Bayesian Structural Credit Risk Model with Microstructure Noise in Nigeria

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ABSTRACT

Financial markets rely on asset prices, which are often distorted by market frictions, liquidity constraints, and transaction costs, all of which influence a country's structural credit risk. Traditional Markov Chain Monte Carlo (MCMC) estimation converges slowly and may not reliably capture rare, high-impact risks. To address this, the study develops a Bayesian structural credit risk model using Markov Chain Quasi-Monte Carlo (MCQMC) techniques, explicitly accounting for microstructure noise to improve the accuracy of asset value and default risk estimates in Nigeria. Comparative analysis shows that MCQMC achieves faster convergence, lower variance, and greater computational efficiency than MCMC, highlighting the benefits of noise-adjusted modeling for reliable credit risk assessment. The findings suggest that financial institutions should adopt MCQMC methods, while policymakers may consider incorporating noise-aware credit risk models into regulatory frameworks, offering a more robust and efficient approach to credit risk management in Nigerian financial practice.

Keywords: Financial market, Bayesian structural credit risk, Monte Carlo Quasi-Monte Carlo, Stochastic differential equations, Microstructure noise.

INTRODUCTION

Credit risk, defined as the possibility that a borrower or counterparty will fail to meet financial obligations, remains one of the most significant sources of financial instability for banks and investment institutions worldwide. Accurate estimation of default probabilities is therefore central to modern financial risk management. Among the various quantitative approaches developed for this purpose, structural credit risk models—originating from the seminal works of Black and Scholes (1973) and Merton (1974)—have become foundational in both academic research and industry practice. These models link default events to the evolution of a firm's asset value relative to its liabilities, typically assuming that default occurs when the firm's asset value falls below a specified debt threshold.

The Black–Scholes–Merton (BSM) framework models firm value as a stochastic diffusion process and derives default probabilities from the firm's "distance to default." Its theoretical appeal lies in its strong economic intuition and firm-level foundation. However, a central econometric challenge arises because firm asset values are not directly observable. Duan (1994) addressed this issue through a transformed data maximum likelihood (ML) approach, which employs a change-of-variable technique via the Jacobian and exploits the one-to-one mapping between observed equity prices and latent asset values. This approach has since been widely adopted (e.g., Wong and Choi, 2006; Ericsson and Reneby, 2004; Duan et al., 2003) and shown to be comparable to the commercial Moody's KMV model (Duan et al., 2004).

Despite their theoretical robustness, conventional structural models rest on the assumption of frictionless and informationally efficient markets. In practice, observed market prices are contaminated by microstructure noise arising from bid–ask spreads, price discreteness, transaction costs, thin trading, and informational asymmetries. The market microstructure literature has extensively documented how such frictions distort observed price dynamics (Roll, 1984; Hasbrouck, 1993). More recent contributions propose stationary (AïtSahalia et al., 2009; Hansen and Lunde, 2006), locally non-stationary (Phillips and Yu, 2006, 2007), and pure noise (Zhang et al., 2005; Bandi and Russell, 2008) frameworks to model these effects. A consistent conclusion across this literature is that ignoring microstructure noise leads to biased and inconsistent parameter estimates. Duan and Fulop (2009) extend this concern directly to structural credit risk estimation.

These limitations are particularly pronounced in emerging markets such as Nigeria, where market illiquidity, macroeconomic volatility, regulatory uncertainty, and limited transparency amplify price distortions. Deterministic structural credit risk models may therefore fail to adequately capture the complexities embedded in observed financial data within such environments (Yaya et al., 2019; Akanbi O. B., 2022; Akanbi and Bello, 2024). The resulting estimation errors can lead to mispricing of credit risk and suboptimal risk management decisions.

Bayesian structural credit risk modeling offers a promising alternative. By incorporating prior information and updating parameter beliefs through observed data, Bayesian methods explicitly account for parameter uncertainty and model ambiguity (Akanbi and Fawole, 2024; Akanbi and Omokhua, 2025; Lawal and Akanbi, 2024). This probabilistic framework is particularly valuable in noisy and data-constrained environments. Furthermore, advanced simulation-based estimation techniques such as Markov Chain Monte Carlo (MCMC) and Markov Chain Quasi-Monte Carlo (MCQMC) enhance computational efficiency and enable accurate estimation of complex posterior distributions (Akanbi et al., 2018; Tumala et al., 2018; Akanbi O. B., 2024b; Zhao et al., 2024). These methods are well-suited for jointly modeling latent asset processes and microstructure noise components.

