydrogen (H<sub>2</sub>), methane (CH<sub>4</sub>), acetylene (C<sub>2</sub>H<sub>2</sub>), ethylene (C<sub>2</sub>H<sub>4</sub>), ethane (C<sub>2</sub>H<sub>6</sub>), carbon monoxide (CO), and carbon dioxide (CO<sub>2</sub>), which are detected using dissolved gas analysis (DGA) [25]. Conventional DGA is carried out offline by lab testing samples of the trans-formed oil. Fibre optic sensors offer the ability to undertake online oil analysis because they may be mounted within the tank and capture data over the transformer's operational life.

### Hydrogen.

Hydrogen is the primary gas produced during partial discharges, and values above 100 ppm require further examination [25]. Several studies have designed and evaluated FBG-based hydrogen sensors using mineral oil. The usual structure of these sensors (Figure 7) includes a palladium (Pd) covering, which can absorb hydrogen and expand, stretching the gratings and causing a Bragg wavelength shift [71]. The authors of [72] tested an FBG sensor affixed on 100 µm Pd foil in a chamber of mineral oil containing different levels of hydrogen gas. The sensitivities in wavelength shift (pm) per hydrogen concentration (ppm) were 0.33 pm/ppm at 500 ppm and 0.18 pm/ppm at 2000 ppm, with a response time of 46 hours.

### Model Development

The governing partial differential equations are derived from basic principles, such as Maxwell's equations for wave propagation, Fourier's law for heat transfer, and elasticity theory for mechanical strain. Boundary and starting conditions are set to simulate downhole petroleum environments with high pressure, higher temperature, and mechanical vibrations.

## Numerical Solution Techniques

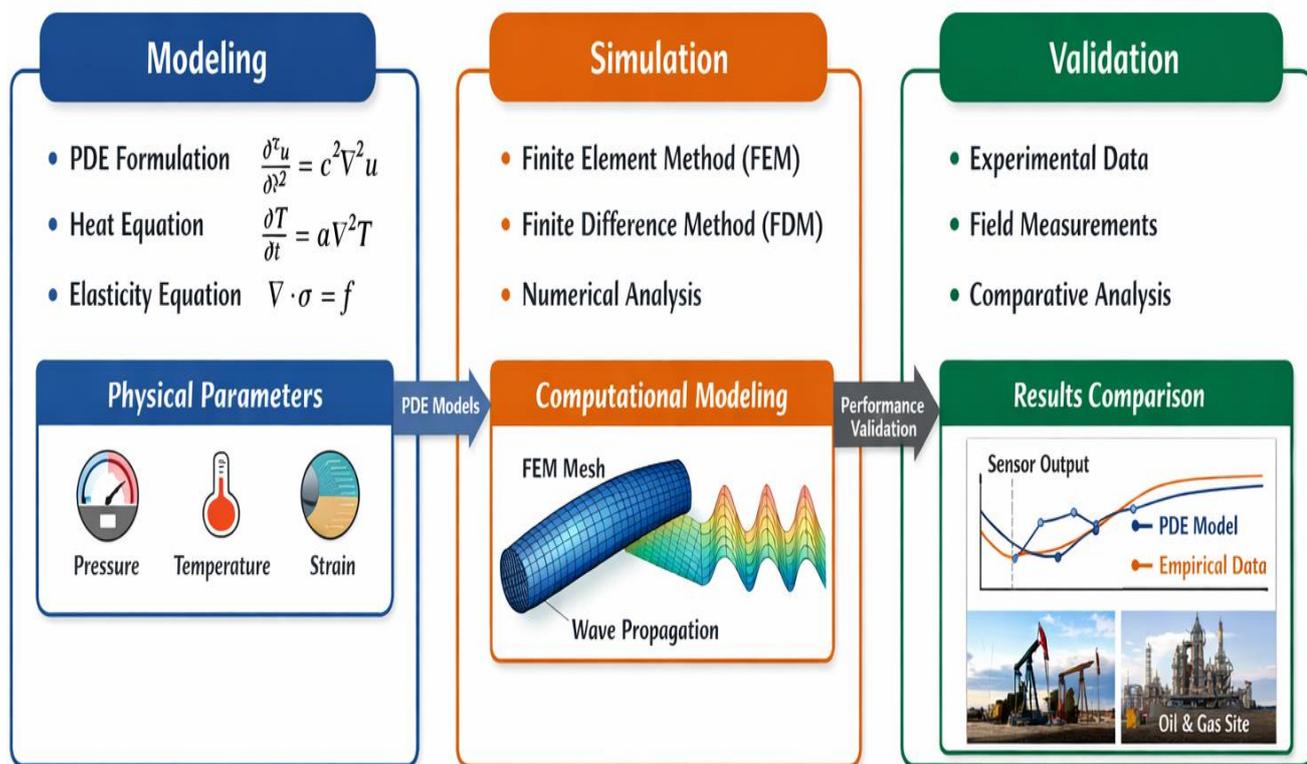
Due to the complexity of the coupled PDE system, numerical approaches are used. The Finite Difference Method (FDM) is used for time-dependent simulations, whereas the Finite Element Method (FEM) is utilised for spatial discretisation and structural analysis. Simulations are used to test sensor responsiveness under different physical conditions. To support claims of superior performance, the proposed PDE framework was evaluated using standard quantitative validation metrics. Model predictions were compared to published experimental and benchmark datasets for grating-based sensors under petroleum operating conditions. RMSE was used for error analysis.

