Modern Applications of Mathematics

Mrs. Sharmila A K

Assistant Professor, Usha Mittal Institute of Technology, Santacruz, Mumbai, India

DOI: https://dx.doi.org/10.51584/IJRIAS.2026.110200081

Received: 21 February 2026; Accepted: 27 February 2026; Published: 14 March 2026

ABSTRACT

Mathematics is the backbone of the innovations in the fields of Artificial Intelligence, Quantum Computing and Epidemiology. Linear algebra, probability and optimization form core tools enabling precise modeling, analysis and computation in these developing technologies. Examples from Machine Learning, Blockchain technology and quantum computing, MRI/CT imaging illustrate recent advancements as of 2026. [1][2].

Keywords: Linear Algebra, Calculus, Probability, Stochastic processes, Fourier transforms, complex vectors and unitary matrices.

INTRODUCTION

Advancements in computational power have amplified Mathematics' role in solving complex problems. Pure concepts like linear algebra now power practical systems in AI and quantum tech. This paper examines applications in AI/ML, cryptography, quantum computing, finance, medicine, climate science, autonomous systems, and graphics, with concrete examples with use of Mathematical concepts in these applications. [2][3]

AI and Machine Learning

Linear algebra represents data in high dimensions, while calculus optimizes models via gradient descent. Probability handles uncertainty in predictions. [1][4]

For instance, backpropagation in neural networks uses calculus derivatives to minimize errors, enabling systems like Tesla's Full Self-Driving to process 1,000 calculations per second for safe navigation. [1][5] Optimization algorithms refine learning rates, improving accuracy in image recognition. [4]

Mathematical Tool	Example	Application
Linear Algebra	Data matrices, neural nets	Pattern recognition in ML [1]
Calculus	Gradient descent	Model training [4]
Probability	Uncertainty modeling	Predictive analytics [1]

Cryptography and Blockchain

Bitcoin's blockchain uses cryptographic hashes for immutability, preventing tampering in records. Zero-knowledge proofs like zk-SNARKs enable private transactions in ZCash. [6]

Homomorphic encryption allows computations on encrypted data, advancing privacy in cryptocurrencies. [6]

Elliptic curve cryptography and hash functions ensure secure, decentralized transactions.

Elliptic curve cryptography:

ECC relies on the math of elliptic curves over finite fields. Here's how keys are generated:

i) Elliptic Curve (E): Defined by: $y^2 = x^3 + ax + b \ (mod p)$, a and b are curve parameters, p is a large prime number for security.

ii) **Base Point (G)**: A point on E with large order.
iii) **Private Key (d):** Random integer (1 < d < n-1), where n is the order of large prime number.
iv) **Public Key (Q):** Q= dG Point multiplication (repeated addition) on E.

Key Generation Steps:

1. Choose curve parameters (E, p, a, b, G, n)

2. Pick random d (private key)

3. Compute Q = dG (public key)

Security:

Hard to find 'd' given Q, G (ECDLP - Elliptic Curve Discrete Log Problem)

Smaller keys vs RSA is efficient & strong

Quantum Computing

In Quantum computing qubits are used instead of the bits (that are 0 or 1) used in classical computing.

Linear algebra describes qubit states via complex vectors and unitary matrices for gates. Algorithms like Shor's exploit superposition for factoring. [8][9]

Quantum gates, as 2x2 or 4x4 matrices, entangle qubits for parallelism. Quantum Support Vector Machines use kernel methods for high-dimensional classification, outperforming classical ML in data-heavy tasks. [8][9]

Qubit state: It is a Vector in $\mathbb{C}^2$ (complex vector space)

$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$ where $|\alpha|^2 + |\beta|^2 = 1$, where $|\varphi\rangle$ is the state of the qubit, $|0\rangle$ and $|1\rangle$ are the basis states. α and β are the complex numbers called probability amplitude.

Gates: They are operations on qubits represented by Unitary matrices (preserve vector norm)

Finance and Risk Management

Stochastic processes and time-series analysis model markets; Value at Risk (VaR) quantifies losses. [10][11]

Stochastic processes and time-series models describe financial markets as systems that evolve over time under random "noise," while still having statistically predictable patterns.

Simple example: **Stock prices as a random walk**

Imagine a stock price S_t that changes each day by a small random step around a drift (average trend).

Illustration of **Geometric Brownian motion (GBM),** is a key model in financial risk management.

$dS_t = \mu S_t dt + \sigma S_t dW_t$, where μ is the average return, σ is volatility, and dW_t is Wiener "noise" increment. In discrete time, this becomes: $S_{t+1} \approx S_t \exp(\mu\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon_t)$, ε_t ~Normal (0, 1). Simulating this over many days produces a jagged price path that looks like a real stock chart, capturing both trend and randomness.

