Advancing Solutions for Fractional Volterra–Fredholm Integro-Differential Equations: A Comprehensive Review of Recent Numerical Methods

Ali Karim Lelo Alobaidi

Department of Mathematics, College of Science, University of Al Qadisiyah, Iraq.

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ABSTRACT

Fractional Volterra–Fredholm integro-differential equations (FVFIDEs) combine fractional derivatives with nonlocal Volterra and Fredholm operators, making them a central tool for modeling systems with memory and spatial interactions. Analytical solutions are rare, and numerical methods have become essential for practical applications. This review synthesizes recent advances in numerical approaches, including predictor–corrector schemes, spectral methods, iterative algorithms, and hybrid techniques. Emphasis is placed on convergence analysis, computational efficiency, and comparative performance. By consolidating developments from 2020–2025, the article provides a structured overview of current solution strategies and highlights open challenges, offering guidance for researchers seeking effective tools to advance the study of FVFIDEs.

Keywords

Fractional calculus; Volterra–Fredholm equations; Integro-differential equations; Numerical methods; Spectral techniques; Predictor–corrector schemes; Convergence analysis; Applied mathematics

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ASJC (Scopus) Classification:2604; 2605

INTRODUCTION

Fractional calculus extends classical differentiation and integration to arbitrary orders, offering a powerful framework for modeling systems with hereditary and memory-dependent behavior. Unlike traditional Volterra–Fredholm equations, the fractional variants introduce non-local operators that significantly increase mathematical complexity. This complexity explains their growing relevance in diverse applications such as viscoelasticity, circuit dynamics, and diffusion processes. However, despite their broad applicability, analytical solutions remain scarce, which has motivated intensive research into numerical methods. A critical challenge lies in balancing accuracy, convergence, and computational efficiency across different approaches.. They also emerge in modeling of oscillations, vibrations, nano Systems, heat conduction, and bio-systems, among other phenomena. The increased complexity compared to either the classical or the purely fractional frameworks—including non-local operators, boundary conditions that do not affect the turnpike property and are not compatible with regular discretization, and fully non local fractional variational principles with kernel functions of unbounded support gives rise to a correspondingly rich variety of numerical methods. (Heydari et al.2026) (Filali et al., 2025) (Gunasekar et al.2024) (Raghavendran et al.2024) (Gunasekar et al.2024) (Raja et al.2025) (Yadav & Mohapatra, 2026)

Fractional calculus provides an efficient mathematical tool to describe systems that exhibit hereditary behavior in various fields of science and engineering. At present, fractional calculus is still an active area of research, as new concepts and models are continuously emerging. Special attention is given to the numerical solution of fractional-order differential equations, and numerous works exist that focus on initial value problems at the fractional-order level. These studies usually rely on classic definitions of fractional derivatives, such as Riemann–Liouville, Caputo, and others. Sufficiently rich kernel functions permit the immediate use of classical techniques from linear integral equations, leading to the emergence of a large number of papers on the numerical treatment of integer non local and non local derivative problems. By contrast, the preliminary investigation of time-fractional Volterra–Fredholm integro-differential equations of mixed order clearly indicates the absence of suitable modeling equations at the classical level (A. Elbeleze et al., 2016). (Vieira et al., 2023) (Diethelm et al.2022) (Baleanu et al.2023) (Ashok & Sayed2024) (Fernandez et al., 2025) (Chen et al., 2022) (Karaca & Baleanu, 2022) (Amilo et al.2024)

FOUNDATIONS OF FRACTIONAL CALCULUS AND VOLTERRA–FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS

Fractional calculus extends differentiation and integration to arbitrary orders, offering a mathematical framework for systems with hereditary and memory effects. Although its origins date back to the seventeenth century, practical applications only gained momentum in the 1990s, when engineers and physicists recognized its potential for modeling real-world processes. The transition from theory to application revealed significant challenges: while fractional Volterra–Fredholm equations provide a richer description of boundary and non-local phenomena, their numerical treatment remains underdeveloped. Existing methods often lack robustness or accuracy when applied to irregular kernels or multidimensional problems. This gap highlights the urgent need for comparative studies that evaluate convergence, stability, and computational efficiency across different numerical strategies. (Heydari et al.2026) (Tunç & Tunç, 2024) (Yadav & Mohapatra, 2026) (Alam & Rohen, 2025) (Sharif et al.2025) (Alabdala et al.2023) (Syam & Hashim2024) (Hamoud et al.2023)

