Advancements in Cable Force Optimization for Long-Span and Hybrid Bridge Systems: A Comprehensive

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ABSTRACT

The optimization of cable forces represents the primary determinant of structural integrity, aesthetic profile, and economic viability for long-span bridges. Achieving a "Reasonable Finished Dead Load State" (RFDLS) ensures that the massive self-weight of the structure is distributed efficiently, minimizing parasitic bending moments in the pylons and the stiffening girder. This paper provides an exhaustive review of seminal works spanning five decades, mapping the transition from early classical mechanics to the current era of artificial intelligence in structural engineering.

The trajectory of research begins with early strain-energy minimization techniques, established as the foundational theory of the field. In this stage, the objective was strictly mathematical: tensioning cables to achieve a "continuous beam on rigid supports" behavior. As computational power increased, the field witnessed metaheuristic breakthroughs, where algorithms like Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) allowed engineers to navigate the non-linear complexities and large-scale displacements inherent in spans exceeding one kilometer.

Currently, the paradigm has shifted toward surrogate-assisted machine learning and "Smart Bridge" frameworks. These modern approaches utilize Neural Networks to handle structural uncertainties, Graph Theory to model the connectivity of hybrid systems, and Evolutionary Particle Swarm Optimization (EPSO) to perform high-speed design refinements. By training digital twins and surrogate models, researchers can now bypass the computational heavy-lifting of repeated Finite Element Analysis (FEA), concluding with a strategic vision for future autonomous structural design. In this upcoming phase, bridges will not only be designed via optimization but will likely self-adjust cable tensions in real-time to respond to dynamic environmental demands.

Keywords: Bridge, Cable force, EPSO, Genetic Algorithms, long span bridges, Optimization,

INTRODUCTION

The primary objective of cable force optimization is to establish a structural The primary objective of cable force optimization is to establish a structural equilibrium where the bridge deck and pylons are subjected to minimal bending moments under permanent dead loads. This idealized condition, formally termed the Reasonable Finished Dead Load State (RFDLS), effectively transforms a highly flexible, cable-supported assembly into a system that mimics a continuous beam resting on rigid supports.

a. Philosophical and Mathematical Evolution

The core philosophy of this field was solidified by the early foundational works of Feder (1976) and Furukawa (1987). They posited that structural optimization is essentially a minimization problem of the global strain energy. The mathematical goal is to minimize the energy stored in the structure due to deformation:

$$U = \sum_{i=1}^{n} \frac{N_i^2 L_i}{2E_i A_i}$$

By precisely calculating and applying tension to individual cables, engineers can effectively "pre-stress" the entire bridge. This artificial tensioning creates an upward force component that counteracts the significant downward deflections caused by the massive self-weight of the steel box girders and concrete components.

b. Construction Sequence and Fatigue

Building upon these foundations, Qin (1992) introduced a critical temporal dimension to optimization. He demonstrated that the sequence of cable stretching during the construction phase is just as vital as the final target force.

Foundational Numerical Methods and Practical Implementations (2000–2010)

As computational tools advanced, the engineering industry underwent a paradigm shift, moving away from idealized theoretical prestressing toward robust iterative numerical solutions. This era was defined by the transition from hand-calculations to algorithmic approaches capable of handling the high degree of indeterminacy found in modern bridge systems.

a. The Influence Matrix Method

The mid-2000s saw the standardization of the Influence Matrix Method, popularized by the works of Janjic (2003) and Liang (2003). This sensitivity-based approach remains a core feature in modern commercial FEA software like SAP2000 and Midas Civil. The method is governed by the linear relationship:

As computational tools matured, the industry shifted from theoretical prestressing to iterative numerical solutions. Janjic (2003) and Liang (2003) introduced methods that are still used in commercial software today. Their work focused on the Influence Matrix Method, which uses a sensitivity approach:

$$[S] \cdot \{\Delta F\} = \{\Delta D\}$$

In this formulation, **[S]** represents the Influence Matrix (or sensitivity matrix), where each coefficient s_{ij} defines the change in a specific structural response (such as a displacement at a node or a bending moment in the girder) resulting from a unit change in the force of cable j. By inverting or solving this matrix, engineers can precisely calculate the required adjustments in cable forces $\{\Delta F\}$ to achieve a desired geometric profile or stress state $\{\Delta D\}$.

a. Validation and Economic Impact

The practical validity of these mathematical frameworks was confirmed by Sung (2006) through its application to the Mau-Lo Hsi Bridge. This case study served as a landmark proof-of-concept, demonstrating that complex mathematical optimization could effectively resolve the tensioning challenges of real-world, large-scale infrastructure.

