

Deflected Space–Time–Mass Theory Based on the Space–Time Exchange Rate $\sqrt{1 - (v/c)^2}$

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ABSTRACT

Special relativity expresses the observer-dependence of space and time through the Lorentz factor, but it keeps space and time as separate coordinates rather than treating them as exchangeable quantities. This paper presents the space–time exchange rate $\sqrt{1 - (v/c)^2}$ as an organizing geometric factor for the special-relativistic scaling laws. The treatment is deliberately an interpretation of standard special relativity, not an alternative to it: the relevant transformation is the Lorentz boost, a hyperbolic rotation in Minkowski space parametrized by the rapidity, and the exchange-rate factor is shown to be exactly the reciprocal Lorentz factor $1/\gamma$. Length contraction and time dilation are re-derived from the boost and expressed as the reciprocal pair $L/L_0 = \tau_0/\Delta t = \sqrt{1 - (v/c)^2}$, from which the product identity $L \Delta t = L_0 \tau_0$ follows as a bookkeeping statement. Because relativistic contraction acts only along the direction of motion, the volume of a moving body contracts by one factor $\sqrt{1 - (v/c)^2}$, not by its cube, and area elements contract only when they contain the direction of motion; a body's rest-mass density likewise grows by the reciprocal single factor $1/\sqrt{1 - (v/c)^2}$. Rest mass is an invariant; energy and momentum satisfy $E^2 = (pc)^2 + (m_0 c^2)^2$, and the apparent increase of mass is correctly read as an increase of energy, $E = \gamma m_0 c^2$. The three-coordinate object $f(t, s, m)$ is re-defined rigorously as the mass-shell hyperboloid of special relativity: boosts act as hyperbolic rotations of the energy–momentum pair and leave the mass axis fixed, which corrects the earlier Euclidean-rotation reading. The paper closes by re-examining Cui's Analytical Space-Time Theory, distinguishing vacuum propagation from propagation in material media and showing that two of its predictions rest on internally inconsistent premises.

Keywords: space–time exchange rate; Lorentz factor; rapidity; length contraction; time dilation; energy–momentum; analytical space-time; special relativity.

INTRODUCTION

Relativistic physics rests on the observation that measured intervals of space and time depend on the observer's inertial system. In special relativity (SR) this dependence is carried by the Lorentz factor $\gamma = 1/\sqrt{1 - (v/c)^2}$, which relates the measurements of two inertial systems in relative motion [1]. SR nevertheless treats space and time as separate coordinates joined by the Lorentz symmetry, not as quantities that can be exchanged with one another.

Earlier attempts to describe relativistic effects through rotation include the flat-space-time (Minkowskian) formulation of Zhang and Chen [2–6] and the Analytical Space-Time Theory of Cui [7], which introduced space-time area invariance and coordinate deflection. Cui's derivations, however, contain internal inconsistencies, in particular in the prediction of a “0.71c light-cone vertex” and in the claim of double refraction of light [7]. These two points are examined in Section 4, where the analysis is made more explicit than in earlier summaries and is carefully distinguished from the physics of light propagation in material media.

The purpose of the present paper is to place the idea of a “space–time exchange rate” on a footing that is fully consistent with standard SR. The central quantity is

$$\sqrt{1 - (v/c)^2} = \frac{1}{\gamma} = \text{sech } \eta,$$

where η is the rapidity defined by $\tanh \eta = v/c$. Because $1/\gamma$ is the reciprocal Lorentz factor, every appearance of the exchange rate below is a standard SR result written in a unified notation. In particular, the paper:

- derives the reciprocal length–time relations from the Lorentz boost and states the precise operational meaning of every measured quantity;
- shows that contraction is directional, so that volume contracts by a single factor $1/\gamma$ and only area elements containing the direction of motion are reduced;
- reformulates the mass increase as an energy–momentum statement with invariant rest mass;
- gives the three-coordinate object $f(t, s, m)$ a precise algebraic meaning as the mass shell of SR, thereby removing the earlier Euclidean-rotation interpretation;
- re-examines Cui’s two predictions with the full equation chain, while flagging the parts that depend on the original source.

The remainder of the paper is organized as follows. Section 2 states the operational definitions and the kinematic assumptions, and derives the exchange-rate relations from the Lorentz boost. Section 3 collects the resulting scaling laws for length, time, area, volume, density, mass, and energy. Section 4 is the discussion, including the re-examination of Cui’s theory. Section 5 concludes.

METHOD

Systems and operational definitions

Let S_{obs} denote the observer’s (laboratory) frame and let S_{mov} be a frame that moves with constant velocity v along the common x -axis relative to S_{obs} . A body is said to be **comoving** with S_{mov} when it is at rest in that frame.

