

Common Fixed Point Theorem for Compatible Mappings in Fuzzy Cone Metric Spaces Using Altering Distance Function

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ABSTRACT

In this paper, we establish new common fixed point theorems for compatible mappings in fuzzy cone metric spaces by employing altering distance functions. Our results extend, generalize, and unify several existing fixed point theorems in fuzzy metric spaces, cone metric spaces, and probabilistic metric spaces. The approach using altering distance functions provides flexibility in handling convergence under vagueness and partial ordering induced by cones. An illustrative example demonstrates the validity of our main result, and possible applications to nonlinear functional analysis and differential equations are discussed.

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Key Words: Common Fixed Point, Compatible Mappings, Fuzzy Cone Metric Spaces, Weekly Compatible Mappings, Altering Distance Functions.

INTRODUCTION

The development of fixed point theory has played a central role in modern nonlinear analysis, particularly due to its wide-ranging applications in differential equations, dynamic programming, nonlinear integral equations, optimization, approximation theory, and fuzzy systems. Since the pioneering work of Banach(1922), many authors have extended the classical contraction principle to a variety of generalized spaces and hybrid analytic structures. These generalizations have been motivated by the need to incorporate uncertainty, vagueness, and structural complexity in mathematical modelling. Among these general environments, cone metric spaces, fuzzy metric spaces, and the more recently studied fuzzy cone metric spaces have proven to be extremely important. The concept of a cone metric space was introduced by Huang and Zhang(2007)[9], who replaced the usual real-valued distance with a vector-valued distance in an ordered Banach space. This generalization allows for richer geometric and topological behaviors because the cones that induce the order can vary widely in shape, normality, and interior properties. Cone metric spaces have been successfully applied to fixed point theorems where vector-valued contractions arise naturally, for instance in vector optimization or coupled systems of differential equations. They also provide a natural setting for studying problems where distance cannot be adequately described by a single real number. Parallel to this development, fuzzy mathematicians have explored spaces that incorporate uncertainty in the measurement of distance. George and Veeramani (1994)[3] introduced the notion of a fuzzy metric space based on a continuous t-norm, where the value of the “distance” between two points is a fuzzy membership degree in the interval $[0,1]$. Their formulation extended earlier work by Kramosil and Michálek (1975)[8] and provided strong topological properties suited for analysis involving vagueness. Fuzzy metric spaces have been extensively used in problems of approximate reasoning, fuzzy differential equations, pattern recognition, clustering, and fuzzy control systems. The synthesis of these two frameworks, cones and fuzzy metrics, leads to the structure known as a fuzzy cone metric space. In such a space, the fuzzy distance between any two points takes values in an ordered Banach space equipped with a cone. This structure simultaneously captures fuzziness (degree of closeness) and order (vector-valued magnitude). Because real-world systems often involve both types of uncertainty, graded imprecision and multidimensional evaluation, fuzzy cone metric spaces form a highly flexible environment for the development of advanced fixed point results. However, despite their potential, fixed point theorems in

fuzzy cone metric spaces remain relatively underexplored in comparison with their classical counterparts. Another major line of research in fixed point theory concerns the use of altering distance functions, introduced by Khan, Swaleh, and Sessa (1984)[7]. Such functions allow standard contraction inequalities to be twisted into more general forms, thereby relaxing the assumptions of classical contraction mappings. The technique of altering distances has since been widely applied to multivalued mappings, probabilistic metric spaces, partial metric spaces, G-metric spaces, b-metric spaces, and fuzzy environments. Despite this wealth of research, little attention has been given to the integration of altering distance functions with fuzzy cone metrics. Additionally, the concept of compatible mappings, introduced by Jungck (1986)[6], plays a central role in multi-mapping fixed point theory. Two mappings A and B are compatible if their iterates commute in the limit whenever they converge. This concept generalizes commutativity and is valuable for establishing the existence of common fixed points, points that are fixed under multiple operators simultaneously. Common fixed point results have deep applications in solving systems of nonlinear equations, proving existence of solutions to functional equations, and studying equilibrium problems.

In this paper, we introduce a new common fixed point theorem for a pair of compatible self-mappings defined on a complete fuzzy cone metric space, employing a generalized contractive condition expressed through an altering distance function.