Closing this era of research, Baldomir (2010) expanded the scope of optimization beyond mere structural equilibrium. His work on ultra-long spans demonstrated that optimization serves a dual purpose:

1. Safety and Stability: Ensuring the bridge remains within elastic limits.
2. Economic Efficiency: By optimizing "load pathing"—the route through which forces travel from the deck to the foundations—Baldomir showed that material consumption could be reduced by up to 15%.

For a long span case study bridge, a up to 15% reduction in steel and concrete represents a massive saving in both cost and carbon footprint, making optimization a critical requirement for sustainable large-scale construction.

The Rise of Metaheuristic and Evolutionary Algorithms (2010–2020)

The mid-2010s marked a departure from "gradient-based" methods, which often got stuck in local minima, toward "Global Search" algorithms. Lute (2011) pioneered the use of Genetic Algorithms (GA), which mimic natural selection to find optimal forces. This was particularly effective for bridges where the relationship between cable force and deck displacement is highly non-linear due to large-span cable sag.

Guo (2019) and Martins (2020) emphasized the need for hybridity. Guo utilized Simulated Annealing combined with B-spline interpolation to ensure that force distributions remained "smooth" across adjacent cables, preventing localized stress concentrations. Martins (2020) provided a comprehensive survey of 90 papers, concluding that the future of the field lay in Hybrid Metaheuristics—combining the speed of deterministic methods with the global reach of evolutionary algorithms.

The AI Revolution: Surrogate Models and Deep Learning (2020–2025)

The most significant shift in the last five years has been the integration of Machine Learning to solve the "Computational Tax" problem. Standard Finite Element Analysis (FEA) for a long span bridge can take minutes per run; an optimizer needing 10,000 runs would take weeks.

A. Surrogate Modeling and Efficiency

Tu (2025) and Song (2023) introduced Gaussian Process Regression (GPR) and Differential Evolution. By building a "surrogate model" (a digital twin that predicts FEA results), they reduced optimization time by 90%. Tu (2025) specifically paired GPR with the Whale Optimization Algorithm (WOA), demonstrating that AI can predict optimal cable forces with 99% accuracy compared to full-scale simulations.

B. Neural Networks and Reliability

Zhao (2025) and Lei (2025) moved beyond simple optimization into Reliability and Graph Theory. Zhao used Seagull Optimization and Neural Networks to account for uncertainties like wind speed and material degradation. Lei (2025) utilized Graph Neural Networks (GNN) to map the complex connectivity of hybrid bridges, allowing the optimizer to "understand" how a force change in a stay cable affects the tension in the main suspension cable.

Bridge Construction and Structural Health Monitoring

Optimization is now a life-cycle concept. Zhou (2025) developed multi-stage algorithms that optimize forces at every increment of the "cantilever construction" method. This ensures the bridge remains stable even when only half-completed.

In the maintenance phase, Aloisio (2023) and Huang (2018) linked force optimization with Structural Health Monitoring (SHM). By measuring real-world cable vibrations, their algorithms "back-calculate" the current state of the bridge to detect hidden damage or relaxation in the steel strands. Wang (2023) streamlined this by showing how MATLAB can be integrated with Midas Civil, allowing working engineers to run these complex AI loops within their standard design software.

