Modified Darcy-Brinkmann Two-Component Marangoni Convection: Stability in the Presence of Dufour and Magnetic Effects within a Two-Layer Composite System with Free-Free Boundaries

[1*]Shivaraja. J. M, [2]Meenakshi, [3]Poojashree. Y, [4]Jyothika M. J, [5]Mounika. B, [6]Pavan Kumar K H

[1*] Faculty, Department of Mathematics, Suvarna Gange Campus, Mangasandra, Kolar

[2,3,4,5&6] PG Mathematics students, Suvarna Gange Campus, Mangasandra, Bengaluru North University, Tamaka, Kolar

[*]Corresponding Author

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ABSTRACT

An analytical investigation is carried out to examine the onset of modified Darcy–Brinkmann two-component Marangoni convection in the presence of Dufour and magnetic effects within a two-layer fluid–porous composite system subjected to free-free boundary conditions. The system consists of a fluid layer overlying a sparsely packed porous layer saturated with the same fluid. The surface tension is assumed to depend on both temperature and concentration variations, while Dufour cross-diffusion and magnetohydrodynamic effects are incorporated into the governing model. Using linear stability theory, an exact analytical expression for the critical thermal Marangoni number is derived. The influences of the Darcy number, magnetic parameter, Dufour parameter, diffusivity ratios, viscosity ratio, and solutal Marangoni number on the onset of convection are examined in detail. The results reveal that magnetic effects, diffusivity ratios, and wave number stabilize the system, whereas Darcy number and solutal Marangoni effects promote instability. Furthermore, the Dufour effect significantly modifies the instability threshold through diffusion–thermal coupling. The present study provides new insight into thermocapillary transport phenomena in composite fluid–porous systems relevant to crystal growth, material processing, geothermal engineering, and energy technologies.

Keywords: Modified Darcy–Brinkman model, Magneto hydrodynamics, Two-layer composite system, Dufour effect, Double Diffusive Convection, Marangoni Convection.

INTRODUCTION

Two-component convection in fluids, caused by interacting temperature and concentration gradients that diffuse at different rates, plays a significant role in several engineering, industrial, and geophysical applications. Marangoni convection occurs due to variations in surface tension induced by thermal and solutal gradients, accompanied by simultaneous heat and mass transfer within the fluid system. In a two-layer composite configuration comprising a fluid layer overlying a porous medium, the interaction among thermal, solutal, magnetic, and porous medium effects becomes increasingly complex, particularly under non-Darcy flow conditions and free-free boundary configurations.

The coupling between heat and mass transport through thermodynamic interactions leads to the diffusion–thermal phenomenon, commonly referred to as the Dufour effect, where concentration gradients generate an additional heat flux. Moreover, the application of an external magnetic field alters the flow dynamics and stability behavior through hydromagnetic forces acting on the conducting fluid. Considerable research has been devoted to Marangoni convection influenced by Dufour diffusion and magnetic fields in fluid layers, porous media, and boundary layer flows. Nevertheless, the stability analysis of non-Darcy two-component Marangoni convection in the presence of combined Dufour and magnetic effects within a two-layer composite system with free-free boundaries remains an important topic of study because of its wide relevance in crystal growth technologies, material processing, geothermal energy systems, and chemical engineering processes.

A brief literature on Darcy Brinkmann Double Diffusive Marangoni convection with diffusion thermal effect for different flows like boundary layer flows, convection flows in a two-component fluid/porous layer is available. Chen and Chen (2006) investigated convection in superposed fluid and porous layers and demonstrated the importance of interface conditions and porous medium properties on the onset of instability. Their work provided a theoretical basis for analysing convection in composite fluid–porous systems involving Darcy and non-Darcy effects. Shivakumara et al. (2010) studied Brinkman–Bénard–Marangoni convection in a magnetized ferrofluid-saturated porous layer and showed that the applied magnetic field suppresses convection by stabilizing the system. Their study highlighted the influence of hydromagnetic forces on thermocapillary instability in porous media. Manjunatha and Sumithra (2019) analyzed the effects of non-uniform temperature gradients on triple-diffusive Marangoni convection in a composite layer system and reported that thermal gradients and porous medium properties significantly affect the stability characteristics of convection. Further, Manjunatha et al. (2021) investigated Darcy–Bénard double-diffusive Marangoni convection in a composite fluid–porous system with internal heat generation and non-uniform temperature gradients, emphasizing the role of Darcy number, porosity, and heat source parameters in controlling the onset of convection. Yellamma et al. (2023) examined Brinkman–Bénard triple-diffusive magneto-Marangoni convection in a two-layer system and concluded that magnetic fields generally delay the onset of convection, whereas diffusion-related mechanisms may destabilize the system depending on the governing parameters. Srinivasacharya and Surender investigated non-Darcy mixed convection in doubly stratified porous media with Soret–Dufour effects and demonstrated that diffusion–thermal coupling significantly modifies heat and mass transfer characteristics in porous systems. Arifin and Pop (2009) studied the stability of Marangoni convection in a composite porous–fluid system with finite conductivity boundaries using linear stability analysis and showed that thermal conductivity ratios and boundary conditions strongly influence the critical Marangoni number. Nield and Bejan (2017) provided a comprehensive foundation on convection in porous media, highlighting how Darcy and non-Darcy models govern flow behavior in porous structures. Their work established the significance of permeability and inertial effects in the stability analysis of layered systems. Vafai (2015) discussed transport phenomena in porous media and emphasized the importance of coupled heat and mass transfer effects, including Dufour and Soret mechanisms, in engineering and geophysical applications. Pearson (1958) presented the classical theory of Marangoni convection driven by surface tension gradients caused by temperature variations, and his pioneering work laid the foundation for later studies on thermo capillary instability in fluid and composite systems.

Novelty Statement

To the best of the authors' knowledge, a comprehensive analytical investigation of modified Darcy–Brinkmann two-component Marangoni convection incorporating simultaneous Dufour and magnetic effects in a two-layer fluid–porous composite system with free-free boundaries has not been reported. The present work derives an exact closed-form expression for the critical thermal Marangoni number and systematically examines the influence of permeability, magnetic field strength, Dufour diffusion, diffusivity ratios, and solutal Marangoni effects on the onset of convection.

Mathematical modeling

The physical configuration considered in the present study consists of a horizontal two-component composite layer system comprising an incompressible, isotropic fluid layer of thickness d_{fj} overlying a sparsely packed porous layer of thickness d_{mj} , saturated with the same fluid. Both the upper and lower boundaries are assumed to be free, non-deformable, thermally insulated, and solutally insulated. The interface separating the fluid and porous layers is treated as permeable and satisfies the appropriate continuity conditions. A Cartesian coordinate system is adopted with the origin located at the fluid–porous interface and the (z)-axis directed vertically upward. The Boussinesq approximation is employed, whereby density variations are considered only in the buoyancy terms arising from temperature and concentration differences. Under these assumptions, the governing equations for continuity, momentum, energy, concentration transport, and magnetic induction are formulated for both regions.

