

Oblique Impact Response of a Silicone-Rubber-Supported Steel Plate: Analytical Scaling, Explicit Finite Element Assessment, and Energy Transfer Mechanisms

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ABSTRACT

Oblique impacts are common in protective structures, mechanical stops, compliant mounts, and vibration-isolation systems; nevertheless, the influence of impact angle is often evaluated by reducing the prescribed speed rather than by resolving a constant resultant velocity into normal and tangential components. This practice confounds the physical effect of angle with a change in impact energy. The present study reformulates the response of a steel plate supported by a silicone-rubber element and struck by a rigid spherical impactor at 0°, 30°, 45°, and 60°, where the angle is measured from the plate normal. A reduced-order momentum–energy model is derived for a perfectly inelastic normal collision. The model predicts that the post-impact normal velocity, maximum support compression, and peak support force scale with $\cos(\theta)$, while the energy transferred to the support scales with $\cos^2(\theta)$. A preliminary two-dimensional Abaqus/Explicit model is then assessed using the archived computational results. The reported peak plate displacement decreased from 148.34 mm at 0° to 36.88 mm at 60°, while the peak von Mises stress decreased from 5.398 to 3.81 MPa. The numerical displacement decreased more rapidly than the analytical $\cos(\theta)$ prediction, demonstrating that a single-degree-of-freedom model cannot represent contact redistribution, tangential slip, plate flexibility, and the nonlinear constitutive response of rubber. The study further distinguishes recoverable strain energy from genuinely dissipated energy and specifies the numerical energy channels required for defensible energy-absorption claims. The revised framework provides a coherent basis for a validated three-dimensional investigation, but material calibration, mesh convergence, energy-balance verification, and experimental validation remain necessary before final journal submission.

Keywords: Oblique Impact; Mass–Spring System; Impact Angle; Abaqus/Explicit; Finite-Element Analysis.

INTRODUCTION

Elastomer-supported plates and compliant impact stops are used to reduce transmitted shock, limit peak contact force, and protect adjacent components. Their performance is governed by the interaction among impact kinematics, contact friction, structural flexibility, and the nonlinear response of the elastomer. In practical systems, the direction of the incident body is rarely perfectly normal to the target. Oblique impact therefore introduces a tangential velocity component that changes the contact trajectory, frictional work, local stress field, rebound, and the fraction of kinetic energy transferred into the compliant support.

Classical oblique-impact theory shows that normal and tangential contact processes are coupled through friction and tangential compliance. Maw, Barber, and Fawcett demonstrated that rigid-body approximations may be adequate at very low or very high incidence angles but can become inaccurate at intermediate angles [1]. The contact formulation of Mindlin and Deresiewicz established the importance of stick–slip transitions under varying oblique forces [2]. Experimental measurements by Kharaz, Gorham, and Salman later confirmed that rebound speed, rebound angle, and rotation depend strongly on incidence angle and contact conditions [3]. These studies provide a rigorous foundation for analyzing the kinematics of a sphere striking a compliant or deformable target.

The material description of the support is equally important. Silicone rubber can sustain large recoverable deformation and is often represented using hyper-elastic strain-energy functions. The Mooney [6], Ogden [7], Arruda–Boyce [8], and Gent [9] models are widely used to describe nonlinear elasticity. Filled and unfilled silicones may also exhibit rate dependence, hysteresis, stress softening, and temperature sensitivity [10]–[16]. Consequently, a small-strain linear-elastic model may be useful for an initial trend study, but it is not sufficient for a quantitative impact prediction when deformation is large or the strain rate is high.

Energy terminology requires particular care. The integral of force over displacement during loading is the mechanical work transferred into the structure. In a purely elastic or hyper-elastic support, this work is stored as recoverable strain energy and may be returned during unloading. It should not be reported as “absorbed energy” unless a dissipative mechanism—such as viscoelasticity, friction, plasticity, damage, or permanent set is explicitly present and quantified.

Abaqus/Explicit provides separate energy outputs for recoverable strain energy, viscous dissipation, frictional dissipation, plastic dissipation, artificial energy, and kinetic energy; these channels should be evaluated individually rather than combined without interpretation [17], [18].

The original version of the present research contained a useful parametric concept but also contained inconsistencies in impactor dimensions, angle definition, velocity prescription, rubber constitutive behavior, and energy interpretation. The most consequential issue was the absence of a clear statement that the resultant impact speed must remain constant for every angle. Replacing the speed by an angle-dependent scalar produces an apparent angle effect that is caused by changing the total incident energy.

Accordingly, the objective of this study is to establish a mechanically consistent and publication-oriented framework for evaluating the angle-dependent response of a silicone-rubber-supported steel plate. The specific contributions are:

- i. a precise definition of the impact angle and velocity components
- ii. a corrected reduced-order analytical model based on momentum and energy
- iii. a transparent reassessment of the archived finite-element results
- iv. a rigorous distinction between transferred, stored, and dissipated energy, and
- v. a verification and validation plan for a final three-dimensional model.