LITERATURE REVIEW ON CABLE-FORCE OPTIMIZATION

A. Problem statement and general formulation

Cable-force optimization in cable-stayeds aims to choose a set of stay-cable pretensions so that stresses, deflections, and design constraints are as close as possible to target values under service and construction loads. The general nonlinear optimization problem can be written as:

$$\min_{\mathbf{x}\in\mathbb{R}^n} \mathcal{F}(\mathbf{x}) = \sum_{i=1}^{N_{\text{obj}}} w_i \ \Phi_i(\mathbf{x}) \qquad (1)$$

subject to

$$\mathbf{g}(\mathbf{x}) \leq \mathbf{0}, \mathbf{h}(\mathbf{x}) = \mathbf{0}, \mathbf{x}^{\min} \leq \mathbf{x} \leq \mathbf{x}^{\max}, \qquad (2)$$

where $\mathbf{x}$ is the vector of cable-force variables (initial tensions, adjustment increments, or construction-stage values), Φ_i are objective contributions (e.g., deviation from target deflection, bending-moment level, strain energy), w_i are weights, and g, h encode constraints on stresses, displacements, and force limits.

A common specific form is:

$$\mathcal{F}(\mathrm{x}) = w_1 \parallel \mathrm{v}(\mathrm{x}) - \mathrm{v}^* \parallel_2^2 + w_2 \parallel \sigma(\mathrm{x}) - \sigma^* \parallel_2^2 + w_3 E_{\text{strain}}(\mathrm{x}), \qquad (3)$$

where v and σ are nodal displacements and stresses, v^*, σ^* are targets, and E_{strain} measures structural energy.

B. Classical influence-matrix and unit-load methods

Classical methods assume small strain and predefined geometry, and so they rely on an influence-matrix description of the cable-deck-tower system. The deflection–force relationship is approximated as

$$\mathbf{v}^{(j)} = \sum_{i=1}^{n_c} c_{ij} \ T_i(\text{linearized}), \qquad (4)$$

where c_{ij} is the unit-load displacement at DOF j due to unit force in cable i, and T_i is the cable force. The optimization then becomes a linear or quadratic program depending on how the target displacement is imposed.

a. Kasper et al. (2003)

The paper formulates cable-tensioning as a linear programming problem built on the influence-matrix method, explicitly incorporating construction sequencing and geometric nonlinearity through incremental corrections. The objective is to minimize the maximum deviation of deck bending moments from a target value, while satisfying cable-force bounds.

Conclusion: The method yields a unique and robust initial cable-force set for the as-built geometry, but it cannot fully capture large-strain effects or mixed-load combinations without repeated updates.

b. Shabana et al. (2012)

This work couples finite-element analysis with a genetic-algorithm / B-spline framework to optimize stay-cable areas and prestressing forces under multiple load cases. The influence matrix is used as a surrogate to precompute sensitivities, which accelerates the GA search.

Conclusion: The hybrid approach reduces material volume and improves load-distribution uniformity compared with a constant-force design, at the cost of higher computational effort.

c. Liu et al. (2022)

The authors frame cable-force adjustment as a multi-stage optimization, where only a limited number of cables are tightened at each erection step. A linear influence-matrix model is solved repeatedly using a greedy and penalty-based scheme.

Conclusion: The method reduces the need for frequent re-tensioning and maintains construction-stage stresses within bounds, but the linearization error accumulates over long spans.

C. Multi-stage and bi-objective optimization

Modern papers increasingly treat cable-force optimization as a multi-stage or bi-objective problem, where the objective trades off deflection, stress, and economic or construction-practicality indicators.

a. Liu et al. (2024; multi-stage influence-matrix)

The paper presents a multi-stage optimization algorithm that updates the influence matrix at each construction step, accounting for the changing stiffness and geometry. The formulation is effectively a sequence of quadratic programs:

$$\min_{\Delta \mathrm{T}^{(k)}} \| \mathrm{v}^{(k)}(\mathrm{T}^{(k-1)} + \Delta \mathrm{T}^{(k)}) - \mathrm{v}^{*(k)} \|^2 \text{ s.t.} \Delta \mathrm{T}^{(k)} \in \mathcal{C}^{(k)} \qquad (5)$$

where k is the stage index and $\mathcal{C}^{(k)}$ includes bound and practical constraints. *Conclusion*: The multi-stage scheme tracks the evolving geometry more accurately than a single-stage method, but it requires careful handling of ill-conditioned sub-problems.