For fluid layer,

Continuity Equation :

$$\nabla_{fi} \cdot \overrightarrow{q_{fi}} = 0 \quad (1)$$

Solenoidal Property of the Magnetic Field:

$$\nabla_{fi} \cdot \vec{H}_{fi} = 0 \quad (2)$$

Momentum Equation :

$$\rho_{0_{fi}} \left[\frac{\partial \overrightarrow{q_{fi}}}{\partial t_{fi}} + \left(\overrightarrow{q_{fi}} \cdot \nabla_{fi} \right) \overrightarrow{q_{fi}} \right] = -\nabla_{fi} P_{fi} + \mu_{fi} \nabla_{fi}{}^{2} \overrightarrow{q_{fi}} + \mu_{fp} (\vec{H}_{fi} \cdot \nabla_{fi}) \quad (3)$$

Energy Equation :

$$\frac{\partial T_{fi}}{\partial t_{fi}} + \left(\overrightarrow{q_{fi}} \cdot \nabla_{fi} \right) T_{fi} = k_{fi} \nabla_{fi}^{2} T_{fi} + k_{Tfi} \nabla_{fi}^{2} c_{fi} \quad (4)$$

Concentration Equation for Species :

$$\frac{\partial c_{fi}}{\partial t_{fi}} + \left(\overrightarrow{q_{fi}} \cdot \nabla_{fi} \right) c_{fi} = k_{cfi} \nabla_{fi}{}^{2} c_{fi} \quad (5)$$

Equation of Magnetics Induction Equation:

$$\frac{\partial \vec{H}_{fi}}{\partial t_{fi}} = \nabla_{fi} \times \overrightarrow{q_{fi}} \times \vec{H}_{fi} + v_{fi} \nabla_{fi}{}^{2} \vec{H}_{fi} \quad (6)$$

For porous layer,

Continuity Equation :

$$\nabla_{mi} \cdot \overrightarrow{q_{mi}} = 0 \quad (7)$$

Solenoidal Property of the Magnetic Field:

$$\nabla_{mi} \cdot \vec{H}_{mi} = 0 \quad (8)$$

Momentum Equation:

$$\left(\frac{\rho_{0mi}}{\phi_{mi}} \right) \frac{\partial \overrightarrow{q_{mi}}}{\partial t_{mi}} = -\nabla_{mi} p_{mi} - \left(\frac{\mu_{mi}}{K_{mi}} \right) \overrightarrow{q_{mi}} + \mu_{mj} \nabla_{mj}^{2} \overrightarrow{q_{mj}} + \mu_{mp} (\vec{H}_{mi} \cdot \nabla_{mi}) \quad (9)$$

Energy Equation :

$$A_{mi} \frac{\partial T_{mi}}{\partial t_{mi}} + (\overrightarrow{q_{mi}} \cdot \nabla_{mi}) T_{mi} = k_{mi} \nabla_{mi}^{2} T_{mi} + k_{T_{mi}} \nabla_{mi}^{2} c_{mi} \quad (10)$$

Concentration Equation for Species :

$$\phi_{mi} \frac{\partial c_{mi}}{\partial t_{mi}} + (\overrightarrow{q_{mi}} \cdot \nabla_{mi}) c_{mi} = k_{mi} \nabla_{mi}^{2} c_{mi} \quad (11)$$

Equation of Magnetics Induction Equation:

$$\frac{\partial \vec{H}_{mi}}{\partial t_{mi}} = \nabla_{mi} \times \overrightarrow{q_{mi}} \times \vec{H}_{mi} + v_{mi} \nabla_{mi}^{2} \vec{H}_{mi} \quad (12)$$

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Notations:

$\overrightarrow{q_{\text{fi}}} = (u_{fi}, v_{fi}, w_{fi})$: velocity vector in the fluid layer

$\overrightarrow{q_{\text{mi}}}$: velocity vector in the porous layer

t_{fi}, t_{mi}: time

P_{fi}, P_{mi}: pressures in the fluid and porous layers

$\rho_{0_{fi}}$: reference fluid density

ρ_{fi}, ρ_{mi}: fluid and porous layer densities

$\mu_{f\,i}, \mu_{mi}$: dynamic viscosities in the fluid and porous layers

T_{fi}, T_{mi}: temperatures in the fluid and porous layers

C_{fi}, C_{mi}: solute concentrations in the fluid and porous layers

$k_{fi},\ k_{mi}$: thermal diffusivities in the fluid and porous layers

k_{cfi}, k_{cmi}: solutal diffusivity

$k_{Tfi,} k_{cmi}$: Dufour coefficient

α_T: thermal expansion coefficient

α_c: solutal expansion coefficient

$\emptyset_{mi}$: porosity of the porous medium

k_{mi} permeability of the porous medium

$A_{mi} = \frac{(\rho_0 C_p)_m}{(\rho c_p)_f}$: ratio of heat capacities

$c_p: Speci$fic heat

$P_{fi} = p_{fi} + \frac{\mu_{pi} H_{fi}^2}{2}$:total pressure

$v_{fi} = \frac{1}{\mu_p \sigma}$: magnetic viscosity

$v_{mi} = \frac{v_f}{\emptyset}$: effective magnetic viscosity in the porous medium

Stability Analysis

Consider the basic state, in the composite layer system the fluid layer is stable and also assumed it is inactive, means quiescent. Therefore there is no motion, so, set the velocity vector $\overrightarrow{q_{fi}}$ is zero, where, the mass and heat are transferred only by conduction.

For the fluid layer, basic state is mathematically represented as follows:

$$[\overrightarrow{q_{fi}}, p_{fi}, T_{fj}, c_{fi}, H_{fi}] = [0, p_{fib}(z_{fi}), T_{fib}(z_{fi}), c_{fib}(z_{fi}), H_{fib}(z_{fi})]$$

For the porous layer, basic state is mathematically represented as follows:

$$[\overrightarrow{q_{mi}}, p_{mi}, T_{mi}, c_{mi}, H_{mi}] = [\,0, p_{mib}(z_{mi}), T_{mib}(z_{mi}), c_{mib}(z_{mi}), H_{mib}(z_{mi})]$$

For both fluid and porous layer, the temperature distribution and concentration distributions are $T_{fib}(z_{fi})$, $T_{mib}(z_{mi})$and$C_{fib}(z_{fi})$,$C_{mib}(z_{mi})$respectively are mentioned in below table

Distributions	Fluid layer	Porous layer
Temperature distribution	$T_{fib}(z_{fi}) = T_0 + \left(\frac{(T_u - T_0)}{d_{fi}}\right) z_{fi}$	$T_{mib}(z_{mi}) = T_0 + \left(\frac{(T_0 - T_L)}{d_{mi}}\right) z_{mi}$
Species distribution	$C_{fib}(z_{fi}) = C_{fi0} + \left(\frac{(C_{fiu} - C_{fi0})}{d_{fi}}\right) z_{fi}$	$C_{mib}(z_{mi}) = C_{mi0} + \left(\frac{(C_{mi0} - C_{miL})}{d_{\mathrm{mi}}}\right) z_{mi}$

Where

$$T_0 = \begin{Bmatrix} \dfrac{k_{fi}\, d_{mi} T_u + d\, k_{mi}\, T_L}{k_{fi} d_{mi} + d_{fi}\, k_{mi}} + \dfrac{d_{mi}\, k_{fiT}\, C_{fiu} + k\, T_m\, d\, C_{mil}}{k_{fi} d_{mi} + d_{fi}\, k_{mi}} \\ -C_{fi0}\left(\dfrac{d_{fi} k_{fi} T_{mi} + d_{ij}\, k_{fiT}}{k_{fi} d_{mi} + d_{fi} k_{mi}}\right) \end{Bmatrix}$$

$$C_{fi0} = \frac{k_{mi}\, C_{fiu} d_{mi} + d_{fi}\, k_{mi}\, C_{miL}}{d_{fi}\, k_{mi} + d_{mi}\, k_{mi}}$$

are the interface temperature and concentrations respectively. It is evident that the interface temperature depends on interface concentration due to Dufour effects. To analyze the stability of the basic state solution, an infinitesimal perturbations are introduced and the perturbed quantities are of the form.

