

Application of Linear Space on Triangular Intuitionistic Fuzzy Numbers Based on (α, β) -Cut Method

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ABSTRACT

Triangular Fuzzy number (TFN) and Triangular intuitionistic Fuzzy number (TIFN) play an important role in various fields. TINF is an extension of standard fuzzy numbers that handles uncertainty by using both a membership function and non membership function with a Triangular shape. In this research paper we will discuss some elementary operations on proposed Triangular fuzzy numbers (TFNs). And also Comparison of the Triangular fuzzy number and Triangular intuitionistic fuzzy number as in the formation of Linear space. Membership and non- membership function are calculated in terms of (α, β) -cut method of Triangular intuitionistic fuzzy numbers. Numerical example and application of an electric network model of dark room is provided here.

Keywords: Triangular Fuzzy Numbers, Triangular Intuitionistic Fuzzy Numbers, Triangular Fuzzy Linear space, Membership and non- membership function.

INTRODUCTION

Fuzzy set theory was introduced by Zadeh in 1965 [16]. It has been researched widely in many fields and various theories and generalization have been appeared in different directions. Intuitionistic Fuzzy set (IFS) is one of the generalizations of fuzzy set theory. The concept of intuitionistic fuzzy set (IFS) is first introduced by Atanassov [1], [2] and [3]. IFS can be viewed as an alternative appropriate approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Out of several higher-order fuzzy set, IFS have been found to be compatible to deal with vagueness. In fuzzy sets only the degree of acceptance is considered but IFS is characterized by a membership function and non-membership function so that the sum of both values is less than one [4]. Two theorems for intuitionistic fuzzy sets are explained in [5]. There are many studies about the inter-valued intuitionistic fuzzy sets and systems [6] and Correlation of intuitionistic fuzzy sets is explained in [11]. The relation between the fuzzy operators are defined over the intuitionistic fuzzy sets [14] and Cartesian product over intuitionistic fuzzy sets is explained in [15]. Introduce positive and non-negative fuzzy numbers [12] and distance measures evaluation between intuitionistic fuzzy multisets [10] and application of trace on triangular fuzzy number matrices are discussed [7]. Application of reliability are discussed [13].

In this paper triangular fuzzy linear space is introduced and it is proved that set of all triangular intuitionistic fuzzy numbers does not form a linear space. Also the membership and non-membership functions are discussed. Section 2 deals with some basic definition of fuzzy number and intuitionistic fuzzy number. In section 3, New Modified method of Triangular fuzzy Linear space and its properties are discussed. In section 4, Arithmetic operations of triangular intuitionistic fuzzy number based on (α, β) -cut method are discussed. In section 5, Numerical example and application of an electric network model of dark room is presented. Concluding remarks are given in section 6.

Basic Concept of Fuzzy Set and Intuitionistic Fuzzy Set

The fuzzy set theory was first introduced by Zadeh in 1965 [1]. Let X be universe of discourse defined by $X = \{x_1, x_2, \dots, x_n\}$. The grade of membership of element $x_i \in X$ in the fuzzy set is represented by real value between 0 and 1. It does indicate the evidence for $x_i \in X$, but does not indicate the evidence against $x_i \in X$. Atanassov [3],[5] introduced the concept of IFS, and pointed out that this single value combines the evidence for $x_i \in X$ and the evidence against $x_i \in X$. An IFS $\tilde{A}$ in X is characterized by two functions expressing the degree of membership function $\mu_{\tilde{A}}(x)$ and the degree of non-membership function $\nu_{\tilde{A}}(x)$ respectively. Here $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are associated with each point in X , a real number in $[0,1]$ with the values of $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ at X representing the grade of membership and non-membership of x in $\tilde{A}$. Thus closure the value of $\mu_{\tilde{A}}(x)$ to unity and the value of $\nu_{\tilde{A}}(x)$ to zero; higher the grade of membership and lower the grade of non-membership of x . When $\tilde{A}$ an ordinary set its membership function and non-membership function is can take on only two values zero and one. If $\mu_{\tilde{A}}(x) = 1$ and $\nu_{\tilde{A}}(x) = 0$ the element x does not belongs to $\tilde{A}$, similarly if $\mu_{\tilde{A}}(x) = 0$ and $\nu_{\tilde{A}}(x) = 1$ the element x does not belong to $\tilde{A}$. An IFS becomes a fuzzy set $\tilde{A}$ when $\nu_{\tilde{A}}(x) = 0$ and $\mu_{\tilde{A}}(x) \in [0,1] \forall x \in \tilde{A}$.

Definition: Triangular Fuzzy Number [9]

A fuzzy set A in R is said to be a triangular fuzzy number if its membership function $\mu_A: R \rightarrow [0,1]$ has the following characteristics

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & , a \leq x \leq b \\ \frac{c-x}{c-b} & , b \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

Now if we get any number $\alpha \in [0,1]$ the α -cut A_α obtained as follows

$$\frac{a^\alpha - a}{b-a} = \alpha, \quad \frac{c-b^\alpha}{c-b} = \alpha, \quad \text{From these we get}$$