Time- series model: A R(1) returns
Markets are often modeled in returns $r_t = \log\left(\frac{S_t}{S_{t-1}}\right)$. A simple time-series model is the autoregressive A R(1) process:
$r_t = \emptyset r_{t-1} + \varepsilon_t$, where ε_t is white noise and $\emptyset$ controls how much yesterday's return drags today's return.

If $|\emptyset| < 1$, the series exhibits mean reversion: big moves tend to be followed by smaller moves back toward zero.
If $\emptyset$ is close to 1, the market looks “trendy” for a while but still reverts eventually.
Plotting simulated r_t paths shows alternating periods of volatility and calm, mimicking the empirical behavior of stock-index returns.

Medicine and Epidemiology:

Fourier transforms enhance MRI/CT imaging; compartmental models predict outbreaks. [12][13]

COVID-19 models used SIR equations to estimate R0 and intervention effects, informing lockdowns.
Histogram equalization and linear algebra improve scan accuracy for early diagnosis. [12][14]

Mathematical modeling optimizes drug dosages via body weight fractions. [12]

Fourier Transform as hidden Mathematical process in MRI and CT imaging:

MRI/CT scanners collect raw data in “k-space”, like frequency and phase information, not actual pixels.

Fourier Transform turns that messy k-space data into clean, visual images you see on screen.

Thus Fourier Transforms help the conversion of the signals sensed while scanning, to visual images.

Clear brain slice images can be made and tumors spotted or bones highlighted.

The FT also help the following.

- Speed up scans (under sampling + smart reconstruction)
- Reduce noise & artifacts
- Enable advanced techniques like compressed sensing

MRI: k-Space and Fourier Transform:

MRI doesn’t capture spatial images directly. Instead, it samples data in k-space — the spatial frequency domain.

Each point in k-space corresponds to a spatial frequency component of the image.

- Low k-space frequencies are coarse structure / contrast (big picture)
- High k-space frequencies are fine details / edges

MRI scanner fills k-space line-by-line using gradient coils and RF pulses. After acquisition:

Image is Inverse Fourier Transform (IFT) of k-space data

Mathematically:

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(k_x, k_y) e^{i\,2\pi(k_x x + k_y y)} dk_x dk_y$$

Where:

$f(x,y)$ = reconstructed image intensity at pixel (x, y)

$S(k_x, k_y)$ = complex-valued k-space signal

$e^{i\,2\pi(k_x x + k_y y)}$ = basis functions encoding spatial position

CT: Filtered Back Projection and Fourier Slice Theorem

CT uses X-ray projections from multiple angles. Reconstruction traditionally uses Filtered Back Projection (FBP).

Fourier Slice Theorem: A 1D FT of a projection at angle θ equals a slice through the 2D FT of the object along that angle.

Climate Modeling and Physics

Climate modeling relies heavily on mathematics to simulate the complex interactions within the Earth's climate system. In climate models, the grid represents the resolved scales (~10-100km) while subgrid processes (e.g., clouds, turbulence) are parameterized to capture their aggregate effects on larger scales.

Subgrid reconstruction and Taylor series derive accurate cloud equations, avoiding oversimplifications. [15]

These methods aggregate unresolvable scales, improving long-term cloud statistics in Earth system models. [15]

Atmospheric modeling (Computational Fluid Dynamics for weather/clouds)

Grid resolution: Δx, Δz (finite)

Subgrid processes: Clouds smaller than grid → need modeling

Subgrid Reconstruction:

Represent fields (u, T, q) within grid cells

Use basis functions (e.g., linear, quadratic)

$$f(x) = f_i + {f_i}^I (x - x_i) + \frac{f_i^{II}}{2}(x - x_i)^2 + \cdots$$

Taylor Series Expansion:

Expand fields locally:

$$f(x + \Delta x) = f(x) + \Delta x \frac{\partial f}{\partial x} + \frac{(\Delta x)^2}{2}\frac{\partial^2 f}{\partial x^2} + O(\Delta x^3)$$

Accurate derivatives → better subgrid fluxes

Cloud Equations: Typically refers to equations governing cloud processes

Conservation laws: Mass, momentum, energy, moisture

$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} = S(q) + subgrid\ terms$$

Subgrid terms are Turbulence, cloud microphysics

Avoiding Oversimplifications:

Higher-order schemes: Reduce numerical diffusion

Reconstruction: Capture sharp gradients (e.g., cloud edges)

