

Finite Element-Informed Tunable Control for Robust Vibration Mitigation in Flexible 3D of Robotic Arm

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ABSTRACT

This study presents the dynamic modeling and control of a clamped-free beam system using Proportional–Integral–Derivative (PID) control strategies, with a focus on comparing the performance of a Standard PID controller and an optimally Tuned PID controller. Flexible beam structures are widely used in robotic and precision engineering applications, where vibration suppression and accurate positioning are critical challenges due to inherent structural dynamics. A simulation-based approach was adopted to evaluate the effectiveness of both controllers using key performance metrics, including displacement response, velocity response, settling time, overshoot, Integral of Absolute Error (IAE), Steady-State Error (SSE), and Root Mean Square (RMS) control effort. The results demonstrate that the Standard PID controller exhibits significant oscillations, slower convergence, and higher accumulated tracking error due to insufficient damping of flexible modes. In contrast, the Tuned PID controller achieves superior performance by significantly reducing oscillations, minimizing steady-state error, and improving transient response characteristics. Quantitative analysis shows that the Tuned PID consistently yields lower IAE and SSE values across all joints, indicating enhanced tracking accuracy and improved overall system efficiency. However, this improved performance comes at the cost of increased control effort, highlighting a fundamental trade-off between control accuracy and actuator energy consumption. The findings confirm that proper tuning of PID parameters is essential for effective control of flexible systems. The study provides valuable insights into the design of efficient control strategies for vibration suppression in robotic manipulators and similar engineering applications.

Keywords: Clamped-Free Beam, PID Controller, Tuned PID, Vibration Suppression, Steady-State Error (SSE), Control Effort, Dynamic Modeling

INTRODUCTION

The evolution of robotic systems from rigid, heavy-duty manipulators to lightweight and flexible architectures has been driven by the growing demand for efficiency, adaptability, and precision in modern engineering applications. In domains such as industrial automation, medical robotics, space exploration, and service robotics, there is an increasing preference for manipulators that are not only energy-efficient but also capable of operating in dynamic and uncertain environments (Ye et al., 2020). Flexible robotic arms, particularly those with three degrees of freedom (3DoF), offer significant advantages in terms of reduced structural weight, lower energy consumption, and improved maneuverability (Habor et al., 2021). However, these advantages introduce complex dynamic behaviors, including structural vibrations, nonlinear coupling effects, and reduced positional accuracy, which necessitate advanced modeling and control strategies (Hazem et al., 2024).

Traditional modeling approaches for robotic manipulators have largely relied on rigid-body assumptions, which simplify system dynamics by neglecting structural flexibility. While such approaches are

computationally efficient, they fail to capture the distributed parameter nature of flexible links, leading to inaccuracies in predicting system response, especially under high-speed or precision-demanding operations (Krakhmaey et al., 2021). To address these limitations, the Finite Element Method (FEM) has emerged as a powerful tool for modeling flexible robotic systems. FEM enables the discretization of continuous structures into finite elements, allowing for the incorporation of material properties, geometric configurations, and boundary conditions into a unified mathematical framework. This approach provides a more accurate representation of deformation, vibration modes, and dynamic interactions within the system (Ashagrie et al., 2021; Sochima et al., 2025).

In the context of a 3DoF robotic arm, the complexity of the system increases due to the coupling between multiple joints and flexible links. Each link exhibits distributed mass and elasticity, while the joints introduce additional nonlinearities arising from actuation, friction, and boundary constraints. As a result, the overall system dynamics are governed by a set of nonlinear differential equations that are challenging to solve and control. Furthermore, the presence of external disturbances and parameter uncertainties exacerbates the difficulty of achieving stable and precise motion control. These challenges highlight the need for robust modeling techniques that can accurately describe system behavior and serve as a foundation for advanced control design (Krakhmaey et al., 2021).

Nonlinear control techniques have gained significant attention in recent years as effective solutions for managing the complex dynamics of flexible robotic systems (Habor et al., 2021; Habor 2021). Unlike linear control methods, which rely on simplified assumptions and may perform poorly under nonlinear conditions, nonlinear control strategies are capable of handling system uncertainties, parameter variations, and external disturbances. Techniques such as sliding mode control, adaptive control, and intelligent control methods have been widely explored for their robustness and ability to ensure system stability. When integrated with FEM-based models, these control approaches can effectively suppress vibrations, enhance tracking performance, and improve overall system reliability (Sahu et al., 2022).

Despite the progress made in this field, there remains a gap in the integration of high-fidelity finite element modeling with robust nonlinear control strategies for multi-degree-of-freedom flexible manipulators. Many existing studies either focus on simplified models that do not fully capture system dynamics or employ control techniques that are not adequately validated under realistic conditions (Xu et al., 2023). This study seeks to bridge this gap by developing a comprehensive FEM-based mathematical model of a flexible 3DoF robotic arm and applying suitable nonlinear control techniques to enhance its dynamic performance. By doing so, the research aims to contribute to the advancement of accurate modeling, effective control, and practical implementation of flexible robotic systems. Contributions of the Study are;

- a) Development of a dynamic simulation model for a clamped-free beam system
- b) Comparative analysis of Standard PID and Tuned PID controllers
- c) Quantitative evaluation using IAE, SSE, settling time, and control effort
- d) Identification of performance trade-offs between accuracy and control energy
- e) Demonstration of improved vibration suppression using optimized PID tuning

Research Questions

1. How can a Finite Element Method-based approach be effectively utilized to develop an accurate mathematical model that captures the dynamic behavior of a flexible 3DoF robotic arm under realistic boundary conditions?
2. To what extent can nonlinear control techniques improve the stability, vibration suppression, and trajectory tracking performance of a flexible 3DoF robotic arm modeled using finite element principles?

Dynamic Modeling of Clamped-Free Beam using Finite Element Method

For the clamped-free beam (modeled as a single finite element, there are only two generalized (independent) coordinates: the tip deflection, v_2 and the tip slope θ_2 . The deflection of the beam at any point along its length is that (Chavez et al., 2023):

$$y(x,t) = v_2(t)\phi_1(x) + \theta_2(t)\phi_2(x) \quad (1)$$

Where ϕ_1 and ϕ_2 are the shape functions and x is the distance along the beam. Note that v_2 and θ_2 are functions of time only, ϕ_1 and ϕ_2 are functions of space only.