$$a^\alpha = (b-a)\alpha + a, \text{ and } b^\alpha = c - (c-b)\alpha$$

Thus $A_\alpha = [a^\alpha, b^\alpha]$

$$= [(b-a)\alpha + a, c - (c-b)\alpha]$$

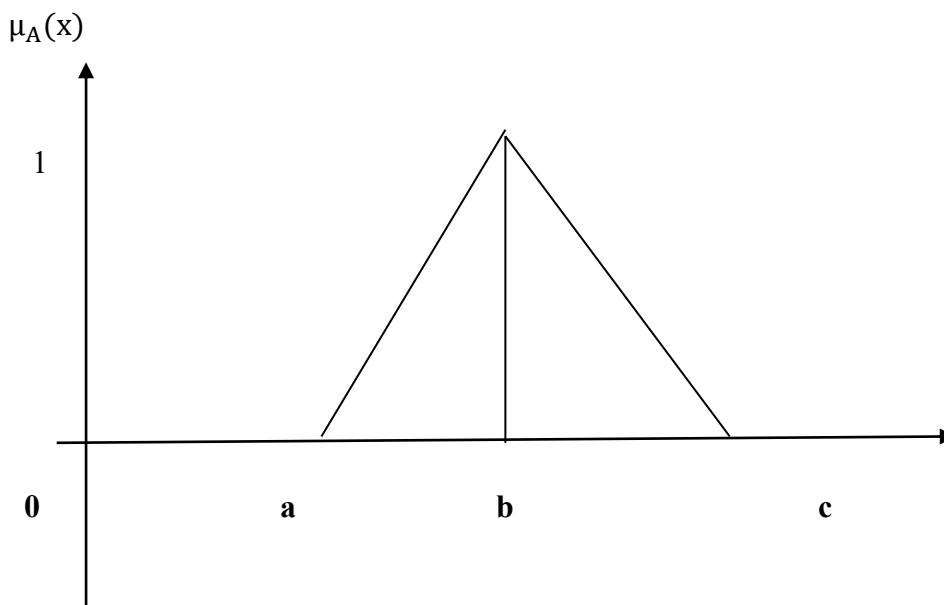


Figure 1: Triangular Fuzzy Number $A = (a, b, c)$

Definition

Positive Triangular Fuzzy Number: A positive triangular fuzzy number is a fuzzy number $\tilde{A}$ is defined as

$$\tilde{A} = (a_1, a_2, a_3) \text{ where all } a_i > 0, \forall i = 1, 2, 3$$

Definition

Negative Triangular Fuzzy Number: A negative triangular fuzzy number is a fuzzy number $\tilde{A}$ is defined as

$$\tilde{A} = (a_1, a_2, a_3) \text{ where all } a_i < 0, \forall i = 1, 2, 3.$$

Remark: A negative triangular fuzzy number can be written as the negative multiplication of a positive triangular fuzzy number.

Definition: (α, β) – Level Intervals or (α, β) –Cuts.

A set of (α, β) –Cut, generated by IFS $\tilde{A}$, where $(\alpha, \beta) \in [0, 1]$ are fixed numbers such that $\alpha + \beta \leq 1$ is defined as

$$\tilde{A}_{\alpha, \beta} = \left\{ (x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)) : x \in X \right. \\ \left. \mu_{\tilde{A}}(x) \leq \alpha, v_{\tilde{A}}(x) \leq \beta : \alpha, \beta \in [0, 1] \right\}$$

(α, β) – Level Intervals or (α, β) –Cut, denoted by $\tilde{A}_{\alpha, \beta}$ is defined as the crisp set of elements x which is belong to $\tilde{A}$ at least to the degree α and which at most to the degree β .

Definition: Triangular Intuitionistic Fuzzy Number

A Triangular Intuitionistic Fuzzy Number (TIFN) $\tilde{A}$ is an intuitionistic fuzzy number in R with the following membership function ($\mu_{\tilde{A}}(x)$) and non –membership function ($v_{\tilde{A}}(x)$) as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & , a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & , a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

$$v_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1} & , a'_1 \leq x \leq a_2 \\ \frac{x-a_2}{a'_3-a_2} & , a_2 \leq x \leq a'_3 \\ 1 & \text{otherwise} \end{cases}$$

Where $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$ and $\mu_{\tilde{A}}(x), v_{\tilde{A}}(x) \leq 0.5$ for $\mu_{\tilde{A}}(x) = v_{\tilde{A}}(x) \forall x \in R$ and triangular intuitionistic fuzzy number is denoted by $\tilde{A}_{\text{TIFN}} = (a_1, a_2, a_3; a'_1, a'_2, a'_3)$.

$\mu_{\tilde{A}}, v_{\tilde{A}}$

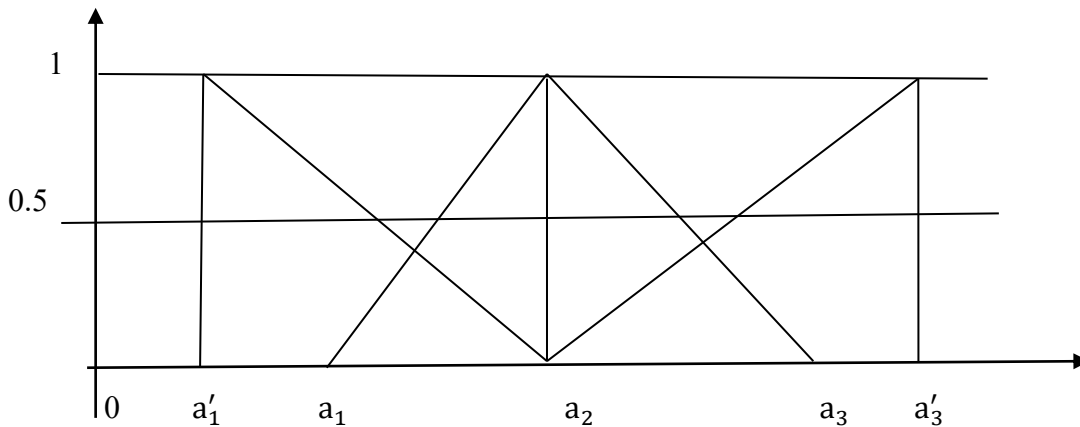


Fig. 2 Membership and non membership functions of TIFN.

Definition: Positive Triangular Intuitionistic Fuzzy Number

A Positive Triangular Intuitionistic Fuzzy Number (TIFN) $\tilde{A}$ is an intuitionistic fuzzy number in $\mathbb{R}$ with the membership function ($\mu_{\tilde{A}}(x)$) and non –membership function ($v_{\tilde{A}}(x)$) with all a_i 's are positive follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & , a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & , a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$v_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1} & , a'_1 \leq x \leq a_2 \\ \frac{x-a_2}{a'_3-a_2} & , a_2 \leq x \leq a'_3 \\ 1 & \text{otherwise} \end{cases}$$

Where $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$ and it is denoted by $\tilde{A}_{\text{TIFN}} = (a_1, a_2, a_3; a'_1, a'_3)$.