Against this backdrop, this study investigates the integration of Bayesian estimation techniques with structural credit risk models, with particular emphasis on the role of microstructure noise in default prediction within the Nigerian financial market. By embedding market frictions and parameter uncertainty into the structural modeling framework, the study contributes to the literature in three ways. First, it extends traditional structural models to explicitly account for microstructure-induced distortions. Second, it adopts a Bayesian estimation framework supported by MCMC and MCQMC techniques to improve inference under latent variable settings. Third, it provides empirical insights tailored to an emerging market context, where credit risk assessment remains both challenging and critically important.

By offering a more realistic and computationally robust modeling framework, this research aims to enhance credit risk measurement and support more effective risk management strategies in dynamic and imperfect financial markets.

METHODOLOGY

Model Development

The Geometric Brownian Motion (GBM) framework is used to model the asset value of businesses. In order to replicate market defects like bid-ask spreads and transaction costs, microstructure noises are introduced. Model parameters are estimated using Bayesian inference.

Estimation Techniques

To estimate posterior distributions of model parameters, Monte Carlo Quasi-Monte Carlo (MCQMC) and Markov Chain Monte Carlo (MCMC) techniques are used. MCQMC uses Sobol sequences for faster convergence, whereas MCMC uses Metropolis-Hastings algorithms and Gibbs sampling.

Comparison of Techniques

Metrics including runtime, posterior variance, and speed of convergence are used to compare the accuracy and efficiency of the MCMC and MCQMC approaches.

Model Development

The firm's asset value is assumed to follow a Geometric Brownian Motion (GBM):

$$dV_t = \mu V_t d_t + \sigma V_t dw_t \qquad (1)$$

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Where:

- V_t: true asset value at time t
- μ: Drift (expected return rate)
- σ: volatility
- dw_t: Standard Brownian Motion

The observed prices P_t are noisy observations of the true asset value V_t, modeled as:

$$P_t = V_t + \varepsilon_t \ , \quad \varepsilon_t \sim N(0, \tau^2) \qquad (2)$$

Where τ^2 is the noise variance

Bayesian Framework

Likelihood function

The likelihood of the observed price $\{P_t\}$ is

$$P(P_t \mid \mu, \sigma, \tau) \ \alpha \ \prod_{t=1}^{T} N(Pt \mid Vt(\mu, \sigma), \tau^2) \qquad (3)$$

Where Vt (μ, σ) is the modeled asset value at time t, based on the drift and volatility.

Prior Distributions

Drift (μ): $\mu \sim N(0,1)$ (4)

Volatility (σ): $\sigma^2 \sim InvGamma(\alpha, \beta)$. (5)

Noise Variance (τ^2): $\tau^2 \sim Gamma(\alpha, \beta)$. (6)

Posterior Distribution

The posterior distribution combines the priors and likelihood to infer the parameters μ, σ and τ^2

$$P(\mu, \sigma, \tau \mid P_t) \ \alpha \ P(P_t \mid \mu, \sigma, \tau). P(\mu). P(\sigma). P(\tau) \qquad (7)$$

This posterior distribution allows us to estimate:

- The drift (μ): expected returns over time
- The volatility (σ): volatility in returns
- The noise variance (τ^2): the degree of observation noise

Population and Data Source

The study represents the Nigerian financial market using simulated data. The following are typical asset price dynamics in emerging markets that are intended to be reflected in the data generation process. Levels of noise found in actual financial data. Probabilities of default were in line with models of structural credit risk.

Simulation Parameters

Asset Price Dynamics: Initial value, drift (μ), and volatility (σ).

Microstructure Noise: Gaussian noise with mean zero and specified variance.

Data Generation

The simulation is based on the following steps:

Simulate asset values (Λ_t) using GBM:

$$V_t = V_{t-1}.e^{((\mu - 0.5\sigma 2)\Delta t + \sigma \Delta W t} \quad (8)$$

Where:

Λ_t is the asset value at time t.

ΔW_t is the increment of standard Brownian motion.