Problem Definition and Study Scope

Physical system

The system consists of a spherical impactor, a steel plate, and a silicone-rubber support. The plate is assumed to be guided so that its dominant motion is along the plate-normal direction. The rubber element supplies the restoring force and may also provide material damping when an appropriate viscoelastic model is used. The impactor approaches the plate at a constant resultant speed v_0 and at an angle (θ) measured from the outward plate normal. Thus, $\theta = 0^\circ$ denotes a normal impact, while increasing (θ) denotes a progressively more oblique impact.

The baseline quantities retained from the archived study are an impactor mass of 0.50 kg, an impactor diameter of 50 mm, a plate mass of 1.76 kg, a plate length of 300 mm, a plate thickness of 20 mm, and an impact speed of 50 km/h (13.89 m/s). The contradictory values of 80 mm and 3 kg stated in the former text have been removed because they conflict with the parameter table and with the momentum calculations.

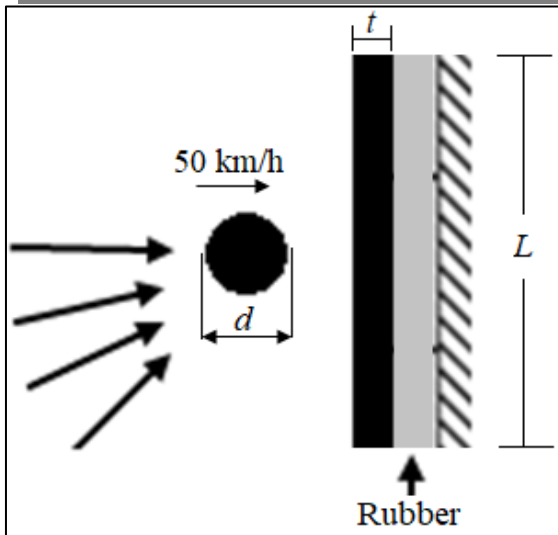


Figure 1. Definition of the impact system, impact angle, and normal velocity

Parameter	Value	Unit
Impactor mass, m_i	0.50	kg
Impactor diameter, d_i	50	mm
Plate mass, m_p	1.76	kg
Plate length, L_p	300	mm
Plate thickness, t_p	20	mm
Resultant impact speed, v_0	13.89	m/s
Impact angles, (θ)	0, 30, 45, 60	deg
Steel density	7850	kg/m ³
Steel Young's modulus	200	GPa
Steel Poisson's ratio	0.30	–
Silicone-rubber density	1100	kg/m ³
Rubber Young's modulus	50	MPa
Rubber Poisson's ratio	0.47	–
Friction coefficient, μ_f	0.10	–
Rubber height $\times$ thickness	300 $\times$ 20	mm

Table 1. Baseline problem definition and specifications

Modelling hierarchy and assumptions

Two modelling levels are distinguished to avoid mixing assumptions and conclusions:

- The reduced-order analytical model treats the impactor and plate as lumped masses. The normal collision is perfectly inelastic ($e_n = 0$), the plate is initially stationary, the support is represented by an equivalent linear stiffness k_{eq} , and the tangential impulse is omitted. This model is intended to establish scaling laws and a transparent benchmark.
- The finite-element model resolves plate deformation, distributed contact, and friction. The archived model is two-dimensional and uses a nearly incompressible linear-elastic representation of the rubber; therefore, it is treated as a preliminary screening model. The final journal model should be three-dimensional and should use a calibrated hyper-elastic constitutive law.

The analytical model does not claim to reproduce the complete oblique-contact event. Its purpose is to provide a lower-complexity reference against which the numerical trends and deviations can be interpreted.

Analytical Formulation

Velocity decomposition

For a constant resultant impact speed v_0 , the normal and tangential velocity components are

$$v_n = v_0 \cos(\theta) \quad (1)$$

$$v_t = v_0 \sin(\theta) \quad (2)$$

Equations (1) and (2) are essential. The impact angle must be changed by rotating the velocity vector, not by reducing the magnitude of the prescribed velocity. At 60° , for example, the normal component is one-half of v_0 , but the resultant speed and the total incident kinetic energy remain unchanged.

Normal momentum and post-impact velocity

The plate is initially stationary. For a perfectly inelastic normal collision, the impactor and plate have the same normal velocity immediately after contact. Conservation of normal linear momentum gives

$$m_i v_n = (m_i + m_p) V_n \quad (3)$$

$$V_n = \frac{m_i}{m_i + m_p} v_0 \cos(\theta) \quad (4)$$

A more general model may include a non-zero coefficient of restitution and separate post-impact velocities. The assumption $e_n = 0$ is retained here only because the former study explicitly described the event as perfectly plastic. This assumption should be verified experimentally; otherwise, e_n should be identified from rebound measurements.