Definition: Negative Triangular Intuitionistic Fuzzy Number

A Negative Triangular Intuitionistic Fuzzy Number (TIFN) $\tilde{A}$ is an intuitionistic fuzzy number in $\mathbb{R}$ with the membership function and non –membership function as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x+a_3}{-a_2+a_3} & , -a_3 \leq x \leq -a_2 \\ \frac{a_1+x}{a_1-a_2} & , -a_2 \leq x \leq -a_1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$v_{\tilde{A}}(x) = \begin{cases} \frac{a_2+x}{a_2-a'_3} & , -a'_3 \leq x \leq -a_2 \\ \frac{x+a_2}{-a'_1+a_2} & , -a_2 \leq x \leq -a'_1 \\ 1 & \text{otherwise} \end{cases}$$

Where $-a'_3 \leq -a_3 \leq -a_2 \leq -a_1 \leq a'_1$ and it is denoted by $A_{\text{TFN}} = (-a_3, -a_2, -a_1; -a'_3, -a_2, -a'_1)$.

Arithmetic operations of triangular fuzzy numbers.

The following are the four operations that can be performed on TFN: Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then,

(i) Addition:

$$\begin{aligned}\tilde{A} (+) \tilde{B} &= (a_1, a_2, a_3) (+) (b_1, b_2, b_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3)\end{aligned}$$

(ii) Subtraction:

$$\begin{aligned}\tilde{A} (-) \tilde{B} &= (a_1, a_2, a_3) (-) (b_1, b_2, b_3) \\ &= (a_1 - b_3, a_2 - b_2, a_3 - b_1)\end{aligned}$$

$$\begin{aligned}\text{Also, } 1 - \tilde{A} &= 1 - (a_1, a_2, a_3) \\ &= (1 - a_1, 1 - a_2, 1 - a_3)\end{aligned}$$

(iii) Multiplication:

$$\begin{aligned}\tilde{A} (x) \tilde{B} &= (a_1, a_2, a_3) (x) (b_1, b_2, b_3) \\ &= (a_1 b_1, a_2 b_2, a_3 b_3)\end{aligned}$$

$$\text{Also, } k\tilde{A} = k(a_1, a_2, a_3) = (k a_1, k a_2, k a_3)$$

Where k is any real number.

(iv) Division:

$$\begin{aligned}\tilde{A} (\div) \tilde{B} &= (a_1, a_2, a_3) (\div) (b_1, b_2, b_3) \\ &= \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right) \text{ where } \forall b_i \neq 0\end{aligned}$$

Triangular fuzzy Linear Space.

In this section, we discuss the Linear Space in the triangular fuzzy form.

Let F is an arbitrary non –empty set of triangular fuzzy number on which two operations addition and multiplication by scalar are defined .If the following postulates are satisfied by all numbers of F then F is called a Triangular fuzzy Linear space.

Taking $\tilde{A}, \tilde{B}, \tilde{C}$ are triangular fuzzy numbers and k, m be any real constants then

Laws of addition.

(i) Closure:

$$\tilde{A} (+) \tilde{B} = (a_1, a_2, a_3) (+) (b_1, b_2, b_3)$$

$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ is again a triangular fuzzy number. So it is closed.

(ii) Associative:

$$\begin{aligned} [\tilde{A} (+) \tilde{B}] (+) \tilde{C} &= [(a_1, a_2, a_3) + (b_1, b_2, b_3)] + (c_1, c_2, c_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) + (c_1, c_2, c_3) \\ &= (a_1 + b_1 + c_1, a_2 + b_2 + c_2, a_3 + b_3 + c_3) \\ &= (a_1, a_2, a_3) + (b_1 + c_1, b_2 + c_2, b_3 + c_3) \\ &= (a_1, a_2, a_3) + [(b_1, b_2, b_3)] + (c_1, c_2, c_3) \\ &= \tilde{A} (+) [\tilde{B} (+) \tilde{C}] \text{ are again a triangular fuzzy number.} \end{aligned}$$

Associativity is satisfied

(iii) Identity:

$$\begin{aligned} 0 (+) \tilde{A} &= (0, 0, 0) + (a_1, a_2, a_3) \\ &= (0 + a_1, 0 + a_2, 0 + a_3) \\ &= (a_1 + 0, a_2 + 0, a_3 + 0) \\ &= (a_1, a_2, a_3) + (0, 0, 0) \\ &= \tilde{A} + 0, 0 \text{ is called the identity element.} \end{aligned}$$

(iv) Inverse:

$$\begin{aligned} \tilde{A} (+) (-\tilde{A}) &= (a_1, a_2, a_3) + (-a_1, -a_2, -a_3) \\ &= (a_1 - a_1, a_2 - a_2, a_3 - a_3) \\ &= (-a_1 + a_1, -a_2 + a_2, -a_3 + a_3) \\ &= (-a_1, -a_2, -a_3) + (a_1, a_2, a_3) \\ &= (-\tilde{A}) (+) \tilde{A} \\ &= (0, 0, 0), (-\tilde{A}) \text{ is called the additive inverse of } \tilde{A}. \end{aligned}$$

(v) Commutative:

$$\begin{aligned} \tilde{A} (+) \tilde{B} &= (a_1, a_2, a_3) + (b_1, b_2, b_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \\ &= (b_1 + a_1, b_2 + a_2, b_3 + a_3) \\ &= (b_1, b_2, b_3) + (a_1, a_2, a_3) \\ &= (b_1, b_2, b_3) + (a_1, a_2, a_3) \end{aligned}$$

$= \tilde{B} (+) \tilde{A}$, Commutativity is satisfied.