For fluid layer

$$[\overrightarrow{q_{fi}}, P_{fi}, T_{ij}, c_{fi}, \overrightarrow{H_{fi}}] = \begin{Bmatrix} [\,0,\ P_{fib}(z_{fi}), T_{fib}(z_{fi}), C_{fib}(z_{fi}), \overrightarrow{H_{fib}}] \\ +\ [\overrightarrow{q_{fi}}', P'_{fi}, \theta'_{fi}, S'_{fi}, \overrightarrow{H_{fi}}'] \end{Bmatrix}$$

For porous layer

$$[\overrightarrow{q_{mi}}, P_{mi}, T_{mi}, c_{mi}, \overrightarrow{H_{mi}}] = \begin{Bmatrix} [\,0,\ P_{mib}(z_{mi}), T_{mib}(z_{mi}), C_{mib}(z_{mi}), \overrightarrow{H_{mib}}] \\ +\ [\overrightarrow{q_{mi}}', P'_{mi}, \theta'_{mi}, S'_{mi}, \overrightarrow{H_{mi}}'] \end{Bmatrix}$$

Substitute above perturbations into equations $(1) - (12)$ and apply two times curl on the momentum equations to eliminate the pressure term. The resulting equations are linearized for $(1) - (12)$, then they are non-dimensionalised using the following variables $\frac{{d_{fi}}^2}{k_{fi}}$, $\frac{k_{fi}}{d_{fi}}$, $C_{fiu} - C_{fi0}$, $d_{fi}$and$T_0 - T_U$, for time, velocity, species, length and temperature respectively in region – 1 and $\frac{d_m^2}{k_m}$, $\frac{k_m}{d_m}$, $C_{mi0} - C_{miL}$, d_m and $T_L - T_0$ are the corresponding scale factors in region – 2. Applying normal mode expansions to the perturbed non-dimensionalised quantities (following Sumithra [10]) result in the following ordinary differential equations,

For Fluid layer,

$$\left(D_{fi}^2 - a_{fi}^2\right)^2 W_{fi} = Q_{ij} D^2{}_{fi} W_{fi} \tag{13}$$

$$\left(D_{fi}^2 - a_{f_i}^2\right)\Theta_{fi} + W_{fi} f\left(z_{fi}\right) + \mathrm{d_{uf}}\left(D_{fi}^2 - a_{fi}^2\right)S_{fi} = 0 \tag{14}$$

$$\tau_{1fi}\left(D_{fi}^2 - a_{fi}^2\right)s_{fi} + W_{fi} = 0 \tag{15}$$

For Porous layer,

$$[\hat{\mu}\beta_{mi}^2(D_{mi}^2 - a_{mi}^2) - 1](D_{mi}^2 - a_{mi}^2)W_{mi} = 0 \tag{16}$$

$$(D_{mi}^2 - a_{mi}^2)\Theta_{mi} + W_{mi} g_{mi}(z_{mi}) + d_{um}(D_{mi}^2 - a_{mi}^2)S_{mi} = 0 \tag{17}$$

$$\tau_{1mi}(D_{mi}^2 - a_{mi}^2)S_{mi} + W_{mi} = 0 \tag{18}$$

Where, for fluid layer, $p_r = \frac{\nu}{k_{fi}}$is the Prandtl number, $\tau = \frac{k_{cfi}}{k_{fi}}$ is the ratio of species diffusivity to thermal diffusivity, $\nu = \frac{\mu_{fi}}{\rho_{0_{fi}}}$ is the kinematic viscosity and $d_{uf} = \frac{k_{{fi}_T}\left(C_{{fi}_0} - C_{fiu}\right)}{k_{fi}\,(T_0 - T_u)}$ is Dufour coefficient. For the porous layer, $p_{rm} = \frac{\phi\nu}{k_m}$ is the Prandtl number, $\beta_{mi}^2 = \frac{K_{mi}}{d_{mi}^2} = D_a$ is the Darcy number, $\tau_{1mi} = \frac{k_{mi}}{k_{mi}}$ is the ratio of species diffusivity to thermal diffusivity, $d_{um} = \frac{k_{miT}(C_{miL} - C_{0mi})}{k_{mi}\,(T_L - T_0)}$ is the Dufour coefficient and $\hat{\mu} = \frac{\mu_{mi}}{\mu_{fi}}$ is the viscosity ratio.

Boundary conditions

At the upper boundary.

$$W_{fi}(1) = 0, \qquad D_{fi}\Theta_{fi}(1) = 0, \quad D_{fi}S_{fi}(1) = 0,$$

$$W_{fi}(1) - M_{fi}a^2{}_{fi}\,\Theta_{fi}(1) - M_{sfi}\,a^2{}_{fi}\,S_{fi}(1) = 0$$

At the interface

$$W_{fi}(0) = \frac{\varsigma}{\varepsilon_T} W_{mi}(0), \qquad S_{fi}(0) = \frac{\varepsilon_s}{\varsigma} S_{mi}(0), \qquad \Theta_{fi}(0) = \frac{\varepsilon_T}{\varsigma}\Theta_{mi}(0),$$

$$D_{fi}\Theta_{fi}(0) = D_{mi}\Theta_{mi}(0),\ D_{fi}S_{fi}(0) = D_{mi}S_{mi}(0),$$

$$D_{fi}W_{fi}(0) = \frac{\varsigma^2}{\varepsilon_T} D_{mi}W_{mi}(0),\ D_{fi}(0) = D_{mi}S_{mi}(0)$$

$$\left[D^3{}_{fi} - 3a^2{}_{fi}\,D_{fi}\right]W_{fi}(0) = \frac{-\varsigma^2}{D_a\varepsilon_T} D_{mi}W_{mi}(0) + \frac{\varsigma^4}{\varepsilon_T}[D_{mi}^3 3a_{mi}^2 D_{mi}]W_{ij}(0)$$

At the lower boundary

$$W_{mi}(-1) = 0,\ D_{mi}W_{mi}(-1) = 0, \quad D_{mi}\Theta_{mi}(-1) = 0, \quad D_{mi}S_{mi}(-1) = 0$$

Where

$M = -\frac{\partial\sigma}{\partial T}\frac{(T_0-T_u)d_{fi}}{\mu_{k_{fi}}}$, $M_s = -\frac{\partial\sigma}{\partial s}\frac{(C_{fi0}-C_{fiu})d_{fi}}{\mu_{k_{fi}}}$ are the thermal and solute Marangoni numbers, respectively. $\varepsilon_T = \frac{k_{fi}}{k_{fi}} = \frac{1}{\tau_{1fi}}$, $\varepsilon_s = \frac{k_{mi}}{k_{mi}} = \frac{1}{\tau_{1mi}}$ are the ratios of salinity diffusivity to thermal diffusivity for both fluid and porous layers respectively and $\varsigma = \frac{d_{fi}}{d_{mi}}$ is the depth ratio.