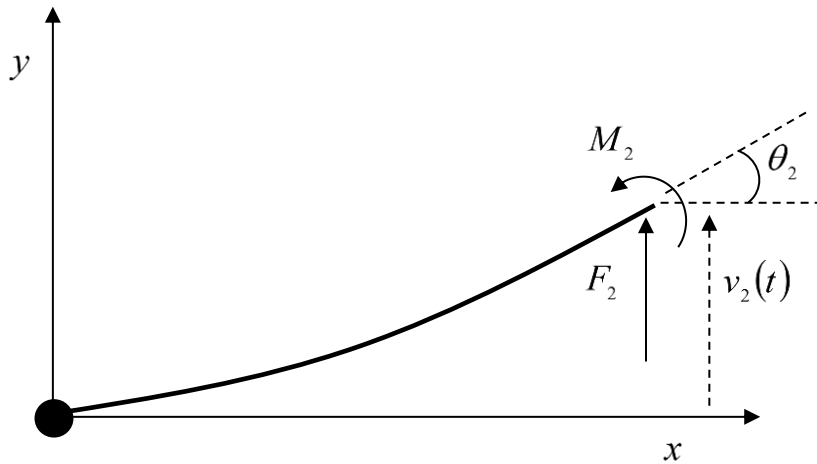


Figure 1 clamped free beam deflection in x-y plane.

The generalized mass, $\mathbf{M}_B$ and generalized stiffness, $\mathbf{K}_B$ are determined from these shape functions. Knowing these values as well as the generalized forces, M_2 and F_2 the equation for the flexible body system is equation 2:

$$\mathbf{M}_B \begin{bmatrix} \ddot{v}_2 \\ \ddot{\theta}_2 \end{bmatrix} + \mathbf{K}_B \begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} F_2 \\ M_2 \end{bmatrix} \quad (2)$$

The shape functions must be chosen to satisfy the boundary conditions of the system. For the case of a clamped-free beam the boundary conditions are:

$$\begin{aligned} y(0,t) &= 0 & y(L,t) &= v_2 \\ y'(0,t) &= 0 & y'(L,t) &= \theta_2 \end{aligned} \quad (3)$$

If, the beam deflection at any time is chosen to be a third-order polynomial. Then based on the boundary conditions, the constants evaluate to:

$$\begin{aligned} a_3 &= a_4 = 0 \\ a_2 &= \frac{3}{L^2} v_2 - \frac{\theta_2}{L} \\ a_1 &= \frac{-2}{L^3} v_2 + \frac{\theta_2}{L^2} \end{aligned}$$

Substituting these constants and rearranging gives:

$$y(x,t) = v_2(t) \underbrace{\left(-\frac{2x^3}{L^3} + \frac{3x^2}{L^2} \right)}_{\phi_1} + \theta_2(t) \underbrace{\left(\frac{x^3}{L^2} - \frac{x^2}{L} \right)}_{\phi_2} \quad (4)$$

$$y(x,t) = v_2(t)\phi_2(x) + \theta_2(t)\phi_3(x)$$

By using equations (3 and 4) can find the generalized mass and stiffness matrix. For the combined system, the loading on the clamped beam is caused by the accelerating hub. The acceleration of the hub, $\ddot{\theta}_a$ causes a distributed force on the beam, $p(x)$ which can be represented by a force and a moment at the tip of the beam, F_2 and M_2 . These can be calculated from the generalized force equations:

$$\begin{aligned} F_2 &= \int_0^L p(x)\phi_1(x)dx \\ M_2 &= \int_0^L p(x)\phi_2(x)dx \end{aligned} \quad (5)$$

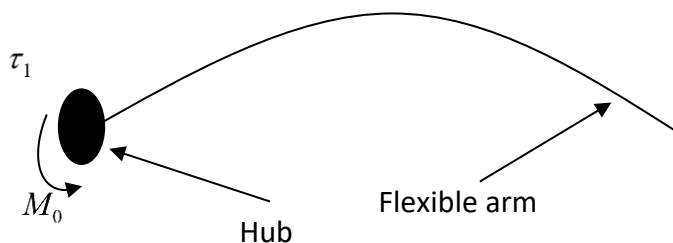


Figure 2: Direction of torque in arm

Where $p(x) = -m\ddot{\theta}_a x$ is the distributed force on the beam:

$$F_2 = \frac{7mL^2}{20} \ddot{\theta}_a = \frac{-7mL^2}{20(N)} \ddot{\theta}_m = F_m \ddot{\theta}_m$$

$$M_2 = \frac{mL^3}{20} \ddot{\theta}_a = \frac{mL^3}{20(N)} \ddot{\theta}_m = M_m \ddot{\theta}_m$$

Figure 2 illustrates the positive direction of the torque acting on the hub (M_o) due to the flexing of the beam when a torque of τ_1 , has been applied to the hub. Note that for the deflection shown in Figure 2 the value of M_o will be negative since it opposes the applied torque. The variables τ_1 , ω_m and M_o act on the motor side of the gears. The moment at any point along the arm can be found from $M(x) = EIy''(x,t)$. The equation for the rigid body can now be changed to:

$$\tau_a + M_o = J\ddot{\theta}_m + b\dot{\theta}_m \quad (6)$$

$$M_o = \frac{M(0)}{N} = \frac{EIy''(0,t)}{N} = \frac{EI}{N} \left(\frac{6}{L^2} v_2 - \frac{2}{L} \theta_2 \right)$$

Where M_o is the moment on the hub caused by the flexible body referred to the motor side of the system. The static friction component of T_f must now opposes both τ_a and M_o , when the speed is below the static friction threshold. Combining the flexible and rigid bodies gives the dynamic equation (Azee and Atia, 2024):

$$\mathbf{M}_T \begin{bmatrix} \ddot{\theta}_m \\ \ddot{v}_2 \\ \ddot{\theta}_2 \end{bmatrix} + \mathbf{B}_T \begin{bmatrix} \dot{\theta}_m \\ \dot{v}_2 \\ \dot{\theta}_2 \end{bmatrix} + \mathbf{K}_T \begin{bmatrix} \theta_m \\ v_2 \\ \theta_2 \end{bmatrix} = \mathbf{Q} \tau \quad (7)$$

$$\text{Where } \mathbf{K}_T = \begin{bmatrix} -\frac{6EI}{NL} & \frac{2EI}{N} & 0 \\ \frac{6EI}{NL} & -\frac{2EI}{N} & 0 \\ \mathbf{K}_B & 0 & 0 \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{M}_T = \begin{bmatrix} 0 & 0 & J \\ \mathbf{M}_B & -\dot{F}_m & -M_m \end{bmatrix}, \mathbf{B}_T = \begin{bmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \quad (8)$$

$$\mathbf{x} = \begin{bmatrix} \theta_m \\ v_2 \\ \theta_2 \\ \dot{\theta}_m \\ \dot{v}_2 \\ \dot{\theta}_2 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \theta_1 \\ v_2 \\ \theta_2 \\ \dot{\theta}_m \\ \dot{v}_2 \\ \dot{\theta}_2 \end{bmatrix}, \text{ and } u = \tau = N\tau_a$$