CORE NUMERICAL TECHNIQUES: DISCRETIZATION AND APPROXIMATION

Developing numerical methods for fractional Volterra–Fredholm integro-differential equations is essential for modeling memory-driven phenomena such as diffusion, viscoelasticity, and population dynamics. However, the choice of fractional operator introduces significant differences in numerical behavior. For example, Caputo derivatives are often preferred in initial-value problems due to their compatibility with physical boundary conditions, whereas Riemann–Liouville derivatives may yield higher accuracy but complicate implementation. This distinction directly affects convergence and stability. Similarly, kernel characteristics—whether weakly singular or nonsmooth—determine the suitability of discretization schemes. While finite-difference approaches offer simplicity, they struggle with irregular kernels, whereas spectral methods achieve superior accuracy but at higher computational cost. A critical comparative analysis of these trade-offs is necessary to guide researchers in selecting appropriate techniques for specific applications. (Pooseh, 2013). (Kumar & Gupta, 2023) (Hamoud et al.2023) (Almhdy et al., 2026) (HamaRashid et al.2023) (Syam & Hashim2024) (Mohseni & Rostamy, 2025) (Lanlege et al.2023) (Yadav & Mohapatra, 2026).

3.1. Fractional Differentiation Operators in Numerical Schemes

Fractional differentiation operators play a central role in numerical schemes for Volterra–Fredholm equations, yet their properties differ substantially. The Grünwald–Letnikov operator offers a straightforward discretization through time-stepping stencils, making it attractive for implementation, but it often suffers from slow convergence and sensitivity to mesh size. In contrast, the Caputo operator aligns better with physical initial conditions and yields more stable results in applied problems, though at the cost of increased computational effort. The Riemann–Liouville operator can achieve higher theoretical accuracy but complicates boundary-value formulations. Comparative studies show that while Grünwald–Letnikov methods are efficient for simple kernels, Caputo-based schemes provide superior robustness in engineering applications. A structured evaluation of convergence order, stability, and computational complexity is therefore essential to guide method selection rather than relying solely on descriptive listings of operator definitions. (Pooseh, 2013). (Cardone et al.2023) (Moghaddam et al., 2024) (Alaia et al., 2025) (Ducharne et al., 2025) (Dar et al.2023) (Granella, 2026) (Owolabi & Pindza2025) (Ghezal et al., 2025)

3.2. Kernel Handling in Volterra–Fredholm Problems

Despite more than a decade of research, kernel handling in fractional Volterra–Fredholm problems remains a major challenge. Weakly singular and nonsmooth kernels often limit the effectiveness of classical discretization strategies, leading to reduced accuracy and slower convergence. While several numerical methods have been proposed, most studies focus on simplified kernel types, leaving complex multi-term frameworks underexplored. For example, time-fractional Volterra equations have received more attention, yet multi-term models involving derivatives of different fractional orders are still poorly understood. Comparative analysis shows that low-order discretization schemes are computationally efficient but fail to capture irregular kernel behavior, whereas high-order spectral approaches improve accuracy but demand significant computational resources. This trade-off underscores the need for hybrid methods that balance efficiency with robustness, especially for real-world applications in viscoelasticity and bioengineering., the better-studied subset of the broad spectrum class. (Gunasekar et al.2024) (Gunasekar et al.2024) (Hamoud et al.2023) (Raghavendran et al.2024) (Yadav & Mohapatra, 2026) (Alsa'di & Long…, 2024) (Mahdy et al.2026)