Taylor series: Accurate time stepping (e.g., RK methods)

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume XI Issue II February 2026

Autonomous Vehicles and Graphics

Calculus computes derivatives for braking and integrals for path distances in self-driving cars.[5]

Tesla Autopilot uses optimization for routes balancing speed and safety. In gaming, vectors and matrices render 3D polygons; physics engines integrate calculus for realistic motion. [16][17]

Derivatives (dx/dt):

Speed, Acceleration:

$$v = \frac{dx}{dt}\ ,\ \ a = \frac{dv}{dt}$$

Steering Control: Curvature $\kappa = d\theta/ds$ (path planning)

Integrals (∫):

Path Distance:

$$s = \int_{t_1}^{t_2} v(t)dt$$

Energy Consumption: ∫ power dt

Self-Driving Applications:

- Localization: Integrate sensor data (IMU, GPS)
- Planning: Optimize paths using calculus of variations
- Control: PID controllers use derivatives for feedback

Other Application Areas:

Game theory models participant incentives. [6][7]. In Economics these are used for modeling market competitions and pricing strategies.

The Black-Scholes model employs partial differential equations for option pricing. Machine learning integrates with econometrics for predictive trading. [10]

Stress testing simulates extremes, using optimization for portfolio resilience. [11]

CONCLUSION

Mathematics remains indispensable for modern challenges, fostering innovations like secure blockchains and precise AI. Future integration with quantum tech promises further breakthroughs, urging interdisciplinary collaboration. [2][3]

Mathematics plays a key role in transformative technologies across industries from AI to climate modeling. This paper surveys and examines modern applications of Mathematics highlighting Mathematical principles and real world examples.

CITATIONS

1. Making AI Fair and... https://www.schiller.edu/blog/how-applied-mathematics-powers-the-next-generation-of-ai-systems/
2. Recent Trends in Mathematics: Emerging Areas of Research and ... https://zenodo.org/records/15542695
3. Current Trends in Mathematics https://jofmath.com/index.php/AJPAM/article/view/173
4. Learn Essential Math Skills... https://www.edx.org/resources/why-math-is-essential-for-ai-ml

5. How Calculus Keeps Self-Driving Cars Safe https://www.linkedin.com/posts/abhigna-upadhyaya_mathinaction-mathematics-autonomousvehicles-activity-7346593578468409344-rT78
6. Advanced Mathematical Application of Cryptographic Protocols in Distributed Ledger Technologies and Digital Currency Systems https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5066790
7. Some Applications of Mathematics in Cryptography https://www.math.auckland.ac.nz/~sgal018/Vietnam.pdf
8. Quantum Math https://quantum.microsoft.com/en-us/insights/education/concepts/quantum-math
9. Mathematical Foundations and Algorithms for Quantum Computing https://www.mathsjournal.com/pdf/2025/vol10issue3/PartB/10-3-15-691.pdf
10. What Math Areas/Topics Are Used in the Following Finance Areas? https://www.reddit.com/r/quant/comments/1oxihwt/what_math_areas_topics_are_used_in_the_following/
11. Mathematical Finance: Modeling and Risk Management https://www.neuroquantology.com/media/article_pdfs/Volume_20_No_7__MATH_Mathematical_Finance_Modeling_and_Risk_Management.pdf
12. The Math Behind Medical Imaging Technology https://skill-lync.com/blogs/the-math-behind-medical-imaging-technology
13. Mathematical Analysis of a COVID-19 Epidemic Model by ... https://pmc.ncbi.nlm.nih.gov/articles/PMC9244063/
14. Modeling the COVID-19 Pandemic: A Primer and Overview of Mathematical Epidemiology https://pmc.ncbi.nlm.nih.gov/articles/PMC8318333/
15. Using Mathematical Rigor to Increase Physical Realism in Models https://climatemodeling.science.energy.gov/research-highlights/using-mathematical-rigor-increase-physical-realism-models
16. Fundamental Math for Game Developers https://pikuma.com/blog/math-for-game-developers
17. The Use of Mathematics in Computer Games http://nrich.maths.org/articles/use-mathematics-computer-games
18. Mathematics for Computer Graphics https://www.reddit.com/r/GraphicsProgramming/comments/1g7axgt/mathematics_for_computer_graphics/
19. Mathematical Techniques for Autonomous Vehicle Navigation https://internationalpubls.com/index.php/pmj/article/download/1895/1209/3435
20. Mathematical Modeling of COVID-19 Epidemic with Effect ... https://www.sciencedirect.com/science/article/pii/S2468042721000154