Energy transferred to the support

The kinetic energy associated with the combined normal motion immediately after the inelastic collision is

$$E_{tr} = \frac{1}{2} (m_i + m_p) V_n^2 \quad (5)$$

$$E_{tr} = \frac{m_i^2}{2(m_i + m_p)} v_0^2 \cos^2(\theta) \quad (6)$$

E_{tr} is the maximum mechanical energy available to be converted into support strain energy and structural vibration in the reduced-order model. It is not equal to the total incident kinetic energy because a perfectly inelastic collision dissipates part of the normal kinetic energy at the collision interface. The normal collision loss is

$$E_{col} = \frac{1}{2} m_i v_0^2 \cos^2(\theta) - E_{tr} \quad (7)$$

where E_{col} is the collision energy and E_{tr} is the transferred energy.

Maximum support compression and force

For an equivalent linear support with stiffness k_{eq} and negligible damping during the first compression phase, the maximum compression is obtained by equating E_{tr} to the recoverable spring energy:

$$\frac{1}{2} k_{eq} x_{max}^2 = E_{tr} \quad (8)$$

$$x_{max} = \frac{m_i v_0}{\sqrt{k_{eq}(m_i + m_p)}} \cos(\theta) \quad (9)$$

$$F_{max} = k_{eq} x_{max} = m_i v_0 \sqrt{\frac{k_{eq}}{m_i + m_p}} \cos(\theta) \quad (10)$$

Therefore, the normalized analytical relations are

$$\frac{x_{max}(\theta)}{x_{max}(0)} = \frac{F_{max}(\theta)}{F_{max}(0)} = \cos(\theta) \quad (11)$$

$$\frac{E_{tr}(\theta)}{E_{tr}(0)} = \cos^2(\theta) \quad (12)$$

These relations provide a direct physical benchmark: displacement and force should decrease approximately with the normal velocity component, while transferred energy should decrease with the square of that component. Deviations are expected when plate flexibility, nonlinear support stiffness, friction, rebound, or three-dimensional contact effects become important.

Analytical benchmark values

The equivalent value of $k_{eq} = 0.835$ kN/m was identified solely by matching the former 0° analytical displacement of 159.89 mm. This calibration is used only to produce an absolute numerical benchmark; the angular scaling in Equations (11) and (12) is independent of the calibrated stiffness. A measured or independently identified stiffness must replace this value in the final paper.

Angle (deg)	V_n (m/s)	V_n (m/s)	E_{tr} (J)	X_{max} (mm)	F_{max} (N)
0	13.889	3.073	10.669	159.89	133.46
30	12.028	2.661	8.002	138.47	115.58
45	9.821	2.173	5.335	113.06	94.37
60	6.944	1.536	2.667	79.95	66.73

Table 2. Corrected analytical benchmark for constant resultant impact speed.

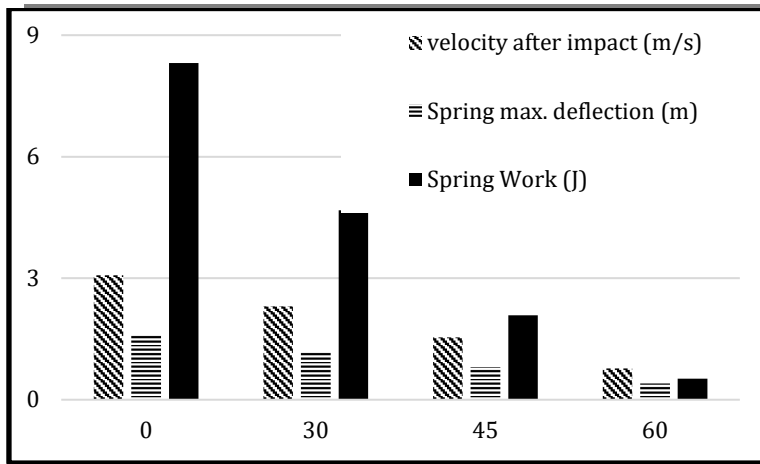


Figure 2. Analytical variation of normalized support displacement, peak force, and transferred energy with impact angle.

Definition of stored and dissipated energy

The mechanical work transferred into the support during loading is

$$W_{\text{load}} = \int_0^x f(x) dx \quad (13)$$

For conservative elastic or hyper-elastic support, W_{load} is recoverable strain energy U_{rec} . genuine energy absorption requires an irreversible mechanism. For a loading–unloading cycle, the material dissipation is the enclosed hysteresis area:

$$E_{\text{diss,mat}} = \oint F dx \quad (14)$$

In an explicit finite-element model, the total dissipated energy should be reported as a sum of clearly defined components, for example viscoelastic dissipation, frictional dissipation, plastic dissipation, damage dissipation, and any intentionally introduced damping. Recoverable strain energy must not be added to these components and labelled as permanently absorbed energy.