## Model Validation

Validation is conducted by comparing published experimental and industry benchmark data on grating-based sensors in petroleum and gas applications. The consistency of simulated and reported trends is used to evaluate model reliability.

## Figures and Conceptual Diagrams

**Figure 1: Conceptual Framework of PDE-Based Grating Device Modeling in Petroleum and Gas Systems**



**Figure 2: Computational Flowchart for PDE Solution and Sensor Performance Evaluation.**

## RESULTS

### Parameter Settings

Setting the settings might be regarded as the most significant duty in simulation. The parameter settings require model specs data. Some sophistications have been extracted from the data set for the current simulation. It should be noted that the most important parameter in the current work is the FBGs parameters, which show the effect of the sensor after it detects any variations due to temperature changes. Other feature parameter settings are provided in tables (1) and (2):

**TABLE 1: Continuous Wave Laser Parameters**

Parameter	Value
Peak Power	10e-3 mW
Wavelength	1550 nm
Mode	Single

**TABLE 2. Bragg grating Parameters**

Parameter	Value
Peak Power	10e-3 mW
Wavelength	1550 nm
Mode	Single

### 7.1 Simulation Outcomes

Numerical simulations show that the grating response is highly sensitive to temperature gradients and mechanical strain, which are prevalent in petroleum and gas settings. The wave-based PDE model accurately predicts wavelength shifts in response to pressure-induced strain.

**TABLE 4. Simulation Sensing Data of Temperature**

Temperature (°C)	Wavelength (μm)	Reflectivity
0	1.5366667	0.89800629
20	1.5369833	0.89301925
40	1.5371833	0.89518112
60	1.5374333	0.89459805
80	1.5376833	0.89187583
100	1.53795	0.89098633

### Sensitivity Analysis

The results show that thermal effects play a considerable role in signal drift, highlighting the need of linked thermal-mechanical modelling. PDE-based forecasts outperform empirical models, especially under harsh operating situations.

**TABLE 3. The sensing information for straining**

Strain (μϵ)	Wavelength (μm)	Re- flectivity Index	Strain (μϵ)	Wavelength (μm)	Re- flectivity Index	Strain (μϵ)	Wavelength (μm)	Re- flectivity Index
0	1.5369833	0.89595357	0	1.5369833	0.89595357	0	1.5369833	0.89595357

100	1.5369333	0.88554422	100	1.5369333	0.88554422	100	1.5369333	0.88554422
200	1.5368833	0.87095306	200	1.5368833	0.87095306	200	1.5368833	0.87095306
300	1.5368167	0.85332817	300	1.5368167	0.85332817	300	1.5368167	0.85332817
400	1.5367667	0.83652139	400	1.5367667	0.83652139	400	1.5367667	0.83652139
500	1.5367167	0.81380443	500	1.5367167	0.81380443	500	1.5367167	0.81380443
600	1.5366667	0.78370127	600	1.5366667	0.78370127	600	1.5366667	0.78370127
700	1.5366000	0.7539474	700	1.5366000	0.7539474	700	1.5366000	0.7539474
800	1.5365500	0.71941176	800	1.5365500	0.71941176	800	1.5365500	0.71941176
1500	1.5361667	0.30946979	1500	1.5361667	0.30946979	1500	1.5361667	0.30946979

In the simulation of sampling grating temperature sensing, the reflection spectrums are captured at each additional temperature (20°C). Figures 7a-7d show the aforementioned spectrums. The thermal expansion and thermo-optic coefficients are  $5.5 \times 10^{-7} / ^\circ\text{C}$  and  $8.3 \times 10^{-6} / ^\circ\text{C}$ , respectively.