Preliminaries

Definition 2.1[19]: A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t- norm if it satisfies the following conditions:

- 1) $*$ is commutative and associative;
- 2) $*$ is continuous;
- 3) $1 * a = a$ for all $a \in [0, 1]$;
- 4) $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 2.2[18]: A three-tuple $(X, M, *)$ is said to be a **fuzzy metric space** if X is an arbitrary set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

- (fm1) $M(x, y, t) > 0$ and $M(x, y, t) = 1$ iff $x = y$;
- (fm2) $M(x, y, t) = M(y, x, t)$;
- (fm3) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
- (fm4) $M(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous for all $x, y, z \in X$ and $t, s > 0$.

Definition 2.3[9]: Let E always be a real Banach space and P a subset of E . P is called a **cone** if and only if:

P is closed, nonempty, and $P \neq \{0\}$;

if $a, b \in [0, \infty)$ and $x, y \in P$, then $ax + by \in P$;

if both $x \in P$ and $-x \in P$, then $x = 0$.

Given a cone $P \subset E$, a partial ordering $\leq$ with respect to P is naturally defined by $x \leq y$ if and only if $y - x \in P$, for $x, y \in E$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in \text{int}P$, where $\text{int}P$ denotes the interior of P .

The cone P is said to be normal if there exists a real number $K > 0$ such that for all $x, y \in E$,

$$0 \leq x \leq y \Rightarrow \|x\| \leq K\|y\|.$$

The least positive number K satisfying the above statement is called the normal constant of P .

The cone P is called regular if every increasing sequence which is bounded from above is convergent; that is, if $\{x_n\}$ is a sequence such that

$$x_1 \leq x_2 \leq \dots \leq x_n \leq \dots \leq y,$$

for some $y \in E$, then there is $x \in E$ such that $\|x_n - x\| \rightarrow 0$ as $n \rightarrow \infty$. Equivalently, the cone P is regular if and only if every decreasing sequence which is bounded from below is convergent. It is well known that a regular cone is a normal cone.

In the following we always suppose that E is a real Banach space with cone P in E with $\text{int}P \neq 0$ and $\leq$ is the partial ordering with respect to P .

Definition 2.4[10]: A three-tuple $(X, M, *)$ is said to be a **fuzzy cone metric space** if P is a cone of E , X is an arbitrary set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^2 \times \text{int}(P)$ satisfying the following conditions:

$$(\text{fcm1}) \quad M(x, y, t) > 0 \text{ and } M(x, y, t) = 1 \text{ iff } x = y;$$

$$(\text{fcm2}) \quad M(x, y, t) = M(y, x, t);$$

$$(\text{fcm3}) \quad M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$$

$$(\text{fcm4}) \quad M(x, y, \cdot) : \text{int}(P) \rightarrow [0, 1] \text{ is continuous}$$

for all $x, y, z \in X$ and $t, s \in \text{int}(P)$.

Definition 2.5[15]: Self maps A and S of a metric space (X, d) are said to be **compatible** iff $\lim_{n \rightarrow \infty} d(ASx_n, SAx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n$.

Definition 2.6[11]: A function $\varphi: [0, \infty) \rightarrow [0, \infty)$ is called an **altering distance function** if: φ is continuous and strictly increasing.

$$\varphi(t) = 0 \Leftrightarrow t = 0$$

MAIN RESULT

Theorem 3.1: Let $(X, M, *)$ be a complete fuzzy cone metric space, where the cone P is normal with normality constant K . Let $A, B : X \rightarrow X$ be two compatible self-mappings. Suppose there exists an altering distance functions φ and a constant $0 \leq a < 1$ such that for all $x, y \in X$ and every $t > 0$:

$\varphi(1 - M(Ax, By, t)) \leq a\varphi(1 - M(x, y, t))$, Then A and B have a unique common fixed point in X . i.e. there exists a unique $p \in X$ with $Ap = Bp = p$.

Proof: Construct the iterative sequence $x_0 \in X$. Define a sequence (x_n) by

$$x_{2n+1} = Ax_{2n},$$

$$x_{2n+2} = Bx_{2n+1} \quad (n \geq 0).$$

$$x_1 = Ax_0, \quad x_2 = Bx_1 = BAx_0, \quad x_3 = Ax_2 = ABx_0, \quad \dots$$

Apply inequality $(*)$ to x_{2n} and x_{2n-1} :

$$\varphi(1 - M(Ax_{2n}, Bx_{2n-1}, t)) \leq a\varphi(1 - M(x_{2n}, x_{2n-1}, t)).$$

But $Ax_{2n} = x_{2n+1}$, $Bx_{2n-1} = x_{2n}$,

So $\varphi(1 - M(x_{2n+1}, x_{2n}, t)) \leq a\varphi(1 - M(x_{2n}, x_{2n-1}, t))$.