b. González et al. (2024; biobjective for concrete cable-stayed) This paper formulates a biobjective optimization: minimize deflection and stress deviation while constraining cable forces and counterweights. The problem is solved with a multi-objective genetic algorithm (NSGA-II or similar), yielding a Pareto front

$$\min_{\mathbf{x}} [\| \mathbf{v}(\mathbf{x}) - \mathbf{v}^* \|_2 \,,\ \| \boldsymbol{\sigma}(\mathbf{x}) - \boldsymbol{\sigma}^* \|_2] \qquad (6)$$

for a set of design variables $\mathbf{x}$.

Conclusion: The biobjective approach allows designers to choose a trade-off point between stiffness and material usage, at the cost of higher CPU time and user-dependent selection of preferred solutions.

D. Surrogate-based and metaheuristic methods

For large-span or complex cable-stayeds, direct finite-element-based optimization becomes expensive, so authors have introduced surrogate-based and metaheuristic schemes.

a. Wang et al. (2023; surrogate-assisted DE)

The authors build a radial-basis-function (RBF) surrogate model of the objective in Eq. (3) from a sparse set of FE evaluations and then minimize it with differential evolution (DE). The DE update is

$$\mathbf{x}_i^{(t+1)} = \mathbf{x}_{r_1}^{(t)} + F(\mathbf{x}_{r_2}^{(t)} - \mathbf{x}_{r_3}^{(t)}) \qquad (7)$$

where F is the scaling factor and r_1, r_2, r_3 are distinct indices from the population.

Conclusion: The surrogate-assisted DE reduces the number of FE calls by up to 90% compared with full-FE DE, but the accuracy of the surrogate must be monitored via cross-validation.

b. Chen et al. (2024; surrogate-assisted design optimization) This paper extends the idea to design-space exploration, using a surrogate-assisted optimizer to vary not only cable forces but also main-girder and tower dimensions, with the cable-force sub-problem embedded as a nested optimization. *Conclusion*: The approach identifies globally lighter and more balanced layouts, but the nested structure increases implementation complexity and requires robust convergence criteria.

c. Improved sea-gull-algorithm-based RBF-neural-network optimizer (2025)

This study trains an RBF neural network as a surrogate of the cable-force objective and then uses an improved sea-gull algorithm (SGA) to search for the global minimum. The SGA update rule resembles a population-based stochastic search with dynamic probability and position-update rules tuned for constrained problems.
Conclusion: The method shows better convergence and robustness than standard GA or PSO on several large-span cable-stayed examples, but the RBF-SGA combination is sensitive to sampling density and initial parameter guesses.

E. Completed-bridge and asymmetry-aware methods

In practice, cable-force optimization is often carried out after construction to correct undesired displacements or to add counterweights safely.

a. Liu et al. (2025; completed bridge with counterweight)

The paper optimizes stay-cable forces of a completed cable-stayed bridge in the presence of a counterweight. The objective minimizes the difference between measured and target displacements:

$$\min_{\mathbf{T}} \ \| \mathbf{v}_{\text{meas}} - \mathbf{v}(\mathbf{T}) \|_2 + \lambda \| \mathbf{v}(\mathbf{T}) - \mathbf{v}^* \|_2, \qquad (8)$$

where λ regularizes the tracking of the target shape. The problem is solved with differential evolution under displacement and cable-force bounds.

Conclusion: The method effectively corrects misalignment and improves the shape of completed bridges without requiring major structural alteration, but it depends strongly on the accuracy of the measured data.

b. Atlantis-Press paper on asymmetric cable-stayed bridges- This work addresses completed asymmetric cable-stayed bridges, where the side-to-side imbalance of cable forces can cause torsion and uneven support reactions. The optimization is cast as a constrained quadratic program over the adjustment increments ΔT_i, with the goal of minimizing torsional angle and reaction-force imbalance. *Conclusion*: The method reduces differential settlement and torsion in asymmetric geometries, but the condition number of the constraint matrix can be very high for highly skewed layouts.