Laws of scalar multiplication.

$$\begin{aligned} \text{(i)} \quad k [\tilde{A} (+) \tilde{B}] &= k [(a_1, a_2, a_3) + (b_1, b_2, b_3)] \\ &= k [(a_1 + b_1, a_2 + b_2, a_3 + b_3)] \\ &= (ka_1 + kb_1, ka_2 + kb_2, ka_3 + kb_3) \\ &= (k a_1, k a_2, k a_3) + (k b_1, k b_2, k b_3) \\ &= k (a_1, a_2, a_3) + k (b_1, b_2, b_3) \\ &= k \tilde{A} (+) k \tilde{B} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (k + m) \tilde{A} &= (k+m) (a_1, a_2, a_3; a'_1 a_2, a'_3) \\ &= ((k+m)a_1, (k+m)a_2, (k+m)a_3; (k+m)a'_1, (k+m)a_2, (k+m)a'_3) \\ &= ((ka_1+ma_1), (ka_2+ma_2)(ka_3+ma_3); (ka'_1+ma'_1), (ka_2+ma_2), (ka'_3+ma'_3)) \\ &= (ka_1, ka_2, ka_3; ka'_1 ka_2, ka'_3) + (ma_1, ma_2, ma_3; ma'_1, ma_2, ma'_3) \\ &= k (a_1, a_2, a_3; a'_1 a_2, a'_3) + m (a_1, a_2, a_3; a'_1 a_2, a'_3). \\ &= (k\tilde{A} + m\tilde{A}) \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (Km) \tilde{A} &= k [m (a_1, a_2, a_3; a'_1 a_2, a'_3)] \\ &= k (ma_1, ma_2, ma_3; ma'_1, ma_2, ma'_3) \\ &= (kma_1, kma_2, kma_3; kma'_1, kma_2, kma'_3) \\ &= km(a_1, a_2, a_3; a'_1 a_2, a'_3) \\ &= k (m\tilde{A}). \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad \tilde{A} 1. &= 1. (a_1, a_2, a_3; a'_1 a_2, a'_3) \\ &= (1. a_1, 1. a_2, 1. a_3; 1. a'_1, 1. a_2, 1. a'_3) \\ &= (a_1, a_2, a_3; a'_1 a_2, a'_3) \\ &= \tilde{A} \end{aligned}$$

So, all the conditions of linear space are satisfied. Therefore, the set of all Triangular Fuzzy numbers form a linear space called Triangular fuzzy linear space.

Arithmetic operations of Triangular intuitionistic Fuzzy Number.

Let $\tilde{A}$ be triangular intuitionistic Fuzzy number, denoted as

$\tilde{A}_{TIFN} = (a_1, a_2, a_3; a'_1 a_2, a'_3)$ where $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$ and Negative Triangular intuitionistic Fuzzy number is defined as $\tilde{A}_{TIFN} = (-a_3, -a_2, -a_1; -a'_3 - a_2, -a'_1)$ with $-a'_3 \leq -a_3 \leq -a_2 \leq -a_1 \leq a'_1$

$$\begin{aligned}\tilde{A} (+) (-\tilde{A}) &= (a_1, a_2, a_3; a'_1 a_2, a'_3) + (-a_3, -a_2, -a_1; -a'_3 - a_2, -a'_1) \\ &= (a_1 - a_3, a_2 - a_2, a_3 - a_1; a'_1 - a'_3, a_2 - a_2, a'_3 - a'_1) \\ &\neq 0\end{aligned}$$

Therefore we cannot find an additive inverse of $\tilde{A}$. Here, property 4 failed.

Therefore the set of all Triangular intuitionistic Fuzzy number does not form a linear Space.

Arithmetic Operations of Triangular Intuitionistic Fuzzy Number Based on (α, β) -Cut Method.

If $\tilde{A}$ is an intuitionistic fuzzy number, (α, β) –level Intervals or (α, β) –Cuts is given by

$$\tilde{A}_{\alpha, \beta} = \{[A_1(\alpha), A_2(\alpha)], [A'_1(\beta), A'_2(\beta)], \alpha + \beta \leq 1, \alpha, \beta \in [0, 1]\} \text{ Where}$$

$A_1(\alpha)(A_2(\alpha))$ and $A'_1(\beta)(A'_2(\beta))$ will be an increasing (decreasing) function of α with $A_1(1) = A_2(1); A'_1(0) = A'_2(0)$.

If $\tilde{A}$ is an intuitionistic fuzzy number then

- (i) $A_1(\alpha)$ and $A'_2(\beta)$ will be continuous, monotonic increasing function of $\alpha, \beta \in [0, 1]$;
- (ii) $A_2(\alpha)$ and $A'_1(\beta)$ will be continuous, monotonic decreasing function of $\alpha, \beta \in [0, 1]$ and
- (iii) $A_1(1) = A_2(1); A'_1(0) = A'_2(0)$.

In other form, if $\tilde{A} = (a_1, a_2, a_3; a'_1 a_2, a'_3)$ is a Triangular Intuitionistic Fuzzy number, then (α, β) level interval or (α, β) -cut is written as

$$\tilde{A}_{\alpha, \beta} = \{[a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]; [a_2 - \beta(a_2 - a'_1), a_2 + \beta(a'_3 - a_2)]\}, \quad \alpha, \beta \in [0, 1] \text{ where } \alpha + \beta \leq 1.$$

Property

- (a) Triangular intuitionistic Fuzzy Number $\tilde{A}_{TIFN} = (a_1, a_2, a_3; a'_1 a_2, a'_3)$ and $x = ka$ here $k > 0$ then $\tilde{x} = k\tilde{A}$ is TIFN $\tilde{A}_{TIFN} = (ka_1, ka_2, ka_3; ka'_1, ka'_2, ka'_3)$.
- (b) If $x = ka$ with $k < 0$ then $\tilde{x} = k\tilde{A}$ is a TIFN $\tilde{A}_{TIFN} = (ka_3, ka_2, ka_1; ka'_3, ka'_2, ka'_1)$.