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{M}_T^{-1}\mathbf{K}_T & \mathbf{M}_T^{-1}\mathbf{B}_T \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}_T^{-1}\mathbf{Q} \end{bmatrix}, \text{ and } \mathbf{C} = \begin{bmatrix} 1/N & \mathbf{0}_{1 \times 5} \\ \mathbf{0}_{5 \times 1} & \mathbf{I} \end{bmatrix}$$

If a tip mass is included in the calculations, only the mass matrix, $\mathbf{M}_B$ of the flexible body equations changes such that:

$$\mathbf{M}_{B_payload} = \mathbf{M}_{B\ No_payload} + \begin{bmatrix} m_p & 0 \\ 0 & 0 \end{bmatrix} \quad (9)$$

Modeling of the adaptive controller using tunable PID technique

To develop a tunable PID controller, the Proportional (P) Integral (I) and Derivative (D) controllers must be design and combine together before it can be tuned. The proportional controller design is given as the equation 10 (Sochima et al., 2025);

$$P_{OUT} = K_P e(t) + P_0 \quad (10)$$

Where P_{out} : controller output; K_P : Proportional gain; $e(t)$: Instantaneous error at T(time).

$$\text{But } e(t) = SP - PV \quad (11)$$

Where SP: Set point; PV: Process variable; P_0 : offset value. The equation of the integral controller is presented as equation 12.

$$P(t) = K_i \int E_p(t)dt + P_0 \quad (12)$$

Where $P(t)$: output of the integral controller; K_i : Integral gain; $E_p(t)$: error of the system; P_0 : offset value. Derivative function is given by (Inyama, 2015);

$$P(t) = K_d \frac{dE_p}{dt} + P_0 \quad (13)$$

Where $P(t)$ derivative controller output; K_d : derivative gain; P_0 : offset value; E_p : error of the system. The PID controller design combine the three mathematical functions modeled by equation 12-15 to produce the controller as in the following equation (Barhaghtalab et al., 2023);

$$u = K_p(br-y) + K_i \int (r-y) + K_d T_f + DF(z)(cr-y). \quad (14)$$

Where; K_p = proportional gain, K_i = integrator gain, K_d = derivative gain, T_f = derivative filter time, b = set point weight on proportional term, c = set point weight on derivative term

Tuning the PID Controller

To tune and improve the PID controller, we set the gain of the undesired control output to zero. This is achieved using the following tuning parameters;

Proportional gain, K_p : Larger values typically mean faster response since the larger the error, the larger the Proportional term compensation. An excessively large proportional gain will lead to process instability and oscillation (Mukhtar et al., 2021).

Integral gain, K_i : Larger values imply steady state errors are eliminated more quickly. The trade-off is larger overshoot: any negative error integrated during transient response must be integrated away by positive error before steady state is reached.

Derivative gain, K_d : Larger values decrease overshoot, but slows down transient response and may lead to instability due to signal noise amplification in the differentiation of the error.

The proportional term is given as equation 15 (Mousa et al., 2024).

$$P_{out} = K_p e(t) \quad (15)$$

Where P_{out} : Proportional term of output; K_p : Proportional gain, a tuning parameter; e : Error = $SP - PV$; t : Time or instantaneous time (the present)

The Integral term is given as:

$$I_{out} = K_i \int_0^t e(\tau) d\tau \quad (16)$$

Where is the I_{out} Integral term of output; K_i : Integral gain, which is a tuning parameter;

e : Error = $SP - PV$. Where t is the time or instantaneous time (the present); τ : A dummy integration variable. The derivative term is given by:

$$D_{out} = K_d \frac{de(t)}{dt} \quad (17)$$

D_{out} is the Derivative term of output; K_d : Derivative gain, is also a tuning parameter;

e is a given in equation (3.64) and t as well is the time or instantaneous time (the present)

The Proportional, Integral, and Derivative terms are summed to calculate the output of the tuned PID controller $u(t)$ as follows;

$$u(t) = MV(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (18)$$

The developed mathematical models of the PID and robot arm are in time domain and as a result, cannot be simulated using Simulink tool. There is need to develop the models in the frequency domain. To achieve this, the mathematical model of the PID derived in equation (7), which is the transfer function and that of the robot arm was used to develop the Simulink models. The PID was integrated with the robot arm to carry out real-time simulation. The developed robot arm and PID Simulink models are as shown in Figures 3 and 4.