When kernels are not non-smooth, prominence has been given to low–high-order space discretization based on the Grünwald–Letnikov or Caputo variants of fractional derivative operators. The diffusion of fractional models to complex engineering systems has shown a particular orientation toward time-fractional derivatives of the Caputo type on fixed and bounded domains, which leverages established numerical methods that rely on polynomial approximations of high-order derivatives. Such models appear naturally within the Volterra–Fredholm framework when memory effects arise from time-dependent sources, as in viscoelastic problems; thus, the need for computationally efficient and easily implementable solutions becomes paramount. (Li et al., 2026) (Ding & Wu, 2024) (Ullah et al., 2023) (Singh et al.2026) (Peng et al., 2025) (Derakhshan & Irandoust Pakchin, 2026) (Yifei et al., 2026) (Qu et al.2025) (Lu & Hou, 2026)

STATE-OF-THE-ART METHODS FOR FRACTIONAL VOLTERRA–FREDHOLM EQUATIONS

Recent advances have improved the numerical treatment of fractional Volterra–Fredholm equations, particularly for irregular kernels and complex domains. However, the diversity of available methods highlights important trade-offs. Finite-difference schemes based on Grünwald–Letnikov derivatives provide simplicity but often require restrictive mesh conditions to ensure stability. Spectral and pseudo-spectral approaches achieve high accuracy and faster convergence, yet they demand smooth kernels and significant computational resources. Variational and collocation techniques offer flexibility in handling mixed boundary conditions but lack comprehensive convergence guarantees. Applications in life sciences, such as blood-glucose modeling or neurophysiological processes, demonstrate the potential of these methods, but also reveal their limitations when extended to multidimensional or highly nonlinear problems. A structured comparison of accuracy, stability, and computational cost across these approaches is therefore essential to guide researchers beyond descriptive listings of available techniques. (Heydari et al.2026) (Hamood et al., 2026) (Georgievskii & Rautian, 2025) (Tunç & Tunç, 2024) (Mohseni & Rostamy, 2025) (Mohan et al.2025) (Bera et al., 2026) (Hamood et al., 2026) (Abbas et al.2025) (Kumar & Tripathi, 2026)

The computational analysis of fractional differential equations, integral equations, Volterra integro-differential equations, non local differential equations, and nonlinear partial differential equations remains active (Hosry et al., 2018). Discretization strategies must address irregular solution profiles in linear and nonlinear cases; classes of such strategies have been documented. The state of the art encompasses a variety of compact and non-compact approaches, with particular focus on the numerical integration of individual auxiliary fractional differential equations and on full discretization directly from the original fractional system. (Bilal et al.2025) (Abid & Shahid2024) (Nooraiepour et al.2026) (Ismail et al., 2025) (Arpaia et al., 2026) (Meloni et al., 2025) (Bae et al.) (Quintana et al., 2025) (Ortiz Ortiz et al., 2025)

4.1. Finite Difference and Grunwald–Letnikov Approaches

Finite-difference schemes based on the Grünwald–Letnikov operator provide a simple and intuitive way to approximate fractional derivatives, but their performance is highly sensitive to mesh design. Stability analysis shows that fine meshes are often required to maintain accuracy, which increases computational cost. In contrast, Caputo-based finite-difference methods align better with physical initial conditions and yield more reliable results in applied problems, though they also demand stricter consistency in time discretization. Comparative studies reveal that Grünwald–Letnikov methods are efficient for problems with smooth kernels but struggle with irregular or singular kernels, while Caputo formulations offer better stability at the expense

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of higher complexity. This trade-off highlights the need for hybrid discretization strategies that combine the simplicity of finite differences with the robustness of spectral or variational approaches. (Odibat, 2026) (Smirnov, 2026) (Kumari et al., 2024) (Bhatter et al., 2025) (Alshammari et al.2025) (Chauhan, 2025) (Olayiwola et al., 2025) (Nyerere & Edward, 2025)

The fractional differential equation can be solved directly to obtain an integral equation that can be subsequently discretized. Indeed, for some forms of the spatial differential operator, including, for instance, the Laplacian, it has been established that the numerical solution converges to the solution of the underlying fractional diffusion equation. It is a property of the classical temporal diffusion equation that a f (t) function with a compact support on (0,1) leads to a solution that remains supported within that interval. This property is lost in the transition to the fractional framework and thus cannot be relied upon to simplify the numerical treatment. (Salama & Fairag, 2024) (Salama et al.2023) (Wang & Barkai2024) (Roul & Sundar, 2025) (Owolabi et al., 2024) (Noor et al.2024) (Roul & Rohil2023)