Proof: Let $k > 0$ and the linear transformation $x = ka$, we can find the membership function and non membership function of TIFN $\tilde{x} = k\tilde{A}$ by (α, β) – cut method.

$$\alpha\text{-cut of } \tilde{A}, A_\alpha = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$$

$$\mu_{\tilde{A}}(x) \geq \alpha \Rightarrow [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)] \text{ for any } \alpha \in [0, 1]$$

$$\text{i.e } x \in [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$$

$$\text{So } x = ka = [ka_1 + \alpha(ka_2 - ka_1), ka_3 - \alpha(ka_3 - ka_2)]$$

Thus the membership function of $\tilde{x} = k\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-ka_1}{ka_2-ka_1} & , ka_1 \leq x \leq ka_2 \\ \frac{ka_3-x}{ka_3-ka_2} & , ka_2 \leq x \leq ka_3 \\ 0 & \text{otherwise} \end{cases} \quad \dots (4.1)$$

Hence the rule is proved for membership function.

For non membership function of, β - cut of $\tilde{A}$ is

$$v_{\tilde{A}}(x) \leq \beta, A_{\beta} = [a_2 - \beta(a_2 - a'_1), a_2 + \beta(a'_3 - a_2)] \text{ for any } \beta \in [0,1]$$

$$\text{i.e } x \in [a_2 - \beta(a_2 - a'_1), a_2 + \beta(a'_3 - a_2)]$$

$$\text{So } x = ka = [ka_2 - \beta(ka_2 - ka'_1), ka_2 + \beta(ka'_3 - ka_2)]$$

Thus the membership function of $\tilde{x} = k\tilde{A}$ is given by

$$v_{\tilde{A}}(x) = \begin{cases} \frac{ka_2-x}{ka_2-ka'_1} & , ka'_1 \leq x \leq ka_2 \\ \frac{x-ka_2}{ka'_3-ka_2} & , ka_2 \leq x \leq ka'_3 \\ 1 & \text{otherwise} \end{cases} \quad \dots (4.2)$$

Hence the rule is proved for non-membership function.

Thus $\tilde{x} = k\tilde{A} = (ka_1, ka_2, ka_3; ka'_1, ka_2, ka'_3)$ is a TIFN when $k > 0$

(b) Similarly we can prove that $\tilde{x} = k\tilde{A}$ if $x = kx, k < 0$.

Here we get the membership and non-membership function as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-ka_3}{ka_2-ka_3} & , ka_3 \leq x \leq ka_2 \\ \frac{ka_1-x}{ka_1-ka_2} & , ka_2 \leq x \leq ka_1 \\ 0 & \text{otherwise} \end{cases} \quad \dots (4.3)$$

$$v_{\tilde{A}}(x) = \begin{cases} \frac{ka_2-x}{ka_2-ka'_3} & , ka'_3 \leq x \leq ka_2 \\ \frac{x-ka_2}{ka'_1-ka_2} & , ka_2 \leq x \leq ka'_1 \\ 1 & \text{otherwise} \end{cases} \quad \dots (4.4)$$

Thus $\tilde{x} = k\tilde{A} = (ka_3, ka_2, ka_1; ka'_3, ka_2, ka'_1)$ is a TIFN when $k < 0$

Property

If $\tilde{A}$ is a TIFN, where k and m are real numbers and the linear transformation $x = (k + m)a$, then $\tilde{x} = (k + m)\tilde{A}$ is also a TIFN.

Proof:

If k and m are positive real numbers then $k + m$ is a positive real number, $k > 0$ (proof is same as property 4.1(a))

If k and m are negative real numbers then $k + m$ is a negative real number, $k < 0$ (proof is same as property 4.1(b))

If k and m are opposite in sign of real numbers then $k + m$ is either positive or negative real number $k+m$ are positive then the property 4.1(a) or property (b) can be applied.

Property

If $\tilde{A} = (a_1, a_2, a_3; a'_1 a_2, a'_3)$ and $\tilde{B} = (b_1, b_2, b_3; b'_1 b_2, b'_3)$ are two TIFN, their sum

$\tilde{A} (+) \tilde{B}$ is also TIFN $\tilde{A} (+) \tilde{B} = ((a_1 + b_1, a_2 + b_2, a_3 + b_3; a'_1 + b'_1, a_2 + b_2, a'_3 + b'_3))$

Proof:

With the transformation $z = x + y$, the membership (acceptance) function and non-membership (rejection) function of IFS $\tilde{Z} = \tilde{A} (+) \tilde{B}$ can be found by (α, β) –cut method.

α –cut for membership function of $\tilde{A}$, $A_\alpha = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$

$\mu_{\tilde{A}}(x) \geq \alpha \Rightarrow [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$ for any $\alpha \in [0, 1]$

i.e $x \in [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$

α –cut for membership function of $\tilde{B}$, $B_\alpha = [b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)]$

$\mu_{\tilde{B}}(x) \geq \alpha \Rightarrow [b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)]$ for any $\alpha \in [0, 1]$

i.e $x \in [b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)]$

So $Z = X + Y \in [a_1 + b_1 + \alpha((a_2 - a_1) + (b_2 - b_1)), a_3 + b_3 - \alpha((a_3 - a_2) + (b_3 - b_2))]$

i.e α - cut of $\tilde{A} + \tilde{B} = [a_1 + b_1 + \alpha((a_2 - a_1) + (b_2 - b_1)), a_3 + b_3 - \alpha((a_3 - a_2) + (b_3 - b_2))]$

So the membership function of acceptance (membership) of $\tilde{Z} = \tilde{A} (+) \tilde{B}$ is given by

$$\mu_{\tilde{A} (+) \tilde{B}}(Z) = \begin{cases} \frac{z - a_1 - b_1}{a_2 - a_1 + b_2 - b_1} & , a_1 + b_1 \leq z \leq a_2 + b_2 \\ \frac{a_3 + b_3 - z}{a_3 - a_2 + b_3 - b_2} & , a_2 + b_2 \leq z \leq a_3 + b_3 \\ 0 & \text{otherwise} \end{cases} \quad \dots (4.5)$$

Hence $\tilde{A} (+) \tilde{B}$ is a TIFN for membership function.