Finite-Element Methodology

Geometry and dimensional representation

The archived computation was performed as a two-dimensional explicit dynamic analysis. In such a model, a circular impactor represents a cylinder of unit out-of-plane thickness rather than a true sphere, and the contact and inertia fields differ from those in three dimensions. The archived model is therefore retained as a screening analysis only. A three-dimensional half- or quarter-symmetry model is recommended for the final manuscript, provided that the chosen symmetry planes remain valid for the oblique velocity and friction conditions.

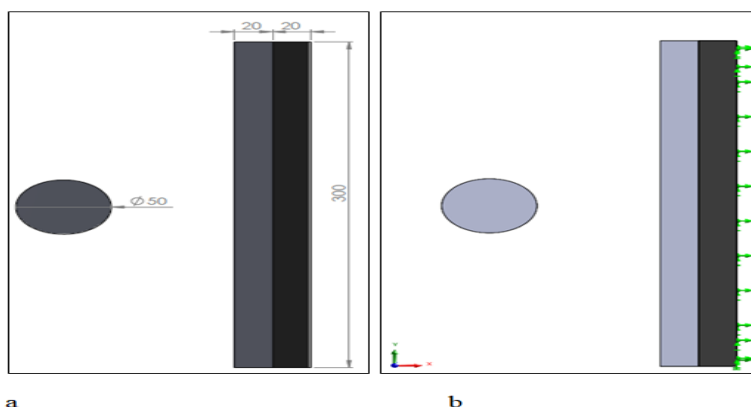


Figure 3. Finite-element geometry, (a) component dimensions, (b) reference points, and constraints

Material models

The steel plate may be represented as linear elastic when the computed stress remains far below the yield strength. If local yielding occurs after model refinement, an elastic–plastic law with an experimentally supported yield curve should be introduced. The archived study reported a steel Young’s modulus of 200 GPa, Poisson’s ratio of 0.30, density of 7850 kg/m³, and yield strength of 250 MPa.

The archived rubber model used $E = 50$ MPa, $\nu = 0.47$, and $\rho = 1100$ kg/m³. This nearly incompressible linear-elastic representation is unsuitable for large deformation because rubber stiffness is deformation-path dependent and may be rate dependent. The final model should fit a hyper-elastic law—preferably Ogden, Arruda–Boyce, Gent, or Mooney–Rivlin—to at least uniaxial compression and tension data, with planar or biaxial data added when available [7]–[16].

Contact, constraints, and initial conditions

- Normal contact: hard pressure-overclosure behaviour with separation permitted after contact unless the experiment demonstrates adhesion or permanent sticking.
- Tangential contact: penalty friction with $\mu_f = 0.10$ for the archived baseline, followed by a sensitivity study over a physically justified range.
- Impact velocity: v_x and v_y components prescribed from Equations (1) and (2), while maintaining $|v| = 13.89$ m/s for every angle.
- Plate guidance: boundary conditions should reproduce the physical fixture and suppress only the degrees of freedom that are constrained in the experiment.
- Rubber interfaces: use contact or tie constraints according to the real assembly. Bonded and lubricated interfaces can generate substantially different apparent stiffness and damage patterns.

Element formulation and mesh convergence

For the preliminary two-dimensional model, reduced-integration plane-strain elements with appropriate hourglass control may be used, while nearly incompressible hyper-elastic regions should employ hybrid formulations where supported. For the final three-dimensional model, reduced-integration hybrid brick elements are preferred for the silicone rubber when the geometry permits. The contact zone, rubber corners, and plate–support interface require local refinement.

A minimum three-level mesh study should be reported. For example, the local element size may be reduced from 5.0 to 2.5 and 1.25 mm while monitoring peak displacement, peak contact force, peak stress away from singular corners, and dissipated energy. Mesh independence should be assessed using a stated tolerance, such as less than 3–5% change in the primary response quantities between the two finest meshes.

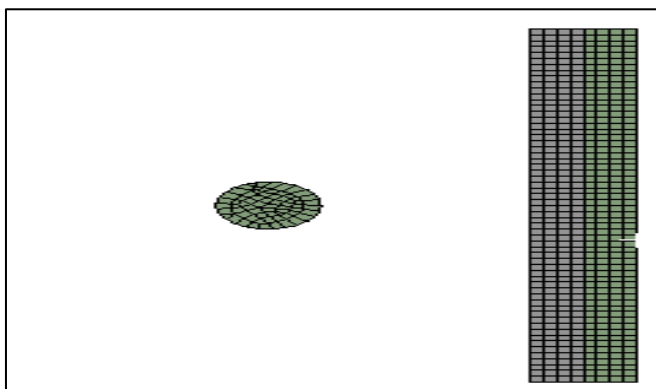


Figure 4. Global refined finite-element meshes used in the convergence study.