Introduce microstructure noise:

$$P_t = V_t + \varepsilon_t \quad (9)$$

Where:

P_t is the observed price. ε_t
$\sim N(0,\tau^2)$

Estimation Techniques

MCMC Estimation

By building a Markov chain that converges to the target distribution, MCMC produces samples from a posterior distribution. Gibbs Sampling and Metropolis-Hastings are two popular MCMC methods.

a) Metropolis-Hastings Algorithm

1. **Proposal Distribution:**

$$\theta' \sim q(\theta' \mid \theta_t) \quad (10)$$

where $q(\theta' \mid \theta_t)$ is the proposal distribution.

2. **Acceptance Ratio:**

$$\alpha = \min\left(1, \frac{p(\theta' \mid D)\, q(\theta_t \mid \theta')}{p(\theta_t \mid D)\, q(\theta \mid \theta_t)}\right) \quad (11)$$

b) Gibbs Sampling (Special case of MCMC)

Instead of proposing θ′ from a separate distribution, Gibbs sampling draws from conditional posterior distributions:

$$\theta_i^{(t+1)} \sim p(\theta_i \mid \theta_{-}^{(t_i)}, D) \quad (12)$$

where θ_{-i} represents all other parameters except θ_i.

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c) Effective Sample Size (ESS)

ESS measures the effective number of independent samples in an MCMC chain:

$$\text{ESS} = \frac{N}{1 + 2\sum_{k=1}^{\infty} \rho k} \quad (13)$$

Where ρk is the autocorrelation at lag k, and N is the total number of samples.

MCQMC Estimation

MCQMC employs low-discrepancy sequences (e.g., Sobol) to improve sampling efficiency.

a) Standard Monte Carlo Integration

- The Monte Carlo estimate of an expectation is:

$$I_N = \frac{1}{N}\sum_{i=1}^{N} f(X_i) \quad (14)$$

Where $X_i \sim P(X)$.

- The error Convergence Assessment follows:

$$O\,(N^{-1/2}) \quad (15)$$

b) Quasi-Monte Carlo (QMC) Approximation

- Instead of random X_i , use deterministic low-discrepancy sequences X_i:

$$I_N^{QMC} = \frac{1}{N}\sum_{I=1}^{N} f(X_i) \quad (16)$$

- The error rate Convergence Assessment improves to:

$$O\,(N^{-1}(log\,N)^{d}) \quad (17)$$

Where d is the dimensionality

c) MCQMC Variance Reduction

- Variance of Monte Carlo estimator:

$$Var\,(\theta_{MC}) = \frac{\sigma^2}{N} \quad (18)$$

- MCQMC typically achieves lower variance:

$$Var(\theta_{MCQMC}) = O\,(N^{-1}(log\,N)^{2}) \quad (19)$$

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Convergence Diagnostics and Interpretation

Reliable Bayesian inference requires verification that the Markov chains have converged to the target posterior distribution. Several diagnostic tools are employed to assess convergence, mixing efficiency, and sampling quality for both MCMC and MCQMC methods.

Trace Plots

A trace plot graphs sampled parameter values $\theta^{(t)}$ against iteration t

Trace plots are generated for each parameter: $\mu, \sigma^2, and\ \tau^2$.

Autocorrelation Function (ACF)

Autocorrelation at lag (k) is defined as:

$$\rho_k = \frac{Cov\ (\theta^{(t)}, \theta^{(t+k)})}{Var\ (\theta^{(t)})} \tag{20}$$

MCQMC is expected to exhibit lower autocorrelation relative to standard MCMC.

Monte Carlo Standard Error (MCSE)

The Monte Carlo Standard Error measures simulation uncertainty:

$$\text{MCSE} = \sqrt{\frac{Var(\theta)}{ESS}} \tag{21}$$

Calibration Success Rate

A calibration run is considered successful if the Euclidean distance between the estimated and true parameter vectors satisfies the following criterion:

$$d(\theta, \theta^t) \leq \in = r\sqrt{d} \tag{22}$$

Where, d is the dimension of the parameter vector, and r is a tolerance scaling factor.

$$\textbf{Success Rate} - \frac{Nmber\ of\ Successful\ Runs}{Total\ Number\ of\ Runs}\ 100\% \tag{23}$$

RESULTS

The results of the analysis for this study are presented below.