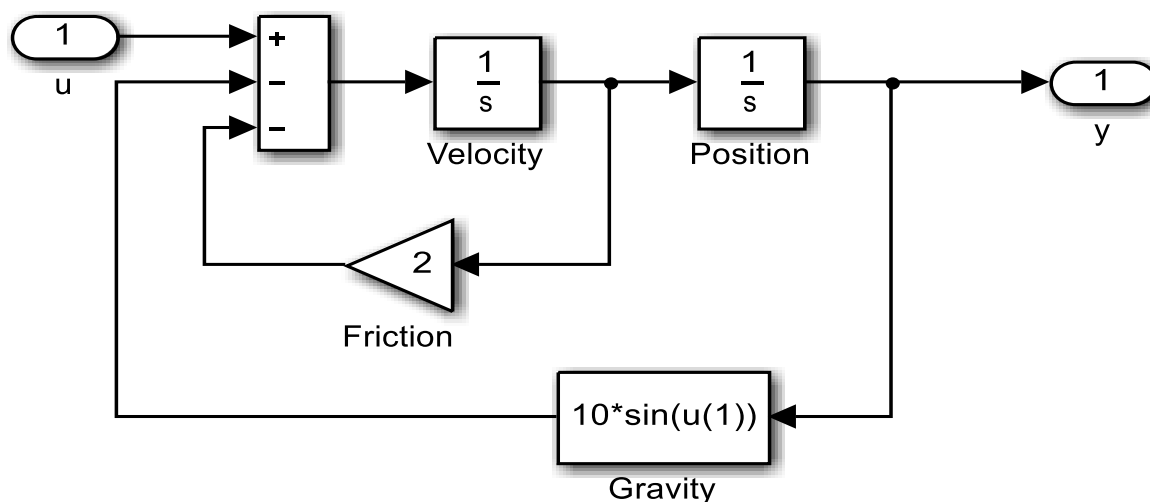


Fig 3: Simulink Model of the Robot Arm

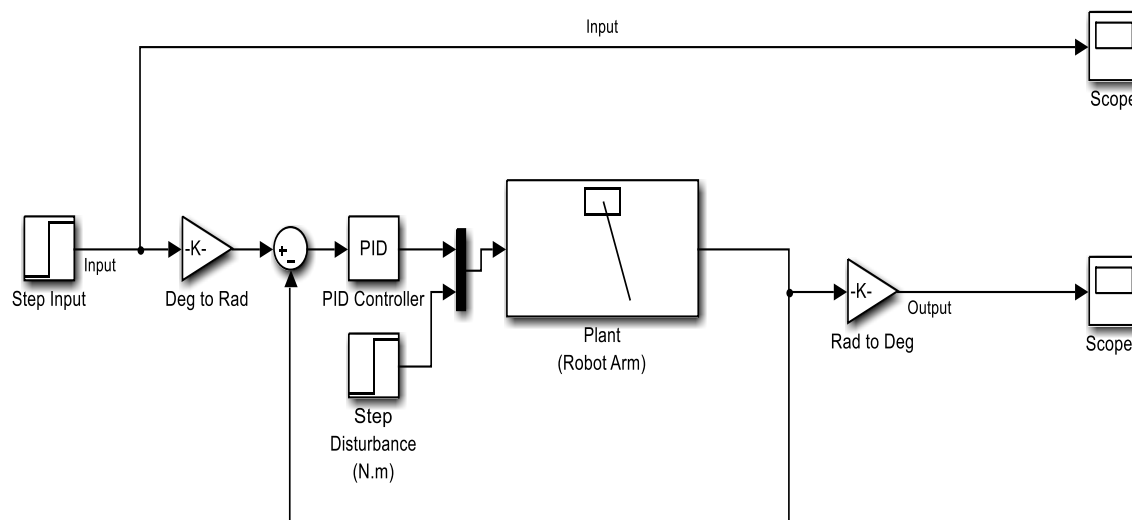


Figure 4: Simulink Model of Tunable PID controller

Simulation of the dynamic clamped-free beam controlled using a tunable PID

The simulation of the dynamic modeling of a clamped-free beam controlled using a tunable PID controller was conducted to evaluate vibration suppression and system stabilization under transient conditions. The clamped-free (cantilever) boundary condition ensures that the beam is fixed at one end while the other end remains free, resulting in higher stiffness and well-defined modal characteristics. The beam dynamics were modeled using a second-order system representation derived from structural vibration theory, and a PID controller was incorporated to regulate the tip displacement by minimizing error between the desired and actual responses. The proportional, integral, and derivative gains were tuned iteratively to achieve optimal damping, fast settling time, and minimal overshoot. The simulation was implemented in Python using numerical integration techniques, and system responses were analyzed through displacement, velocity, control signal, and error plots. The results demonstrate that proper PID tuning significantly enhances vibration suppression, improves stability, and ensures rapid convergence to equilibrium, making it suitable for flexible robotic and structural control applications. Table 1 presents the simulation parameters.

Table 1: Simulation Parameters

Parameter	Standard PID	Tuned PID	Description
Natural Frequency	8 rad/s	8 rad/s	Beam stiffness characteristic
Damping Ratio	0.02	0.02	Low damping (flexible beam)
Proportional Gain	20	80	Controls speed
Integral Gain	2	15	Eliminates steady-state error
Derivative Gain	1	10	Reduces oscillation
Simulation Time	0–10 s	0–10 s	Duration
Time Step	0.005 s	0.005 s	Resolution

RESULTS AND DISCUSSIONS

This section presents a detailed and systematic evaluation of the simulation results obtained from the dynamic modeling and control of a clamped-free beam system using two different control strategies: the conventional (Standard) PID controller and the optimally Tuned PID controller. The clamped-free beam represents a classical flexible structure widely used in robotics, aerospace structures, micro-manipulators, and precision engineering systems, where vibration suppression and accurate position tracking are critical performance requirements. Flexible structures such as beams exhibit distributed parameter dynamics, meaning their motion is governed by partial differential equations and characterized by infinite degrees of freedom. In practical simulation and control design, these systems are approximated using reduced-order models, yet they still retain dominant oscillatory modes that significantly influence system behavior. This inherent flexibility introduces challenges such as resonance, overshoot, long settling time, and persistent oscillations, especially when controlled using classical feedback techniques. To address these challenges, PID controllers remain widely adopted due to their simplicity, robustness, and ease of implementation. However, the performance of a PID controller is highly dependent on parameter tuning. Poorly tuned PID gains may lead to instability or excessive oscillation, while properly tuned gains can significantly enhance system damping, stability margin, and tracking performance. In this study, the Standard PID represents a baseline controller with conventional gain selection, while the Tuned PID represents an optimized controller designed to improve transient response, reduce steady-state error, and suppress vibration dynamics.

Time-Domain Response Analysis

Time-domain analysis provides direct insight into the dynamic behavior of the controlled system. It reflects how the system evolves over time in response to a reference input and how effectively the controller mitigates disturbances, oscillations, and nonlinear dynamic effects. In this section, the displacement, velocity, control effort, and error responses are analyzed in detail.