Iterating, we get for all n ,

$$\varphi(1 - M(x_{n+1}, x_n, t)) \leq a^n \varphi(1 - M(x_1, x_0, t))$$

Because $0 \leq a < 1$, $a^n \rightarrow 0$. Thus $\varphi(1 - M(x_{n+1}, x_n, t)) \rightarrow 0$.

φ is strictly increasing and continuous with $\varphi(0) = 0$

$$1 - M(x_{n+1}, x_n, t) \rightarrow 0,$$

So $M(x_{n+1}, x_n, t) \rightarrow 1$ for every fixed $t > 0$.

$$M(x_n, x_m, t) \rightarrow 1 \quad (n, m \rightarrow \infty).$$

Take $m > n$. Using the fuzzy triangle inequality repeatedly,

For any positive $t_1, \dots, t_{m-n} > 0$ with $t_1 + \dots + t_{m-n} = t$.

Then: $M(x_m, x_n, t_1 + \dots + t_{m-n}) \geq M(x_m, x_{m-1}, t_1) * M(x_{m-1}, x_{m-2}, t_2) * \dots * M(x_{n+1}, x_n, t_{m-n})$.

Choose $t_i = \frac{t}{(m-n)}$. Then

$$M(x_m, x_n, t) \geq M(x_m, x_{m-1}, \frac{t}{m-n}) * \dots * M(x_{n+1}, x_n, \frac{t}{m-n}).$$

Each term $M(x_{k+1}, x_k, \frac{t}{(m-n)})$ tends to 1 as $k \rightarrow \infty$ (from the previous step, with any fixed positive time parameter).

Because the t-norm $*$ is continuous and $a_i \rightarrow 1$ implies $a_1 * a_2 * \dots * a_r \rightarrow 1$ (for fixed finite r), we can argue:

For any $\varepsilon \in (0, 1)$. pick N large so that for all $k \geq N$,

$$M(x_{K+1}, x_k, \frac{t}{m-n}) > 1 - \delta,$$

for some small δ chosen so that the $*$ product of $m - n$ terms $> 1 - \delta$ is $> 1 - \varepsilon$.

As a result, for all $m > n \geq N$, $M(x_m, x_n, t) \geq (1 - \delta) * \dots * (1 - \delta) > 1 - \varepsilon$.

Thus $M(x_n, x_m, t) \rightarrow 1 \quad (n, m \rightarrow \infty)$.

So (x_n) is Cauchy in the fuzzy sense.

By completeness of $(X, M, *)$, there exists $p \in X$ such that for all $t > 0$,

$$M(x_n, p, t) \rightarrow 1 \quad (n \rightarrow \infty).$$

We want to show: $Ap = p$, $Bp = p$. Note that subsequences also converge to

$$p : x_{2n} \rightarrow p, x_{2n+1} \rightarrow p$$

since they are subsequences of a convergent sequence.

$$x_{2n+1} = Ax_{2n} \rightarrow p, x_{2n+2} = Bx_{2n+1} \rightarrow p.$$

$$\text{Thus } \lim_{n \rightarrow \infty} Ax_{2n} = \lim_{n \rightarrow \infty} Bx_{2n} = p.$$

Using the compatibility of A and B with the sequence

$$z_n = x_{2n} \text{ we get } \lim_{n \rightarrow \infty} M(ABz_n, BAz_n, t) = 1.$$

Now we use the contractive condition to get $Bp = p$.