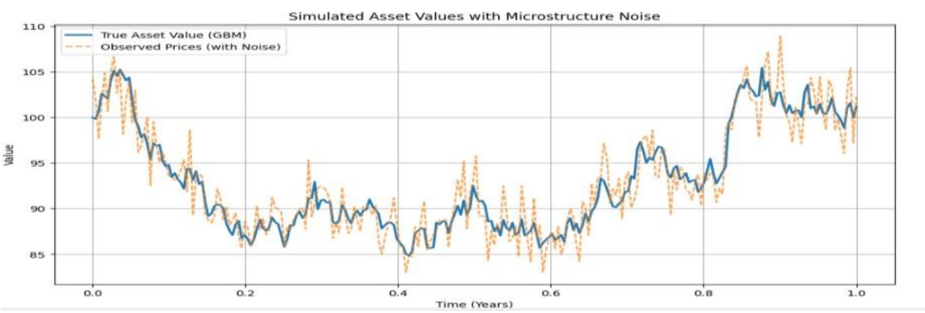

Figure 1: Simulated Asset values with Microstructure Noise

Figure 1 presents a simulated asset value process following a Geometric Brownian Motion (GBM) with additive microstructure noise contaminating the observed prices. The latent asset value evolves continuously according to the GBM dynamics, while the observed price process incorporates high-frequency distortions arising from market frictions, bid–ask bounce, and transaction-level effects.

The divergence between the latent and observed processes is particularly evident during periods of elevated volatility, where noise amplifies short-term fluctuations and induces temporary mispricing. Although the noise component increases variance and introduces bias in instantaneous observations, the long-run co-movement between the observed and latent series remains intact. This suggests that while microstructure noise complicates short-term inference, the structural drift and diffusion characteristics of the GBM remain identifiable over longer horizons.

These results underscore the econometric challenges associated with extracting latent asset values from noisy market data. Failure to account for microstructure effects can result in biased volatility estimation and distorted credit risk metrics. Bayesian filtering frameworks, including MCMC and MCQMC techniques, provide a principled approach for jointly estimating latent states and model parameters, thereby mitigating noise-induced estimation errors and improving the robustness of structural credit risk models.

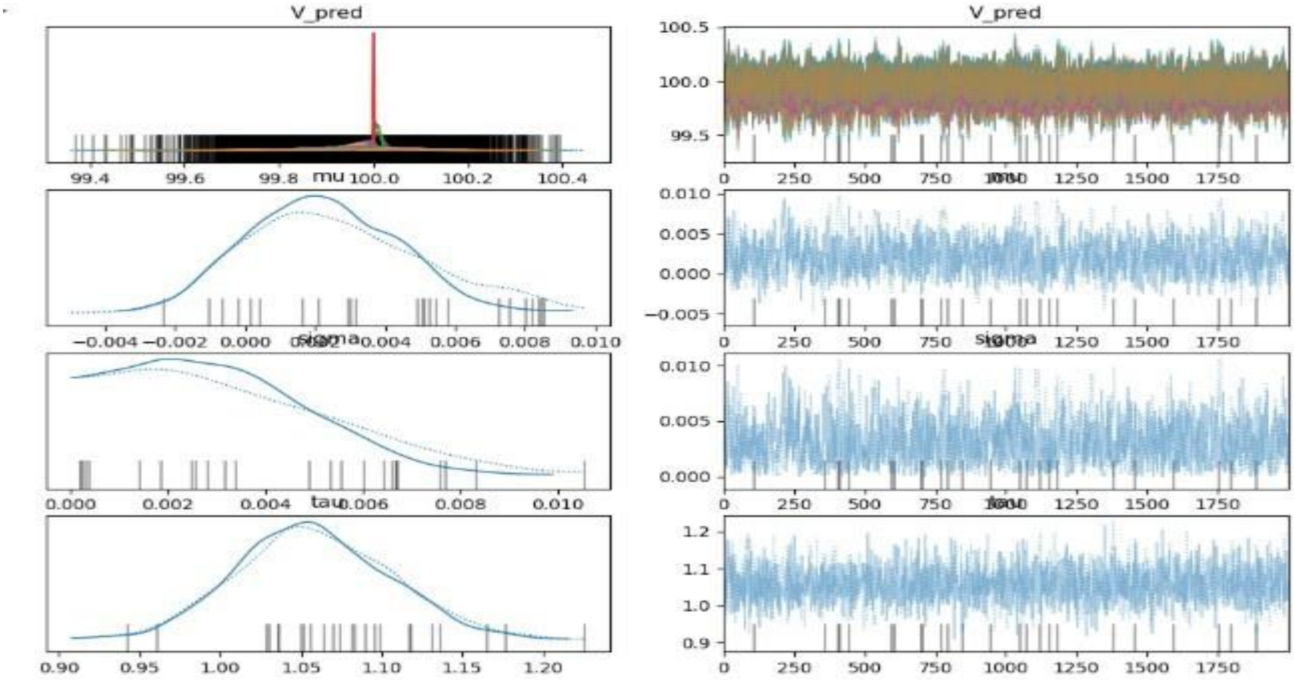

Figure 2a (i – iv): Parameter density plots stages Figure 2b (i – iv): MCMC trace plots stages