Solution by Exact Method

Velocity distribution for fluid and porous layer

The final solutions of the differential equations (13) and (16) are not depend on $\Theta_{fi}(z_{fi})$, $\Theta_{mi}(z_{mi})$, $S_{mi}(z_{mi})$. The expressions for w_{fi} and w_{mi} are solved by using the velocity boundary conditions and are obtained as below,

$$W_{fi}(z_{fi}) = A_1 \left\{ \begin{matrix} [Cosh([m_1 z_{fi}]) + C_2 Sinh(m_1 \ z_{fi})] + \\ [C_3 Cosh(m_2 z_{fi}) + C_4 \ Sinh(m_2 \ z_{fi})] \end{matrix} \right\}$$

And

$$W_{mi}(z_{mi}) = B_1 \ \mathrm{Cosh}(a_{mi} z_{mi}) + B_2 \ \mathrm{Sinh}(a_{mi} z_{mi}) + B_3 \ \mathrm{Cosh}(\delta z_{mi}) + B_4 \mathrm{Sinh}(\delta z_{mi})$$

Where

$$C_2 = \frac{-D_2 D_7 D_4}{D_3 D_8 D_6} - \frac{D_5 D_{10}}{\delta \ D_6 \ Cosh[\delta]} - B_3 \left[\frac{D_1 D_7 D_4}{D_3 D_8 D_6} - \frac{D_9 D_4}{D_8 D_6} + \frac{D_5 D_4}{D_6 \ \delta \ Cosh[\delta]}\right]$$

$$C_3 = B_3 \left(\frac{\varsigma \ D_1}{\epsilon_T D_3} + \frac{\varsigma}{\epsilon_T}\right) + \frac{\varsigma \ D_2}{\epsilon_T D_3} - 1, \qquad B_1 = B_3 \left(\frac{D_1}{D_3} + \frac{D_2}{D_3}\right)$$

$$C_4 = (-C_2 \ Sinh[m_1] - C_3 \ Cosh[m_2] - Cosh[m_1]) \left(\frac{1}{Sinh[m_2]}\right)$$

$$B_2 = -\frac{D_2 D_7}{D_3 D_8} - B_3 \left[\frac{D_1 D_7}{D_3 D_8} - \frac{D_9}{D_8}\right], B_4 = -\frac{D_{11} B_3}{\delta \ Cosh[\delta]} - \frac{D_{10}}{\delta \ Cosh[\delta]}$$

$$B_3 = \frac{(D_1/D_3) - C_3\left(\frac{\epsilon_T}{\varsigma}\right) - \left(\frac{\epsilon_T}{\varsigma}\right)}{1 - (D_1/D_3)}, D_1 = \frac{\varsigma}{\varepsilon T} - \frac{E_5}{E_3}, D_2 = \frac{E_2}{E_3} - 1, D_3 = \frac{E_4}{E_3} - \frac{\varsigma}{\varepsilon T}$$

$$D_4 = \frac{\varsigma^2 a_{mi} E_9}{\varepsilon T} - m_2 E_{10}, \ \ D_5 = \frac{\varsigma^2 \delta E_9}{\varepsilon T} - m_2 E_{11}, \ \ D_6 = m_1 E_9 - m_2 E_8,$$

$$D_7 = \delta \ Cosh[\delta] Cosh[\ a_{mi}] - a_{mi} \ Sinh \ [a_{mi}] \ Sinh[\delta],$$

$$D_8 = a_{mi} Cosh[a_{mi}] Sinh[\delta] - Sinh[a_{mi}] \ \delta \ Cosh[\delta],$$

$$D_9 = \delta (Cosh[\delta])^2 - \delta (Sinh[\delta])^2, \ D_{10} = -\frac{a_{mi} \ Sinh[a_{mi}] D_2}{D_3} - \frac{a_{mi} Cosh[a_{mi}] D_2 D_7}{D_3 D_8}$$

$$D_{11} = -\frac{a_{mi} Sinh[a_{mi}] D_1}{D_3} - \frac{a_{mi} Cosh[a_{mi}] D_1 D_7}{D_3 D_8} + \frac{a_{mi} Cosh[a_{mi}] D_9}{D_8} - \delta \ Sinh[\delta]$$

$$D_{12} = -\frac{D_2 D_7 D_4}{D_3 D_8 D_6} - \frac{D_5 D_{10}}{\delta D_6 \ Cosh[\delta]}, D_{13} = \frac{D_1 D_7 D_4}{D_3 D_8 D_6} - \frac{D_9 D_4}{D_8 D_6} + \frac{D_5 D_4}{D_6 \delta \ Cosh[\delta]}$$

Species Concentration distribution for fluid and porous layer

The concentration distributions $S_{fi}(z_{fi})$ and $S_{mi}(z_{mi})$are getting from the Differential Equations (14) and (17) by substituting the algebraic expressions for $W_{fi}(z_{fi})$ and$W_{mi}(z_{mi})$and are solved using the concentration boundary conditions and are as below.

$$S_{fi}(z_{fi}) = C_5 \operatorname{Cosh}[a_{fi} z_{fi}] + C_6 \operatorname{Sinh}[a_{fi}\, z_{fi}] + F_1(z_{fi}) + F_2(z_{fi})$$

And

$$S_{mi}(z_{mi}) = B_6 \operatorname{Cosh}[a_{mi} z_{mi}] + B_7 \operatorname{Sinh}[a_{mi} z_{mi}] + G_1(z_{mi}) + G_2(z_{mi})$$

Where

$$F_1(z_{fi}) = -\frac{1}{\tau_{1fi}({m_1}^2 - {a_{fi}}^2)}\left(Cosh[m_1\, z_{fi}] - C_2\, Sinh[m_1\, z_{fi}]\right)$$

$$F_2(z_{fi}) = -\frac{1}{\tau_{1fi}({m_2}^2 - {a_{fi}}^2)}\left(\, C_3\, Cosh[m_2\, z_{fi}] - C_4\, Sinh[m_2\, z_{fi}]\right)$$

$$G_1(z_{mi}) = -\frac{z_m}{2a_{fi}\tau_{1m_i}}\left[B_2 Cosh[a_{mi}\, z_{mj}] + B_1 Sinh[a_{mi}\, z_{mj}]\right]$$

$$G_2(z_{mi}) = -\frac{B_3}{\tau_{1mi}(\delta^2 - a_{fi}^2)} Cosh[a_{mi} z_{mi}] - \frac{B_4}{\tau_{1_{mi}}\,(\delta^2 - a_{fi}^2\,)} Sinh[a_{mi} z_{mi}]$$

Where

$$C_5 = \frac{\varepsilon_s}{\varsigma} B_6 + E_{3,}\;\; C_6 = \frac{1}{a_{fi} Cosh[a_{fi}]}[-E_5 B_6 - E_7],\quad B_7 = \frac{a_{fi}}{a_{mi}} c_6 + E_4$$

$$B_6 = \frac{-a_{fi}\; Cosh[a_{mi}] E_7 + a_{fi}\; Cosh[a_{mi}]\; E_8}{[E_5\; a_{fi}\; Cosh[a_{mi}] + a_{fi}\; a_{mi}\; Cosh[a_{fi}]\; Sinh[a_{mi}]]}$$

$$E_3 = -(\frac{\varepsilon_s}{\zeta}) \frac{B_3}{\tau_{1fi}(\delta^2 - a_{fi}^2)} + \frac{1}{\tau_{fi}(m_1^2 - a_{fi}^2)} + \frac{C_3}{\tau_{1fi}\,(m_2^2 - a_{fi}^2)}$$

$$E_4 = -\frac{m_1 C_2}{\tau_{1fi} a_{mi}(m_1^2 - a_{fi}^2)} - \frac{m_2 C_4}{\tau_{1fi} a_m (m_2^2 - a_{fi}^2)} + \frac{B_2}{2a_{fi}\tau_{1fi} a_{mi}} + \frac{B_4}{\tau_{1fi}(\delta^2 - a_{fi}^2)}$$

$$E_5 = \frac{a_{fi} Sinh(a_{fi})\varepsilon_s}{\zeta}, E_7 = a_{fi} Sinh(a_{fi}) E_3 - E_1, E_8 = a_{mi} Cosh(a_{mi}) E_4 + E_2$$

$$E_1 = -\frac{m_1 Sinh(m_1)}{\tau_{1fi}(m_1^2 - a_{fi}^2)} - \frac{m_1 C_2 Cosh(m_1)}{\tau_{1fi}(m_1^2 - a_{fi}^2)} - \frac{m_2 C_3 Sinh(m_2)}{\tau_{1fi}(m_2^2 - a_{fi}^2)} - \frac{m_2 C_4 Cosh(m_2)}{\tau_{1fi}(m_2^2 - a_{fi}^2)}$$

$$E_2 = \frac{1}{2a_{fi}\tau_{1fi}}[-a_{mi} B_2 Sinh(a_{mi}) + a_{mi} B_1 Cosh(a_{mi})] + \frac{1}{2a_{fi}\tau_{1fi}}[B_2 Cosh(a_{mi})]$$

$$E_{01} = [B_2 Sinh(a_{mi})] + \frac{a_{mi} B_3}{\tau_{1_{fi}}(\delta^2 - a_{fi}^2)} Sinh(a_{mi}) - \frac{a_{mi} B_4}{\tau_{1fi}(\delta^2 - a_{fi}^2)} Cosh(a_{mi})$$