Three measurement conventions are used throughout, and all quantities below are defined operationally:

1. **Proper length** L_0 is the distance between the two ends of a body measured in the frame in which the body is at rest, S_{mov} .
2. **Coordinate length** L is the distance between the two ends of the same body measured in S_{obs} **simultaneously** in S_{obs} (Einstein simultaneity convention).
3. **Proper time** τ_0 is the time interval read on a clock that is at rest in S_{mov} (the comoving clock); **coordinate time** Δt is the time interval between the same two events measured with the synchronized clocks of S_{obs} .

Areas and volumes are defined analogously: a proper area A_0 or proper volume V_0 is measured in the rest frame S_{mov} ; the coordinate values A and V are obtained in S_{obs} from a snapshot of the body at a fixed coordinate time t of S_{obs} .

The Lorentz boost and the rapidity

With the x -axis along the relative velocity, the Lorentz transformation from S_{obs} to the comoving frame S_{mov} is

$$ct' = \gamma(ct - \beta x), \quad x' = \gamma(x - \beta ct), \quad y' = y, \quad z' = z,$$

where $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$. The inverse transformation is obtained by $\beta \rightarrow -\beta$. The boost preserves the Minkowski interval

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2,$$

and it is a **hyperbolic rotation** of the (ct, x) plane [8]: introducing the rapidity η by

$$\tanh \eta = \beta, \quad \cosh \eta = \gamma, \quad \sinh \eta = \gamma\beta,$$

the boost matrix reads

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh \eta & -\sinh \eta \\ -\sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}.$$

An ordinary Euclidean rotation in the (t, l) plane would preserve $c^2 t^2 + l^2$; the Lorentz boost instead preserves $c^2 t^2 - l^2$. It is therefore not a Euclidean rotation, and none of the arguments in this paper require it to be one. The Euclidean-angle notation used below, $\cos \theta = \sqrt{1 - (v/c)^2}$, is a convenient parametrization (equivalently $\sin \theta = v/c$), not the generator of the transformation.

The space–time exchange rate

Define the space–time exchange rate as

$$q = \sqrt{1 - (v/c)^2} = \frac{1}{\gamma} = \text{sech } \eta.$$

Because $q = 1/\gamma$, it is simply the reciprocal Lorentz factor; writing it as $q = \cos \theta$ with $\sin \theta = v/c$ is a standard trigonometric parametrization of β that makes several scaling relations compact.

Length contraction and time dilation from the boost

Consider a rod of proper length L_0 at rest in S_{mov} , aligned with the x -axis. Its endpoints satisfy $x'_2 - x'_1 = L_0$. The observer in S_{obs} measures the two endpoint positions at the same coordinate time t . From the inverse boost $x = \gamma(x' + vt')$ and the condition $\Delta t = 0$, one obtains $\Delta x = \Delta x'/\gamma$; hence

$$L = \frac{L_0}{\gamma} = q L_0.$$

The transverse dimensions are unaffected, $y = y'$, $z = z'$, so a body of proper volume V_0 moving along x has the coordinate volume

$$V = \frac{V_0}{\gamma} = q V_0.$$

Next consider a comoving clock at rest in S_{mov} ($\Delta x' = 0$). Two ticks separated by the proper time $\tau_0 = \Delta t'$ are separated in S_{obs} by

$$\Delta t = \gamma \tau_0 = \frac{\tau_0}{q},$$

which is the standard time-dilation formula. Combining the two results gives the reciprocal exchange relations

$$\frac{L}{L_0} = \frac{\tau_0}{\Delta t} = q = \sqrt{1 - (v/c)^2},$$

and therefore the product identity

$$L \Delta t = L_0 \tau_0.$$

This identity is a bookkeeping consequence of the reciprocal factors $1/\gamma$ and γ ; it does **not** assert that an arbitrary measured length multiplied by an arbitrary measured time is a Lorentz invariant. The true invariants of the theory are the interval $(\Delta s)^2$ and, for a closed system, the invariant rest mass m_0 .

Mass and energy: invariant rest mass

Modern SR treats the rest mass m_0 as a Lorentz scalar. The energy and momentum of a free particle are the time and space components of the energy–momentum four-vector $P^\mu = (E/c, \mathbf{p})$,

$$E = \gamma m_0 c^2, \quad \mathbf{p} = \gamma m_0 \mathbf{v},$$

which obey the mass-shell relation

$$E^2 - (pc)^2 = (m_0 c^2)^2.$$

The rest energy is $E_0 = m_0 c^2$, so the total energy scales as

$$E = \frac{E_0}{q} = \gamma E_0.$$

What is sometimes called “relativistic mass,” $m_{\text{rel}} = \gamma m_0$, is simply E/c^2 ; in this paper the invariant rest mass m_0 is used throughout, and the frame-dependent growth is attributed to energy and momentum.