For non membership function of, β - cut of $\tilde{A}$ is

$v_{\tilde{A}}(x) \leq \beta, A_\beta = [a_2 - \beta(a_2 - a'_1), a_2 + \beta(a'_3 - a_2)]$ for any $\beta \in [0, 1]$

i.e $x \in [a_2 - \beta(a_2 - a'_1), a_2 + \beta(a'_3 - a_2)]$

α –cut for non membership function of $\tilde{B}$, $B_\alpha = [b_2 - \beta(b_2 - b'_1), b_2 + \beta(b'_3 - b_2)]$

$\mu_{\tilde{B}}(x) \geq \beta \Rightarrow [b_2 - \beta(b_2 - b'_1), b_2 + \beta(b'_3 - b_2)]$ for any $\beta \in [0, 1]$

i.e $y \in [b_2 - \beta(b_2 - b'_1), b_2 + \beta(b'_3 - b_2)]$

so $Z = X + Y \in [a_2 + b_2 - \beta((a_2 - a'_1) + (b_2 - b'_1)), a_2 + b_2 - \beta((a'_3 - a_2) + (b'_3 - b_2))]$

i.e α - cut of $\tilde{A} + \tilde{B} = [a_2 + b_2 - \beta((a_2 - a'_1) + (b_2 - b'_1)), a_2 + b_2 - \beta((a'_3 - a_2) + (b'_3 - b_2))]$

So the non membership function of acceptance (membership) of $\tilde{Z} = \tilde{A}(+) \tilde{B}$ is given by

$$v_{\tilde{A}(+) \tilde{B}}(Z) = \begin{cases} \frac{a_2 + b_2 - z}{a_2 - a'_1 + b_2 - b'_1} & , a'_1 + b'_1 \leq z \leq a_2 + b_2 \\ \frac{z - a_2 - b_2}{a'_3 - a_2 + b'_3 - b_2} & , a_2 + b_2 \leq z \leq a'_3 + b'_3 \\ 1 & \text{otherwise} \end{cases} \quad \dots(4.5)$$

Hence the addition rule is proved for non-membership function,

Thus $\tilde{A}(+) \tilde{B} = ((a_1 + b_1, a_2 + b_2, a_3 + b_3; a'_1 + b'_1, a_2 + b_2, a'_3 + b'_3))$ is a TIFN.

Property

If $\tilde{A} = (a_1, a_2, a_3; a'_1, a'_2, a'_3)$ and $\tilde{B} = (b_1, b_2, b_3; b'_1, b'_2, b'_3)$ are two TIFN, their product $\tilde{A}(\times) \tilde{B}$ also TIFN
 $\tilde{A}(\times) \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3; a'_1 b'_1, a'_2 b'_2, a'_3 b'_3)$

Proof: With the transformation $z = x \times y$, the membership (acceptance) function and non-membership (rejection) function of IFS $\tilde{Z} = \tilde{A}(\times) \tilde{B}$ can be found by (α, β) -cut method.

α -cut for membership function of $\tilde{A}$, $A_\alpha = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$

$\mu_{\tilde{A}}(x) \geq \alpha \Rightarrow [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$ for any $\alpha \in [0, 1]$

i.e $x \in [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]$

α -cut for membership function of $\tilde{B}$, $B_\alpha = [b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)]$

$\mu_{\tilde{B}}(x) \geq \alpha \Rightarrow [b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)]$ for any $\alpha \in [0, 1]$

i.e $y \in [b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)]$

So $Z = X(\times)Y \in [(a_1 + \alpha(a_2 - a_1))(b_1 + \alpha(b_2 - b_1)), (a_3 - \alpha(a_3 - a_2))(b_3 - \alpha(b_3 - b_2))]$

So the membership function of acceptance (membership) of $\tilde{Z} = \tilde{A}(\times) \tilde{B}$ is given by

$$\mu_{\tilde{A}(\times) \tilde{B}}(Z) = \begin{cases} \frac{-B_1 + \sqrt{B_1^2 - 4A_1(a_1 b_1 - Z)}}{2A_1} & , a_1 b_1 \leq z \leq a_2 b_2 \\ \frac{B_2 - \sqrt{B_2^2 - 4A_2(a_3 b_3 - Z)}}{2A_2} & , a_2 b_2 \leq z \leq a_3 b_3 \\ 0 & \text{otherwise} \end{cases} \quad \dots (4.6)$$

Where $A_1 = (a_2 - a_1)(b_2 - b_1)$

$$B_1 = b_1(a_2 - a_1) + a_1(b_2 - b_1)$$

$$A_2 = (a_3 - a_2)(b_3 - b_2)$$

And $B_2 = -(a_3(b_3 - b_2) + b_3(a_3 - a_2))$

For non membership function of, β -cut of $\tilde{A}$ is

$v_{\tilde{A}}(x) \leq \beta, A_\beta = [a_2 - \beta(a_2 - a'_1), a_2 + \beta(a'_3 - a_2)]$ for any $\beta \in [0, 1]$

$$i.e x \in [a_2 - \beta(a_2 - a'_1), a_2 + \beta(a'_3 - a_2)]$$

$$\alpha - \text{cut for non membership function of } \tilde{B}, B_\alpha = [b_2 - \beta(b_2 - b'_1), b_2 + \beta(b'_3 - b_2)]$$

$$\mu_{\tilde{B}}(x) \geq \beta \Rightarrow [b_2 - \beta(b_2 - b'_1), b_2 + \beta(b'_3 - b_2)] \text{ for any } \beta \in [0,1]$$

$$i.e y \in [b_2 - \beta(b_2 - b'_1), b_2 + \beta(b'_3 - b_2)]$$

$$\text{So } Z = X(\times)Y \in [(a_2 - \beta(a_2 - a'_1))(b_2 - \beta(b_2 - b'_1)), (a_2 + \beta(a'_3 - a_2))(b_2 + \beta(b'_3 - b_2))]$$