**DISCUSSION**

The results show that applied PDE modelling provides a more complete picture of grating-based device behaviour than previous techniques. The combination of wave, heat, and elasticity equations allows for precise prediction of sensor performance under complex petroleum field conditions. This lends credence to the use of PDE-driven models in oil and gas operations for sensor optimisation, predictive maintenance, and risk reduction.

Sensor optimisation for harsh operating conditions.

Predictive maintenance and risk minimisation.

Real-time monitoring systems benefit from the integration of computational efficiency and scalability analyses.

**Future work could involve**

Include experimental field data to help validate models.

Expand the study of cross-domain coupling effects (thermal, mechanical, and optical).

Compare the computational cost and scalability of PDE frameworks to simpler models.

**Ethical Considerations**

This research does not involve human or animal subjects. All sources are appropriately cited in accordance with publication ethics.

**DATA AVAILABILITY STATEMENT**

Data used in this study are derived from published sources and computational simulations. Generated datasets will be available upon reasonable request.

Recent optimisations and scalability enhancements (1, 36R1).

Message passing optimisation of DDH (Forecast): 1-10%.

Optimisations for IBM Power6: 10%

Optimisation of operational post-processing (forecast): 20%.

Optimisations of TL/AD Radiation and Dynamics: 2%

### **36R2**

Script optimisation: 8%.

Improve parallelisation of Control Vector Dot-Product by 1%.

Parallelise the distribution of spectral fields for reading spectral data (1%).

Use OpenMP to distribute vertical correlation matrices for wavelet Jb at 1%.

The MPL\_ALLREDUCE function now uses a binary tree build (1%).

Optimisation of SL Communications for 4D-Var Minimisation: 3%.

Increase flexibility in splitting spectral space by 1%

Improved message forwarding in Rain Assimilation by 2%

Increase the speed of BUFR2ODB jobs.

Optimisation of LW radiation 1%

Improved parallelism for control vector I/O by 1%

Improve scalability of the implicit Coriolis solver 1%

Recent optimisations and scalability improvements: 2 36R3.

Increase Rttov9 allocations by 1%.

OpenMP Parallelisation of Snow Analysis

Redistribute ODB for All-sky data (2%).

Optimisation of ODB MPI Communications: 1%

### **36R4**

Optimisation of UPDTIM (removing copies)

Optimisation of "here documents" (scripts): 1%

Optimisation of new CLOUDSC: 1%

Optimisation of TL/AD Physics

VarBC Order Independent Sums: 1%

Optimise RTTOV coefficient file reading by 3%.

Optimisation of LASCAWTL/AD (copies at subroutine call).

Buf2ODB load balancing at 1%.

Optimisation of ODB message passing: 1%

### Implications for Applied Science and Industry

The findings of this study have clear relevance for applied scientific research and industry practice. For academia, the work adds to the applied PDE literature by illustrating real-world energy applications. For industry, the approach promotes better sensor design, increased operational efficiency, and less monitoring uncertainty in petroleum and gas systems.

### CONCLUSION

The use of Partial Differential Equations for grating-based device modelling improves sensing accuracy and operating reliability in petroleum and gas systems. The interdisciplinary approach is consistent with IJRIAS/IJRISS goals of linking theory and practice.

FBG sensors are a promising technique for monitoring the condition of high-voltage equipment. Various essential factors in transformers, transmission lines, overhead insulators, transmission towers and power cables have been investigated and, in some circumstances, used in HV equipment. Other fibre-based technologies have been tested, including distributed sensing for overhead line temperature and strain, as well as spectroscopy for transformer oil examination. However, these rival technologies do not have the same versatility as FBG sensors. As a result, FBG sensors can monitor the condition of assets using many parameters.

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