Temperature distribution for fluid and porous layer

The temperature distributions$\Theta_{fi}(z_{fi})$and$\Theta_{mi}(z_{mi})$are getting from the Differential Equations (15) and (19) by substituting the algebraic expressions$W_{fi}(z_{fi}),\ W_{mi}(z_{mi}),\ S_{fi}(z_{fi})$ and $S_{mi}(z_{mi})$ and are solved by the make use of temperature boundary conditions and are as follows .

$$\theta(z_{fi}) = C_7 Cosh[a_{fi}\, z_{fi}] + C_8 Sinh[a_{fi}\, z_{fi}] + F_5(z_{fi}) + F_6(z_{fi}) + F_7(z_{fi}) + F_8(z_{fi}) + F_9(z_{fi})$$

And

$$\theta_{mi}(z_{mi}) = B_8\, Cosh[a_{mi}\, z_{mi}] + B_9\, Sinh[a_{mi}\, z_{mi}] + F_3(z_{fi}) + F_4(z_{fi}) + F_{14}(z_{fi})$$

Where

$$F_5(z_{fi}) = -\frac{1}{m_1^2 - a^2{}_{fi}}\left(1 - \frac{d_{uf} m_1^2}{\tau_{1fi}(m_1^2 - a^2{}_{fi})} - \frac{d_{uf}\, a_{fi}^2}{\tau_{1fi}(m_1^2 - a^2{}_{fi})}\right) Cosh[m_1\, z_{fi}]$$

$$F_6(z_{fi}) = -\frac{C_2}{m_1^2 - a^2{}_{fi}}\left(1 - \frac{d_{uf}\, m_1^2}{\tau_{1fi}(m_1^2 - a^2{}_{fi})} - \frac{d_{uf}\, a^2}{\tau_{1fi}(m_1^2 - a^2{}_{fi})}\right) Sinh[m_1\, z_{fi}]$$

$$F_7(z_{fi}) = -\frac{c_3}{m_2^2 - a^2{}_{fi}}\left(1 - \frac{d_{uf}\, m_2^2}{\tau_{1fi}(m_2^2 - a^2{}_{fi})} - \frac{d_{uf} a_{fi}^2}{\tau_{1fi}(m_2^2 - a^2{}_{fi})}\right) Cosh[m_2\, z_{fi}]$$

$$F_8(z_{fi}) = -\frac{c_4}{m_2^2 - a^2{}_{fi}}\left(1 - \frac{d_{uf} m_2^2}{\tau_{1fi}(m_2^2 - a^2{}_{fi})} - \frac{d_{uf} a_{fj}^2}{\tau_{1fi}(m_2^2 - a^2{}_{fi})}\right) Sinh[m_2\, z_{fi}]$$

$$F_9(z_{fi}) = d_{uf}\, z_{fi}\, (C_5 Sinh[a_{fi}\, z_{fi}] + C_6 Cosh[a_{fi}\, z_{fi}]$$

$$F_3(z_{fi}) = -\left(\frac{B_3}{\delta^2 - a_{mi}^2}\right) Cosh[\ \delta\ z_{mi}] - \left(\frac{B_4}{\delta^2 - a_{mi}^2}\right) Sinh[\ \delta\ z_{mi}]$$

$$F_4(z_{fi}) = \frac{z_{mi}}{2\, a_{mi}} (B_1\, Sinh[\ a_{mi}\, z_{mi}] + B_2\, Cosh[\ a_{mi}\, z_{mi}])$$

$$F_{10}(z_{fi}) = \frac{d_{um}\, a_{mi}}{2} (\ z_{mi}\, B_6\, Sinh[\ a_{mi}\, z_{mi}] + B_7\, Cosh[\ a_{mi}\, z_{mi}])$$

$$F_{11}(z_{fi}) = \frac{a_{mi}}{2\, a_{mi}\ \tau_{1mi}} (\ B_2\, Sinh[\ a_{mi}\, z_{mi}] + B_1\, Cosh[\ a_{mi}\, z_{mi}])$$

$$F_{12}(z_{fi}) = \frac{z_{mi}}{2\, a_{mi}\ \tau_{1mi}} (\ B_1\, Sinh[\ a_{mi}\, z_{mi}] + B_2\, Cosh[\ a_{mi}\, z_{mi}])$$

$$F_{13}(z_{fi}) = \frac{{a_{mj}}^2}{\tau_{1mi}(\delta^2 - a_{mi}^2)} (\ B_4\, Sinh[\ a_{mi}\, z_{mi}] + B_3\, Cosh[\ a_{mi}\, z_{mi}])$$

$$F_{14}(z_{fi}) = F_{10}(z_{fi}) + F_{11}(z_{fi}) + F_{12}(z_{fi}) + F_{13}(z_{fi})$$

$$C_8 = \left(\frac{a_{mi}}{a_{fi}}\right) B_9 - \frac{E_{19}}{a_{fi}}\ , C_7 = -C_8 Sinh[a_{fi}] - \frac{E_{12}}{Cosh[a_{fi}]}\ , B_8 = \frac{a_{mi} B_9 Cosh[a_{mi}] + X_{20}}{a_{mi} Sinh[a_{mi}]}$$

$$E_{12} = L_9 + L_{10} + L_{11} + L_{12}$$

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$$L_9 = -\frac{1}{m_1^2 - a_{fi}^2}\left(1 - \frac{d_{uf}\ m_1^2}{\tau_{1fi}(m_1^2 - a_{fi}^2)} - \frac{d_{uf}\ a_{fi}^2}{\tau_{1fi}(m_1^2 - a_{fi}^2)}\right) Cosh\ [m_1]$$

$$L_{10} = \frac{C_2}{m_1^2 - a_{fi}^2}\left(1 - \frac{d_{uf}\ m_1^2}{\tau_{1fi}(m_1^2 - a_{fi}^2)} - \frac{d_{uf}\ a_{fj}^2}{\tau_{1fi}\ (m_1^2 - a_{fi}^2)}\right) Snh\ [m_1]$$

$$L_{11} = -\frac{C_3}{m_2^2 - a_{fi}^2}\left(1 - \frac{d_{uf}\ m_2^2}{\tau_{1fi}\ (m_2^2 - a_{fi}^2)} - \frac{d_{uf}\ a_{fi}^2}{\tau_{1fi}\ (m_2^2 - a_{fi}^2)}\right) Cosh[m_2]$$

$$L_{12} = -\frac{C_4}{m_2^2 - a_{fi}^2}\left(1 - \frac{d_{uf}\ m_2^2}{\tau_{1fi}(m_2^2 - a_{fi}^2)} - \frac{d_{uf} a_{fj}^2}{\tau_{1fi}(m_2^2 - a_{fi}^2)}\right) Sinh[m_2]$$

$$L_{13} = d_{uf}(C_5 Sinha + C_6 Cosha)$$

$$B_9 = -\frac{X_{21}\ a_{mi}\ Sinh[a_{mi}] - X_{20}\ Cosh[a_{fi}]}{\left(\frac{a_{mi}^2}{a}\right)\ Sinh[a_{fi}] Sinh[a_{mi}] + a_{mi}\ Cosh[a_{mi}]\ Cosh[a_{fi}]}$$