So the non-membership rejection function of $\tilde{Z} = \tilde{A}(\times)\tilde{B}$ is given by

$$v_{\tilde{A}(\times)\tilde{B}}(Z) = \begin{cases} 1 - \frac{-B'_1 + \sqrt{B_1'^2 - 4A'_1(a'_1b'_1 - Z)}}{2A'_1} & , a'_1b'_1 \leq Z \leq a_2b_2 \\ 1 - \frac{-B'_2 + \sqrt{B_2'^2 - 4A'_2(a'_3b'_3 - Z)}}{2A'_2} & , a_2b_2 \leq Z \leq a'_3b'_3 \\ 1 & \text{otherwise} \end{cases} \quad \dots (4.7)$$

$$\text{Where } A'_1 = (a_2 - a'_1)(b_2 - b'_1)$$

$$B'_1 = b'_1(a_2 - a'_1) + a'_1(b_2 - b'_1)$$

$$A'_2 = (a'_3 - a_2)(b'_3 - b_2)$$

$$\text{And } B'_2 = -(a'_3(b'_3 - b_2) + b'_3(a'_3 - a_2))$$

Hence the multiplication rule is proved for non-membership function,

Thus $\tilde{A}(\times)\tilde{B} = (a_1b_1, a_2b_2, a_3b_3; a'_1b'_1, a_2b_2, a'_3b'_3)$ is a TIFN.

Numerical Evaluation and an Electric Network Model Presented

1. Let $\tilde{A} = (1,2,3; 0.5, 2,4)$ and $k\tilde{A} = (k,2k,3k; 0.5k,2k,4k)$ where k is a real number then the membership and non-membership function are as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-k}{k} & , k \leq x \leq 2k \\ \frac{3k-x}{k} & , 2k \leq x \leq 3k \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

For non-membership function,

$$v_{\tilde{A}}(x) = \begin{cases} \frac{2k-x}{1.5} & , 0.5k \leq x \leq 2k \\ \frac{x-2k}{2k} & , 2k \leq x \leq 4k \\ 1 & \text{otherwise} \end{cases}$$

2. Let $\tilde{A} = (1,2,3; 0.5, 2,4.5)$ and $\tilde{B} = (2,3,4; 1,3,5.5)$ are two TIFN then the sum $\tilde{A}(+)\tilde{B}$ is defined by $\tilde{A}(+)\tilde{B} = (3,5,7; 1.5,5,10)$ with membership and non-membership function are as follows:

$$\mu_{\tilde{A}(+) \tilde{B}}(x) = \begin{cases} \frac{x-3}{2} & , 3 \leq x \leq 5 \\ \frac{7-x}{2} & , 5 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

For non-membership function,

$$v_{\tilde{A}(+) \tilde{B}}(x) = \begin{cases} \frac{5-x}{3.5} & , 1.5 \leq x \leq 5 \\ \frac{x-5}{5} & , 5 \leq x \leq 10 \\ 1 & \text{otherwise} \end{cases}$$

3. Let $\tilde{A} = (1, 2, 3; 0.5, 2, 4.5)$ and $\tilde{B} = (2, 3, 4; 1, 3, 5.5)$ are two TIFN then their product $\tilde{A}(x) \tilde{B}$ is defined by $\tilde{A}(x) \tilde{B} = (2, 6, 12; 0.5, 6, 24.75)$ with membership and non-membership function are as follows:

$$\mu_{\tilde{A}(x) \tilde{B}}(x) = \begin{cases} \frac{-3 + \sqrt{9 - 4(2-x)}}{2} & , 2 \leq x \leq 6 \\ \frac{-7 - \sqrt{49 - 4(12-x)}}{2} & , 6 \leq x \leq 12 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$v_{\tilde{A}(x) \tilde{B}}(x) = \begin{cases} \frac{8.5 + \sqrt{6.25 - 12(0.5-x)}}{6} & , 0.5 \leq x \leq 6 \\ \frac{37.5 + \sqrt{625 - 25(24.75-x)}}{12.5} & , 6 \leq x \leq 24.75 \\ 1 & \text{otherwise} \end{cases}$$

Above $\tilde{A}(x) \tilde{B}$ is a triangular intuitionistic fuzzy number. It can be approximated to TIFN as $\tilde{A}(x) \tilde{B} = (2, 6, 12; 0.5, 6, 24.75)$ with membership and non-membership function are as follows

$$\mu_{\tilde{A}(x) \tilde{B}}(x) = \begin{cases} \frac{x-2}{4} & , 2 \leq x \leq 6 \\ \frac{12-x}{6} & , 6 \leq x \leq 12 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

For non-membership function,

$$v_{\tilde{A}(+) \tilde{B}}(x) = \begin{cases} \frac{6-x}{5.5} & , 0.5 \leq x \leq 6 \\ \frac{x-6}{18.75} & , 6 \leq x \leq 24.75 \\ 1 & \text{otherwise} \end{cases}$$

For application, an electric network model presented by Dhillon [8] of a dark room is taken. The system models a dark, windowless room containing one master switch and two light bulbs, to develop a desired event in an illuminated dark room. Thus, in this case, the room can only be brightened if there is availability of electricity. If the switch is Ok, the two light bulbs are not burnt out. By using this application is developed. Each event is considered as TIFN.

R_1 represent no fuse failure of electric supply of dark room,

R_2 represent no power failure of electric supply of dark room,

R_3 represent the electric switch of dark room,

R_4 represent the bulb no. 1 of dark room,

R_5 represent the bulb no. 2 of dark room,

R_6 represent the electric supply of dark room,

R_7 represent the bulbs not burnt.

R_8 represent the desired dark room,

All imprecise components R_j are represented by TIFN $(a_{j1}, a_{j2}, a_{j3}; a'_{j1}, a_{j2}, a'_{j3})$ for $j= 1,2, \dots,5$. Let us calculate the desired event i.e dark room with illumination. The value of the event electricity supply,

$$R_6 = R_1(\times)R_2 = (a_{11}a_{21}, a_{12}a_{22}, a_{13}a_{23}; a'_{11}a'_{21}, a_{12}a_{22}, a'_{13}a'_{23}) \dots (5.1)$$

It is an approximated TIFN.