The three-coordinate object $f(t, s, m)$ as the mass shell

The earlier formulation introduced $f(t, s, m) = t \cdot 1 + s \cdot i + m \cdot h$ as a formal linear combination of a time coordinate t , a spatial coordinate s , and a mass coordinate m . To be evaluated rigorously this object must be given a precise algebraic meaning consistent with SR. We define it through the **mass shell** of a free particle.

Let $c = 1$ for notational clarity. For a particle of rest mass m_0 , energy E , and momentum p ,

$$E^2 - p^2 = m_0^2.$$

Now set $t = E$, $s = p$, and keep $m = m_0$ as the third coordinate. Then the state of the particle lies on the two-sheeted hyperboloid

$$t^2 - s^2 = m^2,$$

which is the future mass shell (energy $t > 0$) in the energy–momentum plane decorated by the invariant mass axis. A Lorentz boost of rapidity η acts on the pair (t, s) as the hyperbolic rotation (active convention, i.e. the inverse of the passive frame transformation in Section 2.2)

$$t' = t \cosh \eta + s \sinh \eta, \quad s' = t \sinh \eta + s \cosh \eta,$$

and leaves the mass axis fixed, $m' = m$. This is the precise content of the earlier statement that “rotations in this space reproduce the exchange relations”: the physically correct transformations are the hyperbolic rotations of the boost group, and the mass coordinate is their invariant direction.

Algebraically, the pair (t, s) carries the natural split-complex (hyperbolic) structure: writing $z = t + s i$ with the rule $i^2 = +1$, the boost of rapidity η is multiplication by the unit hyperbolic number $u_\eta = \cosh \eta + i \sinh \eta$, namely $z' = u_\eta z$, and the mass shell is the level set of the norm, $z\bar{z} = t^2 - s^2 = m^2$. The third symbol h in the notation $f(t, s, m) = t \cdot 1 + s \cdot i + m \cdot h$ merely labels this invariant mass axis; it is not rotated into the time or space axes by any boost. In short, f is a point of the future mass-shell hyperboloid, equipped with the split-complex multiplication on its (t, s) part and with the boost-invariant mass label m . Worked examples are given in Section 3.4.

RESULTS

Space–time exchange rate

From Section 2.4, the central relations are

$$\frac{L}{L_0} = \frac{\tau_0}{\Delta t} = \sqrt{1 - (v/c)^2} = \cos \theta,$$

with the invariant product $L \Delta t = L_0 \tau_0$. The exchange angle is defined by $\sin \theta = v/c$, so that $\cos \theta = \sqrt{1 - (v/c)^2}$.

Area and volume: directional contraction

Contraction acts only along the direction of motion. Consider a rectangular box at rest in S_{mov} with proper edges L_{0x} (along x), L_{0y} , L_{0z} , proper volume $V_0 = L_{0x}L_{0y}L_{0z}$, and moving along x . In S_{obs} ,

$$L_x = \frac{L_{0x}}{\gamma}, \quad L_y = L_{0y}, \quad L_z = L_{0z},$$

so the coordinate volume is

$$V = L_x L_y L_z = \frac{V_0}{\gamma} = q V_0.$$

A single factor q appears because only one spatial dimension contracts. This corrects the earlier isotropic assumption $V = V_0 \cos^3 \theta$, which is not a consequence of the Lorentz transformation.

For area elements the orientation matters:

- a face perpendicular to the direction of motion (the y – z face) has unchanged area, $A = L_y L_z = A_0$;
- a face containing the direction of motion (an x – y or x – z face) has area $A = L_x L_y = A_0/\gamma = q A_0$.

Equivalently, an area element contracts by q if its plane contains the direction of motion and is unchanged if its plane is perpendicular to the motion. Oblique elements are handled by resolving the figure into longitudinal and transverse parts.

Density, mass, and energy exchange rates

Let a body of proper volume V_0 contain rest mass m_0 . Its proper (rest-frame) density is $\rho_0 = m_0/V_0$. In S_{obs} the snapshot volume is V_0/γ , and the rest mass m_0 is invariant, so the rest-mass density observed in S_{obs} is

$$\rho = \frac{m_0}{V} = \gamma \rho_0 = \frac{\rho_0}{q}.$$

Equivalently the number density of identical constituents scales as $n = \gamma n_0$. If instead one refers to the energy density of a pressureless distribution of rest mass (dust), the relevant quantity is the T^{00} component, which scales as $\gamma^2 \rho_0 c^2$; this is the standard SR result and is not the isotropic factor $\gamma^4 \rho_0$ obtained by contracting all three dimensions.