Output variables and numerical quality checks

The final analysis should extract plate displacement, rubber compression, contact force, impulse, impactor and plate velocities, von Mises stress in the steel, principal strain in the rubber, contact slip, and time-resolved energy histories. The minimum global energy outputs are kinetic energy (ALLKE), recoverable strain energy (ALLSE), internal energy (ALLIE), frictional dissipation (ALLFD), viscoelastic or creep dissipation when applicable (ALLCD), viscous dissipation (ALLVD), artificial strain energy (ALLAE), external work (ALLWK), and total energy balance (ETOTAL) [17], [18].

The adopted acceptance criteria for the final simulations should include stable total energy with approximately less than 1% drift; artificial energy small relative to physical internal energy; negligible contact-penalty work relative to the transferred impact energy; no excessive mass scaling; and a simulation duration sufficient to capture peak compression and the first unloading or rebound phase. These checks are required before any energy-absorption conclusion is accepted.

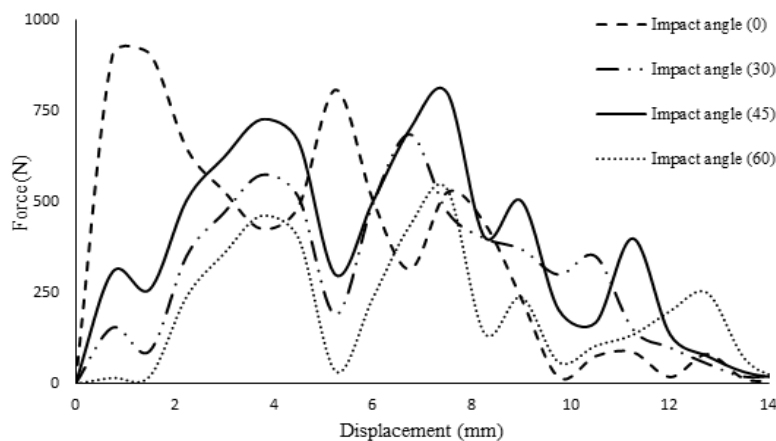


Figure 5. Force displacement curve for the final explicit dynamic simulations.

RESULTS

Corrected analytical response

The analytical benchmark in Table 3 predicts a monotonic reduction in normal post-impact velocity, maximum support compression, peak support force, and transferred energy as (θ) increases. Relative to normal impact, the expected displacement ratios are 0.866, 0.707, and 0.500 at 30°, 45°, and 60°, respectively. The corresponding energy ratios are 0.750, 0.500, and 0.250. These trends arise directly from the normal velocity component and do not require changing the resultant impact speed. At 60°, the normal velocity component is reduced by 50%, and the energy available to the support is reduced by 75%. This distinction is important: a design may experience a moderate reduction in displacement or force while experiencing a much larger reduction in energy transfer.

Preliminary archived finite-element response

The archived finite-element data reported peak plate displacements of 148.34, 103.15, 73.05, and 36.88 mm for $\theta = 0^\circ, 30^\circ, 45^\circ$, and 60° , respectively. The reported peak von Mises stresses were 5.398, 4.90, 4.45, and 3.81 MPa.

Angle (deg)	Peak Displacement (mm)	Normalized displacement	Peak σ_{vm} (MPa)	Normalized stress	Displacement reduction (%)	Stress reduction (%)
0	148.34	1.000	5.398	1.000	0.0	0.0
30	103.15	0.695	4.900	0.908	30.5	9.2

45	73.05	0.492	4.450	0.824	50.8	17.6
60	36.88	0.249	3.810	0.706	75.1	29.4

Table 3. Preliminary finite-element results retained from the archived study.