4.2. Spectral and Pseudo spectral Methods

Spectral and pseudospectral methods provide powerful tools for solving fractional Volterra–Fredholm equations, particularly when high accuracy is required. By reformulating operators into finite-dimensional sparse matrices, these approaches reduce computational complexity while maintaining precision. However, their effectiveness depends strongly on kernel smoothness: Chebyshev and Bernstein polynomial bases perform well for regular kernels, whereas Mittag–Leffler approximations are better suited for fractional dynamics. Comparative studies indicate that pseudospectral methods often outperform classical spectral schemes in terms of stability, especially when combined with Caputo derivatives. Nonetheless, the reliance on smooth kernels limits their applicability to irregular or singular problems. While spectral methods achieve superior convergence rates, they demand significant computational resources and careful grid design. A critical evaluation of accuracy versus cost is therefore necessary, and hybrid approaches that integrate spectral precision with meshless flexibility may offer promising future directions.

In contrast, fractional integral operators can be dealt with efficiently using time-splitting techniques. Practical implementations draw on symbolic software or package libraries to construct appropriate basis functions and grid configurations that enable minimal effort in dealing with non-regularity issues. (Abbasbandy, 2026) (Oloniiju et al., 2024) (Ali & Khan, 2023) (Smith, 2025) (Sahabi & Yazdani Cherati, 2024) (Hafez et al.2025) (Zhang et al., 2024) (Hannani & Ghaderi, 2025) (Ngueabou & Oloniiju2025) (Amal et al.2026)

4.3. Variational and Collocation Techniques

Variational and collocation techniques provide flexible frameworks for solving fractional Volterra–Fredholm equations, particularly when analytical solutions are unavailable. Variational methods reformulate the problem in functional spaces, offering theoretical guarantees of well-posedness, but their accuracy depends heavily on the choice of test functions and interpolation strategies. Collocation approaches, on the other hand, are easier to implement and yield efficient approximations, yet they often lack rigorous convergence proofs in irregular domains. Comparative studies show that variational methods excel in stability and theoretical grounding, whereas collocation techniques are computationally faster but more sensitive to kernel irregularities. The mixed Volterra–Fredholm scenarios further complicate analysis, as they require handling both spatial and temporal operators simultaneously. This trade-off underscores the need for hybrid frameworks that combine the stability of variational formulations with the efficiency of collocation schemes, especially for applications involving complex geometries or space-variant kernels.

arising in the wider literature, where increasingly rich physics consequently interacts with rich free-form geometry. (Harbi et al.2025) (Mahdy et al.2023) (Hamood et al., 2025) (Amin et al.2023) (Jalalian et al., 2025) (Sun et al., 2023) (Ali, 2026)

4.4. Meshless and Collocation-Based Strategies

Meshless and collocation-based strategies represent a promising direction for solving fractional Volterra–Fredholm equations, particularly when structured spatial meshes are impractical. These methods enhance

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flexibility by allowing irregular domains and complex boundary conditions to be handled more naturally. However, their advantages come at a cost: computational requirements are typically higher, and error analysis becomes more intricate compared to classical discretization schemes. Collocation methods based on Riemann–Liouville derivatives have shown efficiency in certain compact kernel problems, yet they struggle with nonsmooth kernels and large-scale systems. Comparative studies suggest that while meshless approaches are well-suited for engineering applications with irregular geometries, they lack the stability guarantees offered by spectral or variational methods. This trade-off emphasizes the need for hybrid frameworks that integrate meshless flexibility with the convergence properties of established numerical techniques, ensuring both accuracy and efficiency in real-world scenarios. (Jiang & Gao, 2024) (Halada et al.2025) (Harbi et al.2025) (Xue et al., 2025) (Çevik et al.2025) (Kansa, 2026) (Dutta & Das, 2025) (Aourir et al.2026)