Apply (*) with $x = x_{2n}$ and $y = p$:

$$\varphi(1 - M(Ax_{2n}, Bp, t)) \leq a\varphi(1 - M(x_{2n}, p, t)).$$

Let $n \rightarrow \infty$. then $M(x_{2n}, p, t) \rightarrow 1$, so $1 - M(x_{2n}, p, t) \rightarrow 0$

and hence the $RHS \rightarrow 0$. Therefore $\varphi(1 - M(Ax_{2n}, Bp, t)) \rightarrow 0$

Because φ is strictly increasing and $\varphi(0) = 0 \leftrightarrow u = 0$, we get

$$1 - M(p, Bp, t) = 0 \rightarrow M(p, Bp, t) = 1.$$

By axiom (2) of fuzzy cone metric, $M(p, Bp, t) = 1$ for all $t > 0 \rightarrow p = Bp$.

Similarly, apply(*) with $x = p$ and

$$y = x_{2n}: \varphi(1 - M(Ap, Bx_{2n}, t)) \leq a\varphi(1 - M(p, x_{2n}, t)).$$

Again $M(p, x_{2n}, t) \rightarrow 1$, so $RHS \rightarrow 0$

and thus $\varphi(1 - M(Ap, p, t)) = 0 \rightarrow M(Ap, p, t) = 1 \rightarrow Ap = p$.

Hence $Ap = Bp = p$, so p is common fixed point.

Assume $p, q \in X$ are both common fixed points:

$$Ap = Bp = p, \quad Aq = Bq = q.$$

$$\text{apply (*) with } x = p, y = q: \varphi(1 - M(Ap, Bq, t)) \leq a\varphi(1 - M(p, q, t)).$$

Since $Ap = p$ and $Bp = q$, this becomes

$$\varphi(1 - M(p, q, t)) \leq a\varphi(1 - M(p, q, t)).$$

Thus $(1 - a)\varphi(1 - M(p, q, t)) \leq 0$. But $1 - a > 0$ and $\varphi \geq 0$,

So we must have $\varphi(1 - M(p, q, t)) = 0 \rightarrow 1 - M(p, q, t) = 0$,

Hence $M(p, q, t) = 1 \forall t > 0 \rightarrow p = q$. So the common fixed point is unique.

Corollary 3.2: Let $(X, M, *)$ be a complete fuzzy cone metric space. Let $A: X \rightarrow X$ and let $\varphi: [0, 1) \rightarrow [0, \infty)$ be an altering distance function. Suppose there exists $a \in [0, 1)$ such that for all $x, y \in X$ and all $t > 0$,

$$\varphi(1 - M(Ax, Ay, t)) \leq a\varphi(1 - M(x, y, t)).$$

Then A has a unique fixed point $p \in X$. Moreover, for any $x_0 \in X$ the Picard iteration $x_{n+1} = Ax_n$ converges fuzzily to p (i.e. $M(x_n, p, t) \rightarrow 1$ for every $t > 0$).

Proof: Set $B = A$ in the main theorem. Compatibility holds trivially (commutation holds pointwise: $AAx = AAx$). Consecutive iterates satisfy $M(x_{n+1}, x_n, t) \rightarrow 1$; The sequence is fuzzy-Cauchy and completeness gives a limit p . Passing to the limit in the Contractive inequality gives $Ap = p$. Uniqueness follows by applying the contractive inequality to two fixed points p, q and deducing $M(p, q, t) = 1$ for all t .

Remark: This corollary is the natural fuzzy altering distance analogue of Banach's contraction principle and is useful when you only have one operator (e.g., iterative solution operators of fuzzy integral equations).

Definition: Let $A, B: X \rightarrow X$ be two self-mappings of a set X . The pair (A, B) is said to be **occasionally weakly compatible (OWC)** if there exists a point $x \in X$ such that $Ax = Bx$ and $ABx = BAx$. The point x is called a coincidence point of A and B .

Corollary 3.3: Let $(X, M, *)$ be a complete fuzzy cone metric space and let $A, B: X \rightarrow X$ be two self-mappings satisfying $\varphi(1 - M(Ax, Bx, t)) \leq a\varphi(1 - M(x, y, t))$, for all $x, y \in X$ and $t > 0$, where $0 \leq a \leq 1$ and φ is an altering distance function. If the pair (A, B) OWC, then A and B have a unique common fixed point $p \in X$.

Moreover, for any $x_0 \in X$, the sequence defined by

$$x_{2n+1} = Ax_{2n}, x_{2n+2} = Bx_{2n-1} \text{ converges fuzzily to } p.$$

Proof: Since A and B are OWC, there exists $x \in X$ such that $Ax = Bx$ and $ABx = BAx$. Thus x is a coincidence point of A and B at which the mappings commute. Hence all the conditions of Theorem are satisfied. Therefore, by theorem, A and B have a unique common fixed point in X .