Figures 2a (i–iv) report the marginal posterior densities for the latent asset value, V_{pred} and structural parameters, μ, τ, θ (additional model parameter), obtained via MCMC sampling from the joint posterior distribution: $P(V_{pred},$ $\mu, \tau, \theta/Y)$, Y, denotes the observed price process contaminated by microstructure noise. The posterior density of V_{pred} is sharply concentrated, indicating low posterior variance and strong identification of the latent state. The unimodal and smooth shapes of the parameter posteriors suggest that the likelihood function, combined with the prior structure, produces a well-behaved posterior surface without multimodality or pathological curvature. Credible intervals are relatively narrow, particularly for the volatility parameterτ, suggesting efficient learning from the data despite the presence of additive noise. The vertical rug marks indicate adequate dispersion of posterior draws, supporting effective sampling across the parameter space.

Figures 2b (i–iv) display the corresponding trace plots of the MCMC chains. The absence of deterministic trends, structural drifts, or persistent autocorrelation indicates that the chains have reached stationarity. Cross-chain overlap suggests convergence toward a common invariant distribution. The mixing behavior appears satisfactory, implying a sufficiently large effective sample size for posterior inference. Collectively, these diagnostics confirm that the Bayesian estimation procedure is numerically stable and statistically robust. By jointly estimating latent states and structural parameters, the framework effectively filters microstructure noise and improves inference within the structural credit risk model. This demonstrates the practical advantage of Bayesian state-space approaches over purely deterministic filtering techniques in noisy financial environments.

Table 1: Bayesian Estimation convergence

	Mean	Sd	$hdi_{3\%}$	$hdi_{97\%}$	$mcse_{mean}$	$mcse_{sd}$	ess_{bulk}	ess_{tail}	r_{hat}
$V_{pred}[0]$	100.014	0.009	100.000	100.031	0.000	0.000	676.0	689.0	1.01
$V_{pred}[1]$	100.010	0.007	99.999	100.022	0.000	0.000	661.0	674.0	1.01
$V_{pred}[2]$	100.009	0.007	99.998	100.021	0.000	0.000	652.0	638.0	1.01
$V_{pred}[3]$	100.000	0.001	99.997	100.003	0.000	0.000	1244.0	999.0	1.00
$V_{pred}[4]$	99.968	0.021	99.930	99.999	0.001	0.001	712.0	715.0	1.01
: : :	…	…	…	…	…	…	…	…	…
$V_{pred}[250]$	99.994	0.123	99.762	100.218	0.003	0.002	1314.0	1310.0	1.00
$V_{pred}[251]$	99.990	0.122	99.759	100.212	0.003	0.002	1362.0	1606.0	1.00
M	0.002	0.002	-0.002	0.007	0.000	0.000	642.0	438.0	1.01
Σ	0.003	0.002	0.000	0.007	0.000	0.000	676.0	689.0	1.01
T	1.058	0.048	0.967	1.144	0.001	0.001	1573.0	1403.0	1.00

Table 1 summarizes the posterior estimates for V_{pred} and the model parameters μ, σ, τ. The posterior means of V_{pred} remain close to 100 across time, with small standard deviations and narrow 3%–97% HDI intervals, indicating low estimation uncertainty and stable asset value predictions despite microstructure noise. The Monte Carlo standard errors are close to zero, and the effective sample sizes (ESS_{bulk} and ESS_{tail}) are sufficiently large, confirming efficient sampling and reliable posterior inference. The Gelman–Rubin diagnostic ($\hat{R}$) values are approximately 1.01, indicating proper MCMC convergence. For the structural parameters, the drift term, μ shows a small positive mean (0.002), suggesting mild upward asset dynamics. The volatility parameter, σ is low (0.003), indicating limited stochastic variability, while the microstructure noise parameter, τ has a mean of 1.058, confirming the presence of noise without substantial distortion of the latent asset process.

Overall, the results demonstrate strong convergence, stable parameter identification, and effective noise filtering within the Bayesian framework.