CONVERGENCE, STABILITY, AND ERROR ANALYSIS: WHAT WORKS AND WHY

Convergence and stability remain central challenges in the numerical treatment of fractional Volterra–Fredholm equations. While Galerkin and collocation approaches show promise, their theoretical guarantees are incomplete, particularly for unbounded kernels and irregular domains. Refinements of Galerkin methods demonstrate improved convergence compared to standard techniques, but these gains are often problem-specific and lack general applicability. Adaptive mesh-free collocation strategies provide flexibility in handling non-local processes, yet their error bounds are not fully established. Comparative studies reveal that Galerkin methods offer stronger theoretical foundations, whereas collocation approaches are computationally more efficient but prone to instability in complex kernels. The absence of rigorous convergence proofs for multistage kernel problems underscores a critical research gap. A systematic evaluation of error behavior, stability conditions, and computational cost across these methods is therefore essential to guide their practical use and to identify opportunities for hybrid or adaptive frameworks., hence work remains to establish reliable quantitative performance bounds. (Round, 2024) (Jiang & Gao, 2024) (Salman2024) (Sreelakshmi et al., 2025) (Weng et al., 2026) (Hafez et al.2026) (Dozva et al.2026) (Krishnan et al.2025) (Aronson & Evans, 2024)

COMPUTATIONAL EFFICIENCY AND IMPLEMENTATION CONSIDERATIONS

Computational efficiency is a decisive factor in the practical use of numerical methods for fractional Volterra–Fredholm equations. While discretization transforms the problem into a solvable system, the choice of solver and implementation strategy determines feasibility in real-world applications. Finite-difference schemes are computationally inexpensive but scale poorly with multidimensional problems. Spectral methods achieve superior accuracy yet demand high memory and careful parallelization to remain efficient. Iterative and hybrid approaches reduce complexity but may sacrifice stability. Comparative evaluations show that time complexity per step, parallelization potential, and memory requirements vary significantly across methods, making direct benchmarking essential. For example, meshless collocation strategies excel in irregular domains but incur higher computational costs compared to structured spectral schemes. A systematic comparison of efficiency, scalability, and solver requirements is therefore critical to guide researchers in selecting methods that balance accuracy with computational feasibility.

(L. MacDonald et al., 2015), (Diethelm, 2021). (Partelow, 2023) (Eaton et al.2023) (Li et al., 2024) (Ediagbonya & Tioluwani, 2023) (Raman et al.2025) (Saad et al.2025) (Zhang et al.2024) (Pang et al., 2023).

Simplified Comparison of Numerical Methods

Method	Advantages	Limitations	Typical Applications
Finite Difference (Grünwald–Letnikov / Caputo)	Easy to implement, suitable for initial-value problems	Sensitive to mesh size, limited accuracy with irregular kernels	Basic models, time-fractional problems
Spectral / Pseudospectral	High accuracy, fast convergence	Requires smooth kernels, high computational cost	Physics simulations, precise scientific modeling

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Method	Advantages	Limitations	Typical Applications
Variational Methods	Strong theoretical foundation, stable	Dependent on choice of test functions, complex in irregular domains	Boundary-value problems, theoretical studies
Collocation Techniques	Simple implementation, computationally efficient	Weak convergence guarantees, sensitive to kernel irregularities	Engineering problems, applied scenarios
Meshless Approaches	Flexible for irregular domains, adaptable	High computational cost, complex error analysis	Bioengineering, structural engineering
Hybrid Methods (Proposed)	Combine accuracy and flexibility	Still under development	Future multidimensional and complex applications

APPLICATIONS ACROSS SCIENCE AND ENGINEERING

Applications of fractional Volterra–Fredholm equations span physics, biology, engineering, and even social sciences, reflecting their versatility in modeling systems with hereditary and memory effects. Compared to classical integer-order models, fractional formulations capture long-term dynamics with greater accuracy, making them particularly valuable in viscoelasticity, diffusion, and biological processes. In engineering, they support advanced tasks such as damage detection, sensor-fault diagnosis, and structural health monitoring. However, despite these successes, practical implementation remains challenging. Many applications demand numerical methods that balance accuracy with computational feasibility, especially in multidimensional or data-intensive settings. Comparative studies reveal that while spectral methods excel in scientific simulations requiring precision, meshless and collocation approaches are better suited for irregular engineering domains. This diversity of applications underscores the importance of structured evaluations of numerical techniques, ensuring that method selection is guided by both domain-specific requirements and computational constraints.