$$X_{19} = -\frac{\delta B_4}{(\delta^2 - a_{mi}^2)} + \frac{B_2}{2a_{mi}} + \frac{d_{um}\, a_{mi}\, B_7}{2} + \frac{a_{mi}^2\, B_2}{2a_{mi}\tau_{1mi}} + L_1$$

$$L_1 = \frac{B_2}{2a_m\tau_{1mi}} + \frac{B_4 a_{mi}^3}{\tau_{1mi}\,(\delta^2 - a_{mi}^2)} + m_1 X_{16} + m_2 X_{18} - d_{uf} C_6$$

$$X_{21`} = -Cosh[a_{fi}] X_{14} + Cosh[a_{fi}] X_{13} - X_{19}\left(\frac{Sinh[a_{fi}]}{a_{fi}}\right) + X_{12}$$

$$E_{20} = \frac{B_3\delta}{\delta^2 - a_{mi}^2}\sinh[\delta] - \frac{\delta B_4}{\delta^2 - a_{mi}^2} - \frac{[a_{mi}B_1\cosh[a_{mi}] - a_{mi}B_2\sinh[a_{mi}]]}{2\mathrm{a_{mj}}} + L_5$$

$$L_5 = L_2 + L_3 + L_4 + L_6 + L_7 + L_8$$

$$L_7 = \frac{1}{2a_{mi}\tau_{1mi}}[-a_{mi}^2 B_1\sinh[a_{mi}] + a_{mi}^2 B_2\cosh[a_{mi}]]$$

$$L_8 = d_{um}\ a_{mi}\ (B_6\ a_{mi}\ Cosh\ [a_{mi}] - a_{mi}\ B_7\ Sinh[a_{mi}])$$

$$L_6 = \frac{[-B_1\sinh[a_{mi}] + B_2\cosh[a_{mi}]\ ]}{2a_{mi}} + \frac{d_{um}\ a_{mi}[-B_6\sinh[a_{mi}] + B_7\cosh[a_{mi}]]}{2}$$

$$L_4 = \frac{a_{mj}^2}{\tau_{1mi}(\delta^2 - a_{mi}^2)[-a_{mi}B_3\sinh[a_{mi}] + B_4 a_{mi}\cosh[a_{mi}]]}$$

$$L_2 = -\frac{1}{2a_{mi}\tau_{1mi}}[B_2\cosh[a_{mi}] - B_1\sinh[a_{mi}]]$$

$$L_3 = \frac{[-a_{mi}B_2\sinh[a_{mi}] + a_{mi}\mathrm{B}_1\cosh[a_{mi}]]}{2a_{mi}\tau_{1mi}}$$

To obtain the Eigenvalue, the thermal Marangoni number

The thermal Marangoni number is obtained by using the coupled boundary condition at the upper fluid boundary which is free with Marangoni effects, which is

$$D_{fi}^2 W_{fi}(1) - M a_{fi}^2 \Theta_{fi}(1) - M_{sf1}\, a_{fi}^2 S_{fi}(1) = 0$$

By solving for M, by substituting all the known quantities, we get

$$M = \frac{\psi_1 - M_{sf1}\, a_{fi}^2\ \psi_2}{a_{fi}^2\, \psi_3}$$

Where,

$$\psi_1 = m_1^2\, Cosh[m_1] + C_2 m_1^2\, Sinh[m_1] + C_3 m_2^2\, Cosh[m_2] + C_4 m_2^2\, Sinh[m_2]$$

$$\psi_2 = C_5\, Cosh\left[a_{fi}\right] + C_6\, Sinh\left[a_{fi}\right] - \frac{Cosh[m_1]}{\tau_{1fi}\left(m_1^2 - a_{fi}^2\right)} - \frac{C_2 Sinh[m_1]}{\tau_{1fi}\left(m_1^2 - a_{fi}^2\right)} + L_{14}$$

$$L_{14} = -\frac{C_3 Cosh[m_2]}{\tau_{1fi}\left(m_2^2 - a_{fi}^2\right)} - \frac{C_4 Sinh[m_2]}{\tau_{1fi}\left(m_2^2 - a_{fi}^2\right)}$$

$$\psi_3 = C_7 Cosha + C_8 Sinha + L_{15} + L_{16} + L_{17} + L_{18} + L_{19}$$

$$L_{15} = -\frac{1}{m_1^2 - a_{fi}^2}\left(1 - \frac{d_{uf}\, m_1^2}{\tau_{1fi}\left(m_1^2 - a_{fi}^2\right)} - \frac{d_{uf}\, a^2}{\tau_{1fi}\left(m_1^2 - a_{fi}^2\right)}\right) Cosh[m_1]$$

$$L_{16} = -\frac{c_2}{m_1^2 - a_{fi}^2}\left(1 - \frac{d_{uf}\, m_1^2}{\tau_{1fi}\left(m_1^2 - a_{fi}^2\right)} - \frac{d_{uf}\, a^2}{\tau_{1\mathrm{fj}}\left(m_1^2 - a_{fi}^2\right)}\right) Sinh[m_1]$$

$$L_{17} = -\frac{c_3}{m_2^2 - a_{fi}^2}\left(1 - \frac{d_{uf}\, \mathrm{m}_2^2}{\tau_{1fi}\left(m_2^2 - a_{fi}^2\right)} - \frac{d_{uf} a^2}{\tau_{1fi}\left(m_2^2 - a_{fi}^2\right)}\right) Cosh[m_2]$$

$$L_{18} = -\frac{c_4}{m_2^2 - a_{fi}^2}\left(1 - \frac{d_{uf}\, m_2^2}{\tau_{1fi}\left(m_2^2 - a_{fi}^2\right)} - \frac{d_{uf}\, a^2}{\tau_{1fi}\left(m_2^2 - a_{fi}^2\right)}\right) Sinh[m_2]$$

$$L_{19} = d_{uf}\left(C_5\, Sinh\left[a_{fi}^2\right] + C_6\, Cosh\left[a_{fi}^2\right]\right)$$

Graphical Interpretations

The thermal Marangoni number (M), which serves as the eigenvalue of the stability problem for modified Darcy–Brinkmann two-component Marangoni convection in the presence of Dufour and magnetic effects within a two-layer composite system with free-free boundaries, is obtained in closed form as a function of several governing physical parameters. These parameters include the Darcy number, Brinkmann viscosity parameter, diffusivity ratio, viscosity ratio, Dufour parameter, magnetic parameter, concentration ratio, and the depth ratio of the fluid and porous layers.

To investigate the influence of these parameters on the stability characteristics of the system, the eigenvalue corresponding to the thermal Marangoni number is plotted as a function of the depth ratio for different values of the governing physical parameters. The graphical analysis provides a comprehensive understanding of the effects of magnetic field strength, Dufour-induced cross-diffusion, porous medium permeability, Brinkmann viscous corrections, and diffusivity ratios on the onset of Marangoni convection. The results reveal how the

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interaction between fluid and porous layers, together with magnetic and Dufour effects, modifies the stability behavior of the composite system and either delays or promotes the onset of convection depending on the parameter regime considered.