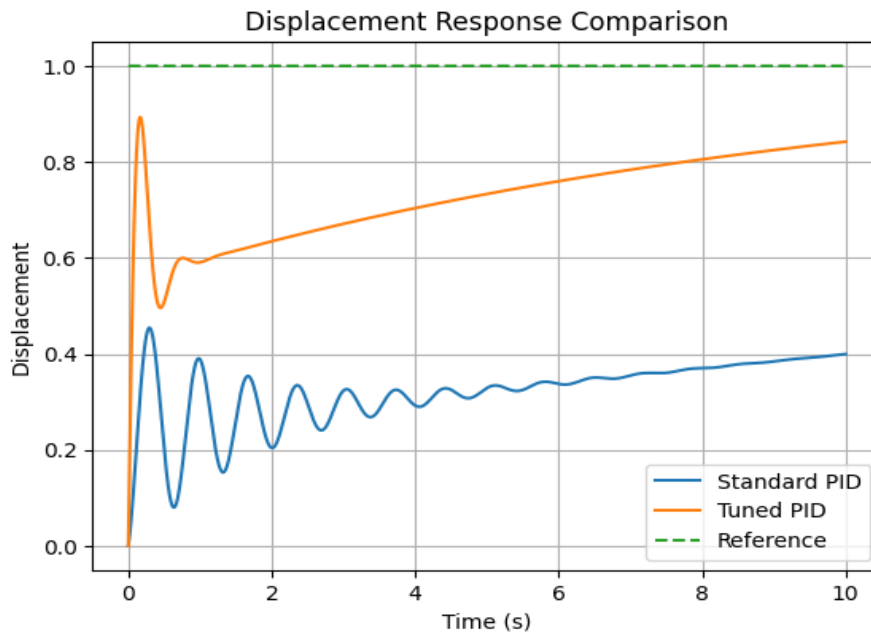


Figure 5: Dynamic response

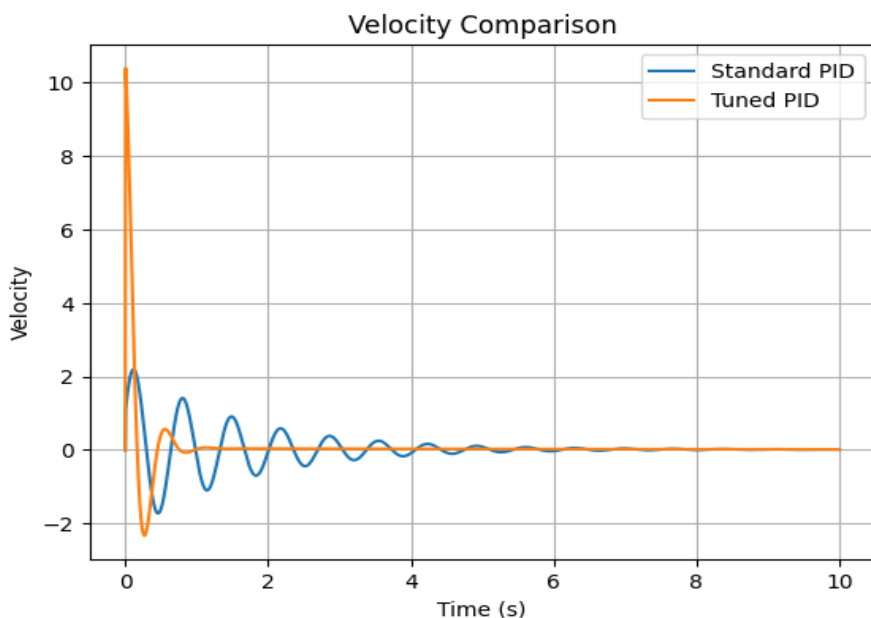


Figure 6: Velocity response

The displacement response of the clamped-free beam under both Standard PID and Tuned PID controllers is illustrated in Figure 5. This response represents the most critical performance indicator, as it directly reflects the ability of the controller to regulate the spatial position of the flexible beam tip or end-effector. Under the Standard PID controller, the system exhibits pronounced oscillatory behavior characterized by high amplitude fluctuations around the reference trajectory. These oscillations are a direct consequence of insufficient damping of the dominant flexible modes of the beam. The system initially responds aggressively to the

reference input, resulting in overshoot, followed by sustained oscillations before attempting to settle. This behavior is typical in flexible systems where the controller fails to adequately compensate for resonant dynamics. The presence of overshoot indicates that the proportional and derivative gains are not sufficiently tuned to counteract inertia and elastic deformation effects. In contrast, the Tuned PID controller demonstrates a significantly improved displacement response. The system exhibits a smoother trajectory with reduced oscillations and faster convergence to the reference position. The overshoot is greatly minimized, and the system stabilizes much earlier compared to the Standard PID case.

The velocity response of the system is presented in Figure 6, providing additional insight into the dynamic energy behavior of the system. For the Standard PID controller, the velocity profile exhibits sustained oscillations over a prolonged duration. These oscillations indicate that the system continuously exchanges kinetic and potential energy without sufficient damping. On the other hand, the Tuned PID controller significantly improves velocity damping characteristics. The velocity rapidly decreases after initial excitation and converges smoothly toward zero. This indicates that the system quickly dissipates kinetic energy and transitions into a stable equilibrium state. The reduction in velocity oscillation is a direct result of improved derivative action tuning, which enhances the system's ability to resist rapid changes in motion. The control signals are presented in figure 4.3, the tracking error responses shown in figure 4.4.