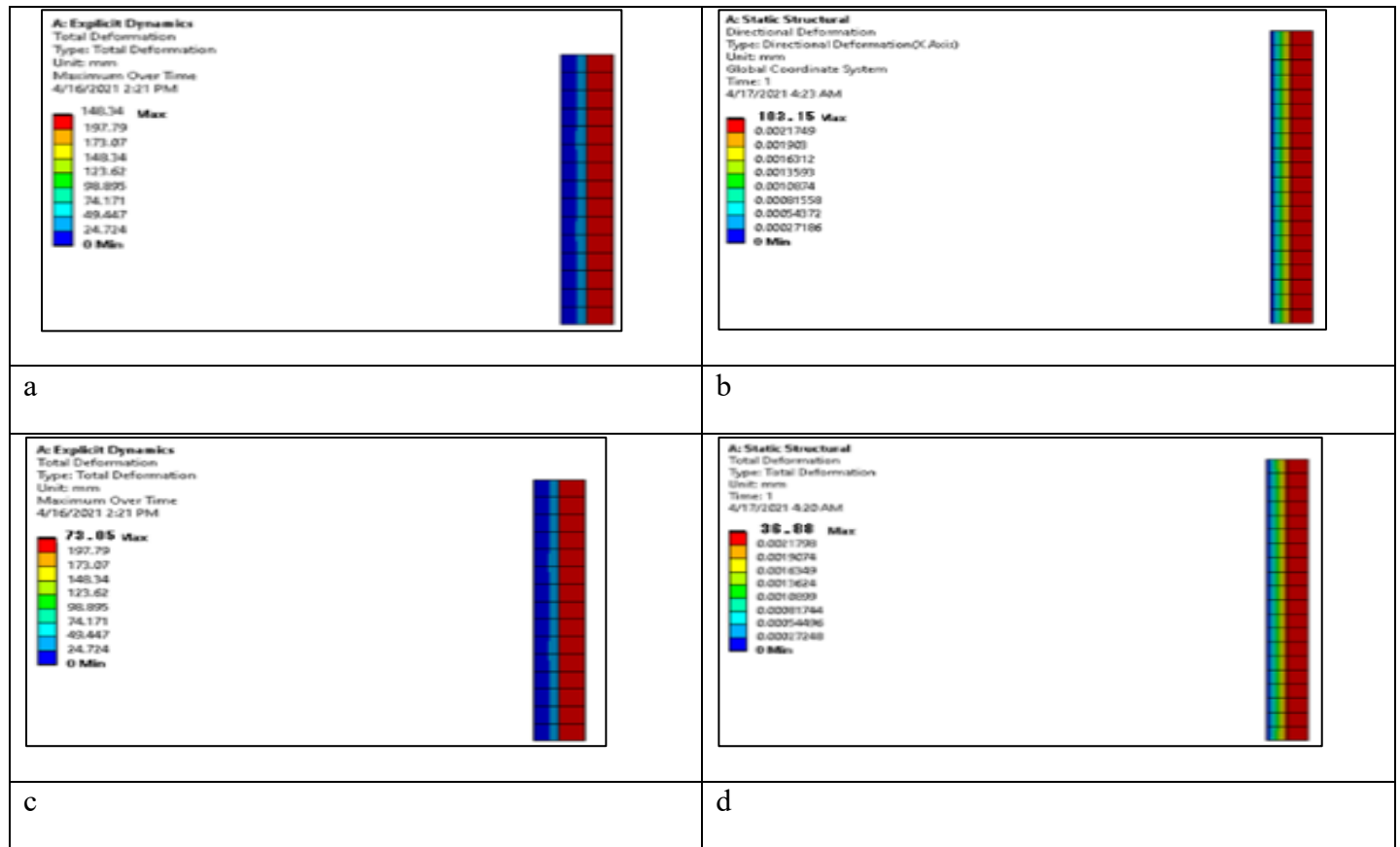


Figure 6. Plate displacement and rubber deformation contours for (a) 0°, (b) 30°, (c) 45°, and (d) 60° impact.