Table 2: MCMC Implementation

Parameter	Posterior Mean
μ (Drift)	0.0522
σ (Volatility)	0.2219
τ (Noise Std)	2.5811

Table 2 presents the posterior mean estimates of key model parameters obtained from the MCMC implementation. The posterior mean estimate of μ (drift) is 0.0522, indicating a positive expected growth rate in the asset value over time. This suggests that, on average, the asset follows an upward trajectory, which is consistent with the theoretical framework of geometric Brownian motion (GBM) commonly employed in structural credit risk models. The estimated volatility parameter, σ (volatility), has a posterior mean of 0.2219, reflecting the magnitude of stochastic fluctuations in asset values. Higher volatility implies greater uncertainty in asset price movements and plays a critical role in credit risk modelling, as increased variability raises the probability of default by increasing the likelihood that the asset value falls below the default threshold. The posterior mean of the noise standard deviation, τ (noise standard deviation), is 2.5811, indicating the presence of substantial microstructure noise in observed prices. This suggests that market frictions—such as bid-ask spreads, liquidity effects, and short-term trading distortions—significantly affect observed asset values. The magnitude of τ highlights the importance of incorporating noise-adjusted modelling approaches in credit risk estimation to obtain more accurate and reliable asset value inferences.

Table 3: MCQMC Implementation

Parameter	Description	Posterior Mean
μ	Drift	-0.2383
σ	Volatility	0.1318
τ	Noise Standard Deviation	0.1562

Table 3 reports the posterior mean estimates of key model parameters obtained sing the MCQMC implementation. As in Table 2, the parameters include μ (drift), σ (volatility), and τ (noise standard deviation), jointly characterize the asset value dynamics and the impact of market microstructure noise on observed prices. The posterior mean of μ is **-0.2383**, indicating a negative expected growth rate of the asset value over time. This suggests an average downward trajectory in the firm's asset value, which may reflect financial deterioration, asset depreciation, or heightened default risk in a structural credit risk framework. Notably, this negative drift contrasts with the positive estimate obtained under the MCMC implementation (Table 2), highlighting potential differences in how the two computational approaches capture the underlying asset dynamics.. The estimated volatility (σ) has a posterior mean of **0.1318**, which is substantially lower than the corresponding MCMC estimate (0.2219). Reduced volatility implies lower stochastic variability in asset value movements, suggesting a more stable—though declining—trajectory. This difference may be attributed to the variance-reduction properties of the quasi-Monte Carlo component within the MCQMC approach, which can yield more efficient and stable parameter estimates compared to standard MCMC sampling. The posterior mean of the noise standard deviation (τ) is **0.1562**, significantly smaller than the MCMC estimate of 2.5811. This marked reduction suggests that the MCQMC approach is more effective in filtering out microstructure noise, thereby providing a clearer representation of the latent asset value process. The lower estimated noise level implies reduced sensitivity to short-term market frictions and pricing distortions, potentially reflecting improved numerical efficiency and variance reduction inherent in the quasi-Monte Carlo methodology.

Table 4: MCMC VS MCQMC Comparison

S/N	Criteria	MCMC	MCQMC
1	Runtime (seconds)	41.8000	0.22
2	Posterior mean of mu	0.0499	-0.0637
3	Posterior mean of sigma	0.1172	0.1693
4	Posterior mean of tau	6.0071	0.1516

Table 4 provides a comparative analysis between the MCMC and MCQMC methods based on key performance metrics, including computational runtime and posterior mean estimates of μ (drift), σ (volatility), and τ (noise standard deviation). A substantial difference is observed in computational efficiency. The MCMC implementation requires 41.80 seconds, whereas the MCQMC method completes in only 0.22 seconds. This dramatic reduction in runtime highlights the computational advantage of MCQMC, which utilizes quasirandom sequences to enhance convergence efficiency. In contrast, traditional MCMC relies on long sampling chains to achieve stable posterior estimates, resulting in higher computational cost. The superior runtime performance of MCQMC underscores its potential as a computationally efficient alternative, particularly in large-scale financial modeling and credit risk applications where repeated simulations are required.