Energy and momentum scale through the same factor:

$$E = \gamma E_0 = \frac{E_0}{q}, \quad pc = \gamma m_0 c v = E_0 \tan \theta.$$

Because $\tan^2 \theta + 1 = \sec^2 \theta$, the two formulas are consistent with $E^2 - (pc)^2 = E_0^2$.

The mass-shell model $f(t, s, m)$: worked examples

Example 1 (rest frame). For a particle at rest, $p = 0$ and $E = m_0 c^2$. With $c = 1$, $(t, s, m) = (m_0, 0, m_0)$, which satisfies $t^2 - s^2 = m^2$.

Example 2 (boosted particle). Boost the particle to speed v (rapidity η). Then

$$t = m_0 \cosh \eta = \gamma m_0, \quad s = m_0 \sinh \eta = \gamma m_0 v, \quad m = m_0.$$

One checks $t^2 - s^2 = m_0^2 (\cosh^2 \eta - \sinh^2 \eta) = m_0^2 = m^2$. The mass coordinate is unchanged: boosts never move the state off its mass shell.

Example 3 (trigonometric form). With $\sin \theta = v/c$, $\gamma = \sec \theta$, $\gamma \beta = \tan \theta$, so $E = E_0 \sec \theta$ and $pc = E_0 \tan \theta$. This is the compact form used throughout the exchange-rate presentation.

DISCUSSION

4.1 Status of the present formulation

The results of Section 3 are standard special relativity expressed in the unified notation of the factor $\sqrt{1 - (v/c)^2}$. Nothing in the paper requires a departure from SR, and the central relations coincide with the usual textbook results: $L = L_0/\gamma$, $\Delta t = \gamma \tau_0$, $V = V_0/\gamma$, $E = \gamma m_0 c^2$, and $E^2 = (pc)^2 + (m_0 c^2)^2$. The contribution of the paper is interpretive and organizational: it exhibits the single factor $q = 1/\gamma$ as a common bookkeeping quantity and corrects three earlier misstatements, namely (i) the treatment of the transformation as an ordinary Euclidean rotation, (ii) the isotropic contraction of volume and density, and (iii) the treatment of mass itself as a frame-dependent quantity.

4.2 Re-examination of Cui's Analytical Space-Time Theory

Cui's Analytical Space-Time Theory [7] is built on two postulates: the invariance of a "space-time area" and the deflection of coordinates with relative speed. Because the original e-print is not universally accessible, the discussion below reproduces the equations as they appear in the cited summary and in the author's own earlier accounts; where the argument depends on the exact text of the original, this is stated explicitly.

4.2.1 Claim 1: the "0.71c light-cone vertex"

Cui's first prediction is that a body moving at $0.71c$ relative to the observer becomes invisible, because the space-time axes are deflected through 45° . In the trigonometric parametrization of Section 2.3, a deflection angle θ with $\sin \theta = v/c$ gives $v = c \sin 45^\circ \approx 0.71c$, so the numerical value $0.71c$ is internally consistent with a 45° angle. The derivation reported by Cui proceeds, however, through the condition

$$x' = y' = c t',$$

i.e. the object's world-line is identified with a light cone. Taken literally, $x' = ct'$ is a light-like trajectory, which requires the object to move at the speed of light, $v = c$; in the trigonometric parametrization that is $\theta = 90^\circ$, not 45° . A massive object cannot move at c , so the two statements are mutually inconsistent: either the vertex occurs at $v = c$ (no massive object reaches it) or it occurs at $0.71c$ and the "light-cone" condition is misapplied.

For $v = 0.71c$ one has

$$\cos \theta = \sqrt{1 - (0.71)^2} = \sqrt{1 - 0.5041} = \sqrt{0.4959} \approx 0.704,$$

i.e. $\theta \approx 45.2^\circ$. The corresponding scaling factors are non-zero: under the single-axis contraction of Section 3 the coordinate volume is $V = 0.704 V_0$, and even the isotropic area factor $\cos^2 \theta \approx 0.496$ used in earlier summaries of the rotational model gives a reduction by about half rather than to zero. The invisibility conclusion is therefore not supported by the equations.