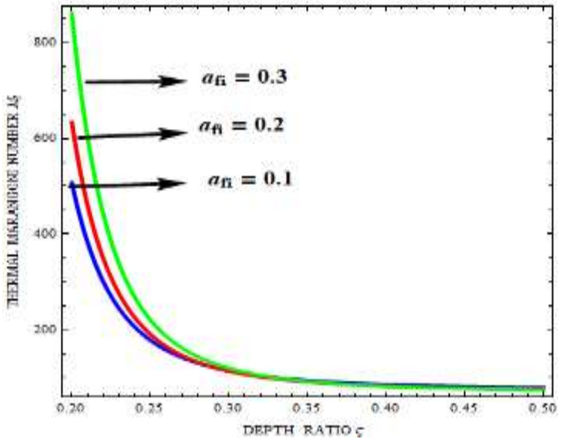

Figure 1: Impact of horizontal wave number a_{fj} on Thermal Marangoni number M

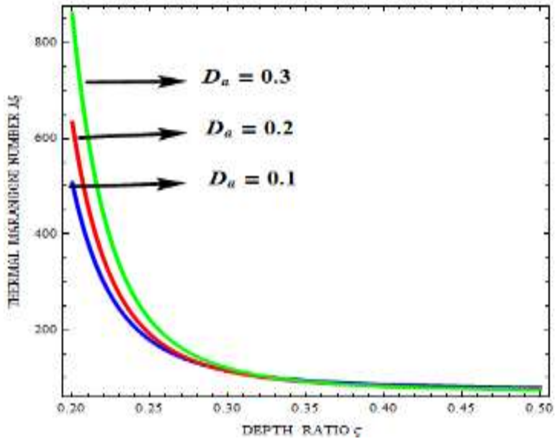

Figure 2: Impact of Darcy number D_a on Thermal Marangoni number M

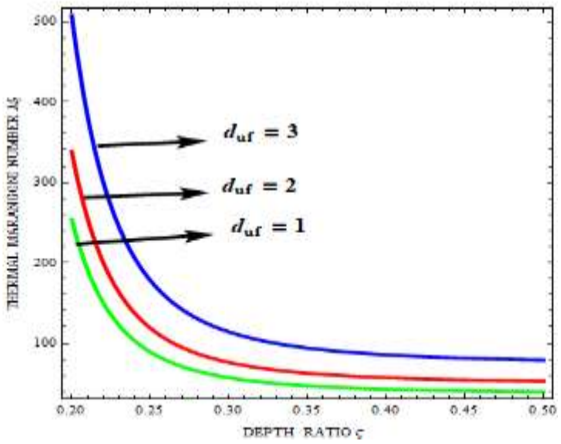

Figure 3: Impact of the Dufour number of the fluid layer d_{uf} on Thermal Marangoni number M

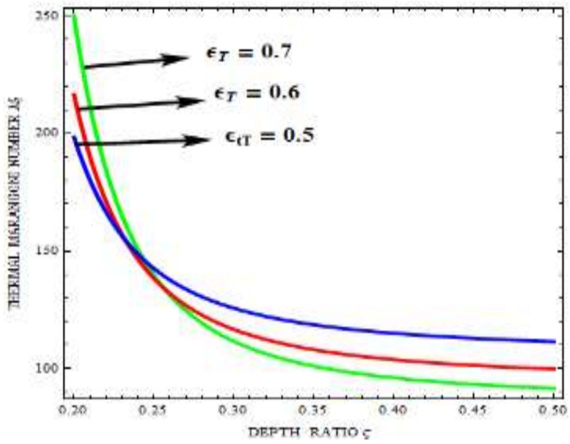

Figure 4: Impact of the ratio of Thermal solutal diffusivities in the fluid layer ϵ_T on Thermal Marangoni number M

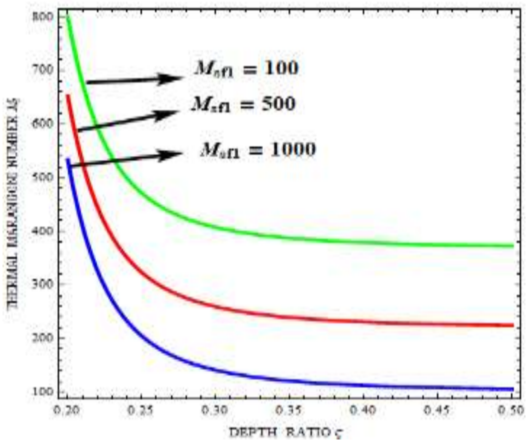

Figure 5: Impact of Solutal Marangoni number M_{sf1} on Thermal Marangoni number M

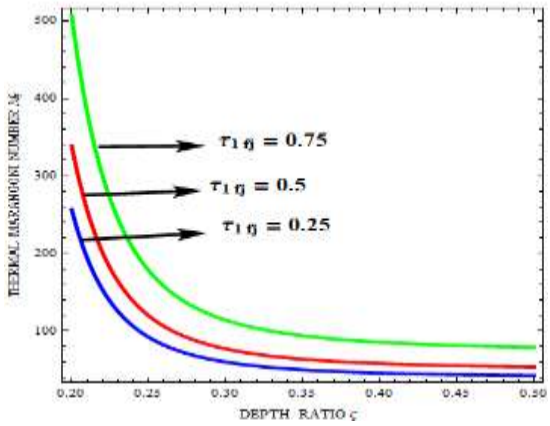

Figure 6: Impact of the ratio of solutal diffusivity to thermal diffusivity τ_{1fi} of on the Thermal Marangoni number M

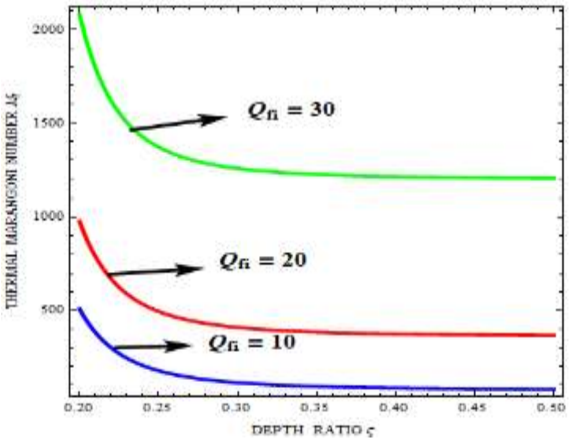

Figure 7: Impact of Chandrashekar number Q_{fi} of fluid layer on the Thermal Marangoni number M

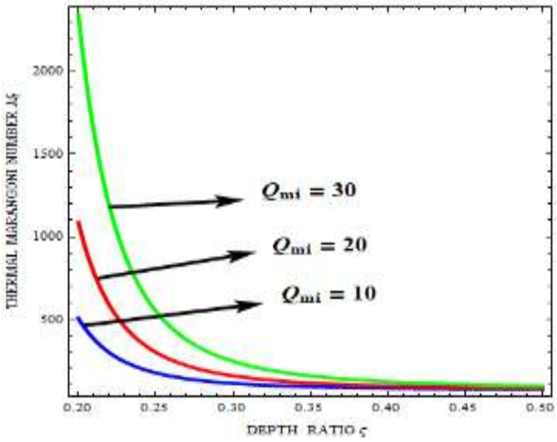

Figure 8: Impact of Chandrashekar number Q_{mi} of the porous layer on the Thermal Marangoni number M

Figure 1 illustrates the variation of the thermal Marangoni number $M$with the horizontal wave number a_{fi}. It is observed that the critical thermal Marangoni number increases with increasing values of a_{fi}, indicating that the system becomes more stable. Physically, a larger wave number corresponds to shorter wavelength disturbances and smaller convection cells. Such disturbances experience stronger viscous dissipation and thermal diffusion, which suppress the growth of convective motions. Consequently, a larger surface-tension force is required to initiate instability. As a result, the onset of convection is delayed, and the system remains stable over a wider range of thermal gradients. This stabilising influence of wave number agrees with the classical work of Pearson (1958), who showed that short-wavelength disturbances require higher Marangoni numbers for instability. Similar trends were reported by Nield and Bejan (2017) and Rudraiah et al. (1985) in porous and layered convection systems.