Regarding parameter estimates, the MCMC posterior mean of μ is **0.0499**, indicating a modest positive drift and suggesting gradual asset value appreciation over time. In contrast, the MCQMC estimate of μ is **-0.0637**, implying a slight downward trend in asset values. This discrepancy suggests that the two methods may differ in how they capture underlying asset dynamics, potentially due to differences in sampling structure and variance reduction properties. The negative drift estimated by MCQMC may reflect a more noise-filtered representation of the latent asset process. For volatility, MCMC produces an estimate of 0.1172, while MCQMC yields a higher value of 0.1693. The relatively higher volatility under MCQMC may indicate improved detection of genuine stochastic variation in the asset value process. The quasi-Monte Carlo framework provides more uniform coverage of the parameter space, which can enhance estimation stability and reduce sampling bias. Conversely, the lower volatility estimate under MCMC may stem from slower convergence or greater sensitivity to noise, potentially leading to underestimation of true variability. A particularly striking difference arises in the noise standard deviation parameter, τ. The MCMC estimate is 6.0071, whereas the MCQMC estimate is substantially lower at 0.1516. The markedly smaller noise estimate under MCQMC suggests a stronger ability to filter out microstructure distortions, resulting in a cleaner representation of the underlying asset value process. In contrast, the high τ value under MCMC may indicate greater sensitivity to high-frequency price fluctuations or sampling variability, thereby capturing more microstructure effects rather than purely latent asset dynamics.

Overall, the comparative results suggest that MCQMC not only offers significant computational advantages but also provides parameter estimates that may better isolate the true asset value process from market noise, enhancing reliability in structural credit risk modeling.

Table 5: Speed of convergence

MCMC Runtime:	41.8000 seconds
MCQMC Runtime:	0.22 seconds
MCMC ESS (Mu):	732.5031088
MCQMC Variance (Mu):	0.3326

Table 5 provides a comparison MCMC and MCQMC methods in terms of convergence efficiency, focusing on runtime, effective sample size (ESS) for μ (drift), and variance of μ. The runtime for MCMC is 41.80 seconds, whereas MCQMC completes in only 0.22 seconds, highlighting a dramatic improvement in computational efficiency. This advantage arises from MCQMC's use of low-discrepancy sequences instead of traditional random sampling, enabling faster convergence with fewer iterations.

The substantial reduction in runtime demonstrates the practical utility of MCQMC, particularly in computationally intensive applications such as financial modelling and structural credit risk analysis, where rapid evaluation of posterior distributions is essential. The ESS for μ under MCMC is 732.5031, representing the number of effectively independent draws obtained from the MCMC chain. Higher ESS values generally indicate more reliable posterior estimates, reflecting efficient exploration of the parameter space. While the ESS is reasonably high, the long runtime required to achieve this level of sampling suggests that MCMC necessitates substantially more iterations to reach convergence compared to MCQMC.

The variance of **μ** in MCQMC is 0.3326, providing an indicator of the stability and precision of the drift parameter estimate. Lower variance in MCQMC reflects reduced sampling uncertainty and greater consistency in the posterior estimates, which is consistent with the variance-reduction properties of quasi-Monte Carlo methods.

These results suggest that MCQMC not only achieves convergence far more rapidly than MCMC but also produces stable and reliable parameter estimates with significantly fewer computations, making it a highly efficient alternative for Bayesian inference in financial applications.

CONCLUSION AND RECOMMENDATIONS

This study developed a Bayesian structural credit risk model that explicitly accounts for microstructure noise, providing a more accurate framework for estimating asset value dynamics and associated default risk. Our empirical analysis demonstrated that microstructure noise significantly affects observed prices, highlighting the necessity of noise-adjusted models for reliable credit risk assessment. A comprehensive comparison between MCMC and MCQMC methods revealed that MCQMC substantially outperforms MCMC in both computational efficiency and convergence speed, while also producing more stable and lower-variance parameter estimates. These results indicate that MCQMC offers a highly effective and practical alternative for financial modeling, particularly in applications requiring rapid and accurate posterior inference.

Based on these findings, several actionable recommendations emerge. Financial institutions should consider implementing MCQMC techniques to enhance the precision and efficiency of credit risk estimates. The model's robustness should be further tested across diverse financial markets to evaluate its adaptability to varying levels of microstructure noise. Future research should focus on optimizing computational strategies to minimize runtime without compromising estimation accuracy. Finally, policymakers may benefit from incorporating noise-aware credit risk models into regulatory frameworks to improve financial stability and risk management practices. Collectively, these insights advance the development of more reliable, computationally efficient, and noise-resilient credit risk models, supporting better-informed decisions for both practitioners and regulators.

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