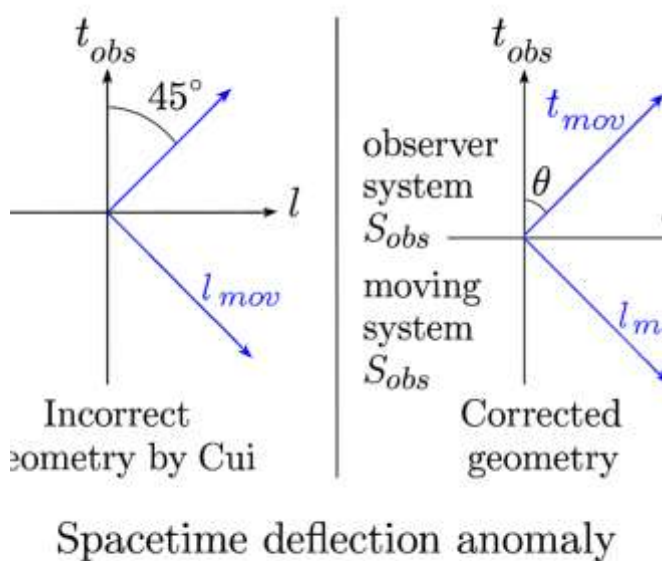


Figure 1. Space-time diagrams for the discussion of Section 4.2. **Left:** Cui's geometry with a 45° deflection of the axes, the configuration underlying the claimed "0.71c light-cone vertex". **Right:** the exchange-rate picture used in the present paper, in which the angle θ satisfies $\cos \theta = \sqrt{1 - (v/c)^2}$, so that $\theta \approx 45.2^\circ$ when $v = 0.71c$.

Claim 2: double refraction of light

Cui's second prediction is that space-time deflection makes light split into an "ordinary ray" of speed c and an "extraordinary ray" of speed $c_e < c$. In the derivation he explicitly assumes $c = c'$, i.e. that the speed of light is the same in both systems. That assumption is consistent with SR but incompatible with a second light speed $c_e < c$ in the same vacuum: if $c = c'$, every light ray in vacuum propagates at the single invariant speed c , and there is no room for a subluminal vacuum ray. If, instead, one wishes to describe an extraordinary ray with $c_e < c$, the relevant physics is propagation in a material medium, where ordinary/extraordinary nomenclature belongs to birefringence (for example in anisotropic crystals) and requires specifying the medium's refractive structure and optical axis. The present discussion therefore distinguishes vacuum propagation, where the double-refraction prediction contradicts the stated postulate $c = c'$, from optical propagation in media, where the prediction would be a statement about material optics rather than about space-time itself. This distinction is essential for evaluating the claim.

4.3 Implications for the present theory and limitations of the critique

The two difficulties identified in Cui's derivations do not affect the present formulation, which introduces no Euclidean rotation, no isotropic contraction, and no frame-dependent mass. The corrected theory preserves the standard SR interval, derives volume contraction from the single longitudinal factor, and expresses the mass increase through the energy–momentum relation.

Two caveats apply to the critique of Cui. First, because the original e-print [7] is difficult to access, the reconstruction of the two predictions relies on the equations as quoted in the secondary sources; a definitive adjudication would require the original text. Second, the double-refraction claim can be read in two ways (vacuum vs. medium), and only the vacuum reading is contradicted by the postulate $c = c'$. These caveats are stated here so that the re-examination is not overstated.

CONCLUSION

The space–time exchange rate $\sqrt{1 - (v/c)^2}$ is exactly the reciprocal Lorentz factor $1/\gamma$. Presented in this way, it organizes the standard special-relativistic scaling laws — length contraction, time dilation, single-axis volume contraction, and the energy–momentum relations — into one consistent geometric picture without departing from special relativity. The transformation is a Lorentz boost (a hyperbolic rotation) parameterized by rapidity, not an ordinary Euclidean rotation; rest mass is invariant; and the three-coordinate object $f(t, s, m)$ is given a precise meaning as the mass-shell hyperboloid on which boosts act by hyperbolic rotation while leaving the mass axis fixed. The re-examination of Cui's Analytical Space-Time Theory shows that its “0.71c light-cone vertex” and vacuum double-refraction predictions are internally inconsistent, while distinguishing the material-medium reading of the second claim. The corrected formulation offers a unified, SR-consistent account of the relations among space, time, mass, and energy.

Declarations

Ethical compliance. This work is purely theoretical and involves no human participants, animals, or personal data; no ethics approval was required.

Consent. Not applicable; the study involved no participants.

Data availability. This is a theoretical derivation and uses no experimental data. All quantities and relations are stated explicitly in the text; there are no data files to share.

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