Similarly, the value of event both bulbs not burnt,

$R_7 = 1(-) \{(1(-)R_4)(1(-)R_5)\}$. It is an approximated TIFN as follows

$$\cong \{1 - \prod_{j=4}^5 1 - a_{j1}, 1 - \prod_{j=4}^5 1 - a_{j2}, 1 - \prod_{j=4}^5 1 - a_{j3}; 1 - \prod_{j=4}^5 1 - a'_{j1}, 1 - \prod_{j=4}^5 1 - a_{j2}, 1 - \prod_{j=4}^5 1 - a'_{j3}\} \dots (5.2)$$

By substituting the above two calculated values and the given data R_3 , we get the value for the, dark room,

$$R_8 = R_1(\times)R_2(\times)R_3(\times)[1(-)\{(1(-)R_4)(1(-)R_5)\}].$$

It is an approximated TIFN as follows

$$\cong \{a_{11}a_{21}a_{31}(1 - \prod_{j=4}^5 1 - a_{j1}), a_{12}a_{22}a_{32}(1 - \prod_{j=4}^5 1 - a_{j2}), a_{13}a_{23}a_{33}(1 - \prod_{j=4}^5 1 - a_{j3}); a'_{11}a'_{21}a'_{31}(1 - \prod_{j=4}^5 1 - a'_{j1}), a_{12}a_{22}a_{32}(1 - \prod_{j=4}^5 1 - a_{j2}), a'_{13}a'_{23}a'_{33}(1 - \prod_{j=4}^5 1 - a'_{j3}) \dots, (5, 3)$$

Let the component of events are

$$R_1 = (0.70, 0.75, 0.80; 0.65, 0.75, 0.85),$$

$$R_2 = (0.80, 0.85, 0.90; 0.75, 0.85, 0.95),$$

$$R_3 = (0.82, 0.85, 0.88; 0.80, 0.85, 0.96),$$

$$R_4 = (0.70, 0.80, 0.85; 0.65, 0.80, 0.95)$$

$$R_5 = (0.65, 0.75, 0.85; 0.60, 0.75, 0.90)$$

So results of equation (5.1) and (5.2) are as follows,

$$R_6 = (0.56, 0.6375, 0.72; 0.4875, 0.6375, 0.8075) \text{ and}$$

$$R_7 = (0.895, 0.95, 0.9775; 0.86, 0.95, 0.995),$$

By substituting the above R_6 and R_7 the given data value is (5.3), the value of desired dark room,

$$R_8 = (0.410984, 0.51478125, 0.619344; 0.3354, 0.51478125, 0.771324).$$

CONCLUSION

In this paper, definition of positive triangular intuitionistic fuzzy number and negative triangular intuitionistic fuzzy number are proposed. Also arithmetic operation proposed TIFN is evaluated based on intuitionistic fuzzy (α, β) -cut method. The set of all triangular fuzzy numbers form a linear space, but the set of all triangular intuitionistic fuzzy numbers does not form a linear space are proved. Membership and non-membership function are discussed using the (α, β) -cut method. Later we will try to calculate the reliability evaluation using triangular intuitionistic fuzzy numbers with the help of triangular fuzzy linear space.

REFERENCES

1. Atanassov, K.T. (1983). Intuitionistic Fuzzy sets, VII ITKR's session ,Sofia (deposed in central science-Technical library of Bulgarian Academy of science,,1697/84)
2. Atanassov, K.T. (1986). Intuitionistic Fuzzy sets, Fuzzy sets and systems, 20 ,87-96.
3. Atanassov, K.T. (1989). More on Intuitionistic Fuzzy sets, Fuzzy sets and systems, 33(1) ,37-46.
4. Atanassov, K.T. (1999). Intuitionistic Fuzzy sets, Physica-Verlag, Heidelberg, New York.
5. Atanassov, K.T. (2000).Two thermo for Intuitionistic Fuzzy sets, Fuzzy sets and systems, 110 ,267-269.
6. Atanassov, K.T., & Gargov, G. (1989). Interval-valued Intuitionistic Fuzzy sets, Fuzzy sets and systems, 31(3), 343-349.
7. Chamuah, H.C.(2026). Application of Trace on Triangular Fuzzy Number Matrices, International Journal of computer Application, vol 187-No.78,0975-8887.
8. Dhillon, B.S., (2007). Human reliability and error in transportation system. (Springer series in reliability engineering), Springer-Verlag, London.
9. Dutta, P., Boruah, H., & Ali, T. (2011).Fuzzy Arithmetic with and without using α -cut method.International journal of latest trend in computing. (IJLTC)2,99-107.
10. Ejegwa, P.A., & Edibo, O.P. (2014). Some distance measures between intuitionistic Fuzzy multisets (IFMSS). International journal of scientific and Technology, Research vol 3, issue 4.
11. Gerstenkorn, T., & Manko, J. (1991) .Correlation of intuitionistic fuzzy sets, Fuzzy sets and systems, 44,39-43.
12. Nasser, Hadi (2008) Fuzzy Number: Positive and Non –negative, International Mathematical Forum, 3, no. 36,1777-1780.
13. Rosario, Michael G. (2015). Intuitionistic Fuzzy Equations and its Application on Reliability Equation, International Journal of Advanced Technology Engineering and science, vol. 03, Number 03.
14. Stoyanova, D., & Atanassov, K.T. (1990) Relation between operations defined over intuitionistic fuzzy sets, IM-MFAIS, Sofia, Bulgaria, 1,46-49.
15. Stoyanova, D. (1993). More on Cartesian product over intuitionistic fuzzy sets, BUSEFAL.54,9-13.
16. Zadeh, L.A. (1965). Fuzzy Sets, information and Control, 8,338-353.