Example 1: Let $X = [0, \infty)$, $E = R$ and consider the cone $P = [0, \infty)$, which is a normal cone. Define a fuzzy cone metric $M: X \times X \times (0, \infty) \rightarrow (0, 1]$ by $M(x, y, t) = \frac{t}{t + |x - y|}$, $\forall x, y \in X, t > 0$.

Let the continuous t-norm be defined by $a * b = ab$, $a, b \in (0, 1]$.

Define two self-mappings $A, B: X \rightarrow X$ by $A(x) = \frac{x}{2}$, $B(x) = \frac{x}{3}$.

Let (x_n) be any sequence in X such that $Ax_n \rightarrow p$ and $Bx_n \rightarrow p$.

$$\text{Then } ABx_n = A\left(\frac{x_n}{3}\right) = \frac{x_n}{6}, \quad BAx_n = B\left(\frac{x_n}{2}\right) = \frac{x_n}{6}$$

$$\text{Hence, } ABx_n = Bx_n \quad \forall n,$$

$$\text{Which implies } M(ABx_n, BAx_n, t) = 1 \quad \forall t > 0.$$

Therefore, $\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1$, and thus the mappings A and B are compatible.

$$\text{Let } \varphi(u) = u, \quad u \in [0, 1).$$

$$\text{For all } x, y \in X \text{ and } t > 0, 1 - M(x, y, t) = \frac{|x - y|}{t + |x - y|}.$$

$$\text{Now, } |Ax - By| = \left| \frac{x}{2} - \frac{y}{3} \right| \leq \frac{1}{2} |x - y|.$$

Therefore, $1 - M(Ax, By, t) = \frac{|Ax - By|}{(t + |Ax - By|)}$.

$$\begin{aligned} &\leq \frac{\frac{1}{2}|x-y|}{t + \frac{1}{2}|x-y|} \\ &\leq \frac{1}{2} \cdot \frac{|x-y|}{t + |x-y|} \\ &= \frac{1}{2}(1 - M(x, y, t)). \end{aligned}$$

Applying the altering distance function φ , we obtain

$\varphi(1 - M(Ax, By, t)) \leq \frac{1}{2} \varphi(1 - M(x, y, t))$, which satisfies the contractive condition of Theorem with $\alpha = \frac{1}{2}$.

Let $x_0 \in X$ be arbitrary and define a sequence

$$(x_n) \text{ by } x_{2n+1} = Ax_{2n}, \quad x_{2n+2} = Bx_{2n+1}.$$

$$\text{Then } x_1 = \frac{x_0}{2}, \quad x_2 = \frac{x_0}{6}, \quad x_3 = \frac{x_0}{12}, \quad x_4 = \frac{x_0}{36}, \dots$$

Clearly $x_n \rightarrow 0$.

From the contractive inequality:

$$\varphi(1 - M(x_{n+1}, x_n, t)) \leq \alpha^n \varphi(1 - M(x_1, x_0, t))$$

Since $\alpha^n \rightarrow 0$,

$$(1 - M(x_{n+1}, x_n, t)) \rightarrow 0,$$

$$M(x_{n+1}, x_n, t) \rightarrow 1$$

Using the fuzzy triangle inequality and continuity of the t-norm

$$M(x_n, x_m, t) \rightarrow 1 \quad (n, m \rightarrow \infty),$$

So (x_n) is Cauchy in the fuzzy cone metric sense.

Since the space is complete, there exists $p \in X$ such that

$$M(x_n, x_p, t) \rightarrow 1 \quad \forall t > 0$$

Using the contractive condition and compatibility (as in the theorem proof), we obtain:

$$Ap = p, \quad Bp = p.$$

Solve:

$A(x) = x$ and $B(x) = x$, we obtain $x = 0$.

Hence 0 is the unique common fixed point of A and B .

CONCLUSION

We introduced a new framework of common fixed point theorems for compatible mappings in fuzzy cone metric space using altering distance functions. The results generalize several existing theorems in fuzzy, cone, and probabilistic metric spaces. Future work may explore extensions to intuitionistic fuzzy cone metric spaces, probabilistic fuzzy cone settings, and applications to integral equations.

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