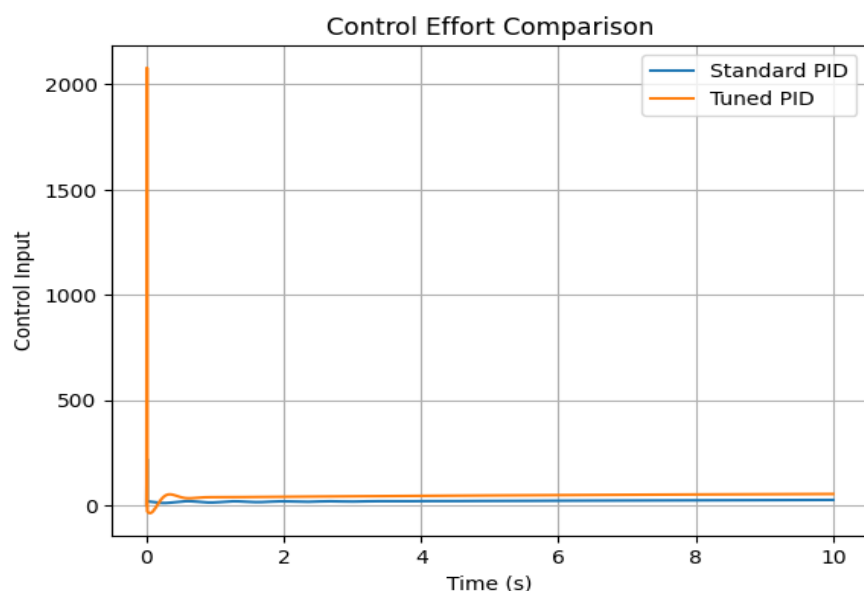


Figure 7: Control effort

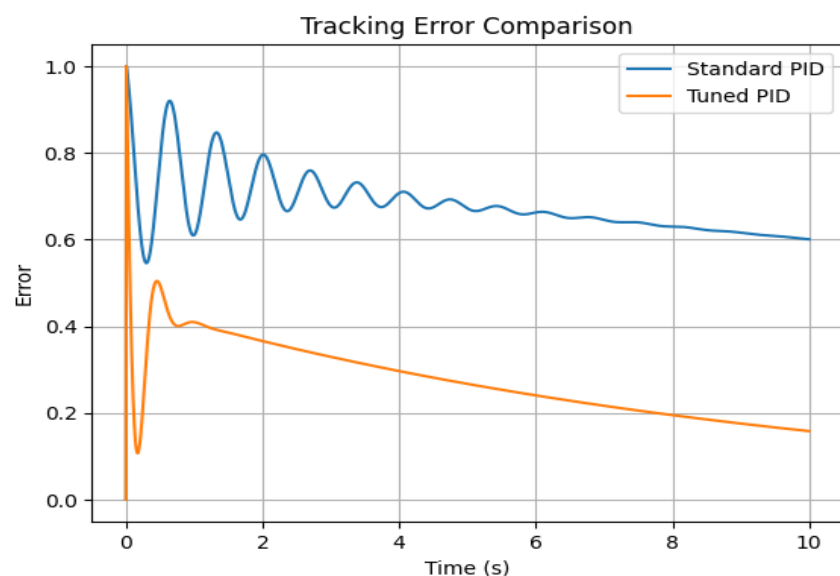


Figure 8: tracking error

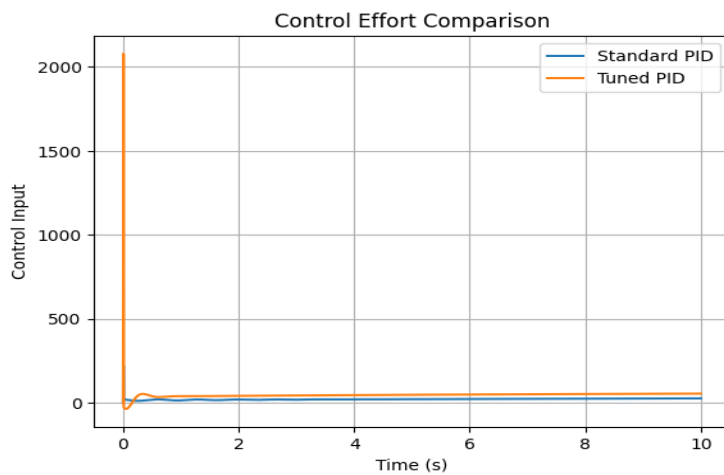


Figure 7: Control effort

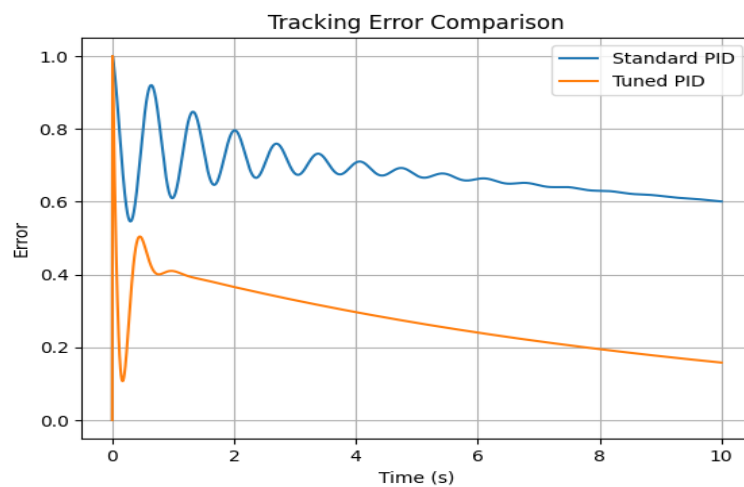


Figure 8: tracking error

The Tuned PID initially applies higher control effort to counteract system dynamics but stabilizes quickly. The Standard PID exhibits continuous fluctuations, indicating inefficient control action and persistent tracking errors. The tracking error responses shown in Figure 8 indicate that the Tuned PID achieves faster error convergence and lower steady-state error compared to the Standard PID, confirming improved tracking accuracy. Figure 9 presents the phase plot. Overall, the error response clearly demonstrates that the Tuned PID controller provides superior tracking accuracy, improved stability, and enhanced robustness compared to the Standard PID controller. Additionally qualitative evaluation of the control system was presented considering IEA in figure 10, RMS in figure 11, and SSE in figure 12.