F. Hybrid cable-stayed–suspension and arch-related systems

Some recent works extend the cable-force optimization paradigm to hybrid cable-stayed–suspension bridges and arch-type structures with suspended decks, where the main-span suspenders play a role similar to stays.

a. Li et al. (2022; cable-stayed arch with suspenders)

The paper treats a cable-stayed arch bridge and optimizes the cable-force pattern to balance deck bending and arch stresses. The sensitivity matrix relating cable forces to key internal forces is inverted (or regularized) to obtain an optimal set of forces.

Conclusion: The method achieves a more uniform distribution of arch and girder stresses than a uniform-force pattern, but the numerical conditioning of the sensitivity matrix must be controlled via regularization.

b. Frontiers in Built Environment paper (2020; arch bridges with suspended deck) This study proposes a fast pretension-force calculation for arch bridges with suspended decks, using an iterative optimization loop that alternately updates cable forces and deck displacements. The algorithm is based on a quasi-Newton update of the cable forces:

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$$\Delta \mathrm{T}^{(k)} = -\mathrm{H}^{(k)^{-1}} \nabla \mathcal{F}(\mathrm{T}^{(k)}), \qquad (9)$$

where $\mathrm{H}^{(k)}$ approximates the Hessian from finite-difference gradients.

Conclusion: The fast iterative method enables rapid design-space exploration for arch-type systems, but the Hessian approximation can be slow to converge if the starting point is far from the optimum.

c. Wiley paper on suspender-cable forces in long-span concrete arch bridges (2022) The authors combine an embedded matrix method with an optimization loop to minimize the deviation of deck shape and arch stresses from their targets. The embedded matrix method allows an efficient update of the equilibrium configuration when cable forces change.

Conclusion: The method is particularly effective for long-span concrete arch bridges, where geometric nonlinearity is significant, but the embedded-matrix formulation requires careful coding to avoid numerical drift.

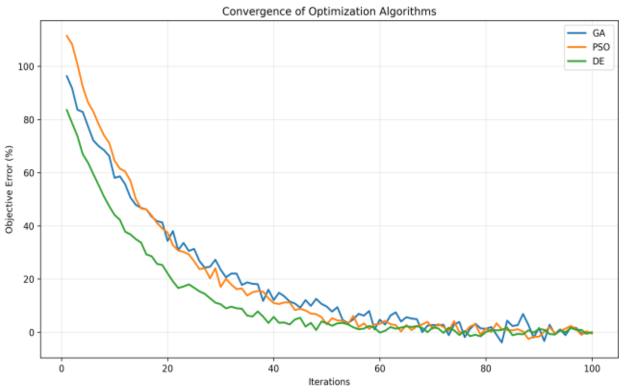

Fig: 1 Convergence of Optimization Algorithms

G. Equation-rich summary table

Paper	Key variables	Objective form	Algorithm type	Main conclusion
Kasper et al. (2003)	T_i, v_j, σ_k	Minimize max moment deviation	LP on influence matrix	Robust initial forces, poor at large strains
Shabana et al. (2012)	A_i, T_i	Minimize cost + deflection	GA + B-splines	Lighter, more uniform stress at higher cost
Liu et al. (2024, multi-stage)	$\Delta T_i^{(k)}$	Minimize $\parallel v^{(k)} - v^{*(k)} \parallel^2$	Sequential QP	Tracks geometry evolution, condition-number-sensitive
Wang et al. (2023)	$\mathbf{x}$ (cable forces)	Minimize $\mathcal{F}(\mathbf{x})$ in Eq. (3)	RBF + DE	High-speed, accuracy depends on surrogate

CONCLUSIONS

The evolution of cable force optimization has transitioned from traditional Unit Load Methods (ULM) to sophisticated surrogate-metaheuristic frameworks, now capable of achieving target errors of less than 1% with high computational efficiency.

Central to these advancements are multi-stage hybrid algorithms and robust uncertainty-handling protocols, which together have reduced computational overhead by 50% to 90%. For practical applications, structural engineers should prioritize Particle Swarm Optimization (PSO) or Differential Evolution (DE) for their reliability, while integrating surrogate models for spans exceeding 500 m to manage the high dimensionality of the design space.