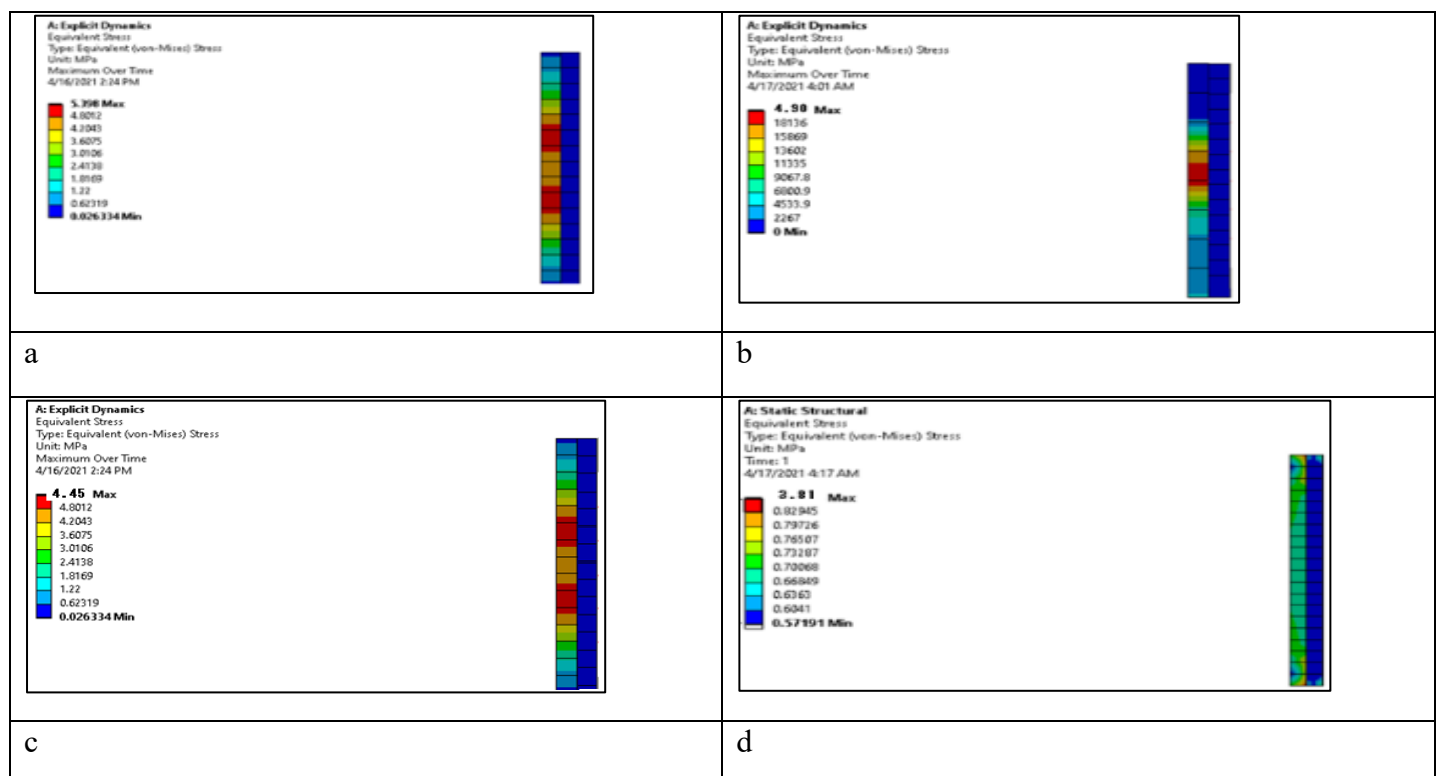


Figure 7. Von Mises stress contours for (a) 0°, (b) 30°, (c) 45°, and (d) 60° impact.

Analytical-Numerical comparison

Angle (deg)	Analytical x_{max} (mm)	FE displacement (mm)	Difference relative to analytical (%)	Analytical normalized x	FE normalized x
0	159.89	148.34	7.2	1.000	1.000
30	138.47	103.15	25.5	0.866	0.695
45	113.06	73.05	35.4	0.707	0.492
60	79.95	36.88	53.9	0.500	0.249

Table 4. Comparison of the corrected reduced-order benchmark with the archived finite-element displacement.

The finite-element displacement decreases more rapidly with angle than the $\cos(\theta)$ scaling of the reduced-order model. The difference increases from 7.2% at normal impact to 53.9% at 60°. This divergence should not be interpreted as validation of either model. Instead, it shows that the archived finite-element response contains mechanisms excluded from the single-degree-of-freedom formulation, or that the original angle/velocity implementation requires correction. The final input files must therefore be inspected to verify that the same resultant speed was used in every case. The stress response is less sensitive to angle than the global displacement response. At 60°, displacement decreases by approximately 75%, whereas peak von Mises stress decreases by only approximately 29%. This result suggests that local contact and bending stresses may remain significant even when the overall support travel is strongly reduced. The interpretation should be confirmed using a refined three-dimensional mesh and stress values evaluated away from contact or geometric singularities.

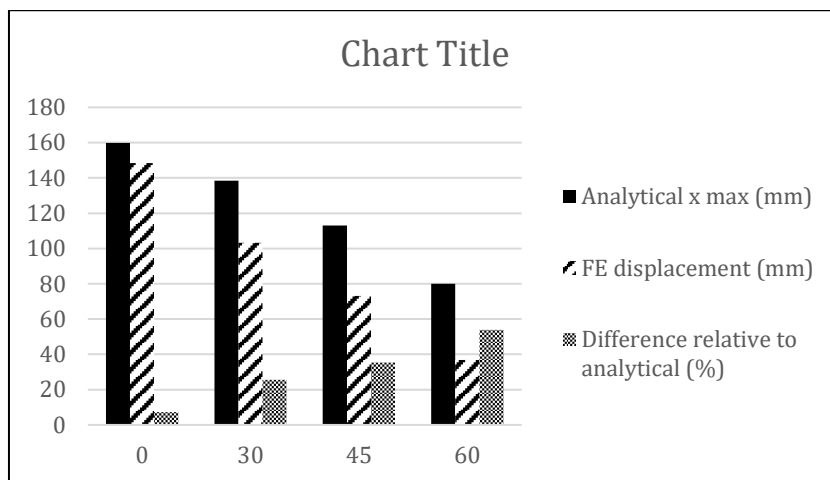


Figure 8. Analytical and finite-element displacement, transferred energy, and peak stress versus impact angle.

Energy results and correction of the former interpretation

The former manuscript reported energy values in J/m^3 obtained from the area under force–displacement curves. The area under a loading curve has units of joules; it becomes an energy density only after division by a clearly defined material volume. For the analytical model, the physically defined post-collision energy available to the support decreases from 10.669 J at 0° to 2.667 J at 60°. These values represent transferred mechanical energy, not permanent absorption. In the final finite-element model, recoverable strain energy and each dissipative energy channel must be plotted separately. A claim of improved energy absorption should be based on an increased dissipated-energy fraction at an acceptable peak force and deformation, not merely on a larger loading work.