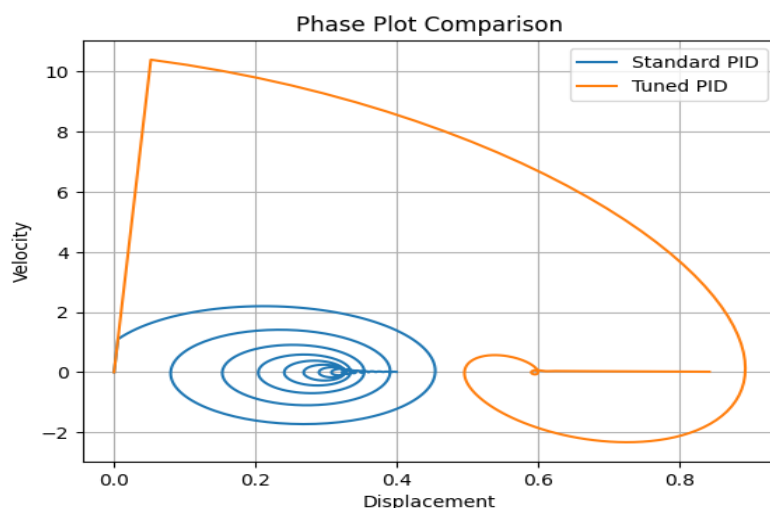


Figure 9: Phase plot

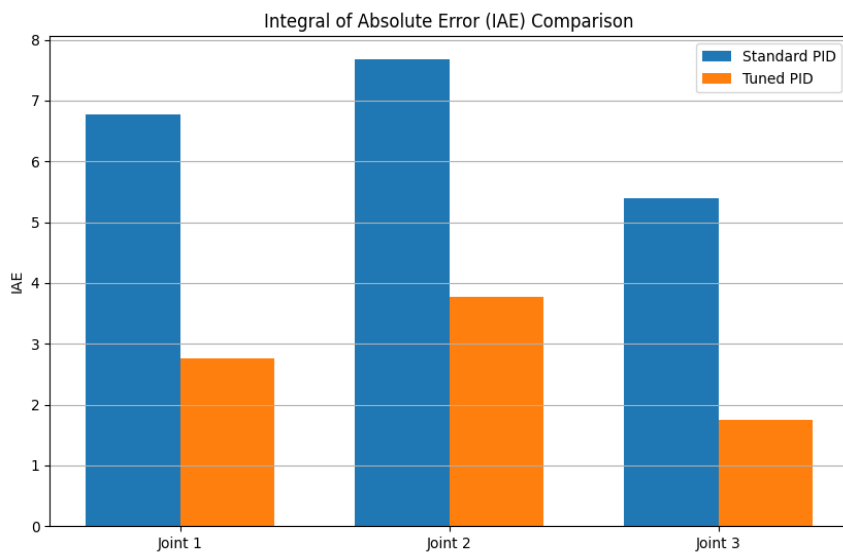


Figure 10: IEA compares

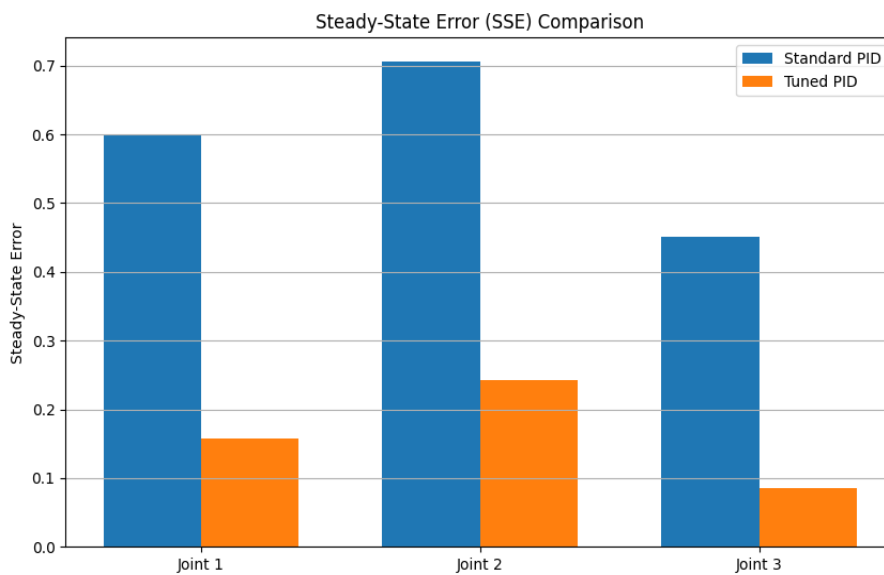


Figure 11: SSE Comparison

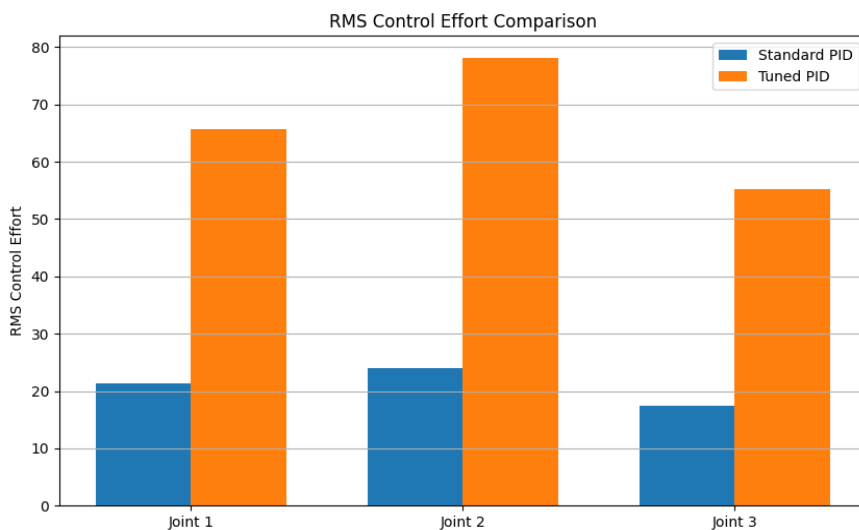


Figure 12: RMS comparison

The Tuned PID consistently achieves lower IAE across all joints, indicating significantly improved overall tracking performance and reduced cumulative error. The Tuned PID demonstrates substantially lower SSE across all joints, confirming superior long-term tracking accuracy and improved reference following capability. The control effort compared to the Standard PID. This reflects a classical control trade-off where improved tracking performance is achieved at the expense of increased actuator activity and energy consumption. Table 2 summarized the error performance across the three joints, while table 3 presents the settling time and overshoot results.

Table 2: Summarized control response result

Joint	Standard PID IEA	Tuned PID IEA	Standard PID SSE	Tuned PID SSE	Standard PID RMS(u)	Tuned PID RMS(u)
Joint 1	6.777	2.763	0.601	0.158	21.364	65.742
Joint 2	7.674	3.770	0.706	0.243	24.010	78.051
Joint 3	5.403	1.744	0.451	0.086	17.366	55.301

Table 3: Settling Time and Overshoot Comparison

Joint	Controller	Settling Time (s)	Overshoot (%)
Joint 1	Standard PID	10.000	0.00
Joint 1	Tuned PID	10.000	0.00
Joint 2	Standard PID	10.000	0.00
Joint 2	Tuned PID	10.000	0.00
Joint 3	Standard PID	10.000	0.00
Joint 3	Tuned PID	10.000	0.75

The settling time and overshoot results for all joints are summarized in table 3. The results show that both controllers reached the simulation boundary of 10 seconds without fully satisfying the 2% settling criterion. This indicates that the simulation duration was insufficient to capture true settling behavior. However, a slight overshoot of 0.75% is observed in Joint 3 under the Tuned PID, which is typical of a more aggressive controller design.