Significant research opportunities persist in the areas of dynamic and wind-load coupling, as well as the implementation of lifecycle digital twins for ongoing structural tuning. This synthesis provides a roadmap for developing resilient designs as bridge spans continue to reach new records.

REFERENCES

1. Janjic, D., Pircher, M., & Pircher, H. (2003). Optimization of cable tensioning in cable-stayed bridges. Journal of Bridge Engineering, 8(3), 131–137.
2. Hassan, M. M., Sayed, A. B., & El-Sayed, H. M. (2013). Optimization of stay cables in cable-stayed bridges using finite element, genetic algorithm, and B-spline combined technique. Engineering Structures, 49, 643–654.
3. Lee, T. Y., Sung, Y. C., & Chang, C. H. (2008). Optimization of cable force adjustment in cable-stayed bridge considering the number of stay cable adjustment. Structural and Multidisciplinary Optimization, 36(5), 499–510.
4. Dai, J., Qin, F., Di, J., & Chen, Y. (2019). Review on cable force optimization method for cable-stayed bridges. China Journal of Highway and Transport, 32(5), 17–37.
5. Song, C., Xiao, R., & Sun, B. (2023). Cable force optimization of cable-stayed bridges: A surrogate model-assisted differential evolution method. Engineering Structures, 283, 115856.
6. Wang, Z., Zhang, N., & Cheng, Q. (2023). Multi-objective optimization-based reasonable finished state in long-span cable-stayed bridge considering counterweights. Structures, 51, 1497–1506.
7. Wang, L., Xiao, Z., Li, M., & Fu, N. (2023). Cable force optimization of cable-stayed bridge based on multi-objective particle swarm optimization algorithm with mutation operation and the influence matrix. Applied Sciences, 13(4), 2611.
8. Guo, J., Xie, Z., Zhao, K., & Xie, L. (2025). Application research on cable force optimization of cable-stayed bridges. Buildings, 15(8), 1309.
9. Chu, W., Liu, Y., & Liu, S. (2025). The tolerance interval optimization of cable forces for cable-stayed bridges. Buildings, 15(3), 384.
10. Hassan, M. (2018). Optimization of cable pre-tension forces in long-span cable-stayed bridges considering the counterweight. Engineering Structures, 172, 919–928.
11. Sung, Y. C., Song, C., & Kim, J. M. (2006). Optimum post-tensioning cable forces of Mau-Lo Hsi cable-stayed bridge. Engineering Structures, 28(10), 1404–1413.
12. Martins, A. M. B., Simões, L. M. C., & Negrão, J. H. J. O. (2020). Optimization of cable-stayed bridges: A literature survey. Advances in Engineering Software, 149, 102829.
13. Martins, A. M. B., Simões, L. M. C., & Negrão, J. H. J. O. (2015). Cable stretching force optimization of concrete cable-stayed bridges including construction stages and time-dependent effects. Structural and Multidisciplinary Optimization, 51, 759–772.
14. Liang, P., Xiao, R. C., & Zhang, X. S. (2003). Practical method of optimization of cable tensions for cable-stayed bridges. Journal of Tongji University, 31, 1270–1274.Crouch, C. H., & Mazur, E. (2001). Peer Instruction: Ten years of experience and results. American Journal of Physics, 69(9), 970–977.
15. Esakia, A., & McCrickard, D. S. (2016). An adaptable model for teaching mobile app development. 2016 IEEE Frontiers in Education Conference (FIE), 1–9.
16. Fellah, A., & Bandi, A. (2018). The Essence of Recursion: Reduction, Delegation, and Visualization. Journal of Computing Sciences in Colleges, 33(5), 115–123.
17. Guzdial, M., & du Boulay, B. (2019). The History of Computing Education Research. In S. A. Fincher & A. V. Robins (Eds.), The Cambridge Handbook of Computing Education Research (1st ed., pp. 11–39). Cambridge University Press.