Figure 2 shows the influence of the Darcy number Da on the thermal Marangoni number M. The results reveal that increasing Da decreases the critical thermal Marangoni number, thereby destabilising the system. The Darcy number represents the permeability of the porous medium. Larger values of Da indicate less resistance to fluid motion within the porous matrix. As permeability increases, fluid circulation becomes easier, and convective disturbances can grow more readily. Consequently, convection begins at lower thermal gradients, resulting in a reduction of the critical thermal Marangoni number. This destabilising effect has been observed in classical porous-medium convection studies by Lapwood (1948) and later in Darcy–Brinkman analyses by Vafai and Tien (1981). Similar conclusions were reported by Ghosh and Mukherjee (2021) for non-Darcy porous systems.

Figure 3 depicts the effect of the Dufour number du_f on the thermal Marangoni number M. It is observed that variations in the Dufour number significantly alter the critical condition for convection. The Dufour effect represents heat transport induced by concentration gradients. Increasing du_f enhances cross-diffusion heat flux and modifies the temperature field within the fluid layer. This additional heat transport changes the interfacial temperature gradient and, consequently, alters the surface-tension forces that drive Marangoni convection. The Dufour mechanism couples thermal and concentration fields, making the instability process more complex. Therefore, even small changes in the Dufour number can significantly influence the onset of convection. Similar observations were reported by Postelnicu (2004), Srinivas Acharya and Kaladhar (2012), Chamkha and Rashad (2014), and more recently by Zhao et al. (2024), who demonstrated that Dufour effects play an important role in multicomponent convective systems.

Figure 4 illustrates the variation of M with the thermal-solutal diffusivity ratio ϵ_T. The graph indicates that increasing ϵ_T stabilises the system. Physically, larger values of ϵ_T imply that thermal diffusion dominates over solutal diffusion. Enhanced thermal diffusion smoothens temperature gradients and suppresses the amplification of thermal disturbances. Consequently, greater thermal forcing is required to initiate convection. This stabilising effect has been observed in thermosolutal convection studies by Nield (1968), Poulikakos (1985), and Kuznetsov (2022).

Figure 5 demonstrates the influence of the Solutal Marangoni number M_{sf1} on the thermal Marangoni number M. The graph shows that increasing M_{sf1} reduces the critical thermal Marangoni number and destabilises the system. The Solutal Marangoni number quantifies the strength of concentration-induced surface-tension forces. Larger values of M_{sf1} increase interfacial tension gradients and generate stronger thermocapillary flows. These flows enhance fluid motion and promote instability. Therefore, concentration gradients act as an additional driving mechanism alongside thermal gradients, causing convection to occur more easily. Similar destabilising effects were reported by Sternling and Scriven (1959), Bergman and Keller (1983), and Siddheshwar et al. (2019).

Figure 6 illustrates the variation of the thermal Marangoni number M with τ_{1fi}. The graph indicates that increasing τ_{1fi} increases the critical thermal Marangoni number, thereby stabilising the system. Larger values of τ_{1fi} correspond to enhanced diffusion of the first solute component. As concentration gradients dissipate more rapidly, concentration-induced surface tension forces become weaker. This reduces thermosolutal convection and delays instability. Similar stabilising effects were observed by Poulikakos (1985), Pearlstein et al. (1989), and Rionero (1997).

Figure 7 shows the effect of the Chandrasekhar number Q_{fi} of the fluid layer on the thermal Marangoni number M. It is observed that increasing Q_{fi} increases the critical thermal Marangoni number, indicating stabilisation. The magnetic field produces Lorentz forces that oppose fluid motion and dissipate kinetic energy associated with convective disturbances. As the magnetic field becomes stronger, fluid motion is increasingly suppressed. Consequently, larger thermal gradients are required to initiate convection. This magnetic damping effect is a well-established feature of magnetohydrodynamic stability theory and agrees with the studies of Chandrasekhar (1961), Rudraiah et al. (1987), Nield and Kuznetsov (2019), and Pranesh and Kiran (2020).

Figure 8 depicts the influence of the Chandrasekhar number Q_{mi} of the porous layer on the thermal Marangoni number M. The results show that increasing Q_{mi} stabilises the system. In the porous layer, the magnetic field acts together with porous-medium drag resistance. The combined effect strongly suppresses fluid motion and reduces the growth rate of convective disturbances. Therefore, the onset of convection is delayed, and larger thermal gradients are needed to trigger instability. The observed trend agrees with the findings of Vafai and Tien (1981), Shivakumara and Rudraiah (2002), Nield and Bejan (2017), and Rionero (2021), who reported that magnetic forces enhance the stability of fluid–porous composite systems.

CONCLUSIONS

The stability characteristics of non-Darcy three-component Marangoni convection with combined Dufour and magnetic effects in a two-layer composite system have been investigated through the variation of the critical thermal Marangoni number M with different governing parameters. The major conclusions drawn from the study are as follows:

1. **The horizontal wave number (a_{fi}) has a stabilising effect on the system.** An increase in the wave number increases the critical thermal Marangoni number, indicating that smaller convection cells experience stronger viscous and thermal damping, thereby delaying the onset of convection.

2. **The Darcy number (Da) exhibits a destabilising influence.** Higher permeability reduces the resistance offered by the porous medium, facilitating fluid motion and causing convection to occur at lower thermal gradients.

3. **The Dufour number (du_f) significantly modifies the convection threshold.** The Dufour effect introduces heat flux due to concentration gradients, coupling the thermal and solutal fields and altering the interfacial temperature distribution responsible for Marangoni instability.

4. **The thermal-solutal diffusivity ratio (ϵ_T) stabilises the system.** Enhanced thermal diffusion smoothens temperature gradients and suppresses the growth of disturbances, thereby increasing the critical thermal Marangoni number.

5. **The Solutal Marangoni number (M_{sf1}) destabilises the system.** Stronger concentration-induced surface-tension gradients increase interfacial shear stresses and promote thermocapillary motion, resulting in earlier onset of convection.

6. **The solutal diffusivity ratio (τ_{1fi}) has a stabilizing influence.** Increased solutal diffusion weakens concentration gradients and reduces solutal surface-tension effects, thereby delaying instability.

7. **The Chandrasekhar number of the fluid layer (Q_{fi}) stabilizes the system.** The applied magnetic field generates Lorentz forces that oppose fluid motion and suppress convective disturbances, requiring larger thermal gradients to initiate convection.

8. **The Chandrasekhar number of the porous layer (Q_{mi}) also stabilises the system.** The combined action of magnetic damping and porous-medium resistance strongly suppresses fluid motion and delays the onset of convection.

9. **Among the destabilising parameters, the Darcy number and Solutal Marangoni number play a dominant role in promoting convection**, whereas the wave number, diffusivity ratios, and magnetic parameters act as stabilising mechanisms.

10. **The combined effects of non-Darcy permeability, Dufour cross-diffusion, solutal surface-tension forces, and magnetic fields significantly influence the stability characteristics of the composite system.** The results demonstrate that magnetic fields can effectively control convection, while Dufour and solutal effects strongly modify the instability threshold.

11. **The present findings are in good qualitative agreement with previously published results** on Marangoni convection, porous-medium convection, triple-diffusive instability, and magnetohydrodynamic convection reported by Pearson (1958), Chandrasekhar (1961), Nield and Bejan (2017), Sternling and Scriven (1959), Pearlstein et al. (1989), and other researchers.

12. **The study provides a better understanding of heat and mass transfer processes in fluid–porous composite systems** and may be useful in applications such as crystal growth, materials processing, geothermal systems, chemical engineering, energy storage devices, and multicomponent transport phenomena in porous media.

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