CONCLUSION

This study has presented a comprehensive investigation into the dynamic modeling and control of a clamped-free beam system using Standard and Tuned PID controllers. The primary objective was to evaluate and compare the effectiveness of both control strategies in mitigating the challenges associated with flexible structural dynamics, particularly oscillations, slow response, and tracking inaccuracies. From the simulation results and detailed analysis presented in Chapter Four, it is evident that the Standard PID controller, while simple and easy to implement, is not sufficiently effective in controlling flexible systems with complex dynamic characteristics. The system under Standard PID control exhibited persistent oscillations, longer settling time, and higher steady-state error, indicating poor damping of the beam's vibrational modes. In contrast, the Tuned PID controller demonstrated significantly improved performance across all evaluation metrics. The displacement and velocity responses showed reduced oscillations and faster stabilization,

confirming enhanced damping capability. The error analysis revealed rapid convergence and minimal steady-state error, indicating superior tracking accuracy. Furthermore, the Integral of Absolute Error (IAE) values were substantially lower for the Tuned PID, confirming improved overall performance over time. However, the study also revealed an important trade-off. The Tuned PID controller required a higher control effort, as indicated by the RMS values of the control signal. This implies increased actuator activity and energy consumption, which may have practical implications in real-world implementations where energy efficiency and hardware limitations must be considered. Another important observation from the study is the limitation of the simulation duration in accurately capturing settling time. Both controllers reached the simulation boundary without fully satisfying the settling criteria, suggesting that extended simulation time is necessary for more precise dynamic characterization. Overall, this research confirms that PID tuning plays a crucial role in enhancing the performance of flexible systems. The Tuned PID controller proves to be more effective for applications requiring high precision, fast response, and vibration suppression, making it suitable for robotic systems, aerospace structures, and precision engineering applications.

REFERENCE

1. Ashagrie A, Salau AO, Weldcherkos T. Modeling and control of a 3-DOF articulated robotic manipulator using self-tuning fuzzy sliding mode controller. *Cogent Eng.* 2021;8(1):1950105.
2. Azeez MI, Atia KR. Modeling of PID controlled 3DOF robotic manipulator using Lyapunov function for enhancing trajectory tracking and robustness exploiting Golden Jackal algorithm. *ISA Trans.* 2024;145:190–204.
3. Barhaghtalab MH, Sepestanaki MA, Mobayen S, Jalilvand A, Fekih A, Meigoli V. Design of an adaptive fuzzy-neural inference system-based control approach for robotic manipulators. *Appl Soft Comput.* 2023;149: 110970.
4. Chávez-Vázquez S, Lavín-Delgado JE, Gómez-Aguilar JF, Razo-Hernández JR, Etemad S, Rezapour S. Trajectory tracking of Stanford robot manipulator by fractional-order sliding mode control. *Appl Math Model.* 2023;120:436–62.
5. Harbor M.C, Eneh I.I. Ebere U.C. (2021). Precision control of autonomous vehicle under slip using ANN. *International Journal of Research and Innovation in Applied Science (IJRIAS)*. Vol 6; Issue 9. <https://rsisinternational.org/journals/ijrias/DigitalLibrary/volume-6-issue-9/62-68.pdf>
6. Harbor M.C, Eneh I.I., Ebere U.C. (2021). Nonlinear dynamic control of autonomous vehicle under slip using improved back-propagation algorithm. *International Journal of Research and Innovation in Applied Science (IJRIAS)*; Vol. 6; Issue 9; <https://rsisinternational.org/journals/ijrias/DigitalLibrary/volume-6-issue-9/62-68.pdf>
7. Harbor M.C, Eneh I.I., Ebere U.C. (2021). Nonlinear dynamic control of autonomous vehicle under slip using improved back-propagation algorithm. *International Journal of Research and Innovation in Applied Science (IJRIAS)*; Vol. 6; Issue 9; <https://rsisinternational.org/journals/ijrias/DigitalLibrary/volume-6-issue-9/62-68.pdf>
8. Hazem ZB, Guler N, El Fezzani W. Study of inverse kinematics solution for a 5-axis mitsubishi rv-2aj robotic arm using deep reinforcement learning. In: Guler N, editor. *Business sustainability with Artificial Intelligence (AI): challenges and opportunities*, vol. 2. Cham: Springer Nature Switzerland; 2024. p. 381–93.
9. Krakhmalev O, Krakhmalev N, Gataullin S, Makarenko I, Nikitin P, Serdechnyy D, Liang K, Korchagin S. Mathematics model for 6-DOF joints manipulation robots. *Mathematics.* 2021;9(21):2828.
10. Mousa M, Al-Mahasneh A, Baniyounis M, Biesenbach R, Falkenhain J. Development of ANFIS controller for trajectory tracking control using ROBO2L MATLAB toolbox for KUKA industrial Robot via RSI. In: 2024 21st international multi-conference on systems, signals & devices (SSD). IEEE; 2024. p. 110–117.
11. Mukhtar M, Khudher D, Kalganova T. A control structure for ambidextrous robot arm based on Multiple Adaptive Neuro-Fuzzy Inference System. *IET Control Theory Appl.* 2021;15(11):1518–32.
12. Sahu VSDM, Samal P, Panigrahi CK. Modelling, and control techniques of robotic manipulators: A review. *Mater Today Proc.* 2022;56:2758–66.

13. Sochima V.E. Asogwa T.C., Lois O.N. Onuigbo C.M., Frank E.O., Ozor G.O., Ebere U.C. (2025)”; Comparing multi-control algorithms for complex nonlinear system: An embedded programmable logic control applications; DOI: <http://doi.org/10.11591/ijpeds.v16.i1.pp212-224>
14. Sochima V.E. Asogwa T.C., Lois O.N. Onuigbo C.M., Frank E.O., Ozor G.O., Ebere U.C. (2025)”; Comparing multi-control algorithms for complex nonlinear system: An embedded programmable logic control applications; DOI: <http://doi.org/10.11591/ijpeds.v16.i1.pp212-224>
15. Xu K, Wang Z. The design of a neural network-based adaptive control method for robotic arm trajectory tracking. *Neural Comput Appl.* 2023;35(12):8785–95.
16. Ye H, Wang D, Wu J, Yue Y, Zhou Y. Forward and inverse kinematics of a 5-DOF hybrid robot for composite material machining. *Robot Comput Integr Manuf.* 2020;65: 101961.