in various application classes. Examples covering the fields of science and engineering demonstrate how options in algorithms and numerical methods significantly affect modeling capability and decisions when dealing with physical partial differential equations and data emerge from physical experiments. (Almhdy et al., 2026) (Raghavendran et al.2024) (Syam & Hashim2024) (Heydari et al.2026) (Mohseni & Rostamy, 2025) (Zada et al., 2026) (Ali et al.2024) (Bera et al., 2026) (Kumar & Tripathi, 2026)

CHALLENGES, GAPS, AND OPPORTUNITIES FOR FUTURE RESEARCH

Despite significant progress, several unresolved challenges continue to limit the advancement of fractional Volterra–Fredholm integro-differential equations. At the modeling level, capturing the interplay between physical phenomena and fractional memory effects remains complex, particularly in multidimensional systems. At the numerical level, rigorous analysis of convergence and stability for nonlinear operators is still incomplete, leaving many methods without theoretical guarantees. Furthermore, the scarcity of comparative studies across different numerical frameworks creates uncertainty about their relative strengths and weaknesses. Future research should prioritize: (1) developing hybrid algorithms that combine spectral accuracy with meshless flexibility, (2) establishing error bounds for irregular and unbounded kernels, and (3) exploring data-driven or machine-learning-assisted solvers to reduce computational cost. Addressing these gaps will not only strengthen theoretical foundations but also expand the applicability of fractional models in engineering, biology, and computational sciences.

(Zhou, 2015). (Syam & Hashim2024) (Yadav & Mohapatra, 2026) (Hamoud et al.2023) (Raghavendran et al.2024) (Gunasekar et al.2024) (Filali et al., 2025) (Hamood et al., 2026) (Kumar & Gupta, 2023)

CONCLUSION

Recent advances in numerical methods for fractional Volterra–Fredholm equations have significantly expanded their applicability across physics, biology, and engineering. Fractional calculus operators remain powerful modeling tools, yet their numerical implementation continues to face challenges related to convergence, stability, and computational efficiency. While innovative techniques and benchmark studies confirm the feasibility of solving complex models, the review highlights that no single method offers a universally optimal solution. Finite-difference schemes provide simplicity but limited accuracy, spectral methods achieve superior precision at high computational cost, and meshless or collocation approaches offer flexibility but lack rigorous error guarantees. A critical comparative perspective is therefore essential: future progress will depend on hybrid frameworks that integrate the strengths of different methods, alongside systematic evaluations of efficiency and accuracy. By emphasizing unresolved challenges and opportunities, this review aims to guide researchers toward more robust and versatile solution strategies for fractional Volterra–Fredholm integro-differential equations.

work for describing phenomena exhibiting memory or hereditary characteristics. Fractional-order derivatives and integrals account for incomplete information from past states and offer flexible ways to represent temporal or spatial non-locality. As a result, ordinary and partial fractional-order differential equations are actively studied. Volterra–Fredholm integro-differential scenarios represent an important subclass, further generalizing the integer-order theory and catering to difficult fractional models. The corresponding integral-differential formulations of the desired equations either contain a derivative of order between zero and one or involve fractional integral operators combined with non local terms. Essential input data can be a single initial value (Volterra-type) or a combination of initial and boundary specifications (Fredholm-type). Knowledge of the system state at one moment determines the subsequent evolution of the process, which materially distinguishes the governing equations from standard delay or other integral-differential equations.The fractional calculus of variations focuses on the fractional integral of a functional involving time derivatives of order less than or equal to one and time solely entering via a fractional derivative of order between zero and one; the corresponding models thus constitute state-of-the-art formulations for Volterra and Volterra–Fredholm systems in a broader context.Models are omnipresent in science and engineering, yet knowledge of modeling guidelines and pragmatic advice for solving fractional Volterra–Fredholm integro-differential equations is spread across disparate publications. To promote awareness and encourage adoption of promising techniques, a review integrating the relevant concepts and providing illuminating examples was deemed welcome. (Garrappa & Popolizio, 2013) (M. Mustafa & N. Ghanim, 2014) (Zhou, 2015)

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