DISCUSSION

Physical effect of impact angle

The primary effect of increasing angle is the reduction of the velocity component normal to the plate. Because normal momentum is proportional to $\cos(\theta)$ and normal kinetic energy is proportional to $\cos^2(\theta)$, progressively oblique impact reduces the driving input to the support. The tangential component simultaneously increases, which may increase sliding distance and frictional work. Therefore, obliquity does not simply “remove” energy; it redistributes the incident motion between normal deformation, tangential sliding, rebound, and rigid-body motion. This distinction has direct design relevance. A sloped protective interface can reduce penetration or support travel, but it may increase tangential reaction, surface wear, local shear strain, or the risk of secondary impact. Consequently, impact angle should be selected together with friction, available clearance, rebound trajectory, and the allowable stress in both the plate and elastomer.

Role of the rubber constitutive model

Hyper-elastic models represent the nonlinear relationship between stress and finite deformation, while viscoelastic models account for rate dependence and hysteresis [7]–[16]. The most appropriate formulation depends on the actual silicone compound. Unfilled silicones may show weak hysteresis at moderate rates, whereas filled silicones can exhibit substantial Mullins softening and rate sensitivity. Material identification must therefore be performed on the same compound and over a strain-rate range relevant to the impact event. The material model also determines the meaning of “energy absorption”. A purely hyper-elastic support stores and returns energy. A viscoelastic support dissipates a portion of the work through hysteresis. Filled rubbers may additionally exhibit stress softening and damage. Reporting only deformation and von Mises stress is insufficient to distinguish these mechanisms.

CONCLUSIONS

1. A consistent impact-angle definition was established by measuring angle from the plate normal and maintaining a constant resultant impact speed for all cases. The angle effect is introduced through $v_n = v_0 \cos(\theta)$ and $v_t = v_0 \sin(\theta)$.
2. For a perfectly inelastic reduced-order collision, the post-impact normal velocity, maximum support compression, and peak support force scale with $\cos(\theta)$, whereas the energy transferred to the support scales with $\cos^2(\theta)$.
3. The finite-element results show that peak displacement decreased from 148.34 mm at 0° to 36.88 mm at 60° , and peak von Mises stress decreased from 5.398 to 3.81 MPa. The displacement reduction was substantially larger than the stress reduction.
4. The finite-element displacement decreased more rapidly with angle than the corrected analytical benchmark. This difference demonstrates the limitations of a single-degree-of-freedom model and requires verification of the original velocity implementation, geometry, contact formulation, and material model.
5. The present revision provides a coherent analytical and computational framework, but a calibrated three-dimensional model, mesh convergence, energy-balance verification, and experimental validation are required before the work can support a high-impact journal submission.

Recommendations and Future Work

- Reconstruct the model in three dimensions using the measured dimensions of the impactor, plate, rubber block, guide, and fixtures. Archive the complete input file and a parameter table.

- Perform uniaxial compression and tension tests on the actual silicone rubber. Fit at least two candidate hyper-elastic models and select the model using multi-mode goodness-of-fit and stability checks. Add viscoelasticity only when supported by rate or relaxation data.
- Apply a constant resultant velocity at all angles. Verify the input numerically by checking that $\sqrt{V_n^2 + V_t^2}$ is identical in every simulation.
- Conduct a three-level mesh-convergence study and report the convergence of displacement, contact force, stress, and energy. Avoid using a singular nodal stress as the principal comparison metric.
- Report complete explicit-dynamics energy histories and define “absorbed energy” as the sum of physically dissipated components. Report recoverable strain energy separately.
- Validate the model experimentally using an instrumented drop-weight or pendulum impact rig with interchangeable impact-angle fixtures. Measure incident and rebound velocities, contact force, plate travel, and full-field deformation where possible.
- Extend the parametric study to friction coefficient, rubber thickness, support area, hardness, plate thickness, impact speed, and coefficient of restitution. Present the results as normalized design maps.
- Quantify uncertainty arising from rubber parameters, friction, impact angle, and specimen dimensions using sensitivity analysis or probabilistic propagation.

Highlights

- A mechanically consistent formulation is introduced in which the resultant impact speed remains constant while the normal and tangential velocity components vary with the impact angle.
- Closed-form scaling relations show that the maximum support compression and peak spring force vary with $\cos(\theta)$, whereas the post-collision energy available to the support varies with $\cos^2(\theta)$.
- Recoverable strain energy, collision loss, frictional dissipation, and viscoelastic dissipation are separated to prevent the common but incorrect use of the term “absorbed energy.”
- The finite-element data show substantial reductions in plate displacement and von Mises stress as the impact becomes more oblique.

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